

Jan. 5, 1954

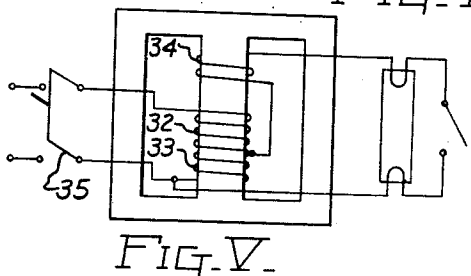
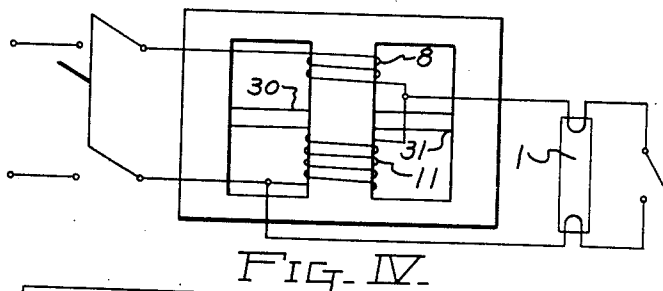
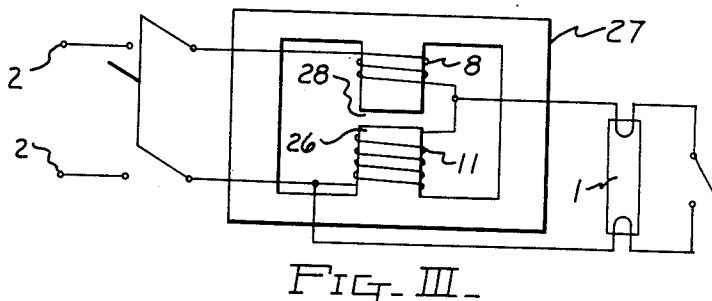
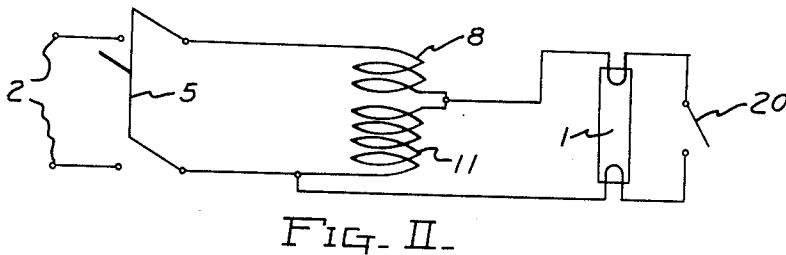
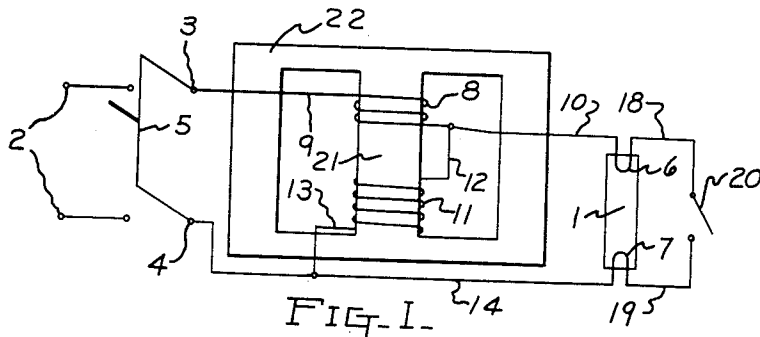
T F. CARMICHAEL

2,665,406

HIGH-POWER FACTOR CURRENT LIMITER

Filed April 1, 1946

3 Sheets-Sheet 1



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3 Sheets-Sheet 2

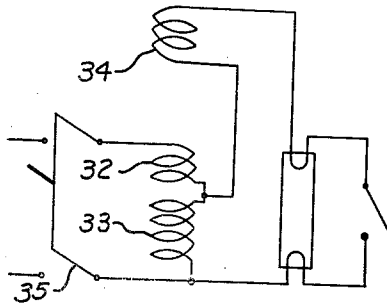


FIG. VI.

