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7-10-73 XR 3,745,317

United States Patent [19]

Berthier et al.

[11] **3,745,317**

[45] **July 10, 1973**

[54] **SYSTEM FOR GENERATING THE FOURIER TRANSFORM OF A FUNCTION**

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[73] Assignee: **Commissariat A L'Energie Atomique**, Paris, France

[22] Filed: **May 21, 1971**

[21] Appl. No.: **145,689**

Related U.S. Application Data

[63] Continuation-in-part of Ser. No. 841,702, July 5, 1969, abandoned.

[30] **Foreign Application Priority Data**

July 30, 1970 France 70161135
May 26, 1970 France 7019221

[52] **U.S. Cl.** ..... **235/156**, 235/150.53, 235/152, 235/197, 324/77 B

[51] **Int. Cl.** **G06f 7/38**, G06g 7/19

[58] **Field of Search** ..... 235/156, 152, 181, 235/183, 151.51, 151.52, 151.53; 324/77 A, 77 B

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Primary Examiner—Felix D. Gruber
Attorney—Cameron, Kerkam & Sutton

[57] **ABSTRACT**

A system generates the Fourier transform of a real function $z(t)$ which is known in a time interval 2T and which is given in this interval in the form of N samples at equal time intervals Θ, i.e., in the form $Z(p\,\Theta)$ in which p is an integer such that $T/\Theta \leq p \leq T/\Theta$. The product $z(p\Theta)x \cos \pi/T\, Kp\Theta$ or the product $z(p\Theta) \times \sin \pi/T\, Kp\Theta$ generated in the system is computed for values of the integer K increasing from 1 inclusive to T/Θ inclusive. The sum of these successive products, is computed; each of the sums represents a sampling point of the harmonic at frequency K/2T of the Fourier transform.

7 Claims, 23 Drawing Figures

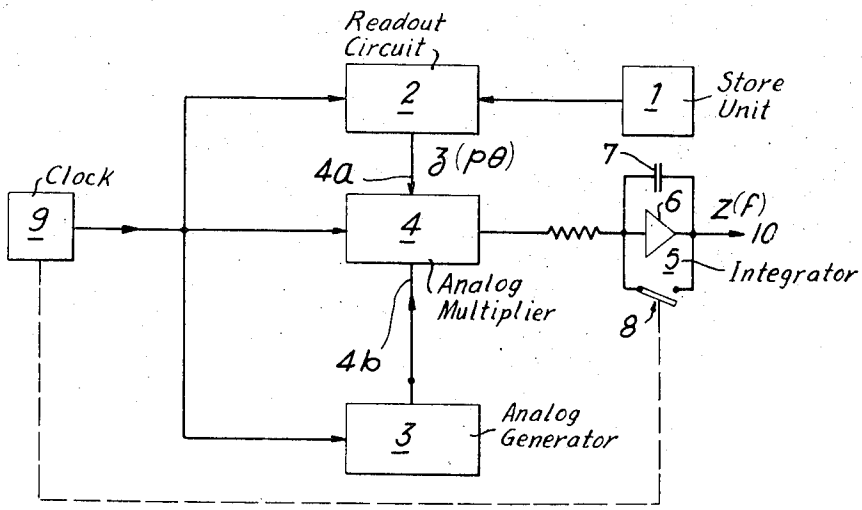


FIG. 1

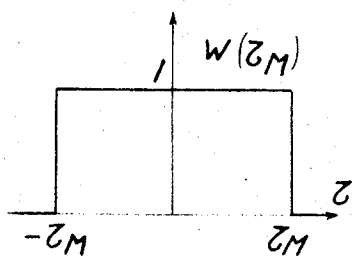


FIG. 2

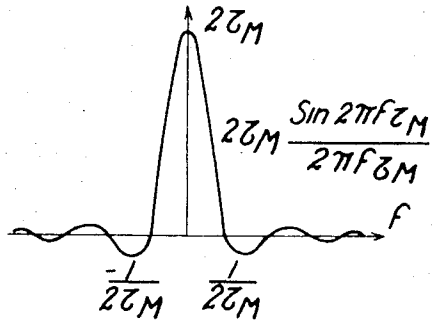


FIG. 3

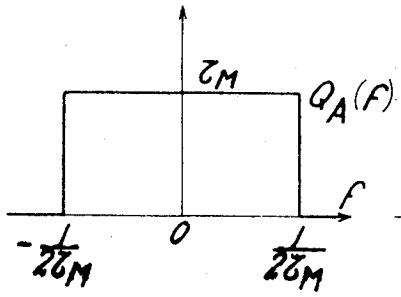
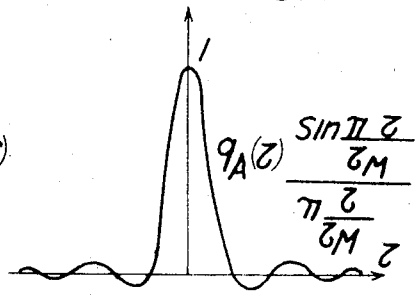


FIG. 4



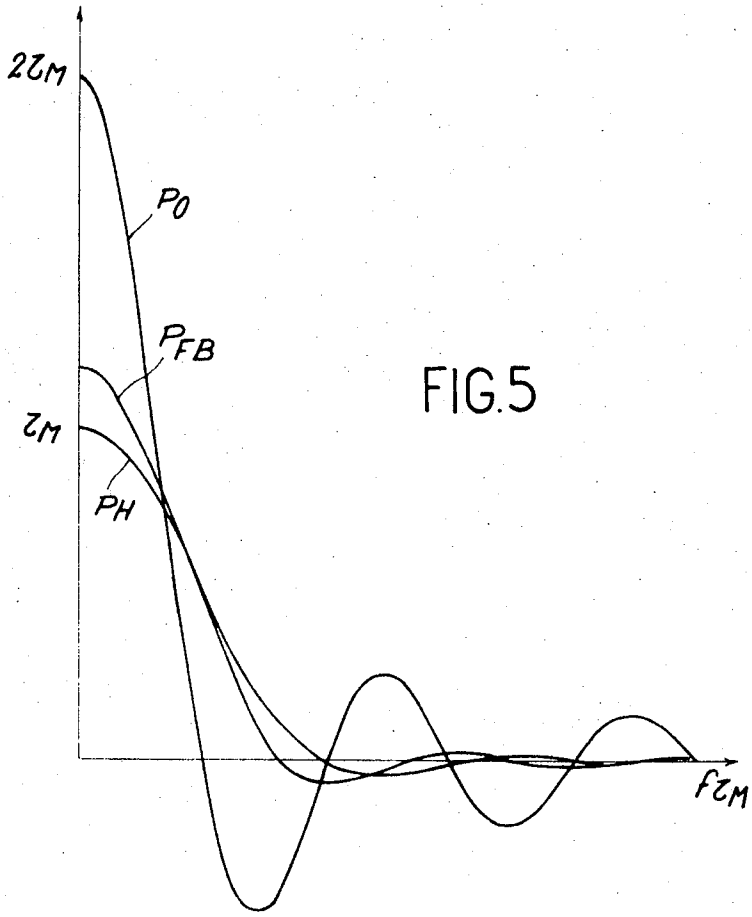


FIG.5

FIG.6

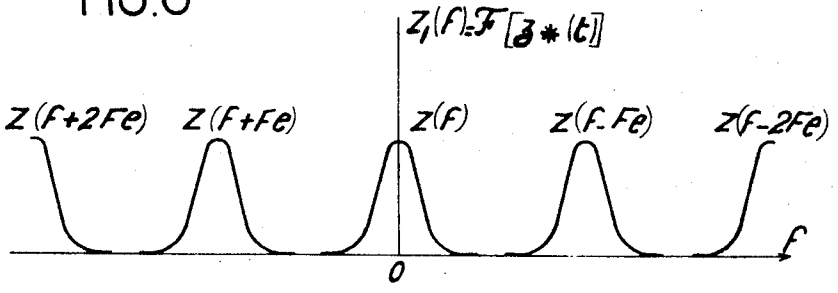


FIG.7

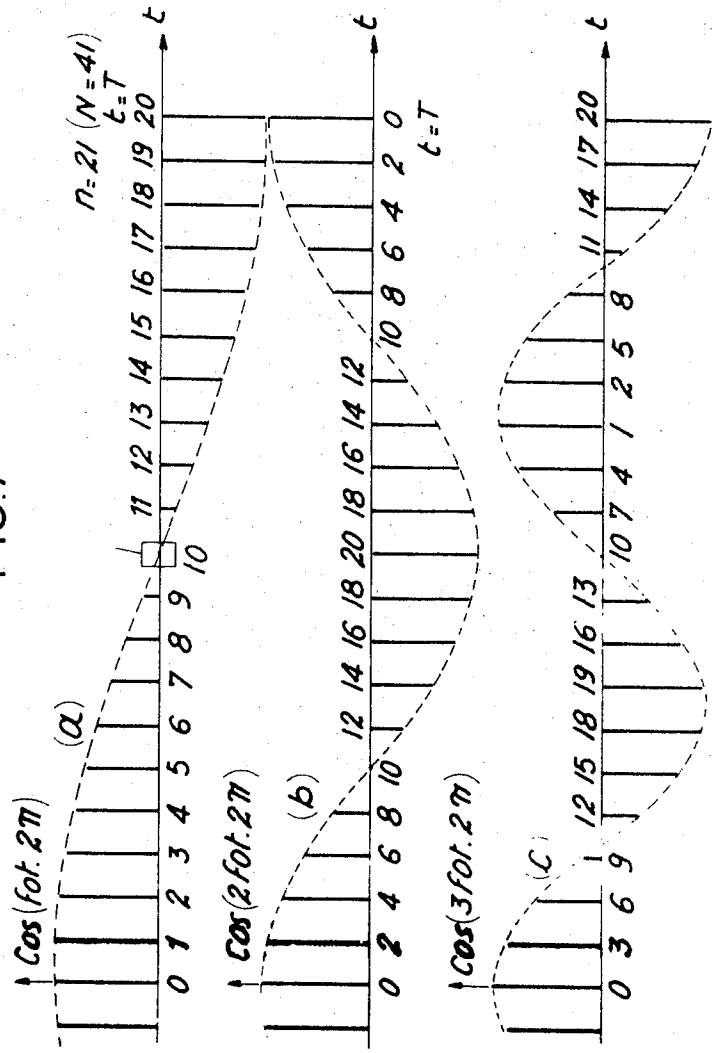

n=21 (N=41)
t=T
t
(a)
cos(fot.2π)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
(b)
cos(2fot.2π)
0 2 4 6 8 10 12 14 16 18 20 18 16 14 12 10 8 6 4 2 0
t=T
(c)
cos(3fot.2π)
0 3 6 9 12 15 18 19 16 13 10 7 4 1 2 5 8 11 14 17 20

