R. S. HOYT.

ARTIFICIAL LINE.

APPLICATION FILED AUG. 7, 1915.

1,167,693. Patented Jan. 11, 1916.

Fig. 1

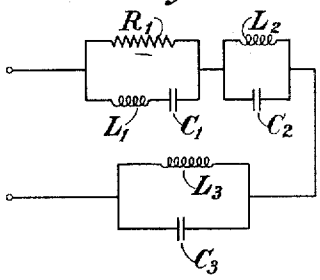


Fig. 2

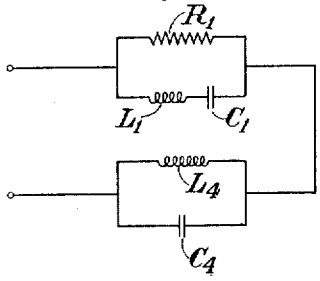


Fig. 3

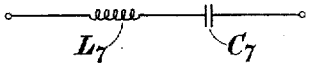


Fig. 4

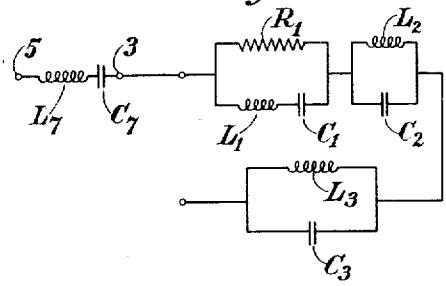
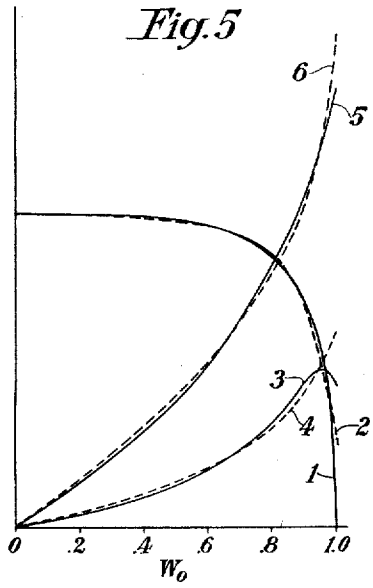


Fig. 5



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UNITED STATES PATENT OFFICE.

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ARTIFICIAL LINE.

1,167,693. Specification of Letters Patent. Patented Jan. 11, 1916.

Application filed August 7, 1915. Serial No. 44,173.

To all whom it may concern:

Be it known that I, RAY S. HOYT, residing at Brooklyn, in the county of Kings and State of New York, have invented certain Improvements in Artificial Lines, of which the following is a specification.

My invention relates to an artificial line for simulating the impedance of a long periodically loaded transmission line over a preassigned range of frequencies, as, for example, over the range of frequencies necessary for the telephonic transmission of speech. It constitutes an improvement over my previous invention disclosed in Patent #1,124,904, dated January 12, 1915, in that my present invention takes into account the distributed character of the line inductance, that is the fact that the line inductance is distributed, and, as a further refinement, it also takes into account the effect of the line and load resistance on the impedance of the loaded line. My present invention, therefore, simulates to a still higher degree of precision the impedance of the actual transmission line.

In the accompanying drawing, Figure 1 illustrates diagrammatically a basic form of artificial line arranged in accordance with my present invention; Fig. 2 shows a modification of the invention, providing what may conveniently be termed a consolidated form; Fig. 3 illustrates a supplementary structure for taking into account the effect of line and load resistance; Fig. 4 shows the complete structure obtained by combining the structures shown in Figs. 1 and 3; and Fig. 5 presents a series of curves showing the precision with which my invention, as embodied in Fig. 1, simulates the impedance of the actual loaded line.

By comparing Fig. 1 of the present application with Fig. 1 of my above mentioned patent, it will be noted that in lieu of the resistance element R_0 shown in the drawing of the patent, my present invention provides a resistance R_1 shunted by an inductance coil L_1 and a condenser C_1 in series. The function of the L_1 C_1 shunt is to give the resistance component of the impedance the drooping characteristic, shown by Fig. 3 of my previous patent, which the resistance component of the actual line impedance has. The L_1 C_1 shunt gives rise, however, to a reactance component, and this reactance component is neutralized by the L_2 C_2 portion of my present invention. The L_3 C_3 portion of my present invention simulates the reactance component of the actual line impedance; its function is therefore the same as that of the L_0 C_0 portion of my previous invention. In addition to these differences between my previous and my present invention, the latter is more precisely proportioned to take into account the distributed character of the line inductance, and, furthermore, the supplementary structure of Fig. 3, which comprises an inductance coil L_7 in series with a condenser C_7, takes into account the effect of line and load resistance, an effect entirely ignored in my previous invention.