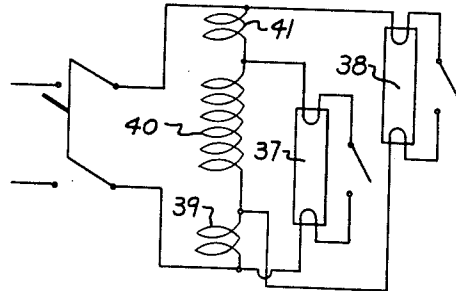


FIG. IX.

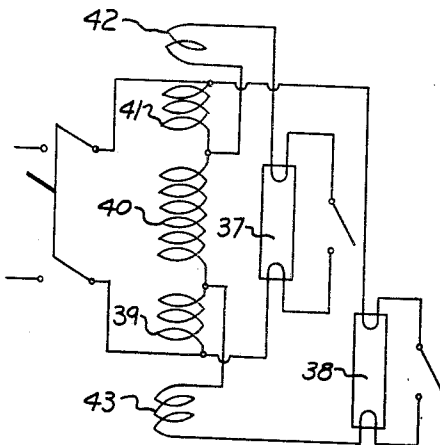


FIG. VII.

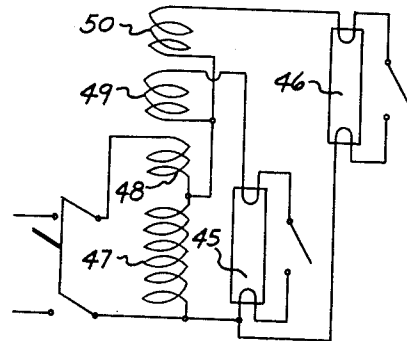


FIG. X.

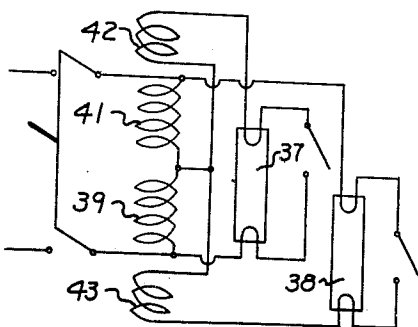


FIG. VIII.

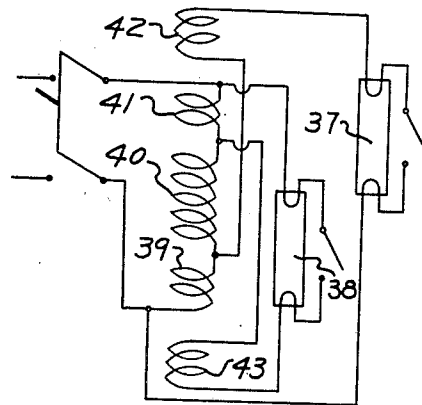


FIG. XI.

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3 Sheets-Sheet 3

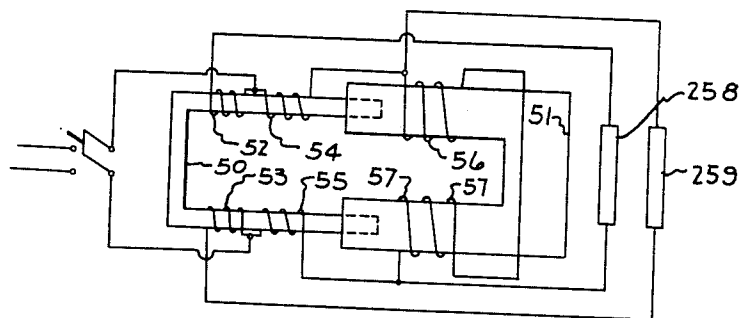


FIG. XII.

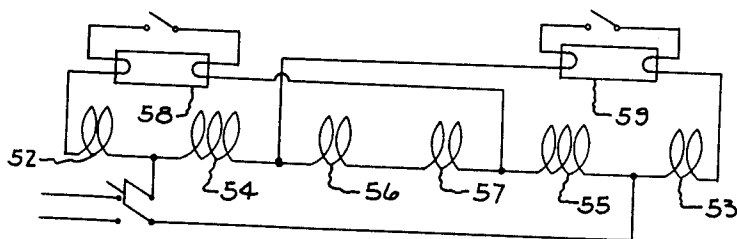


FIG. XIII.

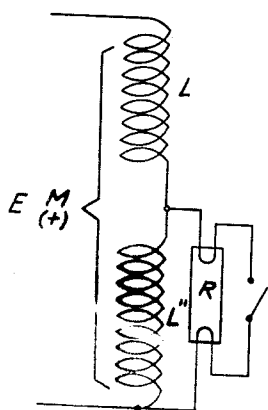


FIG. XIV.

