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- (71) Applicant (for GB, JP, NL only): **SCHLUMBERGER HOLDINGS LIMITED**; Craigmuir Chambers, PO Box 71, Road Town, Tortola (VG).
- (71) Applicant (for AL, AU, AZ, BG, CO, CZ, DE, DK, GQ, GR, HU, ID, IE, IL, IT, KR, KZ, LT, MX, MY, NO, NZ, OM, PL, RO, RU, SI, SK, TD, TM, TN, TR, TT, UZ, ZA only): **SCHLUMBERGER TECHNOLOGY B.V.** [NL/NL]; Parkstraat, 83-89, NL- 2514JG The Hague (NL).
- (71) Applicant (for CA only): **SCHLUMBERGER CANADA LIMITED** [CA/CA]; 7th Floor 525-3rd Avenue S.W., Eau Claire Place 1, Alberta T2P OG 4, Calgary (CA).
- (71) Applicant (for FR only): **SERVICES PETROLIERS SCHLUMBERGER** [FR/FR]; 42, rue saint Dominique, F-75007 Paris (FR).
- (71) Applicant (for all designated States except AL, AU, AZ, BG, CA, CO, CZ, DE, DK, FR, GB, GQ, GR, HU, ID, IE, IL, IT, JP, KR, KZ, LT, MX, MY, NL, NO, NZ, OM, PL, RO, RU, SI, SK, TD, TM, TN, TR, TT, US, UZ, ZA): **PRAD RESEARCH AND DEVELOPMENT LIMITED**; Craigmuir Chambers, PO Box 71, Road Town, Tortola (VG).
- (72) Inventors; and
- (75) Inventors/Applicants (for US only): **CHEVERDA, Vladimir Albertovich** [RU/RU]; Detsky proezd, 3-22, Novos-

ibirsk, 630090 (RU). **RESHETOVA, Galina Vitalyevna** [RU/RU]; Detsky proezd, 3-22, Novosibirsk, 630090 (RU). **BELONOSOV, Mikhail Andreevich** [RU/RU]; Morskoy prosp., 16-6, Novosibirsk, 630090 (RU). **LISITSA, Vadim Viktorovich** [RU/RU]; Uchenykh str., 8-9, Novosibirsk, 630090 (RU). **KOSTOV, Clement** [FR/GB]; 8 Reed Close, Trumpington CB29NX (GB).

(74) Agent: **ARKHIPOVA, Vera Nikolaevna**; LLC "Technology Company Schlumberger", Pudovkina str., 13, Moscow, 119285 (RU).

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(54) Title: METHOD FOR COMPUTING WAVEFIELDS

(57) Abstract:



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METHOD FOR COMPUTING WAVEFIELDS

Field of the invention

This invention relates to methods for numerical simulations of different kinds of wavefields namely, acoustic, elastic, visco-elastic, poro-elastic, or electro-magnetic wavefields. More particularly, the invention relates to methods for solving the acoustic or elastic wave equations for applications of survey design, imaging, interpretation and wavefield inversion of seismic and sonic data.

Background art

Currently, the state-of-the-art in industrial simulations of seismic data is represented by computations of 3D acoustic wavefields (at relatively low frequencies) for applications of survey design and seismic data interpretation, including imaging and wavefield inversion of seismic data. Increasing the bandwidth of the data and solving elastic wave equations, as required for more realistic simulations, increases significantly the computational requirements. The development of algorithms and computer implementations for solving efficiently the elastic wave equations is a very active field of research.

The conventional approaches to solving the acoustic or elastic wave equations are finite-difference methods applied in the time-space domain. Typically, the problem is too large to fit in a single memory space, and therefore various strategies need to be implemented to deal with memory limitations, while solving the wave equation. The two most common approaches are domain decomposition and out-of-core computations (Etgen,

J., O'Brien, M., 2007, Computational methods for large-scale 3D acoustic finite-difference modeling: A tutorial, *Geophysics*, vol 72, pp SM223-SM230.). Such approaches complicate and slow down the algorithms (dealing with memory limitations) and require either intensive communication between processors, or large amounts of disk access during the computations. In the other major approach, leading to solution of Helmholtz equations after Fourier transform, parallelization is also difficult to achieve (Operto, S., Virieux, J., Amestoy, P., L'Excellent, J-Y., Giraud, L., Hafehd Ben Hadj, A., 2007, 3D finite-difference frequency-domain modeling of visco-acoustic wave propagation using a massively parallel direct solver: A feasibility study; *Geophysics*, 72.).