FIG. 8

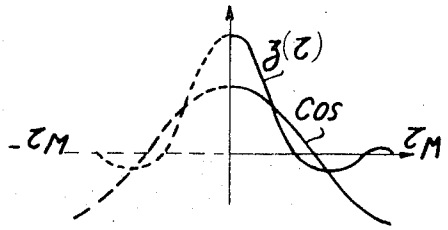


FIG. 9

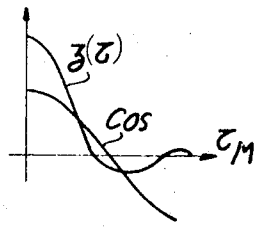


FIG. 10

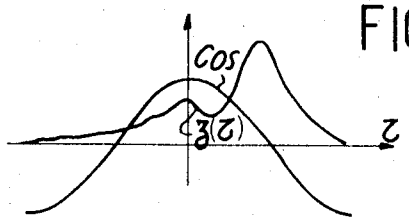


FIG. 11

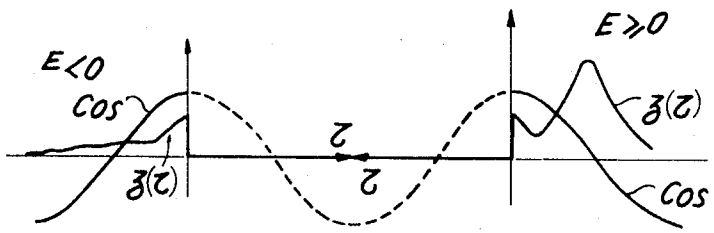


FIG. 12

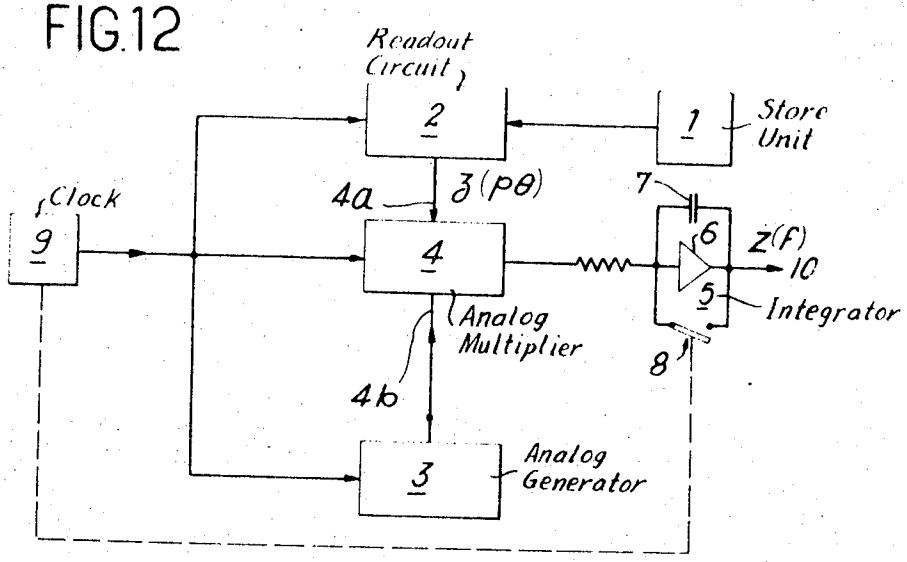
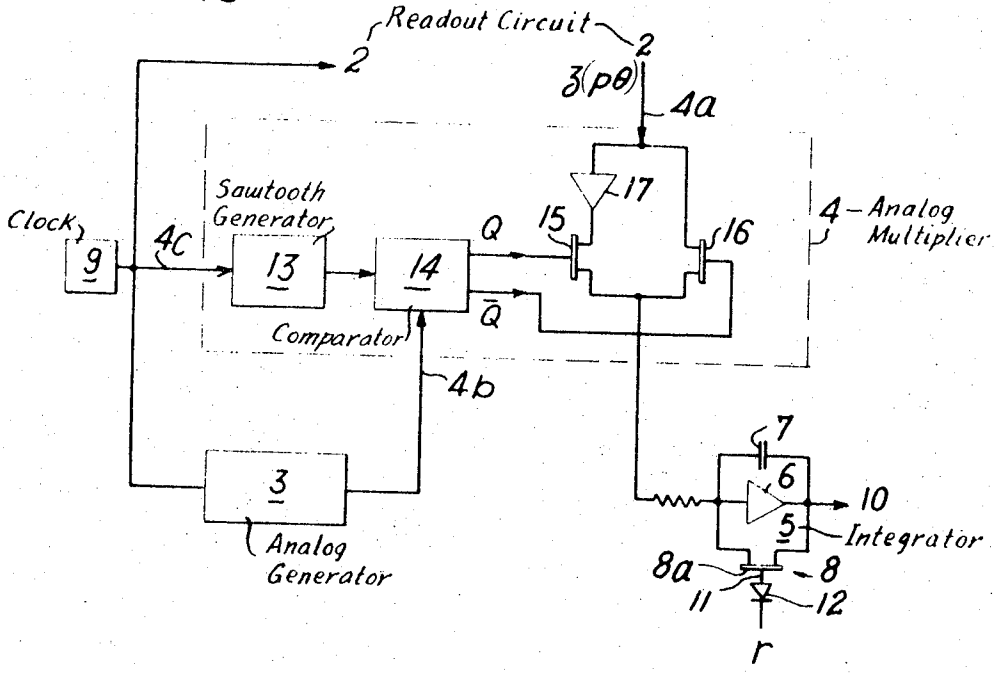
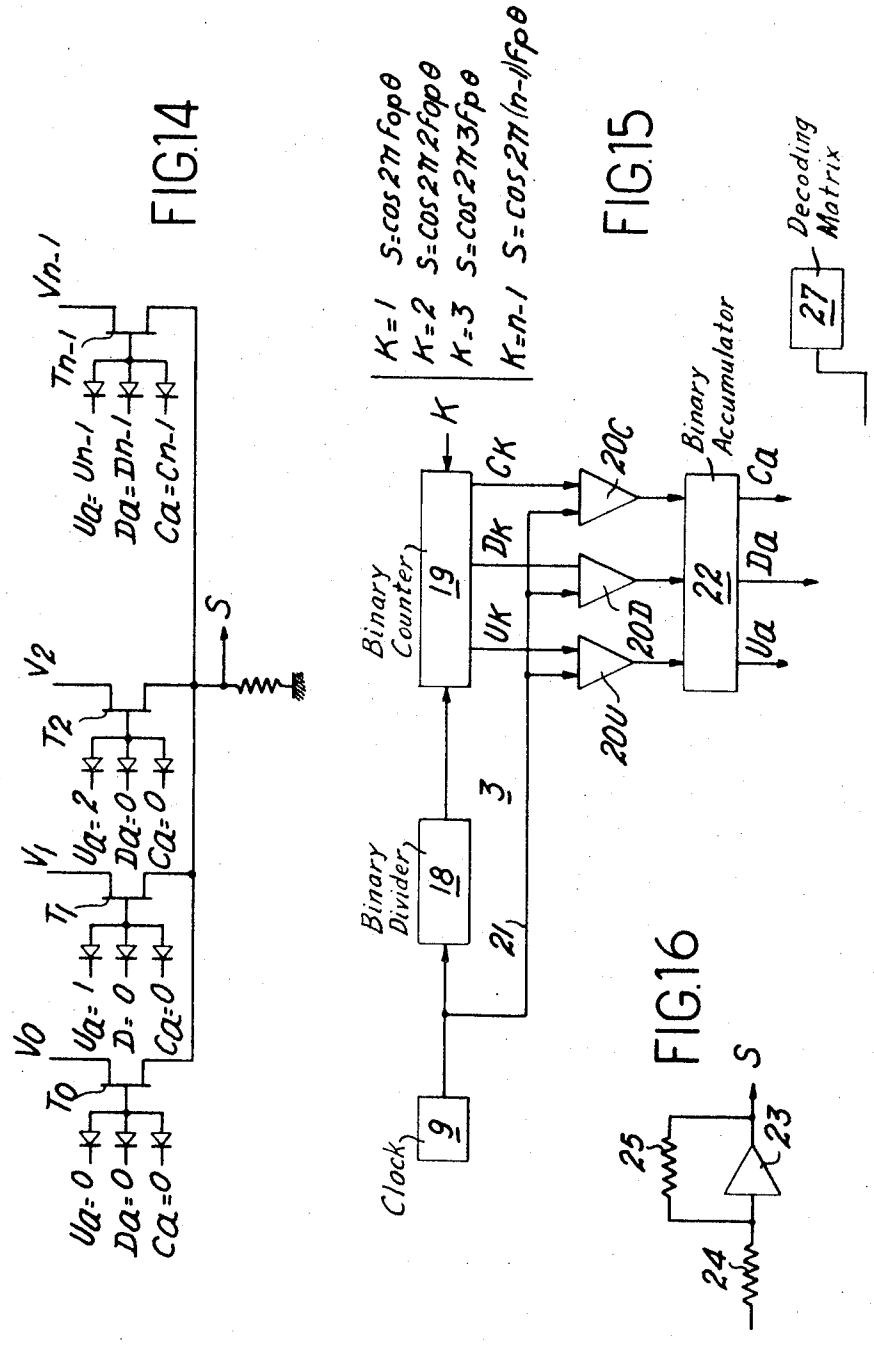

Readout Circuit
2
1
Store Unit
Clock
9
4a
3(pθ)
7
6
Z(f)
10
4
5
Integrator
Analog Multiplier
8
4b
3
Analog Generator

FIG. 13


Readout Circuit
2
2
3(pθ)
4a
17
Sawtooth Generator
Clock
9
4C
13
14
Q
15
16
4 - Analog Multiplier
Comparator
Q̄
4b
7
6
10
5
Integrator
3
Analog Generator
8a
8
11
12
r


FIG14
Vn-1
Tn-1
Ua= Un-1
Da=Dn-1
Ca=Cn-1
V2
T2
Ua=2
Da=0
Ca=0
V1
T1
Ua=1
D=0
Ca=0
V0
T0
Ua=0
Da=0
Ca=0
S
FIG15
K=1 S=cos2πfopθ
K=2 S=cos2π2fopθ
K=3 S=cos2π3fpθ
K=n-1 S=cos2π(n-1)fpθ
K
CK
DK
UK
Binary Counter 19
Binary Divider 18
Clock 9
3
2
20c
20D
20U
Binary Accumulator
22
Ca
Da
Ua
Decoding Matrix
27
FIG16
S
25
24
23

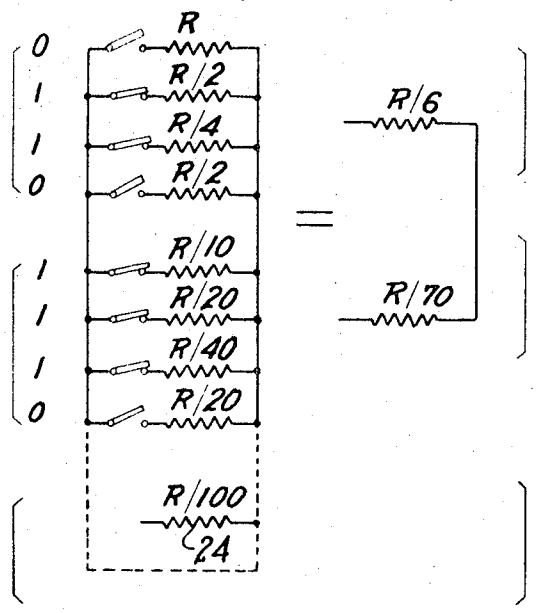

R
R/2
R/4
R/2
R/10
R/20
R/40
R/20
R/100
24
0
1
1
0
1
1
1
0
=
R/6
R/70
x=76

FIG.17

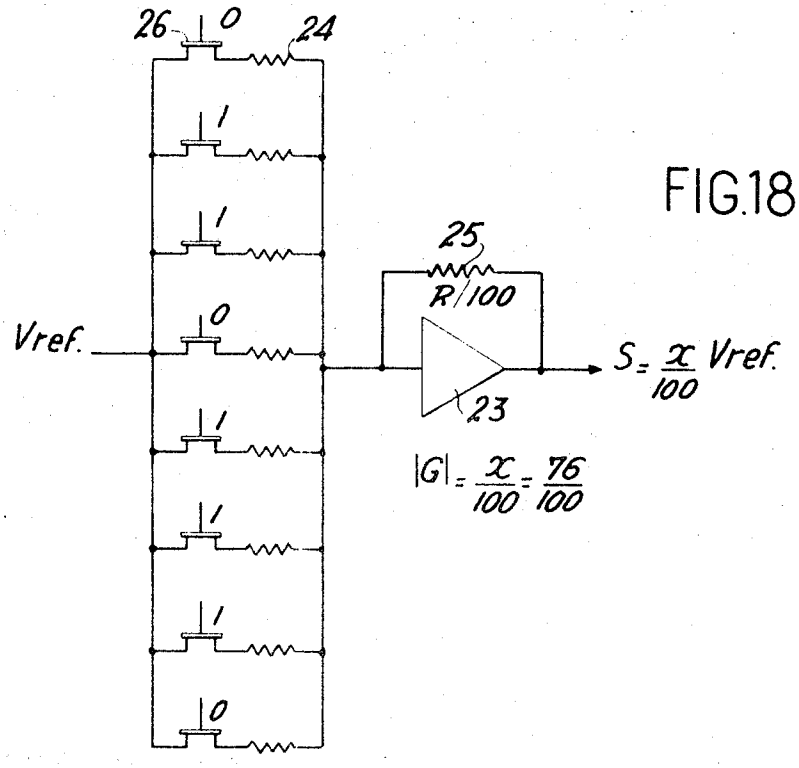

26
0
24
1
1
0
1
1
1
0
Vref.
25
R/100
23
S= x/100 Vref.
|G|= x/100 = 76/100

FIG.18

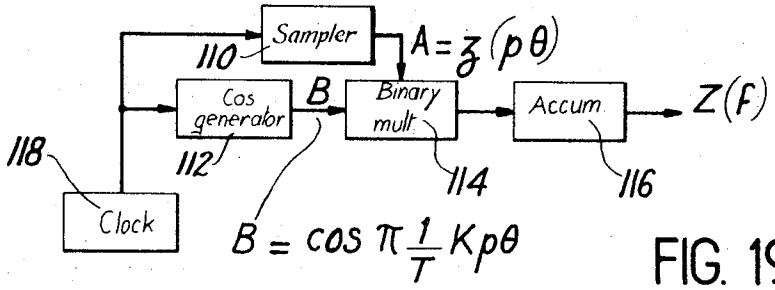

Sampler
110
A = z (pθ)
Cos generator
112
B
Binary mult
114
Accum
116
Z(F)
118
Clock
B = cos π 1/T Kpθ