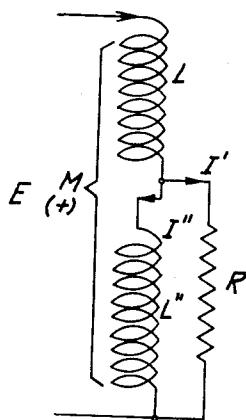


FIG. XV.

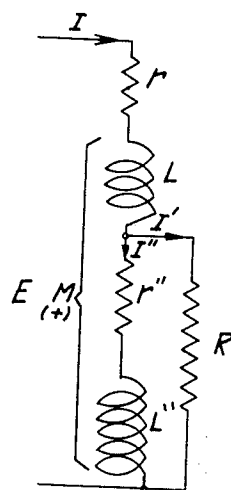


FIG. XVI.

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UNITED STATES PATENT OFFICE

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HIGH POWER FACTOR CURRENT LIMITER

Thomas Frazer Carmichael, Plymouth, Mich.

Application April 1, 1946, Serial No. 658,623

6 Claims. (Cl. 323-45)

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The present invention relates to ballasts for gaseous discharge loads, and more particularly to ballasts for fluorescent lamps to maintain a high power factor load. This is a continuation-in-part of my copending application Serial No. 400,136, filed June 27, 1941, for High Power Factor Current Limiter, now abandoned.

The characteristics of fluorescent lamps are such that when connected to a well regulated voltage source, they will draw an excessive amount of current. In order to limit the amount of current drawn, there is ordinarily included in series with each lamp and its electric power source, a choke coil. However, due to its inductive nature, the choke coil and the lamp with which it is associated, form a low power factor load. Since for reasons well known in the art a high power factor load is desirable, the low power factor load obtained by employing a choke coil is raised to a high power factor load by connecting a capacitor across the conductors of the input power line. The provision of both a choke coil and a capacitor as an auxiliary, which in the trade are combined as a unit known as a ballast, not only increases the expense of fluorescent lighting units but also the ballast is somewhat bulky requiring a substantial amount of space. According to the present invention there is provided a fluorescent lamp high power factor ballast consisting solely of inductances on a core, which not only costs less to manufacture than the combination choke coil and capacitor high power factor ballast, but also takes up less space.

Briefly, this invention in its basic form consists of a core providing a magnetic circuit having disposed therein, two coils connected in series across the electric power source and the fluorescent lamp load shunted across one of the coils on the core. The arrangement is such that the current drawn by the fluorescent lamp load is limited and at the same time the line power factor is not excessively lowered from the unity power factor of the lamp itself which is equivalent to a resistance.

An object of the invention is to provide a gaseous discharge lamp ballast for limiting the current drawn by the lamp without substantially decreasing the power factor thereof.

Another object of the invention is to provide for a fluorescent lamp a ballast for limiting the current drawn thereby without substantially lowering the power factor thereof and without employing a capacitor.

Still another object of the invention is to provide, for a fluorescent lamp, a ballast for limit-

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ing the current drawn thereby, in the form of a transformer with predetermined leakage inductances.

A still further object of the invention is to provide a high power factor current limiting device for gaseous discharge loads.

A still further object of the invention is to provide a fluorescent lamp ballast in the form of an autotransformer having poor regulation.

A still further object of the invention is to provide a fluorescent lamp ballast in the form of an autotransformer having regulation decreasing with load.

A still further object of the invention is to provide a fluorescent lamp ballast which maintains a uniform lamp current throughout the current cycle thus operating the lamp at its most efficient condition.

A still further object of the invention is to provide a ballast which minimizes changes in lamp current due to variations in line voltage.

A still further object of the invention is to provide a high filament current for preheating gaseous discharge lamp filaments for starting.

A still further object of the invention is to provide more leakage reactance than is practical in an ordinary autotransformer of nearly one to one turns ratio.

