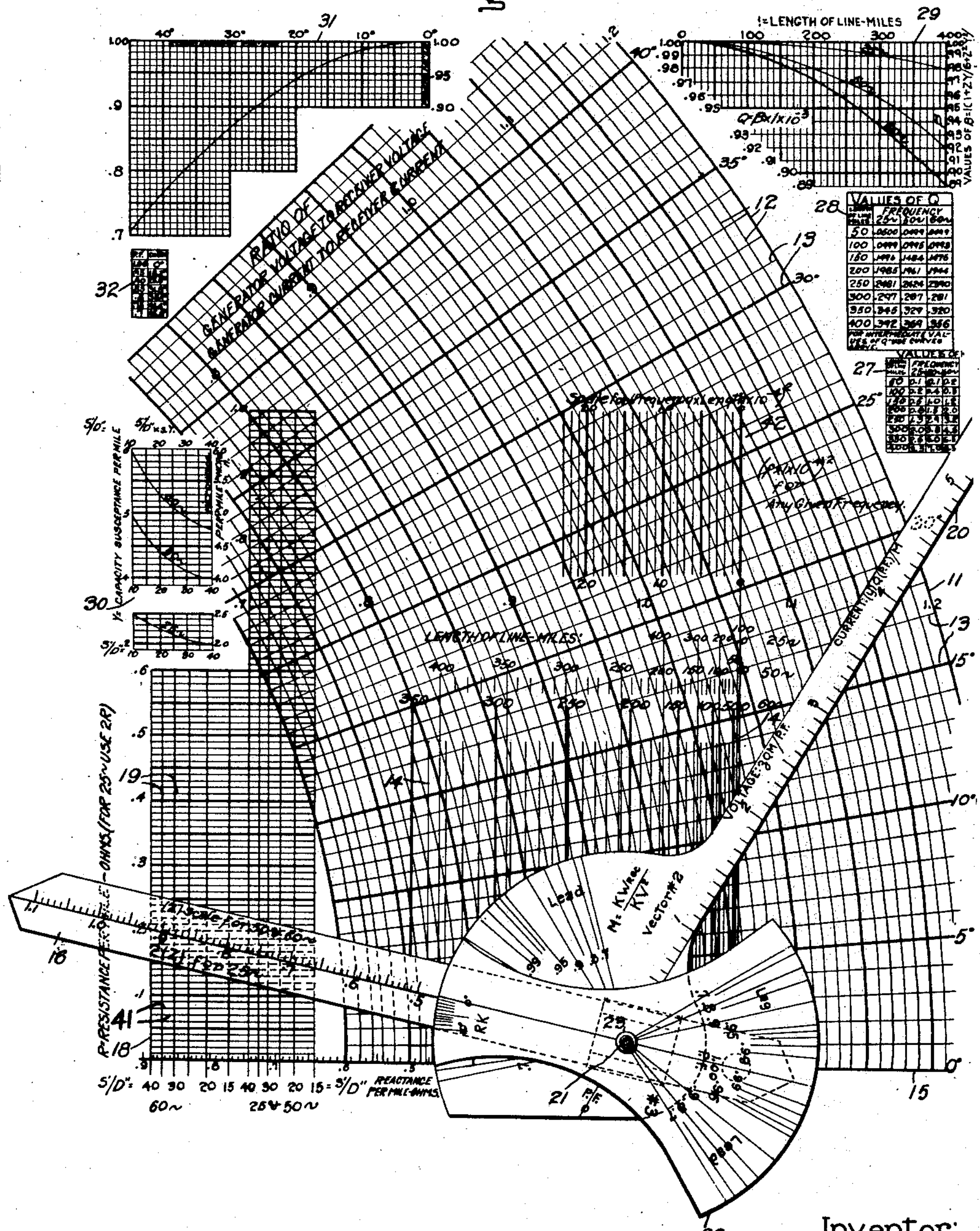
Sept. 1, 1925. 1,552,113

E. CLARKE

CALCULATOR

Filed June 8, 1921 3 Sheets-Sheet 1

Fig.1.



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Her Attorney.

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CALCULATOR

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Fig. 2.

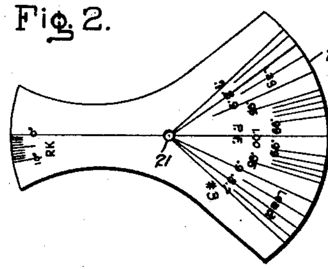


Fig. 3.

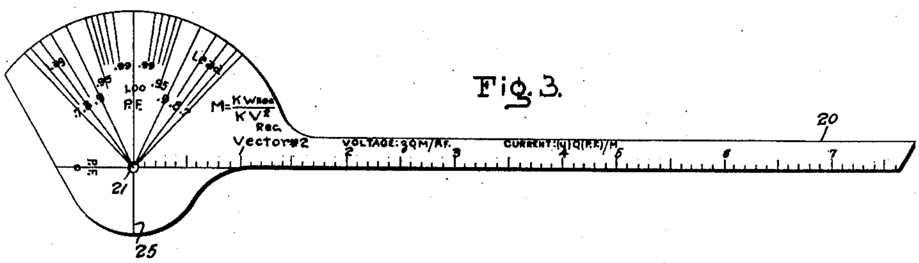


Fig. 4.

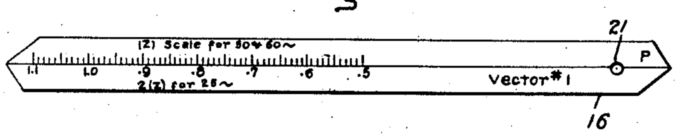


Fig. 5.

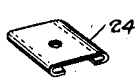
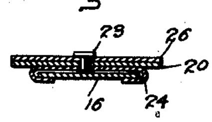


Fig. 6.



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1,552,113

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CALCULATOR

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Fig. 7.

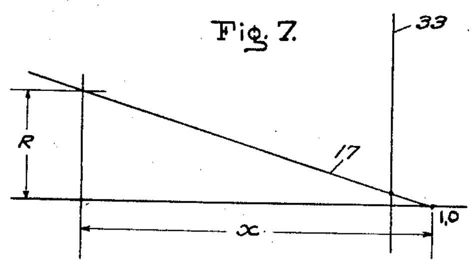


Fig. 8.

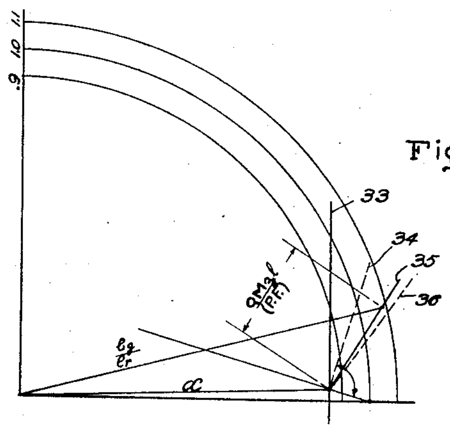
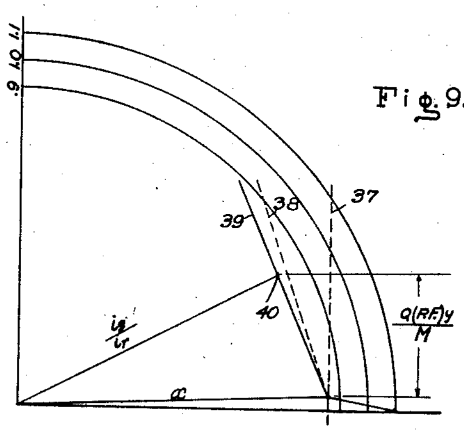


Fig. 9.