FIG. 19

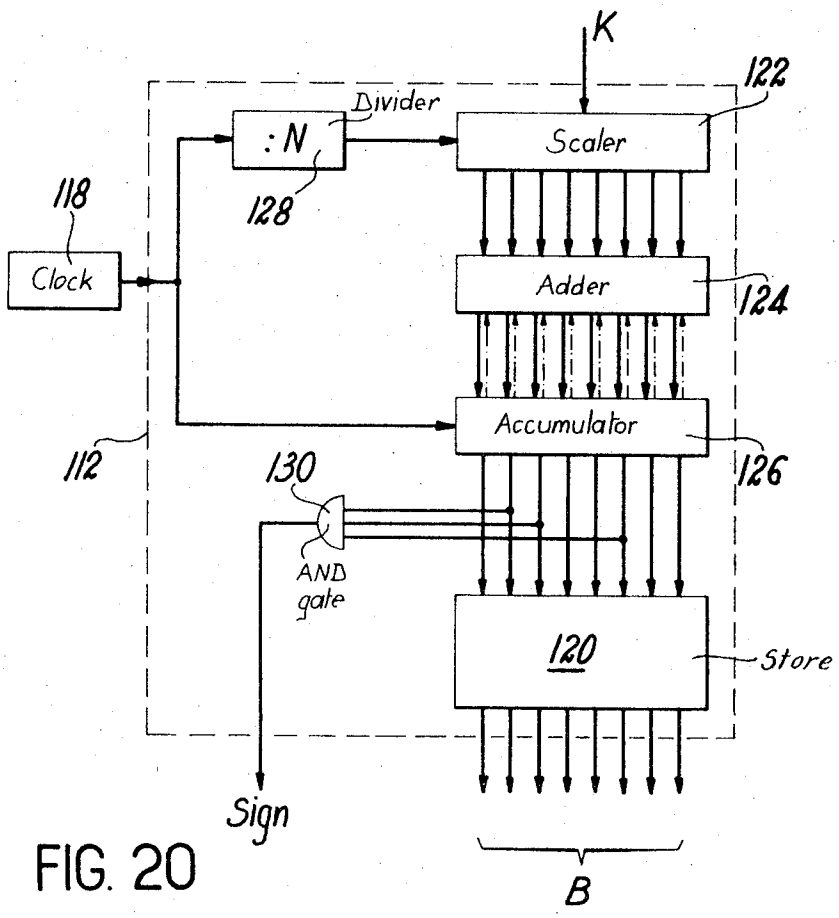

K
122
Divider
:N
128
Scaler
118
Clock
Adder
124
Accumulator
112
130
126
AND gate
120
Store
Sign
B

FIG. 20

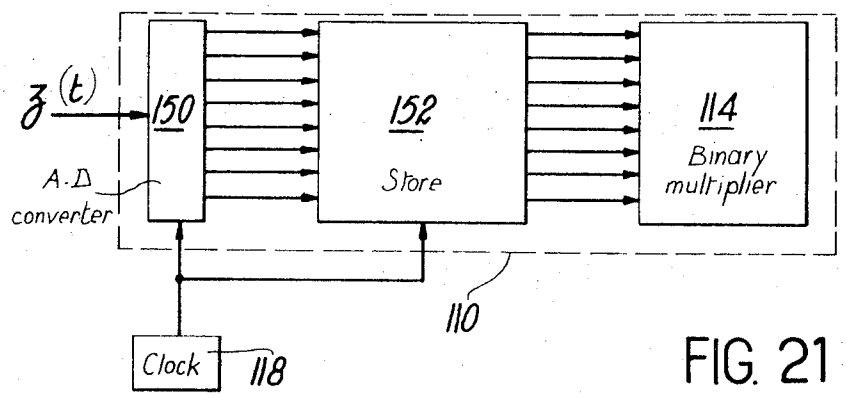

z (t)
150
A-D converter
152
Store
114
Binary multiplier
110
Clock
118

FIG. 21

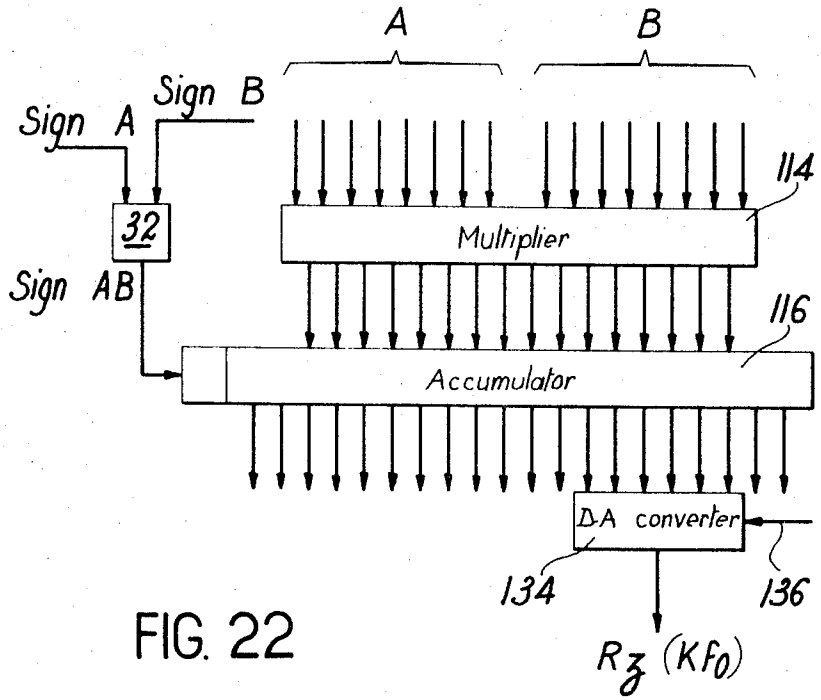

A
B
Sign A
Sign B
32
Sign AB
114
Multiplier
116
Accumulator
D-A converter
134
136
Rz (Kf0)

FIG. 22

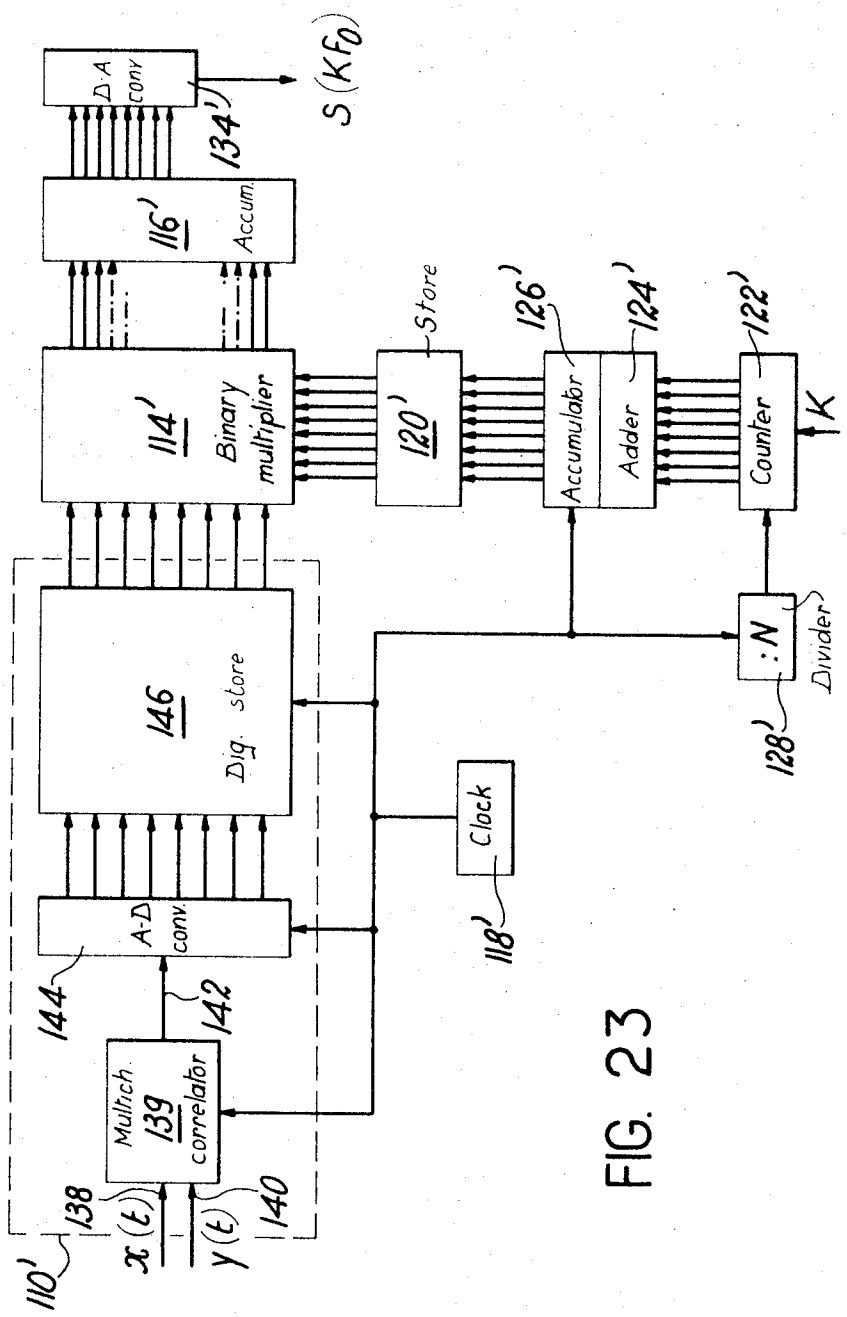

D-A conv
134'
S(KF0)
116' Accum.
114' Binary multiplier
120' Store
126'
124'
122'
Accumulator
Adder
Counter
K
146 Dig. store
:N
Divider
128'
Clock
118'
A-D conv.
144
142
Multich. 139 correlator
138
140
x(t)
y(t)
110'

FIG. 23

SYSTEM FOR GENERATING THE FOURIER TRANSFORM OF A FUNCTION

CROSS REFERENCE

This application is a continuation in part of U.S. Pat. application Ser. No. 841,702 filed July 15, 1969, now abandoned.

BACKGROUND AND SUMMARY OF THE INVENTION

This invention relates to equipment for generating the Fourier transform of a function, suitable for use in conjunction with correlators in order to analyze an energy spectral density.

It is an object of the invention to provide improved equipment for generating the Fourier transform of a real function known by its physical representation; it is another object to provide equipment which is adapted to process signals over a wide spectrum band and which is simple in construction.