Having thus briefly pointed out certain differences between the present invention and that over which it is an improvement, I will next point out the fundamental principles underlying my present invention and the manner of applying said principles so as to provide a structure having the aforementioned distinguishing characteristics.

A general formula for the mid-load impedance of a long periodically loaded transmission line (see equation 19 of G. A. Campbell's paper "On Loaded Lines in Telephonic Transmission", *Philosophical Magazine*, March, 1903) is:

$$K = \sqrt{k^2 + q^2/4 + qk \coth ms} \quad (1)$$

The above formula is the same as that given in the above mentioned paper except for a slight change of notation consisting in substituting the symbol q for Hd, m for γ, and s for d. It is also equivalent to the formula given in my previous patent, as is shown in the said Campbell paper. This particular formula is chosen as more convenient in deriving approximations. In the notation thereof, K denotes the mid-load characteristic impedance of a periodically

loaded line; k, the characteristic impedance of the same line when non-loaded; q, the impedance of the load coil of inductance L; m, the propagation constant per unit length of the line when non-loaded; and s, the length of a loading section or the distance between two consecutive load coils.

The value of the abovementioned characteristic impedance k is expressed by the following equation:

$$k=\sqrt{\frac{R+ipJ}{G+ipC}} \quad (2)$$

in which R, J, G and C denote the resistance, inductance, leakage conductance and capacity, respectively, of the line per unit length;

$$i=\sqrt{-1}\text{; and } p=2\pi f,$$

where f denotes the frequency in cycles per second.

The value of m in equation (1) is expressed by the following equation:

$$m=\sqrt{(R+ipJ)(G+ipC)} \quad (3)$$

The above equations (2) and (3) are familiar ones and are found in standard treatises on the theory of transmission lines.

By modifying the above-given fundamental formula (1) in the manner now to be described, approximate formulæ of high precision may be derived, by the aid of which the artificial line of my present invention may be constructed. In my above-mentioned patent it is stated that the mid-load characteristic impedance of the loaded line is very closely equal to the impedance of a loaded line having no leakage conductance, no resistance and no distributed inductance, but having the same total inductance L and total capacity sC per section as the actual line. In accordance with these approximations the mid-load impedance K is very closely expressed by the equation

$$K=T\sqrt{(1-w^2)} \quad (4)$$

where T is equal to

$$\sqrt{\frac{L}{sC}}\text{, and } w=p\sqrt{\frac{sCL}{4}}.$$

It is also shown in my previous patent that the impedance of a line beginning not at mid-load but at a distance xs from the first loading coil can be approximately written:—

$$Z=\frac{T\sqrt{1-w^2}+iTw(1-2x)}{1-4x(1-x)w^2} \quad (5)$$

where x denotes the fractional length of the first section.

Formulæ (4) and (5) are in substance identical with the formulæ given in lines 16 and 39, respectively, of page 2 of my abovementioned patent, differing slightly therefrom in form owing to certain changes and improvements in notation.

It will be observed that the impedance formulæ expressed by equations (4) and (5) ignore entirely the small effect produced by the actual resistance of the line and of the load coils, and further that they neglect the distributed character of the line inductance in that the total distributed inductance of each loading section is lumped and added to the inductance of the corresponding load coil. The errors introduced by these assumptions, while small, are not always negligible and in the artificial line of my present invention I desire to take into account the effects produced by the resistance of the line and of load coils and also the distributed character of the line inductance. To this end I have derived more closely approximate formulæ in a manner now to be set forth. Since the impedance effects produced by the distributed character of the line inductance and by the resistance of the line and of the load coils are small, in deriving formulæ to take into account said effects, I have considered each of said effects separately and have obtained the joint effect by superposition.