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Patented Sept. 1, 1925. **1,552,113**

UNITED STATES PATENT OFFICE.

EDITH CLARKE, OF SCHENECTADY, NEW YORK.

CALCULATOR.

Application filed June 8, 1921. Serial No. 476,075.

To all whom it may concern:

Be it known that I, EDITH CLARKE, a citizen of the United States, residing at Schenectady, county of Schenectady, State of New York, have invented certain new and useful Improvements in Calculators, of which the following is a specification.

My invention relates to a calculator, and more particularly to a calculator for investigating the electrical characteristics of long lines for the transmission of electrical energy.

In the ordinary methods of calculation employed for short transmission lines, such as 50 miles or so, it is usually assumed that the capacity and inductance of the line are each concentrated at a single point or at a few isolated points along the line. Such assumptions however are not justifiable when the transmission line which is being investigated becomes comparatively long, say a few hundred miles. Even for such distances as 200 miles, errors would result if such assumptions be made. If accuracy is desired in calculations for long lines, it is absolutely necessary to take into consideration the uniformly distributed inductance and capacity of the line. Although formulas have been derived for such conditions, their application involves a great deal of work. Especially is this the case where it is necessary to investigate the behavior of transmission lines upon varying the conditions by small increments at the receiver end or at the generator end. It is the object of my invention to make it possible to investigate the characteristics of transmission lines of varying construction in a simple manner.

With my invention, laborious calculations are obviated for transmission lines of any length desired, and acceptably accurate results obtained. For example, the error obtained by using my calculator does not exceed a small fraction of one percent for lines of about five hundred miles. Lines of such length are being considered, and there is little doubt that in the future such long distance lines will be quite common. With the aid of my invention it is possible to obtain such values as the current and E. M. F. at the generator end when the conditions at the receiver end are known. For example, if at the receiver end the E. M. F. which it is desired to obtain is known as well as the load in kilowatts and the power factor, it is possible to obtain with my calculator in the short space of a few minutes the current at the generator end as well as its vector relation with the current at the receiver end and the E. M. F. at the generator end as well as its vector relation with respect to the E. M. F. at the receiver end. It is also possible to obtain other values when other factors are assumed than those mentioned, as will appear from the description given hereinafter.

The results obtained with my calculator as stated heretofore are surprisingly accurate. Furthermore, my calculator is arranged in such a way that it may obtain the values mentioned heretofore, although such characteristics as the length of the line, frequency, size of conductor and spacing between conductors are varied. This result is possible due to the fact that certain combination of elements involved in the calculation remain substantially constant upon a variation of some of these factors.

The calculator is based upon evaluations of infinite series to a sufficient degree of accuracy, which infinite series take into consideration the distributed inductance and capacity of the line. By making the assumption that certain of the elements involved in the calculations remain substantially constant, it is possible to perform the operations with the calculator by the aid of the combination of only a few pivoted arms representing vectors.

The infinite series chosen to represent the transmission line characteristics are those involving the hyperbolic sines and cosines. The fundamental equations which my calculator is adapted to solve are as follows:

$$e_g = e_r \cosh\sqrt{ZY} + i_r\sqrt{\frac{Z}{Y}} \sinh \sqrt{ZY} \quad (1)$$

$$i_g = i_r \cosh\sqrt{ZY} + e_r\sqrt{\frac{Y}{Z}} \sinh \sqrt{ZY} \quad (2)$$

These equations are those which are given in Heaviside's "Electro-Magnetic Theory," vol.

1, page 450. In these relations, e_g is the generator E. M. F. to neutral expressed as a vector, e_r is the receiver E. M. F. to neutral expressed as a vector, Z and Y are vectors representing the total impedance and admittance respectively of one conductor to neutral, i_g is the generator current, and i_r is the receiver current. The derivation of these relations will not be entered into in this specification, since they are well known.

All of the terms used in relation 1 and 2 are vectors; that is, they may be represented by lines having definite length and directions. The common way of writing vectors is by making use of the quality unit j, which unit represents the value $\sqrt{-1}$. Thus a vector V may be written as $v+jv'$. This means in effect that to plot the vector V with rectangular coordinates, it would be necessary to mark off on the X-axis a distance corresponding to v, to erect at that point a perpendicular and to measure off on this perpendicular a distance equal to v'. The vector extends from the origin to this point. If v' is positive, then the distance is measured upwardly; if negative, it is measured downwardly from the X-axis. Another way of plotting vectors is by the aid of polar coordinates. In that case, the length of the vector and its angle with respect to the zero angle line are made use of. The angle may be positive or negative; if positive, it is measured off in a counterclockwise direction from the zero angle line; if negative, in the clockwise direction. Although both methods of plotting are used in my calculator, its explanation becomes simpler if for the present we consider only the first method of representing vectors; i. e. by rectangular coordinates.

The addition, subtraction, multiplication and division of vectors represented this way may be easily performed, since there are definite rules which tell us how these processes may be performed. The part v of the vector is usually termed the "real" component, and the part v' is usually termed the "imaginary" component of the vector V. To add two vectors it is merely necessary to add their real parts together and their imaginary parts together and write the result as a vector the real part of which is represented by the sum of the real parts of the vectors and the imaginary part by the sum of the imaginary parts of the vectors. Thus if the sum of the two vectors $a+ja'$ and $b+jb'$ is required, this sum may be written as $(a+b)+j(a'+b')$. An analogous method is used if vectors are subtracted from each other. In this case the difference between the real parts as well as the difference between the imaginary parts are taken for the corresponding parts of the resultant vector. When two vectors are multiplied together, the new vector has a length equal to the product of the lengths of the two vectors which are multiplied together and an angular displacement from the zero angle line equal to the sum of the angles of the two vectors. The length of the vector is equal to the square root of the sum of the squares of its two components. When one vector is divided by another, the resultant is a vector, the length of which is equal to the length of the first vector divided by the length of the other, and whose angular displacement from the zero angle line is equal to the angular displacement of the first vector minus the angular displacement of the second vector. From these fundamental relationships it is possible to understand the processes of transformation which will be described hereinafter.

The expansions of the hyperbolic functions give the following results:

$$\sinh x = x + \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} + \frac{x^7}{\underline{|7}} + \text{--------} \qquad (3)$$

$$\cosh x = 1 + \frac{x^2}{\underline{|2}} + \frac{x^4}{\underline{|4}} + \frac{x^6}{\underline{|6}} + \text{--------} \qquad (4)$$

These expansions are given, for example, in B. O. Peirce, A Short Table of Integrals, formulas 790 and 791. Substituting these values in relations 1 and 2, these relations become

$$e_g = e_r\left(1 + \frac{ZY}{\underline{|2}} + \frac{Z^2Y^2}{\underline{|4}} + \text{--------}\right) + i_rZ\left(1 + \frac{ZY}{\underline{|3}} + \frac{Z^2Y^2}{\underline{|5}} + \text{--------}\right) \qquad (5)$$

and

$$i_g = i_r\left(1 + \frac{ZY}{\underline{|2}} + \frac{Z^2Y^2}{\underline{|4}} + \text{--------}\right) + e_rY\left(1 + \frac{ZY}{\underline{|3}} + \frac{Z^2Y^2}{\underline{|5}} + \text{-------}\right) \qquad (6)$$