According to an aspect of the invention, there is provided a system for generating the Fourier transform of a real function $z(t)$ which is known in an interval in the form of data sampled at N points equally spaced by the interval in the form $Z(p\Theta)$ in which p is an integer such that $T/\Theta \leq p \leq T/\Theta$, comprising multiplier for selectively forming the Θ in of $z(p\Theta)$ and cos π/T $Kp\Theta$ for values of the integer K increasing from 1 inclusive to T/Θ inclusive and the product of $z(p\Theta)$ and sin π/T $Kp\Theta$ for values of the integer K increasing from 1 inclusive to T/Θ inclusive and an integrator for summation of the successive products, each of the sums representing a sampling point associated with the frequency K/2T of the Fourier transform wherein to obtain the function cos $\pi/TKp\Theta$, the function cos$2/T\pi t$ is sampled for N points of the interval 2T which are spaced by the interval Θ and 1/K of these sampled values is sampled. A device of the above type may cooperate with a real-time correlator, for instance of the type described in U.S. Pat. No. 3,555,258, for automatically providing the automatic analysis of an energy spectral density.

BRIEF DESCRIPTION OF THE DRAWINGS

The invention will be more readily understood from the following description and accompanying drawings, which are of course given solely by way of example.

FIGS. 1 to 4 of the drawings show the corresponding time and spectral windows respectively without weighting (FIGS. 1 and 2) and with weighting of the recent Fauque-Berthier type (FIGS. 3 and 4) for determining the Fourier transform of a correlation function.

FIG. 5 represents the comparison of two types of weighting and without weighting.

FIG. 6 shows the Fourier transform of a sampled function.

FIG. 7 shows three cosine functions respectively for values of K equal to 1, 2 and 3.

FIGS. 8 and 9; and 10 and 11 respectively illustrate the application of the method according to the invention to the specific case of an auto-correlation function and a cross-correlation function (for a definition of the auto-correlation and cross-correlation functions reference may be had to French Pat. No. 1,473,006).

FIG. 12 diagrammatically illustrates a device according to an embodiment of the invention for generating the Fourier transform of a function known by its physical representation.

FIGS. 13, 14 and 15 are representations of various components illustrated in block form in FIG. 12, namely, the multiplier, the cosine or sine function generator and the control unit for this generator.

FIGS. 16, 17 and 18 show modified embodiment of the read-out of the cosine function.

FIG. 19 is a simplified block diagram of a digitally working embodiment of the device for forming the Fourier transform of a function.

FIG. 20 is a simplified block diagram of the cosine function sample generator of the embodiment of FIG. 1.

FIG. 21 is a block diagram of a sample generator suitable for use in the device of FIG. 19 when the function is available in analog form.

FIG. 22 is a block diagram of the integrating multiplier of the device of FIG. 19, and

FIG. 23 is a simplified block diagram of a spectral analyser comprising a correlator and a device for producing the Fourier transform.

The invention will now be described with particular reference to the generation of the Fourier transform of a function known by its physical representation, more particularly for automatic analysis of the spectral density.

It should first be pointed out that the analysis of an energy spectral density is a problem which has numerous applications, of which the main ones are accurate analysis of vibration phenomena and noise analysis.

To solve problems of this type there are at present two types of instrument available :

An ordinary heterodyne spectral density analyser, which has the serious disadvantage that it is not a real-time instrument. To integrate the measurement over a time T_1, the operation must be repeated for each point of the spectrum, so that to obtain N_1 points of the spectrum the measuring time must be greater than the product $N_1 T_1$; also, in this case, the entire signal to be processed must be stored beforehand.

A "pulse-compression" spectral density analyser, which although it may be considered as a real time instrument is limited by the capacity of the memory used for the time compression, since it generally does not allow a sufficient "slice" T_1 of the signal to be studied. This instrument has to be combined with a relatively complex digital integrator which makes it much more expensive. Also, as a result of its construction, this instrument is limited as regards frequency and cannot process signals having a spectrum extending beyond some thousands Hz.

In contradistinction thereto, the invention provides real-time analysis of the spectral density over a time T_1 of any magnitude; signals can be processed in a much wider band (e.g. from 0 to 4 MHz), and this band can be extended because it is limited solely by present-day performances of switching transistors (in the embodiment concerned, these performances have voluntarily been taken as the limit so that these switching transistors can be used in very simple circuits). The simplicity of design results in an analyser which is much cheaper than the analysers of the above-mentioned two types which are at present available.

Before explaining the invention in detail, the main properties of Fourier transforms will be given, since they will be useful for an understanding of the invention.

It will be assumed that a real function $z(t)$ is known not by its mathematical formulation but by its physical representation for values of the variable t ranging between $-T$ and $+T$.

This function will initially be assumed to have a value of zero for any value of the variable outside the interval $-T$, $+T$.

A function of this type admits of a Fourier transform denoted by $Z(f)$ and defined by the formula:

$$Z(f)=\int_{-\infty}^{+\infty} z(t)\,e^{-2\pi jft}dt \tag{1}$$

Since $z(t)$ is zero outside the interval $(-T, +T)$, equation (1) can be written as follows:

$$Z(f)=\int_{-T}^{T} z(t)\,e^{-2\pi jft}dt \tag{2}$$

i.e.,

$$Z(f)=\int_{-T}^{T} z(t)\,\cos 2\pi ftdt-j\int_{-T}^{T} z(t)\,\sin 2\pi ftdt \tag{3}$$

Generally, therefore, $Z(f)$ is a complex function of the variable f comprising a real part (in phase):

$$R_z(f)=\int_{-T}^{T} z(t)\,\cos 2\pi ftdt \tag{4}$$

and an imaginary part (in quadrature):

$$I_z(f)=\int_{-T}^{T} z(t)\,\sin 2\pi ftdt \tag{5}$$

An amplitude spectrum

$$A_z(f)=\sqrt{[R_z(f)]^2+[I_z(f)]^2} \tag{6}$$

and a phase spectrum

$$\phi_z(f)=\text{Arc } tg\ (I_z(f)/R_z(f) \tag{7}$$

can also be defined.

It is known that $Z(f)$, which is the frequency spectrum of a physical function $z(t)$, has by its nature a limited spectrum, because the pass-band of any real process is limited. It will therefore be assumed that F_o is the frequency so that for $|f| > F_o$, $Z(f)$ is zero (or less than a predetermined value ϵ, which may be as small as desired).

Generally, and according to the definition given above, $Z(f)$ is a function of f which is zero outside the interval $-F_o$, $+F_o$. In practically every case, it is unnecessary to know the spectrum $Z(f)$ continuously; on the contrary, it is sufficient to know just the values of $Z(f)$ for discrete values of the variable f.

We therefore have instead of $Z(f)$ a series of values $Z(O), Z(f_o), Z(2f_o), \ldots Z(Kf_o), \ldots Z(pf_o)$ with integer $p < F_o/f_o$.

We may therefore write the following

$$Z^*(f)=f_0\sum_{K=0}^{K=n} Z(Kf_0)\,\delta(f-Kf_0) \tag{8}$$

where $Z^*(f)$ denotes the sampled function $Z(f)$, f_o being the interval between two successive points of the spectrum $Z^*(f)$ and $\delta(f-Kf_o)$ denotes the Dirac delta distribution or function, i.e., the extension to the range of functions or distributions of Kronecker's coefficient; by definition, the Dirac function $\delta(x)$ is such that for an arbitrary function $c(x)$ of a single continuous variable,

$$\int c(x)\delta(x)dx=c(0) \text{ and therefore } \int c(x)'\delta(x-y)dx=c(y)$$

for any value of y; the Dirac function or distribution may be "represented" by a vertical line of zero width and infinite height, of finite and non-zero area.

If the function $z(t)$ is known in the interval $-T$, $+T$, it is unnecessary to try to obtain the spectrum with a "definition" better than $1/2T$ as regards the frequency.

We may therefore take $fo = 1/2T$, and hence

$$Z^*(f)=\frac{1}{2T}\sum_{K=0}^{K=n} Z\left(\frac{K}{2T}\right)\delta\left(f-\frac{K}{2T}\right) \tag{9}$$

The following remarks will be concerned with the determination of

$$R_z^*(f)=\frac{1}{2T}\sum_{K=0}^{K=n} R_z\left(\frac{K}{2T}\right)\delta\left(f-\frac{K}{2T}\right) \tag{10}$$

The remarks which will be made concerning the real part of $Z(f)$ will be readily transferable to the imaginary part of $Z(f)$.

Having dealt with these general features of Fourier transforms, we shall now pass to the calculation of the Fourier transform. Since, by definition,

$$R_z(f)=\int_{-T}^{T} z(t)\,\cos 2\pi ftdt \tag{11}$$

we also have:

$$R_z(Kf_0)=\int_{-T}^{T} z(t)\,\cos 2\pi Kf_0tdt \tag{12}$$

We must therefore caculate

$$\int_{-T}^{T} z(t)\,\cos 2\pi Kf_0tdt.$$

For this purpose, $z(t)$ must be multiplied by the function $\cos 2\pi Kf_o t$, and then this product must be integrated from $-T$ to $+T$, which is the interval within which $z(t)$ is known.

This operation will be repeated for the various values of K from **0** to F_o/f_o.

To this end, the function $z(t)$ must be stored and the most convenient method is to store the function $z(t)$ after sampling, i.e., have $z(t)$ defined by a series of point values

$-m\Theta, -(m-1)\Theta, \ldots -K(\Theta), \ldots -2\Theta, -\Theta, o, \Theta, 2\Theta, \ldots K\Theta \ldots (m-1)\Theta, \ldots m\Theta$

m being an integer between 0 and T/Θ (as will be seen hereinafter, to perform the invention the values of $z(t)$ at each of these times are stored either digitally, e.g. in core or bistable stores, or in analog form, e.g. as capacitor charges). The function $z(t)$ is therefore represented by a series of values $z(-m\Theta), \ldots z(-K\Theta), \ldots z(-\Theta), z(O), z(\Theta), \ldots z(K\Theta), \ldots z(m\Theta)$.

Shannon's theorem (some explanations of which will be given below) shows that to enable these samples to reconstitute the function $z(t)$ it is only necessary for the sampling interval Θ to be such that $\Theta < 1/2F_o$.

We may now therefore assume that instead of $z(t)$ we have

$$z^*(t) = \theta \sum_{p=-\frac{T}{\theta}}^{p=T/\theta} z(p\theta)\,\delta(t-p\theta) \tag{13}$$

The meaning of $p = -T/\Theta$ or $p = T/\Theta$ is obviously that p is an integer between $-\lambda$ and $+\lambda$, λ being the integer immediately below T/Θ.

It is proved (as explained below) that if the function $z(t)$ is known by $z^*(t)$, we shall have an exact value of $R_z(Kf_o)$ by making

$$R_z(Kf_0) = \theta \sum_{p=-\frac{T}{\theta}}^{p=T/\theta} z(p\theta)\,\cos 2\pi K f_0 p\theta \tag{14}$$

We have seen that the lowest value that was possible for fo was $fo = 1/2T$; and hence

$$R_z\left(\frac{K}{2T}\right) = \theta \sum_{p=-\frac{T}{\theta}}^{p=T/\theta} z(p\theta)\,\cos 2\pi \frac{K}{2T} p\theta \tag{15}$$

In other words, to have the exact value of

$$\int_{-T}^{T}$$

$z(t)\cos 2\pi K/2T t\,dt$, it is only necessary to operate from the function $z(t)$ sampled in accordance with Shannon's sampling theorem, by multiplying each sample of $z^*(t)$ by the corresponding sample of $\cos 2\pi K/2T\,t$, i.e., by multiplying each value of $z(p\Theta)$ by the value of the cosine function at the time $p\Theta$, and by forming the sum of these products, and it is precisely this that is the main feature of the invention.

It is quite pointless to make any interpolation between the successive samples.

Generally, the sampling frequency $2F_e$ will be greater than $2F_o$, and we shall have

$$\Theta = 1/2F_e < 1/2F_o;$$

whence

$$R_z^*\left(\frac{K}{2T}\right) = \frac{1}{2F_e} \sum_{p=-2F_eT}^{p=2F_eT} z\left(\frac{p}{2F_e}\right) \cos 2\pi \frac{K}{2T} \frac{p}{2F_e} = R_z\left(\frac{K}{2T}\right) \tag{16}$$

Such a method provides the real-time determination of energy spectral density (power spectra).

As indicated above, existing instruments for the real-time determination of these power spectra are based on time compression followed by narrow-band smoothing. Such instruments are complex, expensive, and are limited to some tens of kHz on the high-frequency side.

On the other hand, it is an easy matter to obtain a power spectrum from the auto-correlation function of the signal by the Wiener-Kinchine theorem.