Referring to equation (1), if the load coil is considered as having inductance L only and the line as having distributed inductance J and distributed capacity C per unit length only, that is if the resistance of line and load, as well as leakage, are neglected, it follows from the definition of symbols hereinbefore given and from equations (2) and (3) that

$$\left.\begin{aligned} &k\sqrt{\frac{J}{C}}\\ &q=ipL\\ &m=ip\sqrt{JC}\end{aligned}\right\} \quad (6)$$

and that equation (1) reduces to

$$K=T\sqrt{v^2-w^2+2vw\cot.2vw} \quad (7)$$

This last equation is derivable by direct transformation from equations (1) and (6). The symbols T and w have already been defined. The symbol v denotes

$$\sqrt{sJ}$$

that is the square root of the ratio of the line inductance per loading section to the inductance per loaded coil. A further direct mathematical transformation reduces equation (7) to the form

$$K=\frac{T}{v}\sqrt{(v^2-vw\tan.vw)(v^2+xw\cot.vw)} \quad (8)$$

Equation (8) shows that K vanishes if

$v=w$ tan. vw, a property which will be made use of hereinafter.

Equation (7) contains the term $2vw$ cot. $2vw$. Inspection of the known infinite series for cot. $2vw$ shows that $2vw$ cot. $2vw$ is closely equal to

$$1-\frac{4}{3}v^2w^2$$

and that, by the introduction of a coefficient b it can be exactly expressed as

$$2vw \text{ cot. } 2vw=1-(bw)^2. \quad (9)$$

The coefficient b is closely, but not exactly, equal to

$$\frac{2}{\sqrt{3}}v.$$

Rigorously it is a function of w and therefore of the frequency. It varies, however, very slowly with the frequency. Moreover, if any particular frequency is specified, b is determined by equation (9) in terms of the value of w corresponding to said frequency and in terms of the parameter v. If the coefficient b is so determined, equation (9) will be very closely, though not exactly, correct for other values of the frequency. The departure from equality of said equation is, however, very small within the range of frequencies of practical importance.

It is convenient and desirable, for reasons of simplicity, to determine the value of b, as given by equation (9), for the smallest real value of w, or of the frequency, for which K vanishes. As hereinbefore stated K vanishes, as shown by equation (8), when

$$v=w \text{ tan. } vw \quad (10)$$

Equation (10) is transcendental. I have computed the roots of this equation which will be denoted by w_c, for different values of the parameter v. The following table gives the values of w_c corresponding to four different values of v^2, covering the range of values of v^2 occurring in practice.

v^2	w_c
0	1.0000
.0529	.9913
.0900	.9852
.1369	.9778
.1936	.9689

(11)

Other values of w_c within this range can be gotten from table (11) by interpolation.

Returning to equation (7), substitution therein of equation (9) gives:—

$$K=T\sqrt{v^2-w^2+1-(bw)^2}$$

$$=T\sqrt{1+v^2}\sqrt{1-w^2\frac{1+b^2}{1+v^2}} \quad (12).$$

The coefficient b has been so chosen that (12) is exact for $w=w_c$; it therefore follows from (12) that

$$w_c=\sqrt{\frac{1+v^2}{1+b^2}}$$

and that:

$$K=T\sqrt{1+v^2}\sqrt{1-\left(\frac{w}{w_c}\right)^2} \quad (13).$$

Denoting

$$T\sqrt{1+v^2} \text{ by } T_o$$

and

$$\frac{w}{w_c} \text{ by } w_o,$$

equation (13) reduces to:—

$$K=T_o\sqrt{1-w_o^2} \quad (14).$$

Equation (14), giving the characteristic mid-load impedance of the loaded line, corresponds in form with equation (4), with the difference that T_o is substituted for T and w_o for w. If two parameters L_o and C_o are introduced, such that

$$L_o=L\frac{\sqrt{1+v^2}}{w_c} \quad (15)$$

$$C_o=C\frac{1}{w_c\sqrt{1+v^2}} \quad (16)$$

it follows from a reference to the expressions which T_o and w_o represent, that

$$T_o=\sqrt{\frac{L_o}{sC_o}} \quad (17)$$

$$w_o=p\sqrt{\frac{sC_oL_o}{4}} \quad (18).$$

Reference to the significance of the previously employed symbols T and w show that T_o and w_o are of the same form as T and w respectively, differing only in the substitution of L_o for L and C_o for C, as given by equations (15) and (16).