It is an object of the invention to provide a method for improving the efficiency of the calculations and the accuracy of the computed wavefields.

The object of the invention is achieved by a method as set forth in the appended claims.

Summary of the invention

The present invention provides a method for computing wave fields based on the Laguerre transform applied to the time variable in the elastic (or acoustic, or visco-elastic) wave equations which leads to Helmholtz equations. After the Laguerre transform, the associated Helmholtz operators are negative definite, which is a favorable situation for further parallelization by domain decomposition. Here however, the domain decomposition occurs before the solution of the Helmholtz wave equations, and the solver of the Helmholtz equation doesn't need to deal with the domain decomposition. Data exchanges occur between domains only after relatively long independent computations, and therefore, they can be done efficiently via simple disk access, decoupled from the computations. The combination of the Laguerre

transform and an effective domain decomposition method such as the Schwartz iterative method is applied to the numerical simulations of sonic and seismic data surveys, for applications of optimizing the survey design and data interpretation workflows.

So the invention comprises the steps of specifying the governing wave equations, setting up the corresponding material parameters, the dimensions of a computational domain, the conditions at the boundaries and the initial conditions. Then the parameters of the Laguerre transform are selected by examining the transform for the initial conditions. Furthermore the invention comprises the steps of decomposing the computational domain into at least two sub-domains, applying Laguerre transform along the time axis for each sub-domain and obtaining Helmholtz equations, solving Helmholtz equations for each sub-domain, exchanging data between sub-domains as per Schwartz iteration method, iterating previous two steps until sufficient level of accuracy is reached, applying inverse Laguerre transform and obtaining the solution of wave equations.

The wave equations can be for acoustic, elastic, visco-elastic, poro-elastic, or electro-magnetic wavefields

Helmholtz equations can be solved using standard finite-difference or finite-element methods.

Helmholtz equations for different sub-domains can be solved by different numerical methods.

Those and other features of the invention will be understood by those skilled in the art from the following detailed description and drawings.

Brief description of the drawings

Fig. 1 shows computational domain for a simple layered medium.

Fig. 2 shows computational domain for more complicated model.

Fig. 3 shows the wave field computed for a simple layered medium shown on Fig.1.

Fig. 4 shows the wave field computed for more complicated model shown on Fig. 2

Detailed description of the invention

The detailed steps for applying the proposed method are as follows. At first the task of computing seismic wave fields is defined – e.g. simulation of data, or wavefield propagation as needed for imaging and inversion. This step is common to all computational approaches, and includes specifying the governing equations (e.g. acoustic wave equation, or elastic, or visco-elastic, etc wave equations), the corresponding material parameters (compressional and shear velocities, density, etc), the dimensions of a computational domain and the conditions at the boundaries, the initial conditions (e.g. source functions), what results from computations are saved.

Then the parameters of the Laguerre transform are selected by examining the transform for the initial conditions (e.g. source time function). The computational domain is decomposed into sub-domains, taking into account variations in the complexity of the material parameters, target sizes for the computational tasks in each sub-domain, as well characteristics of the available computing resources. The Laguerre transform is applied along the time axis for each sub-domain and Helmholtz equations obtained after Laguerre transform are solved for each sub-domain, using standard finite-difference or finite-element methods. Data exchanges is made between sub-domains as per Schwartz iteration method. Last two steps (solving Helmholtz equations and data exchange) are iterated until sufficient level of accuracy is reached for the solution. Then the inverse Laguerre transform is applied and the solution to the defined task is obtained.

Further, it is proposed to apply different numerical schemes within each of the sub-domains (e.g. finite-difference schemes of different orders in space, or finite-element schemes), thus adapting the computational cost to the accuracy requirements and the earth model properties.