These expressions may be written so that they express the ratios between the generator E. M. F. and receiver E. M. F., and between the generator current and receiver current. Thus expression (5) may be divided through by e_r, and expression (6) by i_r; the results are

$$\frac{e_g}{e_r} = \left(1 + \frac{ZY}{\underline{|2}} + \frac{Z^2Y^2}{\underline{|4}} + \text{--------}\right) + \frac{i_r}{e_r}Z\left(1 + \frac{ZY}{\underline{|3}} + \frac{Z^2Y^2}{\underline{|5}} + \text{--------}\right) \qquad (7)$$

and

$$\frac{i_g}{i_r} = \left(1 + \frac{ZY}{\underline{|2}} + \frac{Z^2Y^2}{\underline{|4}} + \text{--------}\right) + \frac{e_r}{i_r}Y\left(1 + \frac{ZY}{\underline{|3}} + \frac{Z^2Y^2}{\underline{|5}} + \text{--------}\right) \qquad (8)$$

In further derivations, the first term of the right hand member of each of the equations

(7) and (8) will be written in its shorter form as cosh $\sqrt{ZY}$ simply as a matter of convenience, so that the equations become

$$\frac{e_g}{e_r} = \cosh\sqrt{ZY} + \frac{i_r}{e_r}Z\left(1 + \frac{ZY}{\underline{|3}} + \frac{Z^2Y^2}{\underline{|5}} + \text{-------}\right) \quad (9)$$

and

$$\frac{i_g}{i_r} = \cosh\sqrt{ZY} + \frac{e_r}{i_r}Y\left(1 + \frac{ZY}{\underline{|3}} + \frac{Z^2Y^2}{\underline{|5}} + \text{-------}\right) \quad (10)$$

In equations (7) and (9), although E. M. F.'s to neutral are taken, the ratio of these E. M. F.'s is the same as the ratio between lines. Furthermore it must not be overlooked that e_r, i_r, e_g, and i_g are vectors as well as Z and Y, so that the ratios given in equations (9) and (10) are vector ratios.

To simplify expressions (9) and (10) further, it is advisable to obtain equivalent expressions for $\frac{e_r}{i_r}$ and $\frac{i_r}{e_r}$. The vector i_r may be obtained as follows: The load at the receiver end in watts in a three-phase system which is the only kind considered, in this discussion, is

$$\text{watts}_r = |e_r| \times 3 \times |i_r| \times \text{P.F.}_r \quad (11)$$

where $|e_r|$ and $|i_r|$ are the absolute values of e_r and i_r respectively, and P.F.$_r$ is the power factor at the receiver end. From this it follows that

$$|e_r| = \frac{\text{watts}_r}{|e_r| \times 3 \times \text{P. F}_r} \quad (12)$$

If e_r is taken as the zero angle vector, then i_r will be displaced therefrom by an angle depending on the power factor P.F.$_r$. This angle is equal to that of the unit vector

$$\left[\text{P. F}_r + j(\pm\sqrt{1 - \text{P. F}_r^2})\right].$$

The ambiguous sign is placed before the radical expression since it is taken as positive when i_r leads, and negative when i_r lags. The vector i_r may now be written

$$i_r = \frac{\text{watts}_r}{e_r \times 3 \times \text{P.F.}_r} \times \left[\text{P.F.}_r + j(\pm\sqrt{1 - \text{P.F.}_r^2})\right]. \quad (13)$$

The vector e_r being taken as zero angle vector, it is equal to e_r.

We may now divide both the left hand member and the right hand member of relation (13) by this vector e_r. This results in

$$\frac{i_r}{e_r} = \frac{\text{watts}_r}{e_r^2 \times 3 \times \text{P.F}_r} \times \left[\text{P.F.}_r + j(\pm\sqrt{1 - \text{P.F.}_r^2})\right] \quad (14)$$

Now we know that watts$_r$=K.W.$_r \times 10^3$, where K.W.$_r$ is the receiver load in kilowatts; also that

$$e_r^2 = \frac{E_r^2}{3},$$

where E_r is the E. M. F. between lines at the receiver end. We can also put K.V$_r$ as the receiver kilovolts between lines, or $E_r \times 10^{-3}$. When these substitutions are made, we obtain the final form for (14) as

$$\frac{i_r}{e_r} = 10^{-3}\frac{\text{K.W.}_r}{\text{K.V.}_r^2 \times \text{P.F.}_r} \times \left[\text{P.F.}_r + j(\pm\sqrt{1 - \text{P.F.}_r^2})\right] \quad (15)$$

Substitution of expression (15) in expressions (9) and (10) results in

$$\frac{e_g}{e_r} = \cosh\sqrt{ZY} + 10^{-3}\frac{\text{K.W.}_r}{\text{K.V}_r^2 \times \text{P.F}_r} \times \left[\text{P.F.}_r + j(\pm\sqrt{1 - \text{P.F}_r^2})\right] \times z \times 1 \times \beta \quad (16)$$

and

$$\frac{i_g}{i_r} = \cosh\sqrt{ZY} + \text{P.F.}_r \times \frac{\text{K.V.}_r^2}{\text{K.W}_r} \times 10^3 \div \left[\text{P.F.}_r + j(\pm\sqrt{1 - \text{P.F.}_r^2})\right] \times y \times 1 \times \beta \quad (17)$$

In these two equations, z and y were taken as vectors representing the impedance and admittance respectively, per unit length of one conductor to neutral, 1 as the length of the line, and β as the infinite series

$$\left(1 + \frac{ZY}{\underline{|3}} + \frac{Z^2Y^2}{\underline{|5}} + \text{-------}\right).$$

Equations (16) and (17) are the final forms of the equations which the calculator is adapted to solve.

Now bearing in mind that Z equals $1z$ that Y equals $1y$, and that $z = R + j2\pi fL$, and $y = o + j2\pi fC$, since the leakance is neglected, where R is the resistance of the line per unit length of one conductor, f is the frequency, L is the inductance per unit length of one conductor to neutral, and C the capacitance per unit length to neutral, such values as ZY and

Z^2Y^2 may be obtained as vectors which are functions of the above mentioned factors. Thus

$$ZY = 1z.1y = 1^2(R + j2\pi f L)(j2\pi f C) = 1^2\left\{-4\pi^2 f^2 LC + j2\pi f CR\right\}$$

and

$$Z^2Y^2 = 1^4\left\{16\pi^4 f^4 L^2C^2 - 4\pi^2 f^2 C^2R^2 - j16\pi^3 f^3 LC^2R\right\}$$

Substitution of these values in cosh $\sqrt{ZY}$ and in the infinite series β gives

$$1 + \frac{ZY}{|\underline{2}} + \frac{Z^2Y^2}{|\underline{4}} + \ldots = \left\{1 - 2\pi^2 f^2 1^2 LC + \frac{2}{3}\pi^4 f^4 1^4 L^2C^2 - \frac{1}{6}\pi^2 f^2 1^4 R^2C^2 + \ldots\right\} + jR\left\{\pi f 1^2 C - \frac{2}{3}\pi^3 f^3 1^4 LC^2 + \ldots\right\} \quad (18)$$

and

$$1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \ldots = \left\{1 - \frac{2}{3}\pi^2 f^2 1^2 LC + \frac{2}{15}\pi^4 f^4 1^4 L^2C^2 - \frac{1}{30}\pi^2 f^2 1^4 C^2R^2 + \ldots\right\} + jR\left\{\frac{1}{3}\pi f 1^2 C - \frac{2}{15}\pi^3 f^3 1^4 LC^2 + \ldots\right\} \quad (19)$$

It is to be noted that the leakance of the line is neglected in these expressions.