The Wiener-Kinchine theorem states that the energy spectral density of a signal and the auto-correlation function of this signal are Fourier transforms of one another and this may be written as follows

$$C_{xx}(\tau) \rightleftharpoons S_{xx}(f)$$

$$C_{xx}(\tau) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} C_{xx}(\tau')\,e^{2\pi j f(\tau-\tau')}\,d\tau'\,df \tag{17}$$

where

$$C_{xx}(\tau) = \frac{1}{2T}\int_{-T}^{T} x(t)\,x(t-\tau)\,dt \tag{18}$$

An interspectrum $S_{xy}(f)$ is also defined, which is the Fourier transform of the inter-correlation function $C_{xy}(\tau)$ between two signals x and y where

$$C_{xy}(\tau) = \frac{1}{2T}\int_{-T}^{T} x(t)\,y(t-\tau)\,dt \tag{19}$$

This interspectrum $S_{xy}(f)$ is very important in many problems, particularly in the case where the function $C_{xy}(\tau)$ is the pulse response of a system; in that case $S_{xy}(f)$ is the frequency response or transfer function of said system.

There are instruments available which automatically carry out the auto-correlation and inter-correlation functions and provide these functions in a sampled form, i.e., in the form of a series of equally spaced pulses.

$$C^*(\tau) = \theta \sum_{K=\frac{T}{\theta}}^{K=T/\theta} C(k\theta)\,\delta(t-K\theta) \tag{20}$$

Such correlators therefore only have to be connected to a Fourier transform generator operating in accordance with the above-described principle to provide a system for the automatic real-time analysis of a spectral density. Such an instrument is much less complex than current spectral density analysers which operate on other principles and the instrument according to the invention also has better performances.

We shall now examine the precautions required for calculating the Fourier transform of a correlation function.

When it is required to obtain the Fourier transform of a correlation function defined between $-\tau_M$ and $+\tau_M$, two cases must be considered. First case: $C(\tau)$ is zero

when $|\tau| > \tau M$; in this case, it is only necessary to apply the basic definition and the Fourier transform of $C(\tau)$ gives $S(f)$. Second case: $C(\tau)$ differs from zero when $|\tau| > \tau M$; this means that the correlation function $C(\tau)$ is obtained from a correlation function $C_1(\tau)$ by truncation, $C_1(\tau)$ being zero in the case of $|\tau| > \tau_M$ with $1M > \tau M$.

$C(\tau)$ is therefore $C_1(\tau)$ multiplied by a "window" function $W(\tau)_M$, i.e.,

$$C(\tau) = C_1(\tau) \cdot W(\tau_M) \tag{21}$$

where

$$W(\tau) = \begin{cases} 0 \text{ for } |\tau| > \tau_M \\ 1 \text{ for } -\tau_M \leq \tau \leq \tau_M \end{cases}$$

If we calculate the Fourier transform of $C(\tau)$, there corresponds to the product $C(\tau) \cdot W(\tau_m)$ in the time range, the convolution product

$$S(f) = (S_1(f) * 2\tau_M (\sin 2\pi f \tau_M / 2\pi f \tau_M) \tag{22}$$

$$C_1(\tau) \cdot W(\tau_M) \rightleftharpoons S_1(f) * 2\tau_M (\sin 2\pi f \tau_M / 2\pi f \tau_M) \tag{23}$$

in the frequency range; $S(f)$ is therefore the result of convolution of the true spectrum $S_1(f)$ by the function

$$2\tau_M (\sin 2\pi f \tau_M / 2\pi f \tau_M).$$

This convolution will introduce spurious oscillations (or lobes) on $S(f)$. This disadvantage is reduced by multiplying the correlation function by a weighting function. There are three main weighting methods:

a. Von Hann's method (the "hanning" method), which comprises multiplying $C(\tau)$ by $1/2\ (1 + \cos(\pi\tau/\tau_M))$;

b. R.W. Hamming's method (the "hamming" method), which comprises multiplying $C(\tau)$ by $0.54 + 0.46 \cos(\pi\tau/\tau_M)$;

c. the Fauque-Berthier method, which comprises multiplying $C(\tau)$ by:

$$\frac{\sin \pi \frac{\tau}{\tau_M}}{\pi \frac{\tau}{\tau_M}}.$$

The first two methods are conventional but the third is a recent method and will therefore be briefly detailed below.

Let $W(\tau_M)$ be the time window (FIG. 1) which multiplies the function (τ) and let $Q_o(f)$ be the corresponding spectral window (FIG. 2) which convolutes the spectrum when no weighting is applied. This spectral window $Q_o(f)$ is inconvenient, because in introduces side lobes, but on the other hand it has the advantage of being narrow in the frequency range and hence giving a fairly good resolution.

The conventional weighting methods (hanning or hamming) eliminate the side lobes almost completely but substantially double the width of the spectral window and hence halve its height; this greatly reduces the resolution.

The resulting spectral windows (hanning or hamming) are very compressed in relation to $Q_o(f)$.

The attempt must therefore be made in the spectral range to reduce the side lobes of $Q_o(f)$ without excessively reducing the height of the central lobe. For this purpose, $Q_o(f)$ is convoluted by a rectangular window $Q_A(f)$, the width of which is equal to the period of the side lobes of $Q_o(f)$, i.e., $1/\tau_M$ (FIG. 3); corresponding to this window $Q_A(f)$ we have in the time range (FIG. 4) a window

$$q_A(\tau) \frac{\sin \pi \frac{\tau}{\tau_M}}{\pi \frac{\tau}{\tau_M}}$$

; since it is required to convolute $Q_o(f)$ by $Q_A(f)$ in the frequency range, it is only necessary to obtain the product of $W(\tau_M)$ and $q_A(\tau)$ in the time range, because $Q_o(f) * Q_A(f) \rightleftharpoons W(\tau_M) \cdot q_A(\tau)$ (24)

For this Fauque-Berthier weighting, therefore, we shall multiply $C(\tau)$ by

$$q_A(\tau) = \frac{\sin \pi \frac{\tau}{\tau_M}}{\pi \frac{\tau}{\tau_M}}.$$

FIG. 5 shows the representative curves P_H of the case of a weighting of the conventional type (hanning or hamming) P_{FB} of the case of weighting of the recent type (Fauque-Berthier) and P_o of the case without weighting as a function of f_M.

To conclude these general remarks, we shall return to Shannon's sampling theorem.

Let $z(t)$ be a function and a system which samples said function at the frequency Fe, i.e., which samples the value of the function $z(t)$ at times regularly spaced by $\Theta = 1/F_e$.

The function $z(t)$ then becomes the sampled function $z^*(t)$ represented by a series of values $\ldots Z(-n/F_e), \ldots z(-1/F_e), z(O), z(1/F_e), \ldots z(n/F_e), \ldots$ which may be written as follows:

$$z^*(t) = \frac{1}{F_e} \sum_{n=-\infty}^{+\infty} z\left(\frac{n}{F_e}\right) o\delta\left(t - \frac{n}{F_e}\right) \tag{25}$$

or

$$z^*(t) = \frac{1}{F_e} \sum_{n=-\infty}^{+\infty} z(t)\,\delta\left(t - \frac{n}{F_e}\right) \tag{26}$$

or alternatively

$$z^*(t) = \frac{1}{F_e} z(t) \sum_{n=-\infty}^{+\infty} \delta\left(t - \frac{n}{F_e}\right) \tag{27}$$

It is known that for a Fourier transform

$$\sum_{n=-\infty}^{+\infty} \delta\left(t - \frac{n}{F_e}\right)$$

admits

$$F_e \sum_{K=-\infty}^{+\infty} \delta(f-Kf_e)$$

i.e., that

$$\sum_{n=-\infty}^{\infty} \delta\left(t-\frac{n}{F_e}\right) \rightleftharpoons F_e \sum_{K=-\infty}^{\infty} \delta(f-Kf_e) \quad (28)$$

Assuming that $z(t)$ admits of a Fourier transform $Z(f)$, the Fourier transform of $z^*(t)$ which is expressed as $F[z^*(t)]$, may be written as equal to the convolution product of the Fourier transform of $z(t)$ and the Fourier transform of

$$\sum_{n=-\infty}^{\infty} \delta\left(t-\frac{n}{F_e}\right) \text{ i.e.}$$

$$\mathcal{F}[z^*(t)] = \mathcal{F} \frac{1}{F_e} z(t) \sum_{n=-\infty}^{\infty}$$

$$\delta\left(t-\frac{n}{F_e}\right) = Z(f) * \sum_{K=-\infty}^{\infty} \delta(f-Kf_e) \quad (29)$$

and hence

$$\mathcal{F}[z^*(t)] = \sum_{K=-\infty}^{\infty} z(f-Kf_e) \quad (30)$$

$F[z^*(t)]$ is therefore a sum of the functions . . . $Z(f - Kf_e)$, . . . $Z(f - F_e)$, $Z(f)$, $Z(f + F_e)$, . . . $Z(f + Kf_e)$. . .

We have a sum of functions derived from one another by F_e translations along the frequency axis.

If $z(t)$ has a limited frequency spectrum, i.e., if $Z(f)$ is substantially zero for $|f| \geq F_o$, these translations will not affect the form of the elements $Z(f - Kf_e)$ provided that Fe is greater than or equal to $2F_o$; there will then be no overlap of the spectra, as shown in FIG. 6, in which f is plotted on the abscissae and $F[z^*(t)]$ on the ordinate.

In these conditions, the spectrum $Z_1(f)$ of $z^*(t)$ enables the spectrum $Z(f)$ of $z(t)$ to be found; for this purpose it is only necessary to multiply $Z_1(f)$ by a window function $Q(F_e/2)$ so that

$$Q(Fe/2) = O \text{ for } f < -F_e/2 \text{ and } f > F_e/2$$

$$Q(Fe/2) = 1 \text{ for } -F_e/2 < f < F_e/2.$$

Then $Z(f) = Z_1(f) \cdot Q(f)$.

This window $Q(f)$ will be produced in practice by considering $Z_1(f)$ only between $F_e/2$ and $F_e/2$. We therefore have $Z(f) = Z_1(f) \cdot Q(f)$.

Taking the Fourier transforms of the two numbers, we have

$$z(t) = z^*(t) * Fe\,(\sin \pi F_e t/\pi F_e t) \quad (31)$$

$$z(t) = \frac{1}{F_e} \sum_{n=-\infty}^{\infty} z\left(\frac{n}{F_e}\right) \delta\left(t-\frac{n}{F_e}\right) = F_e \frac{\sin \pi F_e t}{\pi F_e t} \quad (32)$$

and hence

$$z(t) = \sum_{n=-\infty}^{\infty} z\left(\frac{n}{F_e}\right) \frac{\sin \pi F_e\left(t-\frac{n}{F_e}\right)}{\pi F_e\left(t-\frac{n}{F_e}\right)} \quad (33)$$

Shannon's sampling theorem shows that if a function $z(t)$ has a frequency spectrum limited by the values $-F_o$, F_o, $z(t)$ can be found from samples taken at the frequency $F_e > 2F_o$.

Returning to the Fourier transform of $z(t)$ which is $Z(f)$, then by definition

$$Z(f) = \int_{-\infty}^{+\infty} z(t)\, e^{-2\pi i f t} dt \quad (34)$$

The Fourier transform of $z^*(t)$ is

$$Z_1(f) = \int_{-\infty}^{+\infty} z^*(t)\, e^{-2\pi i f t} dt \quad (35)$$

We have just seen that Z(f) can be found by considering $Z_1(f)$ between only $-F_e/2$ and $F_e/2$.