Equation (14) is a highly precise expression for the characteristic mid-load impedance of a loaded line. The precision thereof is higher than that of the corresponding equation (4) in that in equation (14) the distributed character of the line inductance is taken into account. The characteristic impedance of a loaded line having an initial section of length xs is derivable from equation (14) precisely as equation (5) is derivable from (4) and is given by

$$Z=\frac{T_o\sqrt{1-w_o}+iT_ow_o(1-2x)}{1-4x(1-x)w_o^2} \quad (19).$$

This will be written for convenience as

$$Z=D+iQ \quad (20).$$

where D and Q are the real and imaginary

parts of the right-hand side of equation (19).

Equations (14) and (19) express the characteristic impedance of a loaded line having no resistance. The effect of the actual resistance in modifying said formulæ is now to be shown. In investigating this effect, which is known to be quite small, as hereinbefore stated, it is allowable, at a very small sacrifice of precision, to assume that the loaded line to be simulated has no distributed line inductance but a load coil inductance L_o and a distributed capacity C_o per unit length of line. It is further allowable to assume that the resistance R′ of the load coil is added to the distributed resistance R of the line, giving an apparent distributed resistance r per unit length of line,

$$r = R + \frac{R'}{s} \quad (21).$$

Referring to equations (2) and (3), substituting r for R, C_o for C, and setting J and G equal to zero, the values of k and m reduce to

$$k = \sqrt{\frac{r}{ipC_o}} \quad (22)$$

$$m = \sqrt{irpC_o} \quad (23).$$

Returning to equation (1) and replacing q by ipL_o and giving k and m the values expressed by equations (22) and (23), the expression for K reduces to

$$K = T_o\sqrt{-w_o^2 - i\frac{g}{4w_o} + \sqrt{igw_o}\coth\sqrt{igw_o}} \quad (24).$$

This equation, in which the symbol g denotes

$$\frac{2sr}{T_o},$$

is directly derivable from equation (1) in accordance with the assumptions hereinbefore stated. If

$$\coth\sqrt{igw_o}$$

is replaced in (24) by its equivalent series, and if negligible terms thereof are dropped, equation (24) reduces to

$$K = T_o\sqrt{1 - w_o^2 + \frac{g^2w_o^2}{45} - i\left(\frac{g}{4w_o}\right)\left(1 - \frac{4}{3}w_o^2 + \frac{8g^2w_o^4}{(45)(21)}\right)} \quad (25)$$

For ordinary loaded lines the parameter g is of the order 0.1 or less. It will then be observed that the third term,

$$\frac{g^2w_o^2}{45}$$

under the radical is very small compared with the first two, and further that the imaginary part is small compared with the real part. Equation (25), therefore, may be approximately written as

$$K = T_o\sqrt{1 - w_o^2} - i\left(\frac{gT_o}{8w_o}\right)\left(1 - \frac{4}{3}w_o^2\right) \quad (26)$$

Equation (26) differs from equation (14), only in the addition of the imaginary term. It will be seen that the actual resistance of the line and load coil produces a quite negligible effect on the real or resistance component of the impedance, but that it does produce an appreciable effect on the imaginary or reactance component of said impedance. It may be then stated that the effect of line and load resistance on the characteristic impedance of the loaded line is taken into account if to equations (14) and (19) a term iy is added, such that

$$iy = -i\left(\frac{gT_o}{8w_o}\right)\left(1 - \frac{4}{3}w_o^2\right)$$

$$= -i\left(\frac{sr}{4w_o}\right)\left(1 - \frac{4}{3}w_o^2\right). \quad (27)$$

In equation (27) sr, as hereinbefore defined, is the total resistance per loading section; that is the resistance of the line of length s plus the resistance of one load coil.

My invention in the light of the foregoing discussion will now be understood to consist in providing a simple circuit arrangement which shall have over the range of frequencies between the limits corresponding to $w=o$ and $w=w_c$ the impedance expressed by equation (19) and, as a higher refinement, that given by adding the expression (27) to (19), as will now be more fully described.