Expression (18) shows that in the first three terms of the real part of the expression the inductance L and capacity C of one line to neutral enter into the equation to the same degree; i. e., in each of these first three terms L and C are raised to the same power. In the calculator it is assumed that this product is a constant for all lines and that therefore the real part of expression (18) is a constant for all lines having the same length and frequency. Further, a reasonable value is assigned to R, and since it appears first in the fourth term, which is always very small, the error introduced has been found to be extremely small. The product of L and C also remain substantially constant. The formulas for L and C are as follows:

$$L = \left[0.74113 \log\left(2 \times 12 \times \sqrt[3]{2}\frac{S'}{D''}\right) + 0.08047\right] 10^{-3} \text{ henries per mile}$$

$$C = 0.03883 \times 10^{-6} \div \log\left(2 \times 12 \times \sqrt[3]{2}\frac{S'}{D''}\right) \text{ farad per mile}$$

Therefore the formula for the product of L and C is

$$LC = .02878 \times 10^{-9} + .003125 \times 10^{-9} \div \log\left(2 \times 12 \times \sqrt[3]{2}\frac{S'}{D''}\right) \quad (20)$$

S' is the flat spacing of the conductor in feet and D'' is the diameter of the conductor in inches. It is seen from this formula that the product of L and C is a function solely of S' and D''. When the expression

$$\frac{S'}{D''}$$

varies from 15 to 35 the product of L and C varies to a very small extent; that is, between $.02996 \times 10^{-9}$ and $.02981 \times 10^{-9}$. It is thus possible to plot the real part of expression (18) against length of lines at any given frequency and obtain surprisingly close results for this value, especially since a large majority of transmission lines are so constructed that the value

$$\frac{S'}{D''}$$

lies between 15 and 35. In the calculator, such a value of

$$\frac{S'}{D''}$$

can be assumed that it represents the average of a large number of lines actually constructed. Furthermore if the lines have a triangular spacing arrangement, instead of a flat spacing, this may be taken into consideration by taking S' as the

$$\frac{1}{\sqrt[3]{2}}$$

times the distance between adjacent conductors. This formula is based on derivations which it is not necessary here to investigate.

Discussing now expression (19), several features of this expression should be noted: The first is that the imaginary part is always very small as compared with the real part. The error introduced by assuming LC as a constant for obtaining the length of this vector is also very small, for the reasons stated heretofore. It is thus possible to plot the absolute value of $\beta \times 1 \times 10^{-3}$ against given length of line at any given frequency so that the length of such a vector may be taken off a chart directly. This value may be termed Q and will be so designated in the remainder of the specification.

It should also be noted that the slope of the vector of expression (19) is small, since the imaginary component is small as compared to the real component. The angle of vector (19) is the angle the tangent of which is equal to the imaginary component divided by the real component. The tangent is thus proportional to R, since R is a factor in the imaginary component. Without appreciable error, since the angle is small, the value of the angle itself may be considered proportional to R. This angle may then be written R K, where K is the angle the tangent of which is the imaginary component of (19) divided by R and also by the real component. This small angle may also be considered without appreciable error to be constant for all lines of the same frequency and length. In the calculator there is a table giving the value of this angle designated as K for lines having varying lengths and frequencies.

Now returning to the expressions (16) and (17), it will be seen how these assumptions are utilized. The values

$$\frac{e_g}{e_r} \text{ and } \frac{i_g}{i_r}$$

which it is desired to obtain are each equal to the sum of two vectors. It is the function of the calculator to plot both of these vectors expeditiously and to find the extremity of the resultant vector, whereby the numerical value of the ratio, as well as the phase difference between the numerator and denominator of the ratio may be immediately obtained. For reading the ratio and the phase difference there is provided a base chart constructed on the principle of polar co-ordinates. Use is also made of a plurality of pivoted arms or members for adding the vectors. These arms carry appropriate divisions for their proper setting.

For a better understanding of my invention reference is to be had to the following description together with the accompanying drawings, in which Fig. 1 is a plan view of the complete calculator, showing all of its parts assembled, but not necessarily at any setting used in calculations; Fig. 2 is a view of a disc by which certain angles may be set off; Fig. 3 is a view of a pivoted arm; Fig. 4 is a view of another pivoted arm; Fig. 5 is a perspective showing how the slide is constructed for enabling the arm shown in Fig. 3 to slide along the arm shown in Fig. 4; Fig. 6 is a cross-sectional view showing more in detail how the arms and the disc are assembled; Fig. 7 is a diagram showing how the extremity of one of the vectors of expressions (16) or (17) is determined; Fig. 8 is a diagram showing how the voltage ratio of expression (16) is determined with the aid of the chart, and Fig. 9 is a diagram showing how the current ratio of expression (17) is obtained with the aid of my calculator.

Referring now more in detail to the drawings, in which like reference characters refer to like parts throughout, I provide a chart 11 divided for ease in reading polar co-ordinates. In order to have as large a scale as possible for a given size of chart, only the useful part of the polar co-ordinate scale is plotted, and thus the origin o, o does not appear thereon. The divisions for plotting polar co-ordinates comprise the concentric circles 12 and the radial lines 13, each marked with their appropriate values as indicated. The series of vertical lines 14 intersect the horizontal zero angle line 15 at points corresponding to the real component of the expansion of

$$\cosh \sqrt{ZY}$$

as stated heretofore. This real component stays substantially constant for lines of quite varying construction so long as their frequency and length are the same. These division lines which correspond to the real part

$$\cosh \sqrt{ZY}$$

are grouped into three parts; those corresponding to lines of 25 cycles, those corresponding to lines of 50 cycles and those corresponding to lines of 60 cycles. It would have been possible to designate these division lines by the product of frequency and length since both f and l enter into the expansion of the real portion of this function substantially to the same degree. In fact, such lines, plotted as a product of f and l are shown at 42. The distance from the origin of the polar co-ordinates along the zero angle line 15 to any one of these division lines 14 represents the real part of the expansion for that line having the length and frequency corresponding to the division line.

It should also be noted that the real part of expression (18) which is the real part of the hyperbolic function referred to above includes the value unity, from which the next term of the expansion is subtracted. Then the signs change alternately, but the real part stays always less than unity. At the point 1,0 of the polar chart 11, an arm 16 is pivoted by means of which it is possible to obtain the extremity of the vector corresponding to the expression (18), or to the first term of the right hand members of expressions (16) and (17). The particular means for doing this will now be discussed.

Attention is now directed to Fig. 7. In this figure the point 1,0 is shown from which the line 17 extends towards the upper left hand corner. This line 17 corresponds to the center line or graduated edge of the arm 16. For convenience in setting this arm, it may be made of transparent material. If this line 17 be given the proper slope it is evident that it will intersect that particular division line of the set 14 at the extremity of the vector corresponding to expression (18). To obtain this slope the left hand member of this expression is to be considered. If only the first two terms of this expression are considered it is seen that the vector to be added to the vector unity, which extends from the origin to the point 1,0 is

$$\frac{ZY}{2}.$$

It is the slope of this supplementary vector which is determined by the setting of the arm 16. To obtain this slope it should be noted that the vector ZY must have a slope which is equal to the sum of the slopes of the vector Z and the vector Y, from the fundamental theory of vectors discussed heretofore. Furthermore, since the vector Y represents the capacity susceptance of one line, it has an angle of 90°. Therefore all that is necessary to do to set the arm 16 at the proper angle which is the sum of 90° and the angle of the vector Z. The vector Z represents the impedance of one line. If from the point 1,0 the value X or reactance per unit length is plotted along the zero angle line toward the origin and if a perpendicular be erected at the extremity of this line of the length corresponding to R, the resistance per unit length of the line, then the line connecting the point 1,0 with the extremity of the perpendicular last constructed will have the proper slope. To facilitate this setting on the arm 16, I make use of a chart 18 to the left of the main chart, so arranged that the point —X, R as plotted from the point 1,0 may be immediately determined from the constants of the transmission line. Along the horizontal zero angle line 15 there are a series of vertical division lines 41 corresponding to the ratio of spacing of the conductors to the diameter, and to the frequency of the line. There are two sets of these perpendicular lines corresponding to 60 cycles and 50 cycles. As stated heretofore, the reactance X per unit length of the line is a function solely of the ratio of spacing and of the diameter of the conductor. A series of horizontal parallel lines 19 corresponding to R, the resistance per unit length. The horizontal lines 19 are so arranged that the intersection of the proper vertical line with the proper horizontal line will give the extremity of the vector whose co-ordinates are —X, R from the point 1,0. To obtain points corresponding to 25 cycle lines, the 50 cycle values are taken for the reactance, and since this is equal to twice the reactance which the line would have at 25 cycles, the resistance per mile is taken as twice R. In this way, for 25 cycle lines, instead of plotting the point —X, R, the point —2X, 2R is plotted, but the line 17 has exactly the same slope as if the point —X, R had been plotted. The zero angle line 15 may also be graduated directly in reactance per mile in ohms, as shown on Fig. 1.