The integral Laguerre transforms (forward and backward) are provided by the formulae:

$$g_n(h) = \int_0^{\infty} g(t) \cdot (ht)^{\frac{\alpha}{2}} l_n^{\alpha}(ht) dt,$$

$$g(t) = \sum_{n=0}^{\infty} g_n(h) \cdot (ht)^{\frac{\alpha}{2}} l_n^{\alpha}(ht),$$

where $l_n^{\alpha} = \sqrt{\frac{hn!}{(n+\alpha)!}} (ht)^{\frac{\alpha}{2}} e^{-\frac{ht}{2}} L_n^{\alpha}(ht)$, and $L_n^{\alpha}(ht)$ are the classical Laguerre

polynomials of degree n , α is an auxiliary parameter. The most important variable appearing in the formulae is h which is real and positive. As it follows from the representation of the Laguerre transform, h is a scaling parameter, the higher h is the narrower the Laguerre polynomials are if considered as functions of time. This means that the higher h is the less polynomials are needed to approximate a source wavelet.

Having applied Laguerre transform to scalar wave equation one achieves so-called negative defined Helmholtz equation, i.e. all the eigenvalues of the operator are strictly negative:

$$\Delta u_n - \frac{h^2}{4c^2(\bar{x})} u_n = \sum_{k=0}^{n-1} \beta_k u_k + \hat{f}(h, \bar{x}),$$

$$u = \sum_{n=0}^{\infty} u_n(h) \cdot (ht)^{\frac{\alpha}{2}} l_n^{\alpha}(ht)$$

The main advantages of the obtained equation are: the spectra is well separated from zero, hence - good conditionality; the operator does not depend on degree of Laguerre polynomial, hence can be inverted only once and used for different right-hand sides; right-hand side depends on the linear combination of the previously computed coefficients, but β_k do not depend on current number of polynomial (n), hence no need to store all the previous coefficients but only the linear combination.

As it was mentioned above negative-definite Helmholtz equation, obtained by means of Laguerre transform, is elliptic one, hence the well-developed theory of Schwartz domain decomposition method can be applied in straightforward way. Taking into account a number of different ways designed to improve convergence of Schwartz method for elliptical operators one may expect high convergence rate for negative-definite Helmholtz equation.

To illustrate the efficiency of the proposed method a set of numerical experiments was done. For the very beginning 1D problem was considered and the comparison of Schwartz iterations was done for both classical and negative-definite Helmholtz equation. It was observed that for variable velocity Schwartz method converged within 45 iterations for each frequency if applied for classical Helmholtz and within only 5 iterations if done for negative-definite one.

2D simulations were done with the help of Laguerre transform only. Wave fields were computed inside the domain of 1 over 1 kilometer. Two models were considered. The first one was rather simple, i.e. horizontally layered – it is shown on Fig.1. The other one was more complicated and designed to observe refracted waves (see Fig. 2).

In order to apply Schwartz method the domain was divided into four subdomains – one division in each spatial direction. In Figure 3 one can see the wave field computed for the simple layered medium of Fig.1, and the synthetic wave field calculated for the second model (Fig.2) is presented in Figure 3. Reflected waves as well as refracted ones are clearly seen.

It is worth mentioning that for both experiments it took Schwartz algorithm only 7 iterations to converge, while the simple conjugation conditions were used. Applying more specific conditions one may expect significant increase of the convergence rate (number of iterations close to number of divisions along a single spatial direction).

Claims

1. A method for computing wave fields comprising the steps of:

- specifying the governing wave equations,
- setting up the corresponding material parameters, the dimensions of a computational domain, the conditions at the boundaries and the initial conditions,
- selecting the parameters of the Laguerre transform by examining the transform for the initial conditions,
- decomposing the computational domain into at least two sub-domains,
- applying the Laguerre transform along the time axis for each sub-domain and obtaining Helmholtz equations,
- solving Helmholtz equations for each sub-domain,
- exchanging data between sub-domains as per Schwartz iteration method,
- iterating previous two steps until sufficient level of accuracy is reached,
- applying inverse Laguerre transform and obtaining the solution of wave equations.