The circuit arrangement which simulates the impedance expressed by formula (19) may conveniently be termed the basic structure and is shown in Fig. 1. The part consisting of the inductance L_3 in parallel with the capacity C_3 simulates the reactance component Q of Z as given by equations (19) and (20). Referring to said equations it will be seen that the imaginary or reactance component may be written

$$Q = \frac{pL_o\frac{1-2x}{2}}{1 - x(1-x)sC_oL_op^2} \quad (28)$$

The impedance of an inductance L_3 in parallel with a capacity C_3 is well known to be expressible as

$$\frac{pL_3}{1 - L_3C_3p^2}.$$

Comparison of this expression with equa-

tion (28) shows that they are identical provided that L_3 and C_3 be given the following values:

$$L_3=(1/2-x)L_o \quad (29)$$

$$C^3=\frac{x(1-x)}{1/2-x}sC_o \quad (30)$$

It is thus seen that the L_3 C_3 portion of the basic structure simulates the reactance component of the impedance of the loaded line provided that L_3 and C_3 are proportioned in accordance with equations (29) and (30).

Equations (19) and (20) show that the resistance component of the characteristic impedance of the loaded line is given by

$$D=\frac{T_o\sqrt{1-w_o^2}}{1-4x(1-x)w_o} \quad (31)$$

Referring to the curves shown in Fig. 3 of my above mentioned patent, it will be seen that said resistance component for values of x less than .17 has a drooping characteristic and finally becomes zero at $w=1$. Now a resistance element shunted by an inductance element and a condenser in series will have a similar resistance characteristic provided that the inductance-condenser shunt is resonant in the neighborhood of $w=1$. The impedance of a resistance R_1 in parallel with an inductance L_1 and a condenser C_1 in series is well known to be

$$\frac{R_1(1-L_1C_1p^2)}{(1-L_1C_1p^2)+ipR_1C_1}=A_1+iB_1 \quad (32)$$

Here A_1 and B_1 are, respectively, the real and imaginary components of the expression on the lefthand side of equation (32), and are therefore the resistance and reactance components respectively of the impedance of said combination. If an equation (32)

$$\frac{w_o}{\sqrt{\frac{sC_oL_o}{4}}}$$

is substituted for its identity p, the values of A_1 and B_1 are:

$$A_1=\frac{R_1\left(1-4\frac{L_1C_1}{sL_oC_o}w_o^2\right)^2}{\left(1-4\frac{L_1C_1}{sL_oC_o}w_o^2\right)^2+4\frac{R_1^2C_1^2}{sL_oC_o}w_o^2} \quad (33)$$

$$B_1=\frac{-\left(\frac{2R_1^2C_1}{\sqrt{sL_oC_o}}\right)w^o\left(1-4\frac{L_1C_1}{sL_oC_o}w_o^2\right)}{\left(1-4\frac{L_1C_1}{sL_oC_o}w_o^2\right)^2+4\frac{R_1^2C_1^2}{sL_oC_o}w_o^2} \quad (34)$$

Proceeding then on the assumption that the resistance component of the impedance of a loaded line having an initial sectional length of 0.17 or less might be simulated by the resistance component A_1 of a resistance element R_1 in parallel with an inductance L_1 and a condenser C_1, provided proper values were assigned to x and R_1, L_1, C_1, I performed an elaborate series of computations and plottings. The general character of these computations was to assign tentative values to the parameters x, R_1, L_1 and C_1, compute the corresponding values of D as given by equation (31) and A_1 as given by equation (33), plot curves with said values as ordinates and w_0 as abscissa and compare said curves. By such means I discovered that the desired simulation was attained over the required range from $w_0=o$ to $w_0=1$ by assigning the following values:

$$R_1=\sqrt{\frac{L_o}{sC_o}} \quad (35)$$

$$L_1=(2.023)L^o \quad (36)$$

$$C_1=(.1069)sC_c \quad (37)$$

$$x=0.14 \quad (38)$$

Fig. 5 serves to show the high degree of precision with which the resistance component A_1 approximates the resistance component D of the line impedance. In said Fig. 5, the curve 1 is a plot of the resistance component of the actual loaded line with an initial section of $.14s$, while the curve 2 is a plot of the resistance component A_1 of my artificial line, the abscissæ being the values of w_o. Equations (35), (36), (37), (38), are then the design formulæ in accordance with which I proportion the R_1 L_1 C_1 portion of the artificial line.