The intersection of the line 17, corresponding to the center line or graduated edge of the pivoted arm 16 with the vertical division line 33 conforming to the real component of expression (18) gives the extremity of the vector

$$\cosh \sqrt{ZY},$$

which vector will be hereafter designated as $\propto$. Although the terms beyond the second have been neglected, for obtaining the slope of arm 16, it has been found that the extremity of this vector is quite accurately determined by the method described. This is due to the fact that one of the two components of the vector of expression (18) is quite accurately determined in the first place and further that if the imaginary component of the expression (18) be accurately obtained it would practically coincide in length with the distance from the zero angle line 15 to the extremity of the vector obtained by the foregoing method, measured perpendicularly to the zero angle line.

The chart 18 used for determining the slope of arm 16 has certain novel features. It is so arranged that certain electrical properties of the line, such as its reactance, which are dependent upon its physical construction may be determined against the ratio

$$\frac{S'}{D'}$$

and the frequency. This is a useful arrangement and saves a great deal of time in calculating the value of the reactance X.

After the extremity of the vector $\propto$ corresponding to the expression (18) has been obtained as described above it is necessary to add to this vector the other vector occurring in the right hand members of expressions (16) and (17). Considering for the moment the case where the vector corresponding to the ratio of e_g and e_r is desired, as expressed in relation (16), the supplementary vector to be added to the vector already obtained is given by the second term of the right hand member of this expression. It is to be noted that this latter expression is a product of several scalar quantities and of three vectors. The first vector is the unity vector

$$\left[\mathrm{P.F.}_r + j\left(\pm\sqrt{1-\mathrm{P.F.}_r^2}\right)\right]$$

and the others are the vectors z and β. Therefore to obtain the angle of this second vector it is necessary to get the sum of the angles of these three component vectors. For setting this second vector I make use of a second arm 20 so arranged that it slides along the arm 16. This arm 20 is shown in greater detail in Fig. 3. The particular means for obtaining this sliding connection is of no importance so far as this invention is concerned. In the present instance a runner or slider 24 shown in Figs. 5 and 6, is used, to which is pivotally attached the arm 20. The arm 20 is thus free to rotate about a pivot point 21, which pivot point is located in the member 24 in such a way that this pivot point may be accurately placed over any desired point on the chart 11. This may be accomplished for example by utilizing a small metallic bushing 23 passing through arm 20 and the member 24. The arm 20 pivots about the bushing 23.

Returning now to the second term of the right hand member of expression (16), the angle of z may be immediately obtained, since this vector z is perpendicular to the setting of the arm 16, this arm having been set with the aid of chart 18, at an angle equal to 90° plus the angle of z. To assist the setting of arm 20 in this direction use is made of the line 25 drawn perpendicular to the direction of arm 20. The first step then for obtaining the direction of the second vector is to slide the arm 20 up along the arm 16 until the pivot point 21 coincides with the extremity of the first vector. The next step is to rotate the arm 20 about this pivot point until the perpendicular line 25 coincides with the center line of the arm 16. With this setting the slope of arm 20 is equal to the slope of the vector z. The direction of arm 20 is that shown by the dotted line 34 on Fig. 8. Now considering again the expression (16), to this angle of arm 20 must be added the slope of the unity vector

$$\left[\mathrm{P.F.}_r+j\,(\pm\sqrt{1-\mathrm{P.F.}_r^2})\right].$$

This slope is dependent upon the power factor at the receiving end. When the load at the receiving end is lagging then the sign in front of the square root sign is negative. When the load is leading then the sign in front of the square root sign is positive. The conditions having been assumed at the receiving end, this sign is now taken into consideration. The circular portion of arm 20 is graduated then directly in angles of lead or lag. Thus with the power factor corresponding to .95 lead, the arm 20 must be turned through a further angle in a counterclockwise or positive direction. If the power factor is, for example, .95 lag, then the arm 20 must be turned through a similar angle in a clockwise direction. The circular portion of arm 20 is directly graduated to give power factors, and all that it is necessary to do to add to the angle of arm 20 the angle corresponding to the power factor, is to rotate the arm 20 until the proper power factor division coincides with the center line of arm 16. The new setting of arm 20 is shown by dotted line 36 of Fig. 8. To the angle of this arm must also be added the slope of the vector β. The expansion of β is given in expression (19). The additional angle through which the arm 20 must be turned corresponds to the angle whose tangent is equal to the imaginary part of β divided by the real part of β. This tangent is directly proportional to R since the factor R may be divided out of the imaginary part of expression (19). After this factor R is taken out, due to the fact that the angle considered is always small, the other values appearing in expression (19) may be used representing a fair average for all transmission lines, at a given frequency and length of line. It is possible to obtain values of the angles, the tangents of which are equal to the imaginary part of β divided by R and by the real part of β, which angles have values given as K on the supplementary chart 27 for any given frequency and length of line. The values of K as given in this table thus correspond with any given frequency and length of line to the angle whose tangent is equal to the imaginary part of the expression (19) divided by R and also by the real part of expression (19). No great error is introduced by assuming that the angle of β is directly proportional to the value of R. Therefore all that it is necessary to do to obtain the slope of β is to multiply the values of K given in chart 27 by the value R. The arm 20 may now be turned through this angle. Since this angle is always positive the arm 20 is turned always in a counterclockwise direction. To effect this result easily divisions are placed upon the disc 26 also pivoted on bushing 23, which read directly in angles and correspond merely to a protractor. Therefore, the next step to get the proper slope of the second vector is to rotate it to an angle corresponding to RK. To do this the disc 26 is turned so that the zero line of its protractor portion corresponds with the graduated edge or to the center line of arm 20. The arm 20 may now be swung in a counterclockwise direction until its graduated edge corresponds with the proper division on the RK scale of disc 26. The direction of the second vector is now determined, and is shown as the full line 35 on Fig. 8.