2. A method of claim 1 wherein wave equations are acoustic or elastic wave equations.

3. A method of claim 1 wherein wave equations are visco-elastic or poro-elastic wave equations.

4. A method of claim 1 wherein wave equations are electromagnetic wave equations.

5. A method of claim 1 wherein Helmholtz equations are solved using standard finite-difference method.

6. A method of claim 1 wherein Helmholtz equations are solved using standard finite-element method.
7. A method of claim 1 wherein Helmholtz equations for different sub-domains are solved by different numerical methods.

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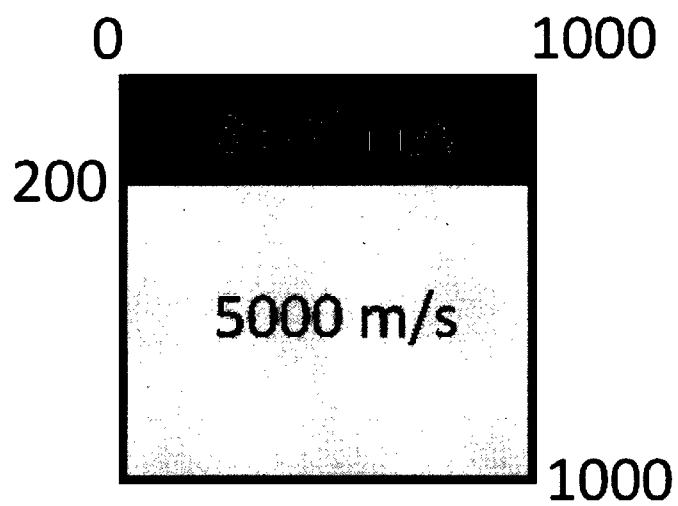