The above two expressions are therefore equivalent; in these conditions

$$Z_1(f) = \int_{-\infty}^{+\infty} z^*(t)\, e^{-2\pi i f t} dt \quad (36)$$

may be written as follows:

$$Z_1(f) \int_{-\infty}^{+\infty} \frac{1}{F_e} \sum_{n=-\infty}^{\infty} z\left(\frac{n}{F_e}\right) \delta\left(t-\frac{n}{F_e}\right) e^{-2\pi i f t} dt \quad (37)$$

or alternatively

$$Z_1(f) = \frac{1}{F_e} \sum_{n=-\infty}^{\infty} z\left(\frac{n}{F_e}\right) \int_{-\infty}^{+\infty} \delta\left(t-\frac{n}{F_e}\right) e^{-2\pi i f t} dt \quad (38)$$

but

$$\int_{-\infty}^{+\infty} e^{-2\pi jft}\delta\left(t-\frac{n}{F_e}\right)=e^{-2\pi jt\frac{n}{F_e}} \quad (39)$$

and hence

$$Z_1(f)=\frac{1}{F_e}\sum_{n=-\infty}^{\infty} z\left(\frac{n}{F_e}\right)e^{-2\pi jf\frac{n}{F_e}} \quad (40)$$

If the function $z(t)$ is known between $-T$ and $+T$, and is assumed to be zero for $|t| \geq T$, n will vary from $-TF_e=-p$ to $+TF_e=+p$

$$Z_1(F)=\frac{1}{F_e}\sum_{n=-p}^{n=p} -z\,\frac{n}{F_e}\,e^{-2\pi jf\frac{n}{F_e}} \quad (41)$$

If we consider only the values of f contained between $-F_e/2$ and $+F_e/2$ (i.e., between O and $F_e/2$ in fact), we have

$$Z_1(f)\Big]_0^{F_e/2}=Z(f) \quad (42)$$

whence

$$Z(f)=\frac{1}{F_e}\sum_{n=-p}^{n=p} z\left(\frac{n}{F_e}\right)e^{-2\pi jf\frac{n}{F_e}}\Big|_{f=0}^{f=F_e/2} \quad (43)$$

If f varies by discrete values Kf_o, then

$$Z(Kfo)=\frac{1}{F_e}\sum_{n=-p}^{n=p} z\left(\frac{n}{F_e}\right)e^{-2\pi jKfo\frac{n}{F_e}}\Big|_{Kf_o=0}^{Kf_o=Fe/2} \quad (44)$$

or, since $\Theta = 1/F_e$,

$$Z(Kf_o=\theta\sum_{n=-p}^{n=p} z(n\theta)\,e^{-2\pi jKf_o n\theta} \quad (45)$$

which can be divided up into

$$R_z(Kf_o)=\theta\sum_{n=-p}^{n=p} z(n\theta)\cos 2\pi Kf_o n\theta \quad (46)$$

and

$$I_z(Kf_o)=\theta\sum_{n=-p}^{n=p} z(n\theta)\sin 2\pi Kf_o n\theta \quad (47)$$

If $f_o = 1/2T$, then we have formula (15) again :

$$R_z\left(\frac{K}{2T}\right)=\theta\sum_{n=-T/\theta}^{n=T/\theta} z(n\theta)\cos 2\pi\frac{K}{2T}n\theta.$$

After these preliminary remarks, we shall now explain the details of the method according to the invention.

The function $z(t)$ is assumed to be given in a sampled form, Θ being the sampling interval.

The function $z(t)$ is therefore represented by a series of values

$\ldots z(-K\Theta), \ldots z(-2\Theta), z(-\Theta), z(O), z(\Theta), z(2\Theta), \ldots z(K\Theta)$, equally spaced over the t axis, the interval being Θ.

These values $z(K\Theta)$ are stored either digitally (e.g. in core or bistable memories) or in analog form (e.g. as capacitor charges).

For calculation of the Fourier transform of $z(t)$, the sampled values of $z(t)$ are finally read out in analog form either by digital-analog decoding of the contents of the memories or stores if they are of the digital type, or by direct read-out thereof if they are of the analog type.

The function $z(t)$ therefore being sampled with the interval Θ, it will be defined by N points, N being an integer immediately below $2T/\Theta + 1$. Since the interval 2T is generally large with respect to the interval Θ, we may write $N = 2T/\Theta + 1$ in order to simplify.

The function $z(t)$ being known over an interval of 2T, the definition over the spectrum cannot be greater than 1/2T; we may therefore take $f_o = 1/2T$; as a result, the lowest frequency of the cosine function will be $1/2T = f_o$. Thus at the end of N points of cos 2π 1/2T $p\Theta$, we shall have

$$\cos(2\pi\ 1/2T\cdot{}^{b}2T)=\cos 2\pi$$

We shall therefore produce a period of the function cos $2\pi/2Tt$ sampled at N points having the interval Θ (FIG. 7*a* gives an example for $N = 41$ and $n = N + 1/2 = 21$), i.e., a programmed cosine function.

Taking the products of $z(p\Theta)$ and cos 2π $(1/2T)p\Theta$ and the sum of these products, we have

$$R_z\left(\frac{1}{2T}\right)=\sum_{p=-\frac{T}{\theta}}^{p=\frac{T}{\theta}} z(p\theta)\cos 2\pi\frac{1}{2T}p\theta.$$ If

we now make $K = 2$ for the cosine function, then for $z(p\Theta)$ we shall have cos 2π 1/2T $2p\Theta$, and if we take the sum to p of the products $z(p\Theta)$ cos 2π 1/2T $2p\Theta$, then we have

$$R_z\left(\frac{2}{2T}\right)=\theta\sum_{p=-\frac{T}{\theta}}^{p=T/\theta} z(p\theta)\cos 2\pi\frac{1}{2T}2p\theta.$$

In practice this is equivalent to reading out the values of the programmed cosine in increments of two, at the addresses **0, 2, 4, 6 . . .** (FIG. 7*b*).

If we now read out the points of the programmed cosine taking the addresses by increments of three (FIG. 7*c*), we shall have for each value of $Z(p\Theta)$ the associated value of the cosine function cos 2π (1/2T) $3p\Theta$.

Taking the products and the sum of the products we shall have

$$R_z\left(\frac{3}{2T}\right)=\theta\sum_{p=-T/\theta}^{p=T/\theta} z(p\theta)\cos 2\pi\frac{1}{2T}3p\theta.$$

Similarly, by reading out the values of the programmed cosine for adresses in increments of K, we shall have generally

$$R_z\left(\frac{K}{2T}\right)=\theta\sum_{p=-T/\theta}^{p=T/\theta} z(p\theta)\ \cos\frac{2\pi}{2T}Kp\theta$$

and this applies to values of K increasing up to $K = n - 1$, with $n = N + 1/2$ or $n - 1 = N - 1/2 = T/\Theta$, and hence $n - 1/2T = 1/2\Theta = F_e/2$ (Fe being the sampling frequency), and from this we have the value

$$Rz\left(\frac{n-1}{2T}\right)=Rz\left(\frac{F_e}{2}\right)=\theta\sum_{p=-\frac{T}{\theta}}^{T/\theta} z(p\theta)\ \cos\frac{2\pi}{2T}(n-1)p\theta$$

There is no point in calculating R_z (K/2T) for $K > n - 1$ since higher values of K would correspond to spectrum frequencies above $F_e/2$; the latter is already greater than or equal to F_o.

This will also give a control, since R_z ($n - 1/2T$) will have to be zero.

We shall now examine the specific case in which $z(\tau)$ is a correlation function known between $-\tau_M$ and $+\tau_M$. We shall distinguish between the following two possible hypotheses:

a. $z(\tau)$ is an auto-correlation function; it is known that the function $z(\tau)$ is even: $z(\tau) = z(-\tau)$; a knowledge of $z(\tau)$ in the case of $\tau > 0$ is thus sufficient and if we know n points of $z(t)$ for $\tau > 0$, that is equivalent to knowing $N = 2n - 1$ points of $z(\tau)$ for $\tau \gtrless 0$; on this hypothesis, a half-period of the cosine function is programmed over n points.

b. $z(\tau)$ is an cross-correlation function; $z(\tau)$ and $z(-\tau)$ are calculated separately:
(for $\tau > 0$, we have n points;
)for $\tau > 0$, we have $n - 1$ points, and we thus
(return to the preceding hypothesis.

Thus where this method is applied to correlation functions, it will only be necessary to program a half-period of the cosine at a number of points n equal to the number of known points of the part of the corresponding correlation function, either at $\tau > 0$ or at $\tau < 0$.

Referring to FIGS. **8** and **9**, it will be apparent that a knowledge of n points of the auto-correlation function is equivalent to a knowledge of $(2n - 1)$ points of this function, these figures illustrating the pairs of $z(t)$ and cosine curves linked by auto-correlation.

The cosine function is therefore programmed at n points and the auto-correlation function is processed at n points, the resultant then having to be multiplied by two.

With regard to the cross-correlation function, in the general case in which it is non-zero between $-\tau_M$ and $+\tau_M$, it is first calculated at n points from 0 to τ_M, and then at n points from 0 to $-\tau_M$.

For calculation of $Z(f)$, the two parts of the cross-correlation function will be processed successively, and the sum of the two Fourier transforms will be formed counting the products associated with $\tau = 0$ just once; FIGS. **10** and **11** show the pairs of $z(t)$ and cos curves linked by inter-correlation, distinction being made between the two cases $E < 0$ and $E > 0$.

In these cases, therefore, the number of points programmed over a half-period of the cosine (at the lowest frequency) will be equal to the number of points of the correlation function taken for $\tau > 0$ or for $\tau < 0$.

In the case where the function $z(t)$ is not even, it is also necessary to calculate

$I_z\,[(K/2T)] = \Theta\,\Sigma\, z\,(p\Theta)\,\sin\,(2\pi/2T)\,Kp\Theta$ i.e., the sine function $(2\pi/2T)\,Kp\Theta$ must be generated.

For this purpose it is only necessary to read out the programmed values of the cosine function with a quarter-period offset, i.e, start reading out at the sample which has zero as its value (sample No. **10** in FIG. **7***a*) instead of commencing with the sample having **1** as its value (sample No. **0** in FIG. **7***a*).

The method and the analog read-out means for the programmed values of the cosine and/or sine function having been explained in detail, reference will now be made to FIG. **12** which illustrates a complete device according to the invention for generating a Fourier transform.

This device comprises in combination store unit **1** whereby the values of the sampled function $z(p\Theta)$ can be stored (in digital or analog form), reading out circuit **2** for reading out the contents of the storage means **1** in analog form, at least one analog generator **3** for the function cos π/T $Kp\Theta$ and/or sin π/T $Kp\Theta$, at least one analog multiplier **4**, one of the inputs **4***a* of which receives the output of the reading out circuit **2**, while the other input **4***b* receives the output of the function generator or generators **3**, an integrator **5** connected to the output of the multiplier or multipliers **4**, said integrator, which advantageously consists of an operational amplifier **6** with a capacitor **7** in parallel, being provided with a normally open switch **8**, for rezeroising, and a clock **9** which will be further considered hereinafter with reference to FIG. **13**, for synchronising the read out of the contents of the store unit **1** by the circuit **2** and the operation of the generator or generators **3**; the required output $Z(f)$ is available at **10**.

In the circuit diagram shown in FIG. **12**, p varies from $-(n - 1)$ to $+(n - 1)$ and f varies from $1/2(n - 1)\Theta$ to $1/2\Theta$ by increments of $f_o = 1/(n - 1)\Theta/2$ with $n = N + 1/2$. The Fourier transform of $z(t)$ will then be obtained point by point as an output at **10**.

The multiplier **4** is advantageously of the type described in French Pat. No. 1,493,450 filed on Apr. 25, 1966 by the Commissariat a l'Energie Atomique (Inventors: Daniel Berthier and Jacques Max).

It is a multiplier in which a pulse is modulated by the multiplicand as to width and by the multiplier as to amplitude (or vice versa), the resulting pulse area being proportional to the product.

In the embodiment shown in FIG. **13**, which also shows some other parts of the complete device: generator **3**, clock **9**, integrator **5** (with operational amplifier **6**, capacitor **7** and rezeroising switch in the form of a field-effect transistor **8***a*, the gate **11** which receives the rezeroising voltage r through a diode **12**). The voltage r for return to zero is produced each time that N products are carried out in multiplier 4. This voltage r can be then obtained by utilizing a divider by N, as for example, element **128** in FIG. **20**. The multiplier **4** comprises the following:

A sawtooth generator **13** (e.g. of the bootstrap circuit type) triggered by the clock **9**, the clock pulses being fed to the input **4c** of the multiplier **4** (i.e., the generator **13**).