The next step, to locate the extremity of the second vector is to mark off along the arm 20 the length of this vector. The length, as may easily be seen from an inspection of the expression (16) has factors corresponding to the length of β, and of z, and also the factor 10^{-3}. The other elements entering into the length of this second vector are 1, the length of the line, KW_r, KV_r and $P.F._r$. The elements KW^r and KV_r may be grouped as a single factor M and may be obtained readily for any given condition at the receiver end. The length of the vector z is equal to the length of the line extending from the point 1,0 along the arm 16 to the point obtained on the chart 18. This is seen to be true since the two perpendicular components corresponding to X and R have been plotted from the point 1,0 on the chart 18. When 25 cycle lines are considered, only half the value of the reading is taken, since as stated heretofore, the values of R and X have been multiplied by 2, for getting the slope of line 17. The arm 16 is appropriately graduated as shown in Fig. 2 so that this value of the length of z is immediately obtained. The length of β multiplied by the length of the line 1 and by 10^{-3} may be plotted against the length of line at a given frequency of the line, as shown in the supplementary table or chart 28. These values have been given the notation Q. As stated heretofore, the value of Q is substantially constant for lines of a given length and frequency. For more accurate determination of the value of Q where odd lengths of lines are used, a curve sheet 29 is provided giving the value of β for 25, 50 and 60 cycle lines plotted against the length of the line in miles. From the pivot point 21 the arm 20 is graduated to the same scale as the polar co-ordinate scale so that it is possible to mark off the value of

$$\frac{Q\times M\times z}{PF_r}.$$

When this is done the extremity of the second vector is obtained and furthermore the sum of the two vectors corresponding to the two vectors of expression (16). Its length and direction may be read off from the polar co-ordinate chart 11. This value represents the ratio of the two vectors e_g and e_r and thus shows the vector relation between the generator E.M.F. and the receiver E. M. F., as well as the ratios of the absolute value of these quantities. The various operations described hereinbefore for obtaining this value do not take more than a few minutes and save an enormous amount of calculation. It is possible to perform this operation quickly because the calculator is based upon the assumption that the real part of

$$\cosh \sqrt{ZY}$$

as well as the length of vector β are variable only with the frequency and length of the line and substantially independent of the other physical constants of the line. These assumptions are entirely justifiable and errors in lines up to four or five hundred miles or even longer correspond to no more than a fraction of one per cent. This degree of accuracy is quite sufficient for all ordinary purposes.

Considering now the expression (17), for obtaining the vector ratio of the generator current to the receiver current, the vector represented by the first term of the right hand member is exactly the same vector as the first term of the right hand member of expression (16), which has just been discussed. It is necessary to add to the first vector as obtained previously a second vector corresponding to the second term of the right hand member of expression (17). Therefore, the arm 16 is left in the same place as before as well as the pivot point 21 which corresponds to the extremity of the first vector. Applying the principles of multiplication and division of vectors, it is evident that the angle of the vector represented by the second term is equal to the sum of the angles of vector of β and of the vector y minus the angle of the unit vector

$$\left[P.F._r+j\,(\pm\sqrt{1-P.\,F_r.^2})\right].$$

The slope of vector y is 90° since it represents the capacity susceptance of the line per unit length. Therefore, to obtain this setting the arm 20 is rotated until its graduated edge is perpendicular to the zero angle line 15. This setting is represented by the dotted line 37 of Fig. 9. From this angle of arm 20 must be subtracted the slope of the unit vector

$$\left[P.F._r+j\,(\pm\sqrt{1-P.F._r^2})\right].$$

Subtraction means that if the slope of unit vector is positive the arm 20 is moved in a clockwise direction. If the slope of the unit vector is, however, negative, then the arm 20 must be rotated in a counterclockwise direction. These slopes correspond to power factors at the receiver end of the line and practically means that for a lagging power factor the arm 20 must be rotated in a counterclockwise or positive direction, whereas for leading power factors it must be rotated in a counterclockwise or negative direction. The amount of this rotation is obtained with the aid of the disc 26, which has divisions corresponding to the power factor, as shown in Fig. 2. The unity power factor line is made to coincide with line 37; disc 26 is now held stationary while arm 20 is rotated to a new position corresponding to the power factor. The new position is shown by the dotted line 38 of Fig. 9. After this setting is obtained it is necessary to add to the angle of arm 20 the angle of β. This is obtained exactly as before and needs no further comment and it is not considered necessary to discuss this operation any further. After the arm 20 is set at the right angle it is merely necessary to scale off the scalar value of the vector represented by the second term of the right hand member of expression (17). This scalar value is equal to the value of

$$\frac{y \times Q \times P.F._r}{M}.$$

The values of Q and M are obtained as before. The scalar value y may be obtained by the aid of another chart which I supply with the calculator, which chart is labeled 30. This chart is plotted to give the absolute or scalar value of y for lines of 25, 50 or 60 cycles against the ratio of flat spacing of the conductor in feet to the diameter of the conductor in inches. In this way this absolute value of y is obtained with the minimum amount of difficulty. This chart 30 is similar to chart 18 in that it makes use of the ratio

$$\frac{S'}{D''}$$

directly to obtain an electrical property of the lines. It is thus unnecessary to use lengthy formulas nor to use the factor $2\pi f$.

Another chart 31 and table 32 are placed for convenience upon the same sheet as the main chart 11. The chart 31 is a curve connecting power factor with the angle of lead or lag. This is merely a sine or cosine curve as will be readily recognized. The table 32 gives values of the power factor for the appropriate angles of lead or lag.

The explanation of the theory of the calculator being now completed, a short review of the method of operation will now be described. Consider first the solution of the equation (16). This expression gives the numerical ratio between the generator E. M. F. and the receiver E. M. F., as well as their phase difference. Although the ratio as given in expression (16) is that between the E. M. F. to neutral in each case, this ratio of course is the same as the ratio between the E. M. F. between lines. To solve this equation having given the conditions at the receiver end of the line and the constants of the line such as its resistance R, the diameter of the conductors D and the spacing of the conductors S, the arm 16 is first turned so that the graduated center line intersects the point on chart 18 which corresponds to the resistance R per mile and also to the ratio of the flat spacing between the conductors in feet to the diameter of the conductors in inches at the proper frequency. This setting of arm 16 is shown in Fig. 7. The absolute value of the impedance or z per mile for each conductor may now be read directly from the graduations on arm 16. The intersection of the graduated line of arm 16 with one of the series of vertical lines 14 which correspond to the length of line and frequency of the line gives the end of the vector

$$\cosh \sqrt{ZY}.$$

The arm 20 and disc 26 are now slid along the arm 16 until their pivot represented by the central point of bushing 23 is directly over the end of this vector. The arm 20 is now moved so that the line 25 perpendicular to the graduated edge of the arm 20 coincides with the direction of arm 16. The direction of arm 20 will then be that of the dotted line 34 of Fig. 8, or perpendicular to the arm 16. The arm 20 is now further turned until the proper power factor line thereon coincides with the direction of arm 16. The new position of arm 20 is then shown by line 36 of Fig. 8. Now after rotating the disc 26 so that its zero line of the graduations corresponding to RK coincides with the graduated edge of arm 20 corresponding to the dotted line 36, the arm 20 is rotated in a counterclockwise direction for the angle equal to RK. The value of R has been used heretofore for the location of the point on chart 18. The value of K may be read off directly from chart 27 for any given length of line and frequency. After this setting is made the graduated edge of arm 20 is now in its correct position for adding the two vectors in the right hand member of expression (16). It is now merely necessary to get the length of the second vector. This may be obtained by obtaining the value of Q from charts 28 and 29 for the proper frequency and length of line. Then this value may be multiplied on the slide rule by the absolute value of z, the impedance per unit length of a single line. This impedance has been noted from the divisions on arm 16. The value M may also be readily computed and the three factors Q, M and z are to be divided by the power factor of the line to obtain the length of the second vector. When this value is obtained a point is made opposite this value on the graduated edge of arm 20 on the base chart 11. This chart gives in polar co-ordinates the length and direction of the ratio which may be immediately read off by the aid of the graduations. The angle read off of the base chart shows how much the generated E. M. F. leads the E. M. F. at the receiver end.