Fig. 1

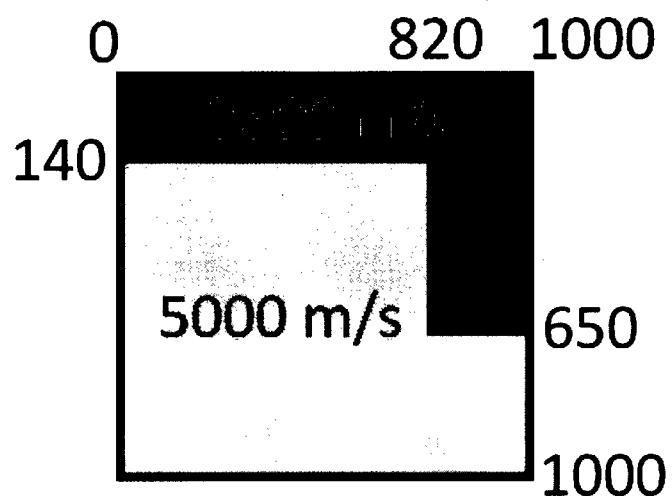


Fig. 2

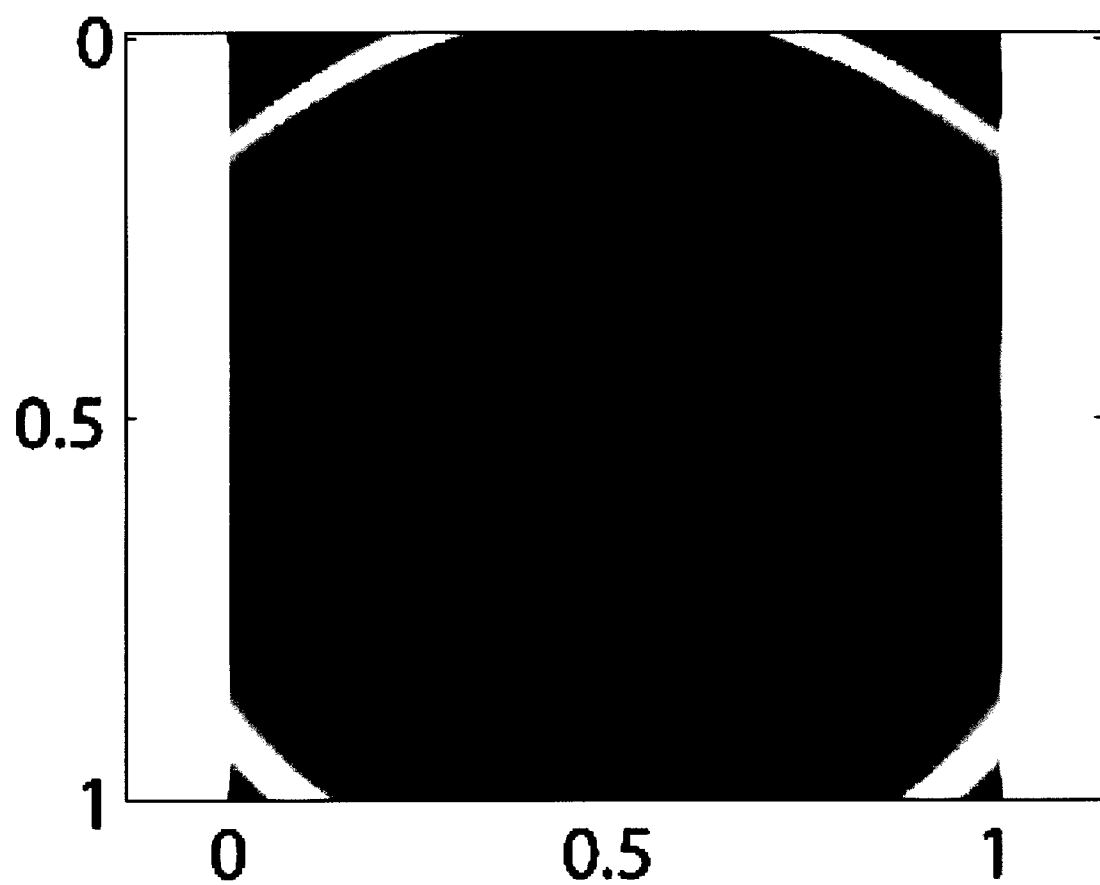


Fig. 3

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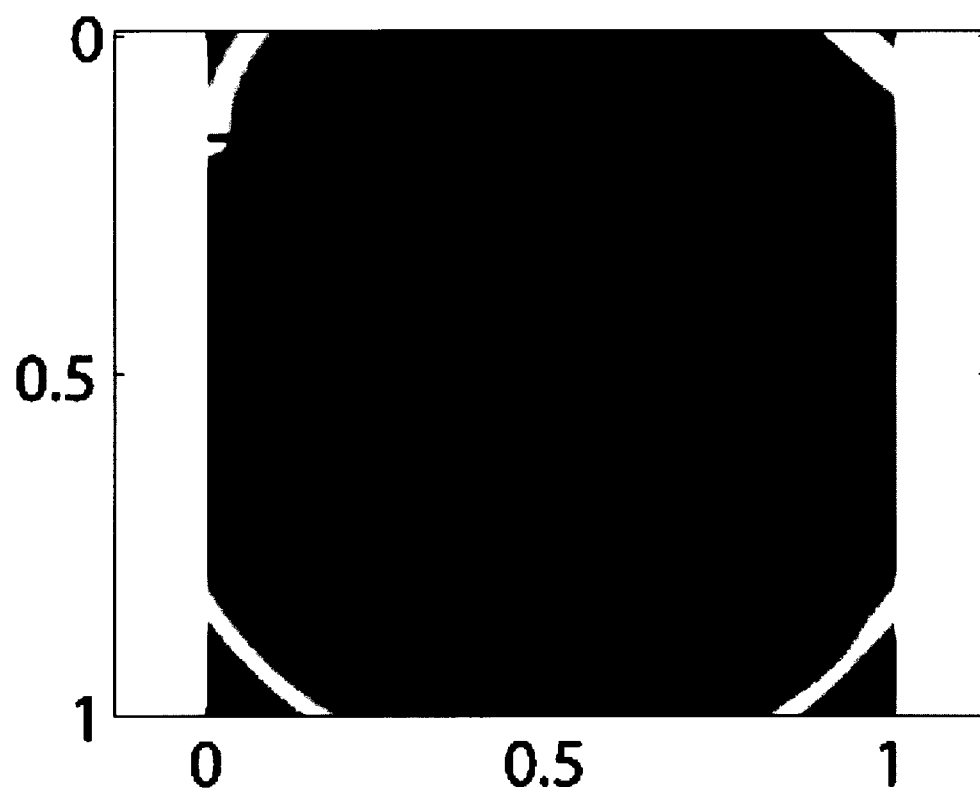