A comparator **14** (comprising a differential amplifier and an inverter) which compares the amplitude of the sawtooth pulses of the generator **13** with the cosine and/or sine function of $2\pi Kf_o p\Theta$ or $\pi/T\ Kp\Theta$ from the generator **3**, and delivering two opposite signals Q and $\overline{Q}$, Q being the resultant of the comparison in the amplifier stage and $\overline{Q}$ being the output of the inverter stage (the input of which is connected to the output of the amplifier), these signals Q and $\overline{Q}$, for example, having the curve shown in FIG. **3** (signals $X(t)$ and $\overline{X}(t)$ of the aforesaid patent.

A pair of field-effect transistors **15, 16** which alternately conduct under the control of Q and $\overline{Q}$, the transistor **16** being fed directly by the output of the read-out means **2**, while the transistor **15** is fed by said means **2** via an inverter **17** ($-1 \times$ multiplier), the transistor **15** thus delivering $-z(p\Theta)$ to the integrator **5** under the control of Q, while transistor **16** delivers $z(p\Theta)$ to the integrator **5** under the control of $\overline{Q}$.

The sum of the N products corresponding to a value of K is finally obtained by the integrator **5** which is rezeroised by a voltage r after each calculation of a point of $Z(f)$.

Thus N products have to be evaluated and summated for one value of $Z(f)$. To obtain n points of $Z(f)$, $n \cdot N$ products must be evaluated (in the case of a correlation function defined at n points, then n^2 products must be evaluated).

We shall now discuss the cosine or sine generator which has to generate the signal

$$C_K^* = \sum_p \cos 2\pi K f_o p\theta\delta(t - p\theta)$$

$$S_K^* = \sum_p \sin 2\pi K f_o p\theta\delta(t - p\theta)$$

for integer K varying from 1 to $n - 1$ and integer p varying from

$$-T/\Theta - N/2 \text{ to } +T/\Theta - N/2$$

The principle of this sampled function generator has been described above with reference to FIG. **7.** An embodiment of such a generator **3** is illustrated in FIGS. **14** and **15.** The operation may be divided up into two parts: firstly, storage of the voltages corresponding to $n = N + 1/2$ values of the cosine for values of the variable regularly spaced between 0 and π; secondly, the read-out programme of these various cosine values.

a. Storage of the values of the cosine (or sine) function.

The successive values of the cosine function are represented by voltages $V_o, V_1, \ldots V_n^{-1}$ (which are in fact voltages corresponding to $n - 1$ values of the cosine distributed equally between 0° and 180°) and are read out in sequence by field-effect transistors (e.g. $T_0, T_1, \ldots T_{n-1}$) (FIG. **14**). The system then behaves as an electronic switch having n inputs and a common output S.

If the transistor T_2 conducts, S gives the voltage V_2, while similarly if T_i is conductive, the voltage V_i is obtained at S, and so on. These field-effect transistors are controlled by diode matrices which decode the "units," "tens," "hundreds" states of the read-out address.

b. Read-out programme in the case where it is desired to obtain the Fourier transform of a correlation function delivered by a correlator.

We have seen that in such cases there were n programmed points of the cosine function for n points of the correlation function where τ is greater than 0 or is less than 0.

The program is obtained (FIG. **15**) by means of a binary divider **18** which divides by n, a decimal binary counter **19,** "AND" circuits **20** (which receives the out-puts: units U_K, tens D_K and hundreds C_K, from the counter **19** and the parallel transfer order from the clock **9** via the line **21**) and a parallel binary accumulator **22.**

The three outputs U_a, D_a, C_a of the accumulator **22** lead to a decoding matrix **27** which allows 0, 1, 2 . . . 199 to be obtained from U_a, D_a, C_a and the field-effect transistors T_o, T_1 . . . etc. in FIG. **14** to be controlled.

The logic states "U_K," "D_K," "C_K" represent the value of K for the units, tens and hundreds.

K thus increases all the n synchronisation pulses by one unit, so that K remains constant during n pulses. The "AND" circuits **20** enable K to be added to the contents of the binary accumulator on each synchronisation pulse. Thus after each pulse the address (U_a, D_a, C_a) of the accumulator is increased by K.

Let us assume an initial state:

$K = 1 \rightarrow U_K = 1$ and $U_a = 0$ $\quad D_K = 0$ $D_a = 0$ $\quad C_K = 0$ $C_a = 0$

V_0 will be obtained at S;

after one pulse:

$K = 1$ $\quad U_K = 1$ $U_a = 1$ $\quad D_K = 0$ $D_a = 0$ $\quad C_K = 0$ $C_a = 0$

V_1 will be obtained at S;

after two pulses:

$K = 1$ $\quad U_K = 1$ $U_a = 2$ $\quad D_K = 0$ $D_a = 0$ $\quad C_K = 0$ $C_a = 0$

V_2 will be obtained at S;

and so on until the $(n-1)^{th}$ pulse; at the n^{th} pulse we shall have:

$K = 2$ $\quad U_K = 2$ $U_a = 0$ $\quad D_K = 0$ D_a 32 0 $\quad C_K = 0$ $C_a = 0$

V_0 will again be obtained at S;

at the $(n+1)^{th}$ pulse we shall have:

$K = 2$ $\quad U_K = 2$ $U_a = 2$ $\quad D_K = 0$ $D_a = 0$ $\quad C_K = 0$ $C_a = 0$

V_2 is obtained at S.

In other words:

for the pulses from 1 to $n - 1$, $k = 1$ and the following will be obtained successively at S:

$V_0, V_1, V_2, V_3 \ldots V_{n-2}, V_{n-1}$;

for the pulses from n to $2n - 1$, $K = 2$ and the following will be obtained successively at S;

$V_0, V_2, V_4 \ldots V_{n-3}, V_{n-1}, V_{n-3} \ldots V_2, V_0$;

for the pulses from $2n$ to $3n-1$, $K = 3$ and the following will be obtained at S:

$V_0, V_3, V_6, V_9 \ldots V_{n-4}, V_{n-1}, V_{n-4} \ldots V_0 \ldots$; and so on,

and finally& for the pulses between $(n-2)n$ and $(n-1)n-1$, we have $K = N-1$ and the following will be obtained successively at S:

$V_0, V_{n-1}, V_0, V_{n-1}, \ldots V_{n-1}, V_0$.

Of course the device may have ancillary devices.

More particularly, when the signal $z(t)$ is not zero in the case of the limits $-T$ and T, the Fourier transform of $z(t)$ has oscillations as indicated hereinbefore.

To reduce these spurious oscillations, the samples of $z(t)$ may be weighted by a time function of the form sin $\Theta p/\Theta p$. This result is obtained by using a variable-gain amplifier. The gain of this amplifier is controlled by the $z(\Theta p)$ read-out synchronisation pulses. The variable gain is obtained by looping an operational amplifier by various resistors whose value in the case of the sample of the order p is proportional to sin $\Theta p/\Theta p$, with integer p ranging between 0 and T/Θ.

The performances of the apparatus are as follows:

— time for one multiplication: 1 microsecond;

— time for calculating one point of the Fourier transform, the function $z(t)$ being known by 200 points; 200 microseconds.

— time for calculating the Fourier transform for 200 points for the spectrum: 200 × 200 microseconds = 40 ms.

— thus to calculate the real part and the imaginary part of the Fourier transform the time required is 80 ms.

If the function $z(t)$ is known by N points between $-T$ and $+T$, the half-period of the cosine being programmed between 0 and T at n points ($n = N - 1/2$), it is easy to generate a full period of the cosine to cover the interval $-T$, $+T$ by a modification of the address programme.

It will be apparent that the circuit shown in FIG. **14** has two disadvantages:

a. n voltages corresponding to n samples of the cosine uniformly spaced between 0 and π must be displayed; this means that n potentiometers are required and $n = 100$ or 200.

b. n field-effect transistors T are required to read out these n samples in sequence.

In a variant, instead of storing n samples V_0, V_1, V_2, and so on, and reading out these n samples in sequence, it is possible to use an amplifier whose gain varies in accordance with the addresses 0, 1, . . .

The variable gain may be obtained with the circuit diagram shown in FIG. **16**, in which reference **23** denotes an amplifier, **24** a resistor equal to R/x and **25** a resistor equal to R/100; the gain is therefore $|G| = x/100$.

The input resistance R/x must therefore be varied to vary the gain of the looped amplifier in accordance with the addresses 0, 1, 2 and so on. R/x is obtained by the parallel connection of a network of resistors **24** having the values R, R/2, R/4, R/2, R/10, R/20, R/40, R/20, R/100. With a resistance network of this type, and by means of combinations, it is possible to obtain integer values of x between 0 and 199.

The dynamic over x is limited only by the accuracy of the resistors. The resistor R/100 connected as a negative feedback resistor to the amplifier acts as a gain limiter.

FIG. **17** shows an example for $x = 76$.

FIG. **18** shows the system illustrated in FIGS. **16** and **17**, i.e., a system adapted to be substituted for that shown in FIG. **14** in the complete apparatus. In this Figure, the resistors **24** are switched by field-effect transistors **26** which are either cut off (state **0**) or conductive (state **1**).

Thus "**76**" corresponds to the logic word **01110110** illustrated in FIGS. **17** and **18**. The same applies to the other values as shown in the following Table:

R/20	R/40	R/20	R/10	R/2	R/4	R/2	R	x
0	1	1	1	0	1	1	0	76
0	1	1	1	0	1	1	1	77
0	1	1	1	1	0	0	0	78

For each address $i = 0, 1, 2 \ldots$ etc. there is a correspondy logic word, the analog equivalent of which would be V_0, V_1, V_2 etc. is made to correspond to each address $i = 0, 1, 2$, etc. In this application, these voltages correspond to the samples of a cosine between 0 and π.

Any other law could be selected to produce a generator of any function, with a voltage V_i corresponding to each address i.

In the case of the Fourier transform generator, this solution is very advantageous, because using eight resistors it is possible to generate as many samples V_1 as desired with an accuracy of 1 percent. The accuracy would be 0.1 percent using 12 resistors.

In the case of a 200-point Fourier transform generator, the solution of FIG. **14** would require 200 potentiometers and 200 field-effect transistors, while the solution shown in FIG. **14** would require only 12 resistors and 12 field-effect transistors.

The use of any of the above embodiment will thus provide a system for generating the Fourier transform of a function, and the operation of such a system is sufficiently clear from the foregoing for any further remarks to be superfluous, and such a system has numerous advantages over existing systems of the type in question, inter alia, as follows:

Firstly, it is simpler, more flexible and provides better performance than the prior-art systems, particularly when coupled to a real-time automatic correlator, to form a spectral density analyser.

It has an extensive band width.

It can operate in real time.

It is highly reliable.

The speed of calculation is very high.

The modified embodiments, illustrated in FIGS. **19–23** operate digitally, but the principle of operation remains similar to that described above. However, some general considerations may be useful to produce a digital sample R_z (K/2T) the following criteria must be met:

a. N samples of the function $z(t)$ must be made available in digital form (N being a predetermined integer);

b. the corresponding N samples of the cosine function must be made available;

c. the N elementary products of sample pairs must be formed, and

d. the N partial products must be added (integration operation).

Since the device of the invention will often be used to determine the Fourier transform of a function supplied as a round number of samples (50, 100, 150, 200 . . .) by a correlator, the fundamental frequency chosen is likely to be not $1/2T = 1/N\Theta$ but the next-lowest value $1/(2T + \Theta)$.

Theoretically, the implication of criterion (b) is that N cosine function samples different from the samples used for any other sample of the transform are available for calculating each sample of the transform. Actually, formula (1) shows that if $1/(2T + \Theta)$ is taken as the fundamental value, there is a return after N points to the same cosine function value. In other words, all that is necessary is for N samples of the cosine function to be available in an addressable store (the stored elements are identified by an address through which they can be read) for recovery at complete 2π intervals; immediately consecutive addresses are called to calculate the fundamental and energy Kth addresses are called to calculate the Kth harmonic for any value K.

The device whose block diagram is shown in FIG. **19,** comprises a generator **110** whose output A delivers the function $z(t)$ in the form of samples $z(p\Theta)$, a generator **112** whose output B delivers the cosine function in the form of samples $\cos 2\pi(1/2T)\ Kp\Theta$, a multiplier **114** receiving the pairs of samples on its two imputs, an integrator **116,** and a clock **18** which supplies the generators **110, 112** with synchronizing pips. The generator **110** corresponds to the function or criterion (*a*), generator **112** to function (b), multiplier **114** to function (c) and integrator **116** to function (d). These various components will now be described consecutively.