Coming now to equation (17) it is seen that this equation gives the ratio of the vectors corresponding to the current through the lines at the generator end to the current through the lines at the receiver end. The first term of the right hand member of this equation is exactly the same as the first term in equation (16). Therefore, the setting of arm 16 and the location of the pivot point 21 is exactly the same as before. The arm 20, however, is set differently. Its method of setting is readily understood with the aid of Fig. 9. The first setting of arm 20 is made so that it is perpendicular to the zero angle line. The position of arm 20 is then shown by the dotted line 37 of Fig. 9. To compensate for the angle of lead or lag at the receiver end it is necessary to rotate the arm 20 through an angle corresponding to the angle of lead or lag. This is done with the aid of disc 26, the zero line of which is made to coincide with the graduated edge of arm 20. The arm 20 is then rotated through the proper angle, clockwise for an angle of lead and counterclockwise for an angle of lag. In the illustration given in Fig. 9 it is assumed that the load is lagging and that the arm 20 has been rotated in a counterclockwise direction so that its graduated edge coincides with the dotted line 38. A further rotation of arm 20 is then performed in the same way as before, corresponding to the angle RK. The final position of the graduated edge of arm 20 is then shown by the full line 39. The length of the second vector of expression (17) must now be scaled off along the arm 20. This numerical value is obtained by multiplying together the value Q, the power factor and the absolute value y, the capacity susceptance per mile of the line, and dividing by M. The value of y is obtained immediately from chart 30 for the proper frequency of the line and for the proper value of the ratios

$$\frac{S'}{D''}.$$

The value of Q is obtained from charts 28 and 29. The value of M is the same as before and is equal to

$$\frac{KW_r}{KV_r^2}.$$

After the value of the length of the vector is obtained by the aid of a slide rule it is properly pointed off along the graduated edge of arm 20. The position of this point corresponds to the position of the end of vector ratio

$$\frac{i_g}{i_r}.$$

The angle by which the current at the generator end is leading the current at the receiver end corresponds to one co-ordinate of this point, shown as 40 on Fig. 9, and the numerical ratio between them is given by the other co-ordinate.

The power factor at the generator end may be obtained by assuming that the receiver E. M. F. is the vector which coincides with the zero angle line. The current at the receiver end of the line may now be plotted, since the power factor at the receiver end is known. From the directions of these two vectors representing the receiver E. M. F. and the receiver current, may be plotted vectors representing the generator E. M. F. and the generator current from the solutions obtained by the aid of the calculator. The cosine of the angle between the vectors representing the current and E. M. F. at the generator end is then the power factor at the generator end.

The power at the generator end may easily be obtained as is self evident from the values at the generator end of the current, E. M. F., and power factor.

For obtaining the generator current and E. M. F. for a condition corresponding to no load at the receiver end, the right hand member of expression (16) reduces merely to the first term, since in such a case KW_r is zero and the second term vanishes. Therefore the ratio of the E. M. F. at the generator end to the E. M. F. at the receiver end is represented by the extremity of the first vector corresponding to the intersection of the graduated line of arm 16 with one of the vertical division lines 14.

There is sometimes utilized at the receiver end, a synchronous condenser for raising the power factor at no load. The synchronous condenser must run with a current lagging by practically 90° to accomplish this result. To calculate the conditions at the generator end when such an apparatus is used, the arm 20 must be set at zero power factor lagging; that is, while obtaining the value

$$\frac{e_g}{e_r}$$

for example the arm 20 is set so that its graduated edge falls along the center line of arm 16 but extending in the opposite direction. When obtaining the value

$$\frac{i_g}{i_r},$$

the arm 20 extends in a horizontal direction. To these settings of arm 20 must be added of course the small angle RK, determined as before. Furthermore, after the proper angular setting is obtained, the length of the vector to be marked off on arm 20 is in the first instance z Q M′ and in the second instance

$$\frac{yQ}{M'};$$

M′ is equal to

$$\frac{KVA}{KV^2}$$

where KVA is the kilovolt ampere load on the condenser. This change is necessary, since the value on M came into the formulas for the evaluation of

$$\frac{i_r}{e_r}.$$

See equations (11), (12), and (13). With zero power factor and no watts load, for equation (11) may be substituted:

$$\text{KVA}=|e_r|\times 3\times|i_r| \quad (20)$$

This leads to

$$|i_r|=\frac{\text{KVA}}{|e_r|\times 3} \quad (21)$$

and equation (15) is accordingly modified to read

$$\frac{i_r}{e_r}=10^{-3}\frac{\text{KVA}}{\text{KV}_r^2}\times\left[o-j\right] \quad (22)$$

Expressions (16) and (17) become

$$\frac{e_g}{e_r}=\cosh\sqrt{\text{ZY}}+10^{-3}\text{M}'\times\left[o-j\right]\times z\times 1\times\beta \quad (23)$$

and

$$\frac{i_g}{i_r}=\cosh\sqrt{\text{ZY}}+\frac{10^3}{\text{M}'}+\left[o-j\right]\times y\times 1\times\beta. \quad (24)$$

The absolute value of the second term of the right hand member in (23) is $\text{M}'\times z\times\text{Q}$, and in (24) is

$$\frac{\text{Q}\times y}{\text{M}'}$$

as stated heretofore.

To find the conditions during short circuit of the line, use is made of formula developed by Prof. Kennelly in his "Application of Hyperbolic Functions to Electrical Engineering, Problem", page 16. This formula is as follows:

$$i_g=\frac{e_g}{\sqrt{\frac{z}{y}}\tanh\sqrt{\text{ZY}}.}$$

The absolute value reduces to

$$|i_g|=\frac{\text{KV}_g\,|\alpha|}{\sqrt{3}\text{Q}\,|z|}.$$

To obtain the charging current for no load on the line, use is made of the formula given on page 15 of Prof. Kennelly's book referred to before. This formula is as follows:

$$i_g=\frac{e_g}{\sqrt{\frac{z}{y}}\tanh\sqrt{\text{ZY}}.}$$

The absolute value reduces to

$$|i_g|=\frac{\text{KV}_g\text{Q}\,|y|}{\sqrt{3}\,|\alpha|}.$$

It is thus evident that my calculator may be used to obtain practically all of the values required in transmission line calculations. All of these values are obtained within a very short time, such as five minutes, by anyone who is fairly familiar with its operation. A great saving of time results when such calculations are to be performed and in fact my calculator has been used and is being used to a large extent by engineers desiring to investigate the characteristics of transmission lines.

While I have shown in the accompanying drawings but one embodiment of my invention, it is evident that it is not limited thereto and I aim to embrace in the appended claims all modifications falling within the spirit and scope of my invention.

What I claim as new and desire to secure by Letters Patent of the United States, is,—

1. In a calculator for investigating the electrical characteristics of transmission lines of varying construction, means for combining vectors of proper length and direction to obtain the desired result, comprising a chart, and a plurality of arms cooperating with said chart for scaling off the lengths of the vectors, the chart being provided with divisions conforming with results obtained from evaluating an appropriate infinite series to a sufficient degree of accuracy, whereby one of two components of one of the vectors may be immediately determined.

2. In a calculator for investigating the electrical characteristics of transmission lines of varying construction, means for combining vectors of proper length and direction to obtain the desired result, comprising a chart, and a plurality of arms cooperating with said chart for scaling off the lengths of the vectors, the chart being provided with divisions whereby one of two perpendicular components of one of the vectors may be immediately determined.