Fig.4

PATENT COOPERATION TREATY

PCT

DECLARATION OF NON-ESTABLISHMENT OF INTERNATIONAL SEARCH REPORT

(PCT Article 17(2)(a), Rules 13^{ter}.1(c) and (d) and 39)

Applicant's or agent's file reference	IMPORTANT DECLARATION	Date of mailing (<i>day/month/year</i>) 04 August 2011 (04.08.2011)
International application No. PCT/RU 2010/000770	International filing date (<i>day/month/year</i>) 21 December 2010 (21.12.2010)	(Earliest) Priority Date
International Patent Classification (IPC) or both national classification and IPC		G01V 1/28 (2006.01) G06F 17/10 (2006.01)
Applicant CHEVERDA Vladimir Albertovich et al.		

This International Searching Authority hereby declares, according to Article 17(2)(a), that **no international search report will be established** on the international application for the reasons indicated below.

1. ☒ The subject matter of the international application relates to:
 - a. ☐ scientific theories
 - b. ☒ mathematical theories
 - c. ☐ plant varieties
 - d. ☐ animal varieties
 - e. ☐ essentially biological processes for the production of plants and animals, other than microbiological processes and the products of such processes
 - f. ☐ schemes, rules or methods of doing business
 - g. ☐ schemes, rules or methods of performing purely mental acts
 - h. ☐ schemes, rules or methods of playing games
 - i. ☐ method for treatment of the human body by surgery or therapy
 - j. ☐ method for treatment of the animal body by surgery or therapy
 - k. ☐ diagnostic methods practised on the human or animal body
 - l. ☐ mere presentations of information
 - m. ☐ computer programs for which this International Searching Authority is not equipped to search prior art

2. ☐ The failure of the following parts of the international application to comply with prescribed requirements prevents a meaningful search from being carried out:

☐ the description
 ☐ the claims
 ☐ the drawings

3. ☐ A meaningful search could not be carried out without the sequence listing; the applicant did not, within the prescribed time limit;

☐ furnish a sequence listing on paper complying with the standard provided for in Annex C of the Administrative Instructions, and such listing was not available to the International Searching Authority in a form and manner acceptable to it.
☐ furnish a sequence listing in electronic form complying with the standard provided for in Annex C of the Administrative Instructions, and such listing was not available to the International Searching Authority in a form and manner acceptable to it.
☐ pay the required late furnishing fee for the furnishing of a sequence listing in response to an invitation under Rule 13^{ter}.1(a) or (b).

4. Further comments: See supplemental sheet.

Name and mailing address of the ISA/RU FIPS, Russia, 123995, Moscow, G-59, GSP-5 Berezhkovskaya nab., 30-1 Fax: (499)243-3337	Authorized officer: A, Grigoryan Telephone № (499)240-25-91
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DECLARATION OF NON-ESTABLISHMENT OF
INTERNATIONAL SEARCH REPORT

International application №

PCT/RU 2010/000770

Supplemental Box (To be used when the space in any of the proceeding boxes is not sufficient)

Continuation of Box IV:

The claimed method for wave field calculation doesn't seem to be a technical matter. The method relates to a mathematical theory (mathematical method) directed at solution of wave equations. That is confirmed by the fact that all actions mentioned as the features of the claimed method for wave field calculation (including the family notion reflecting the purpose of the claimed proposal) represent a mathematical procedure for solution of wave equations characterizing oscillation processes in continuum describing, in particular, acoustic and seismic fields. The said actions of the claimed method for wave fields calculation are brought to the following stages: objective function synthesis comprising wave equation type determination, setting of basic parameters and constraints, obtaining Helmholtz equation with the use of Laguerre transformation, use of reverse Laguerre transformation and obtaining wave equation solution.

It's worth noting that as applied to the claimed proposal no operation of technical character is implemented, and the innovation proposed by the applicant consists merely in transformation of wave equation solution process due to use of calculation tools differing from the ones known from the art by methods of solution of wave equations.

Therefore, the obtained effect is not a technical one as it lies merely in obtaining more precise information on wave seismic and acoustic fields due to use of more improved mathematical method for solution of wave equations within the process of mathematical simulation.