The sample generator **110** shown by way of example in FIG. **21** is a simple analog-to-digital converter **150** which receives the function $z\ (t)$ at its input and which provides digitally encoded samples on its output at times determined by the clock **118.** The samples are stored in a store **152,** which provides long-term storage for low-frequency functions. It will be assumed as an example that generator **110** outputs 200 samples throughout the computation time period, each sample being represented by a 9-bit number consisting of 8 significant bits and 1 extra bit for the sign.

The cosine function sample generator **112,** shown in FIG. **20,** can be considered as comprising store **120** for storing the samples and an address generator FIG. **20.** The N evenly distributed samples (to the number of 200 in the case under consideration) are preferably encoded with the same number of bits as is used for the significant bits for the samples of $z(t)$. For instance, if the last-mentioned samples are in a nine-bit code (eight significant bits and one sign bit), the end values can be as follows:

$\cos 0 = 2^8 - 1 = 255$

$\cos \pi/2 = 0$

$\cos \pi = -255$

These samples are stored in a permanent digital store **120** such as a construction-programmed diode matrix, a L.S.I. dead store or read only store etc. When the read-out address of the store advances from 1 to 1 (consecutively) the store consecutively outputs all the cosine samples corresponding to one-quarter of a period with the plus sign and one-quarter of a period with the minus sign, as Table I indicates.

TABLE I

ADDRESS DECIMAL	ADDRESS BINARY	OUTPUT BINARY	OUTPUT DECIMAL	SIGN
0	000000000	111111111	255	
1	000000001	111111111	255	
2	000000010	111111111	255	
3	000000011	111111110	254	
4	000001100	111111110	254	
5	000001111	111111100	253	
.	.	.	.	
.	.	.	.	
.	.	.	.	+
.	.	.	.	
98	011000010	000001100	4	
99	011000011	000000010	2	
100	000001100	000000000	0	
101	011001101	000000010	2	
102	011001110	000001100	4	
.	.	.	.	
.	.	.	.	
.	.	.	.	
.	.	.	.	−
.	.	.	.	
198	110001110	111111111	255	
199	110001111	111111111	255	

The adress generator associated with store **120** comprises a counter **122** displaying the harmonic K, an adder **124** and an accumulator **126.** The adress generator of FIG. **20** also has a divider-by −N 128 for obtaining the various values of K corresponding to the various harmonics. The address generator then operates as follows: whenever a sync pulse from clock **118** (such pulse being an accumulation instruction) is applied to accumulator **126,** adder **124** totals the previous contents of accumulator **126** and the contents K of counter **122** and re-enters the total in accumulator **126.** The contents of accumulator **126** therefore increase by an amount K at each clock pulse. Upon application of this address to the input of store **120** the output thereof delivers the value of the sample called at its exit during each interval between consecutive clock pulses. To reduce the number of binary positions required of the store **120,** the sign for the output is determined by AND gate **130** so connected to the appropriate outputs (weights 2^2, 2^5 and 2^6) of accumulator **126** as to detect the changeover to **100.**

The divider **128** of the generator of FIG. **20** outputs a pulse to the counter **122** each time it has received N sync pips since the preceding pulse. This pulse provides as one-unit increase of the number K displayed by the counter **122** and changes over from calculating one harmonic to calculatting the next harmonic without manual action by the operator.

The binary multiplier of FIG. **22** comprises a binary multiplier **114** receiving a binary digit A from the generator **110** and a binary digit B from the generator **112.** If the generators **110, 112** supply words consisting of eight significant bits and one sign bit, the digital multiplier must have 16 binary places plus a sign-determining circuit **32.** For the sake of rapidity a series-parallel multiplier is normally used. The multiplier outputs a partial product to the accumulator **116** for each input data pair. The integrator of FIG. **22** is an accumulator which can have **20** binary positions in the case previously considered and which totals consecutive partial products to deliver a sample, the significant digits of each term being supplied by multiplier **114** and the sign of the term being supplied by a circuit **32.**

Since the result is usually required to be available in analog form, the accumulator **116** of FIG. **22** is associated with a digital-to-analog converter **134** whose number of input binary places can be very much less than in the case of the accumulator **116.** In cases in which the generators **110, 112** output words having the same number of bits, this common number can be used for the converter **134** provided of course that the same has a control input **136** for selecting those outputs of the accumulator **116** which are decoded and more particu-

larly, for decoding the significant binary digits of highest weight appearing at the accumulator output.

FIG. 23 shows a spectral analyser comprising in association a correlator and a system outputting the Fourier transform. Such elements of this analyser as correspond to the elements already shown in FIGS. 19 – 22 have for the sake of simplicity the same reference number plus an apostrophe and will not be described again. The function $z\ (p\Theta)$ sample generator **110'** of the spectral analyser is an analog cross-correlator **139** whose two inputs **138, 140** receive analog signals representing functions $x(t)$ and $y(t)$, the output **142** of the correlator **139** indicating the cross-correlation function of the latter functions. The correlator **139** is for instance a multichannel correlator of the kind described and claimed in French Pat. specification n·1 493 450 of the Assignee of the present invention. An analog-to-digital converter **144** converts into binary form the consecutive specimens appearing at output **142.** For example, 100 samples may be delivered in the form of eight-bit words, and stored in a digital store **146.** The pips from the clock **118'** therefore control not only the operation of the Fourier transformer but also the operation of the assembly formed by the correlator **139** and store **146.**

The store **146** can perform various functions. More particularly, it makes it possible to use the system supplying the Fourier transform not only in real time but in delayed time relatively to the correlator **139.** It can also be used as an integrator and accumulate the cross-correlation function over prolonged periods, for instance, of several hours. The words of the store **146** therefore usually occupy binary places than the multiplier **114'**.

This feature is a very useful way of circumventing one of the limitations of analog integrating correlators, namely the limited duration of the integration time constant (a few hundreds of seconds).

Of course, the correlator **136,** instead of providing the cross-correlation function of two functions $x(t)$ and $y(t)$ can of course also be used to provide the auto-correlation function of a physical value, in which event the Fourier transform generator provides the spectrum of such value.

Accuracy is better than in analog operation, for the only limitation on the accuracy of encoding the cosine function samples is the capacity of the dead store or read only store, and multiplication can be performed to any required accuracy by the data introduced into the multiplier. Accuracy e.g., 1 percent o with eight-bit words. Numerical integration of consecutive products provides considerable dynamic range on the value of each point of the Fourier transform; for instance 20 binary places correspond to an amplitude dynamic of something like 10^7. Also, a dead store or read only store comprising 200 specimens can be passed through from generation of the fundamental of a Fourier transform by reading only one address out of a number ; consequently for instance, a dead store having 200 binary places can be associated with a correlator providing only 50 or 100 samples.

We claim:

1. A system for generating the Fourier coefficients of a real function $z(t)$ which is generated in the form of N samples equally spaced by the interval Θ from $t=-T$ to $t=T$ in the form $z(p\Theta)$ where p is an integer such that $-T/\Theta < p < T/\Theta$ comprising a store unit (**1**) for storing N samples equally spaced by the interval Θ, of the function $\cos \pi t/T$ from $t=-T$ to $t=T$ in the form $\cos \pi\ p\Theta/T$; a reader (**2**) connected to said store for reading out every K^{th} samples of the functions $\cos \pi\ t/T$, K being an integer increasing from 1 inclusive to T/Θ inclusive, said reader and said store unit forming a further generator (**3**) of samples $\cos (\pi\ Kp\Theta/T)$; a multiplier (**4**) one of the inputs (**4***a*) of which is connected to said generator of samples $z(p\Theta)$ while the other of the inputs (**4***b*) is connected to said generator (**3**) of samples $\cos (\pi\ Kp\Theta/T)$; said multiplier selectively forming the product of $z(p\ \Theta)$ and $\cos (\pi\ Kp\Theta/T)$; an integrator (**5**) connected to said multiplier for summation of successive products each of the sums over p for which K is constant representing the Fourier coefficient associated with the frequency K/2T; a switch (**8**) for rezeroizing said integrator (**5**) and a clock (**9**) for actuating said reader, said further generator, said multiplier and said switch.

2. A system according to claim **1,** wherein the multiplier provides double amplitude and width modulation.

3. A system according to claim **1,** wherein the integrater is an operational amplifier with a parallel connection of a capacitor and an electronic switch for rezeroising.

4. A system according to claim **1,** wherein the function generator of sample $Z(p\Theta)$ comprises a series of n field effect transistors, in parallel, said transistors being switched by units, tens, and hundreds signals, and delivering to a common output n voltages $V_0, V_1 \ldots V_{n-1}$ corresponding to $(n-1)$ cosine values evenly distributed between 0° and 180° and a divider which divides by $(n-1)$ and which is fed by the synchronisation clock, a decimal counter connected to the output of said divider and delivering units, tens and hundreds pulses, three "AND" gates in parallel, their first input receiving the units, tens and hundreds pulses respectively from the counter, while their second input is connected to the synchronisation clock, and a parallel triple accumulator receiving the outputs of the three "AND" gates and delivering the accumulated units, tens and hundreds signals, the output of said dividers actuating said rezeroing switch.

5. A system according to claim **1,** wherein the function generator comprising in series a circuit comprising in parallel a plurality of field-effect transistors, and resistors of predetermined values, said circuit receiving a reference voltage, and an amplifier shunted by a resistor having a predetermined value, said values of the resistor being selected so that the gain of the series system consisting of the parallel circuit and shunted amplifier increases by $(n-1)$ successive increments in accordance with a cosine law and further includes a divider which divides by $(n-1)$ and which is fed by the synchronisation clock, a decimal counter connected to the output of said divider and delivering units, tens and hundreds pulses, three "AND" gates in parallel, their first input receiving the units, tens and hundreds pulses respectively from the counter, while their second input is connected to the synchronisation clock, and a parallel triple accumulator receiving the outputs of the three "AND" gates and delivering the accumulated units, tens and hundreds signals.

6. A system for generating the Fourier coefficients of the real function $Z(t)$ which is generated by a digital generator (**110**) in the forn of N digital samples equally spaced by the interval Θ from $t=-T$ to $t=T$ in the form $Z(p\Theta)$ where p is an integer such that $-T/\Theta < p$

$< +/\Theta$ comprising a digital store unit (**120**) for storing N digital samples equally spaced by the interval Θ, of the function $\cos \pi t/T$ from $t = -T$ to $t = T$ in the form $\cos \pi t\Theta/T$; a reader connected to said digital store (**120**) for reading every K^{th} sample of the function $\cos \pi t/T$, K increasing from 1 inclusive to $T\Theta$ inclusive, said reader and said digital store unit (**120**) forming a further generator (**112**) of samples $\cos (\pi Kt\Theta/T)$; a digital multiplier (**114**) one of the inputs (A) of which is connected to said generator (**110**) of samples $Z(p\Theta)$, while the other of the inputs (B) is connected to said generator (**112**) of samples $\cos (\pi Kp\Theta/T)$ said digital multiplier (**112**) selectively forming the product of $Z(t\Theta)$ and $\cos (\pi Kp\Theta/T)$; an integrator (**116**) connected to said multiplier (**114**) for summation of successive products, each of the sums over p for which K is constant being the Fourier coefficient associated with the frequency K/2T and a clock (**118**) for actuating said reader, said generators, said multiplier and said integrator.

7. A system as described in claim **6** for generating the Fourier coefficients of a real function $Z(t)$, said digital generator (**110**) giving the function $Z(t)$ including a digital store unit (**152**) for storing the digital values of the sampled function $Z(p\Theta)$, a reader connected to the store unit (**152**) for reading out the contents of the store unit (**152**) in digital form, said clock (**118**) being connected to said reader connected to said digital store unit (**152**) and to said reader connected to said store unit (**120**) for synchronising the read out of the contents of the store units.

8. A system according to claim **7,** wherein the first address generator includes a counter with provision for displaying K, an accumulator whose output defines the store address, and an adder coupled with the counter and accumulator, the adder adding the value K to the accumulator contents whenever a synchronising clock transmits a pulse to the counter and accumulator.

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