3. In a calculator for investigating the electrical characteristics of transmission lines of varying construction, means for plotting the expression $\cosh\sqrt{\text{ZY}}$ comprising a chart having divisions representing the real part of the expansion of

$$\cosh\sqrt{\text{ZY}},\text{ or }1+\frac{\text{ZY}}{\underline{|2}}+\frac{\text{Z}^2\text{Y}^2}{\underline{|4}}+\text{-------}$$

evaluated to a sufficient degree of accuracy for varying frequencies and lengths of line, where Z and Y are vectors representing the total impedance and capacity susceptance respectively of one conductor to neutral, said chart also having additional divisions corresponding to resistance per unit length, and divisions perpendicular to the last mentioned divisions corresponding to reactance per unit length, and an arm pivoted at the point 1,0 of the chart co-operating with the resistance-reactance divisions in such a way that a slope equal to that of the vector $\frac{ZY}{2}$ may be given it, whereby the intersecton of the center line of this arm set at the proper point on the resistance-reactance divisions intersects the division line corresponding to the real part of cosh $\sqrt{ZY}$ at the extremity of this vector.

4. In a calculator for investigating the electrical characteristics of transmission lines of varying construction, means for graphically solving equations of the form of expression (16) or its equivalent, comprising a graduated chart, an arm for obtaining the extremity of the vector cosh $\sqrt{ZY}$ on the chart, and an arm having graduations for setting it with respect to the first arm so that it will be at an angle to the zero-angle line corresponding to the sum of the angles of vector

$$\left[P.F._{r}+j\,(\pm\sqrt{1-P.F._{r}^{2}})\right]$$

and of the vector z.

5. The combination as set forth in claim 4, with means for adding to the angle of the arm an additional angle corresponding to the angle whose tangent is the imaginary part of β divided by the real part of β.

6. In a calculator for investigating the electrical characteristics of transmission lines of varying construction, means for graphically solving equations of the form of expression (17) or its equivalent, comprising a graduated chart, an arm for obtaining the extremity of the vector cosh $\sqrt{ZY}$ on the chart, and an arm having graduations for setting it with respect to the first arm so that it will be at an angle to the zero angle line corresponding to the angle of vector y minus the angle of vector

$$\left[P.F._{r}+j\,(\pm\sqrt{1-P.F._{r}^{2}})\right]$$

7. The combination as set forth in claim 6, with means for adding to the angle of the arm an additional angle corresponding to the angle whose tangent is the imaginary part of β divided by the real part of β.

8. The apparatus set forth in claim 4, together with means for adding to the angle of the second mentioned arm an additional angle corresponding to the angle whose tangent is the imaginary part of β divided by the real part of β and a second chart giving the value of the angle whose tangent is the imaginary part of β divided by the real part of β and by R, for any given length of line and frequency.

9. The apparatus set forth in claim 6, together with means for adding to the angle of the second mentioned arm an additional angle corresponding to the angle whose tangent is the imaginary part of β divided by the real part β and a second chart giving the value of the angle whose tangent is the imaginary part of β divided by the real part of β and by R, for any given length of line and frequency.

10. In a calculator for investigating the electrical characteristics of transmission lines, of varying construction, means for solving graphically expressions of the type

$$\frac{e_g}{e_r}=\cosh\sqrt{ZY}+10^{-3}\frac{K.W._{r}}{K.V._{r}^{2}(P.F._{r})}\times\left[P.F._{r}+j\,(\pm\sqrt{1-P.F._{r}^{2}})\right]\times z\times 1\times\beta,$$

or its equivalent where Z and Y are vectors representing the total impedance and capacity susceptance respectively of one conductor to neutral, e_g and e_r are the generator E. M. F. to neutral vector and receiver E. M. F. to neutral vector respectively, $K.\ W._{r}$ is the load in kilowatts at the receiving end, $K.\ V._{r}$ is the value of the receiver E. M. F. in kilovolts, $P.\ F._{r}$ is the power factor of the load, β is the infinite series

$$1+\frac{ZY}{\underline{|3}}+\frac{Z^2Y^2}{\underline{|5}}+\text{--------},$$

z is the impedance vector per unit length of one conductor to neutral, and 1 is the length of the line, comprising a chart having divisions representing the real part of the expansion corresponding to

$$\cosh\sqrt{ZY},\ \text{or}\ 1+\frac{ZY}{\underline{|2}}+\frac{Z^2Y^2}{\underline{|4}}+\text{--------},$$

evaluated to a sufficient degree of accuracy for varying frequency and length of line, said chart also having additional divisions corresponding to resistance per unit length, and divisions perpendicular to the last mentioned divisions corresponding to reactance per unit length, an arm pivoted at the point 1,0 of the chart cooperating with the resistance-reactance divisions in such a way that a slope equal to that of the vector

$$\frac{ZY}{2}$$

may be given it, whereby the intersection of the center line of this arm set at the proper point on the resistance-reactance divisions intersects the division line corresponding to the real part of

$$\cosh \sqrt{ZY}$$

at the extremity of this vector, and another arm having graduations for setting it with respect to the first arm so that it will be at an angle to the zero angle line corresponding to the sum of the angles of vector

$$\left[P.F._{r}+j\,(\pm\sqrt{1-P.F._{r}^{2}})\right]$$

and of the vector z.

11. The combination as set forth in claim 10, with means for adding to the angle of the second arm an additional angle corresponding to the angle whose tangent is the imaginary part of β divided by the real part of β.

12. In combination the apparatus set forth in claim 10, together with means for adding to the angle of the second mentioned arm an additional angle corresponding to the angle whose tangent is the imaginary part of β divided by the real part of β and a second chart giving the value of the angle whose tangent is the imaginary part of β divided by the real part of β and by R, for any given length of line and frequency.

13. In a calculator for investigating the electrical characteristics of transmission lines of varying construction, means for solving graphically expressions of the type of expression (17) or its equivalent, comprising a chart having divisions representing the real part of the expansion corresponding to

$$\cosh \sqrt{ZY} \text{ or } 1+\frac{ZY}{\lfloor 2}+\frac{Z^2Y^2}{\lfloor 4}+\text{--------},$$

said chart also having additional divisions corresponding to resistance per unit length and divisions perpendicular to the last named divisions corresponding to reactance per unit length, an arm pivoted at the point 1,0 of the chart cooperating with the resistance-reactance divisions in such a way that slope equal to that of the vector

$$\frac{ZY}{2}$$

may be given it, whereby the intersection of the center line of this arm set at the proper point on the resistance-reactance divisions intersects the division line corresponding to the real part of

$$\cosh \sqrt{ZY},$$

at the extremity of this vector, and another arm having graduations for setting it with respect to the first arm so that it will be at an angle to the zero angle line corresponding to the angle of vector y minus the angle of vector

$$\left[P.F._{r}+j\,(\pm\sqrt{1-P.F._{r}^{2}})\right].$$

14. The combination as set forth in claim 13, with means for adding to the angle of the second arm an additional angle corresponding to the angle whose tangent is the imaginary part of β divided by the real part of β.

15. The apparatus set forth in claim 13, together with means for adding to the angle of the second mentioned arm an additional angle corresponding to the angle whose tangent is the imaginary part of β divided by the real part of β and a second chart cooperating with said means giving the value of the angle whose tangent is the imaginary part of β divided by the real part of β and by R, for any given length of line and frequency.

16. In a calculator, a chart having divisions plotted in accordance with polar coordinates, an arm pivoted at a point away from the origin of the chart, and another arm the pivot of which is slidable along the first arm.

17. In a calculator, a chart having divisions plotted in accordance with polar coordinates, an arm pivoted at the point away from the origin of the chart, and another arm the pivot of which is slidable along the first arm, provided with graduations for setting it at any angle with respect to the first arm.

18. In a calculator, a chart having divisions plotted in accordance with polar coordinates, an arm pivoted at the point away from the origin of the chart, another arm the pivot of which is slidable along the first arm, provided with graduations for setting it at any angle with respect to the first arm, and a graduated member pivoted at the same point as the second arm.

In witness whereof, I have hereunto set my hand this 6th day of June, 1921.

EDITH CLARKE.