The reactance component B_1 must be neutralized since it does not usefully simulate any component of the line impedance. This neutralization is accomplished very closely, as hereinbefore stated, by adding to the R_1 L_1 C_1 portion of the structure an inductance coil L_2 in parallel with a condenser C_2, provided that, as determined by computations and comparison of the expression for B_1 and the impedance of an inductance in parallel with a condenser,

$$L_2=(.120)L_0 \quad (39)$$

$$C_2=(1.28)sC_0 \quad (40)$$

The precision with which the reactance of the L_2 C_2 portion neutralizes the reactance of the R_1 L_1 C_1 portion is shown in Fig. 5, in which curve 3 is a plot of $-B_1$, and curve 4 is a plot of the reactance of the L_2 C_2 portion of Figs. 1 and 4, wherein $x=0.14$. If $x=0.14$, the value hereinbefore assigned to it, equations (29) and (30) become

$$L_3=(.3600)L_0 \quad (41)$$

$$C_3=(.3344)sC_0 \quad (42)$$

If then the elements of the structure

shown in Fig. 1 are proportioned in accordance with equations 35–42 inclusive, the structure simulates very closely over the entire range of frequencies necessary for telephonic transmission of speech, the impedance of the loaded line having an initial length of line equal to 0.14 of the length of a loading section. This is illustrated in Fig. 5 in which the resistance simulation is shown in curves 1 and 2, already referred to, and in curves 5 and 6 which are plots of the reactance component of the actual loaded line, and of the reactance component of the structure shown in Fig. 1, respectively.

It will be noted that the basic structure designed in accordance with the design formulæ (35) to (42), inclusive, simulates the characteristic impedance of a loaded line having a particular initial length of line equal to 0.14 of a loading section, since the simulation is most precise for this particular value of the initial length xs. In general, of course, the line may have any length of initial section, so that it becomes necessary to supplement the basic structure to the end that it may simulate the impedance of a line of any initial length xs. This may be accomplished in the manner stated in my above-mentioned patent, as for example by adding to the artificial line an extension, so that the extended artificial line shall be equivalent to a line having an initial length xs. It will be understood that such extension may likewise be proportioned in accordance with L_0 and C_0 whereas in my previous patent the extension was proportioned in accordance with the values of L and C.

If it is desired to take into account the resistance effect as given by equation (27), there should be added in series with the basic structure an inductance coil L_7 and a condenser C_7 in series, whose values are:

$$L_7=\left(\frac{sr}{6}\right)\sqrt{sC_0L_0} \quad (43)$$

$$C_7=(2/sr)\sqrt{sC_0L_0} \quad (44)$$

The impedance of this combination, L_7 in series with C_7, is well known to be

$$-i\frac{1}{C_7p}(1-L_7C_7p^2).$$

Substitution of the above given values of L_7 and C_7, and comparison of the impedance of this arrangement with that given by equation (27) shows that they are equal.

A simplified form, which may conveniently be termed the consolidated artificial line, can be gotten from the basic structure at a very small sacrifice of precision of simulation by combining the L_2 C_2 and L_3 C_3 portions into a single L_4 C_4 portion, as shown in Fig. 2, and this constitutes a particular simplified embodiment of my invention. By computations analogous to those by which the R_1 L_1 C_1 portion is obtained, the most suitable values of L_4 and C_4 are given as follows:

$$L_4=(0.483)L_0 \quad (45)$$

$$C_4=(0.265)sC_0 \quad (46)$$

This invention, as in the case of the invention disclosed in my aforementioned patent, finds its application where the impedance of a loaded line is to be simulated, as for example, for balancing purposes in systems involving two two-way telephone repeaters and in other transmission lines. Suppose, for example, it is desired to design an artificial line in accordance with my present invention, which shall simulate the characteristic impedance of a periodically loaded line whose specifications are as follows: An aerial line consisting of two parallel #8 B. W. G. copper wires, loaded at intervals of 8 miles ($s=8$) with an initial 0.14 section; the distributed inductance, J, per mile of line is 0.0034 henries; the capacity, C, per mile is 0.0092×10^{-6} farads; total resistance per mile including load coil resistance is 5 ohms; and the inductance per load coil is 0.240 henries.

The first step is to compute the value of v^2 or

$$\frac{SJ}{L}$$

which is equal to 0.113. Referring to equation (11) the corresponding value of w_c is, by interpolation, equal to 0.982. L_0 and C_0 then by equations (15) and (16) have the values, $L_0=0.258$ henries,

$$C_0=0.00888\times10^{-6} \text{ farads.}$$

The values of the elements constituting the basic structure, simulating the impedance of the loaded line beginning with a section of length $(.14)\times(8)$ miles are then, by aid of the foregoing computations and equations 35—42, found to be as follows: R=1910 ohms; $L_1=0.522$ henries; $C_1=0.00759\times10^{-6}$ farads; $L_2=0.0310$ henries; $C_2=0.0909\times10^{-6}$ farads; $L_3=0.0929$ henries; $C_3=0.0238\times10^{-6}$ farads.

If particularly high simulative precision is desired, the supplementary part, shown in Fig. 3, consisting of the inductance L_7 and capacity C_7 may be added to the basic structure as shown in Fig. 4. The values of L_7 and C_7 by equations (43) and (44) are: $L_7=0.00090$ henries; $C_7=6.77\times10^{-6}$ farads.

If the consolidated form of net-work of Fig. 2 is desired, the constants L_4 and C_4 are determined by equations (45) and (46): $L_4=0.1248$ henries; and $C_4=0.0188\times10^{-6}$ farads.

I claim:

1. An artificial line simulating the char-

acteristic impedance of an actual loaded line, said artificial line comprising precomputed resistance, inductance and capacity elements having values depending on the load coil inductance, the distance between consecutive load coils, the capacity and inductance of said actual line, and the distributed character of said line inductance.

2. An artificial line simulating the characteristic impedance of an actual loaded line, said artificial line comprising precomputed resistance, inductance and capacity elements having values depending on the load coil inductance and resistance, the distance between consecutive load coils, the capacity, resistance and inductance of said actual line, and the distributed character of said line inductance.

3. An artificial line simulating the characteristic impedance of an actual loaded line, said artificial line including means having an impedance composed of resistance and reactance components, said resistance component simulating the resistance component of the impedance of the actual loaded line, and means for neutralizing the reactance component of the impedance of said first named means.

4. An artificial line simulating the characteristic impedance of an actual loaded line, said artificial line including resistance and reactance elements providing impedance having a resistance component simulating the resistance component of the impedance of the actual loaded line, and means for neutralizing the reactance component of the impedance produced by said resistance and reactance elements; whereby the joint impedance of said resistance and reactance elements and of said neutralizing means has no reactance component but has a resistance component substantially equal to the resistance component of the impedance of said actual loaded line.

5. An artificial line simulating the characteristic impedance of an actual loaded line, the combination of means for simulating the reactance component of said impedance; means composed of resistance and reactance elements providing an impedance having a resistance component simulating the resistance component of the impedance of said actual loaded line; and means for neutralizing the reactance component of the impedance produced by said resistance and reactance elements.

6. An artificial line simulating the characteristic impedance of an actual loaded line, said artificial line including the combination of a resistance element in parallel with an inductance element and a condenser connected in series with each other, said combination having an impedance whose resistance component simulates the resistance component of the impedance of the actual loaded line, and means for neutralizing the reactance component of the impedance of said combination.

7. An artificial line simulating the characteristic impedance of an actual loaded line, said artificial line comprising resistance, inductance and capacity elements simulating the resistance component of the impedance of the actual loaded line and inductance and capacity elements simulating the reactance component of the impedance of the actual loaded line, said elements having values depending on the load coil inductance, the distance between consecutive load coils, the capacity and inductance of said actual line, and the distributed character of said line inductance.

8. An artificial line simulating the characteristic impedance of an actual loaded line, said artificial line having a portion containing a resistance element in parallel with inductance and capacity elements connected in series with each other, and a portion in series with said other portion and consisting of an inductance element in parallel with a capacity element, said resistance, inductance and capacity elements having values depending on the load coil inductance, the distance between consecutive load coils, the capacity and inductance of said actual line, and the distributed character of said line inductance.

9. An artificial line simulating the characteristic impedance of an actual loaded line, said artificial line comprising a portion containing a resistance element in parallel with an inductance coil and a condenser connected in series, said portion being connected as a whole in series with two other portions each consisting of an inductance coil in parallel with a condenser, the values of said resistance, inductance and capacity elements being determined by the load coil inductance, the distance between consecutive load coils, the capacity and inductance of said actual line, and the distributed character of said line inductance.

10. An artificial line simulating the characteristic impedance of an actual loaded line, said artificial line comprising a portion consisting of an inductance coil in series with a condenser, said portion simulating the effect of the actual line and load resistance, a portion consisting of a resistance element in parallel with a serially connected inductance coil and condenser, and a portion consisting of an inductance coil in parallel with a condenser, said three portions of the artificial line being connected in series with each other, and the values of said resistance, inductance and capacity elements being determined by the load coil inductance and resistance, the distance between consecutive

load coils, the resistance, capacity and inductance of said actual line, and the distributed character of said line inductance.

11. An artificial line, simulating the characteristic impedance of an actual loaded line, said artificial line comprising a portion consisting of an inductance coil in series with a condenser, said portion simulating the effect of the line and load resistance, a second portion consisting of a resistance element in parallel with an inductance coil and a condenser in series with each other, and two other portions each consisting of an inductance coil in parallel with a condenser, said four portions of said artificial line being connected in series with one another, and the values of said resistance, inductance and capacity elements being determined by the load coil inductance and resistance, the distance between consecutive load coils, the resistance, capacity and inductance of said actual line, and the distributed character of said line inductance.

12. In an artificial line simulating the characteristic impedance of an actual loaded line, a sub-combination simulating the resistance component of said impedance and comprising a portion consisting of a resistance element in parallel with an inductance coil and a condenser connected in series with each other, and a portion consisting of an inductance coil and a condenser in parallel, said portions being connected in series with each other, and said resistance, capacity and inductance elements having values depending on the load coil inductance, the distance between consecutive load coils, the capacity and inductance of said actual line, and the distributed character of said line inductance.

In testimony whereof, I have signed my name to this specification in the presence of two subscribing witnesses, this 4th day of August 1915.

RAY S. HOYT.

Witnesses:
GEORGE E. FOLK,
JOHN R. CARSON.

load coils, the resistance, capacity and inductance of said actual line, and the distributed character of said line inductance.

11. An artificial line, simulating the characteristic impedance of an actual loaded line, said artificial line comprising a portion consisting of an inductance coil in series with a condenser, said portion simulating the effect of the line and load resistance, a second portion consisting of a resistance element in parallel with an inductance coil and a condenser in series with each other, and two other portions each consisting of an inductance coil in parallel with a condenser, said four portions of said artificial line being connected in series with one another, and the values of said resistance, inductance and capacity elements being determined by the load coil inductance and resistance, the distance between consecutive load coils, the resistance, capacity and inductance of said actual line, and the distributed character of said line inductance.

12. In an artificial line simulating the characteristic impedance of an actual loaded line, a sub-combination simulating the resistance component of said impedance and comprising a portion consisting of a resistance element in parallel with an inductance coil and a condenser connected in series with each other, and a portion consisting of an inductance coil and a condenser in parallel, said portions being connected in series with each other, and said resistance, capacity and inductance elements having values depending on the load coil inductance, the distance between consecutive load coils, the capacity and inductance of said actual line, and the distributed character of said line inductance.

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Corrections In Letters Patent No. 1,167,693.

It is hereby certified that in Letters Patent No. 1,167,693, granted January 11, 1916, upon the application of Ray S. Hoyt, of Brooklyn, New York, for an improvement in "Artificial Lines," errors appear in the printed specification requiring correction as follows: Page 2, line 105, equation 6, for "$k\sqrt{\frac{J}{C}}$" read $k=\sqrt{\frac{J}{C}}$; same page, line 116, for the symbol "$\sqrt{sJ}$" read $\sqrt{\frac{sJ}{L}}$;; page 3, line 72, equation 13, for "$\left(\frac{w}{w_c}\right)^2$" read $\left(\frac{w}{w_c}\right)^s$; page 5, line 7, equation 30, for "C^3" read C_s; same page, line 56, equation 34, for "w^o" read w_o; and that the said Letters Patent should be read with these corrections therein that the same may conform to the record of the case in the Patent Office.

Signed and sealed this 30th day of May, A. D., 1916.

[SEAL.]

J. T. NEWTON,
Acting Commissioner of Patents.

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