These and other objects will be apparent from the following specification when taken with the accompanying drawings, in which

Fig. 1 is a diagram showing one form of unit according to the present invention,

Fig. 2 is a simplified wiring diagram of the unit disclosed in Fig. 1,

Fig. 3 is a form of the invention showing the use of a gap in the iron core,

Fig. 4 is a form of the invention showing the use of magnetic shunts,

Fig. 5 discloses a form of the invention using three coils,

Fig. 6 is a simplified wiring diagram of the unit disclosed in Fig. 5,

Fig. 7 is a wiring diagram for a form of the invention for operating two lamps,

Fig. 8 is a wiring diagram for another form of the invention for operating two lamps,

Fig. 9 is a wiring diagram for another form of the invention for operating two lamps,

Fig. 10 is a wiring diagram for another form of the invention for operating more than one lamp,

Fig. 11 is a wiring diagram for another form of the invention for operating more than one lamp,

Figs. 12 and 13 illustrate another form of the invention, and

Figs. 14, 15 and 16 are simplified circuits for the purpose of mathematically analyzing the invention.

Referring particularly to the drawings, reference character 1 indicates a conventional hot cathode fluorescent lamp lighted from a source 2 of alternating electrical current, ordinarily of about 118 volts. The source 2 is adapted to be connected to terminals 3 and 4 by a switch 5. The lamp 1 is provided at the opposite end thereof with thermionic electrodes or filaments 6 and 7.

A series coil 8 is connected in series between the terminal 3 and one terminal of the filament 6, a conductor 9 providing a connection between the terminal 3 and one end of the series coil 8, and a conductor 10 providing a connection between the other end of the series coil 8 and the filament 6. A shunt coil 11 is shunted across the filaments 6 and 7, a conductor 12 connecting one end of the shunt coil 11 to the conductor 10, and a conductor 13 connecting the other end of the shunt coil 11 to a conductor 14. The conductor 14 connects one terminal of the filament 7 with the terminal 4.

The other terminals of the filaments 6 and 7 are connected by conductors 18 and 19 respectively to a starting switch 20, which may be a conventional glow relay, a delayed action magnetic or thermal switch, or a manually operated switch to preheat the filaments. In any case, however, the starting switch 20 is opened shortly after the main switch 5 is closed to start the lamp 1, as will be well understood by those skilled in the art.

The series coil 8 and the shunt coil 11 are mounted on the leg 21 of a laminated iron core and are arranged in the circuit above described so that their directions of winding are continuous about the leg 21. The core is made up of a stack 22 of E-shaped laminations.

In operation to start the lamp 1, the switch 5 is closed, after first closing the starting switch 20, either automatically or manually. In a typical automatically operating starting switch 20, as soon as the switch 5 is opened to put out the lamp 1, the starting switch 20 closes. Also in an automatic switch 20, after the switch 5 has been closed for starting the lamp, the starting switch 20 opens after a short period of time which is sufficient to permit the filaments 6 and 7 to heat to incandescence. When the starting switch 20 is manually actuated, it is opened manually after a short period of time following the closing of the switch 5. As the starting switch 20 is opened, a voltage surge causes the current from the source 2 to arc across the filaments 6 and 7 providing an ultraviolet radiation which causes the lamp 1 to provide a light due to the fluorescent coating on the inside of the tube thereof.

The characteristics of the lamp 1 are such that without some device to limit the current which it draws from the source 2, the arcing across the filaments 6 and 7 increases the ionization of the gas within the lamp 1, which causes the lamp 1 to draw more current which in turn causes increased ionization within the lamp 1. Accordingly, in order to prevent the lamp from drawing increased amounts of current to a point at which it will burn itself out, some current limiting device must be employed. Some drop in voltage is also necessary since more voltage is required to

start the lamp than to run it. The prior art current limiting device is a choke coil or reactor, associated with a condenser, the condenser being for the purpose of raising the power factor of the load which is lowered by the reactor. According to the present invention, the device above described serves as a current limiting device without providing a low power factor load, notwithstanding the fact that no condenser is required.

The theory of operation of the device according to the present invention is as follows: As will be observed from Fig. 2 constituting a simplified wiring diagram of the circuit in Fig. 1, the series coil 8 and shunt coil 11 constitute a step-down autotransformer with internal leakage reactance to reduce the output voltage with increasing load current. This is necessary in order to provide regulation for the lamp 1 or other discharge device with which the invention is used, to prevent the discharge device from drawing excessive current due to its negative resistance characteristics. The ballast steps down the line voltage as much by its autotransformer action, as will still provide the voltage required to start the lamp, and utilizes only sufficient leakage reactance voltage drop to prevent the lamp from drawing excessive current due to its negative resistance characteristics. By this means the ballast maintains a higher power factor than the ordinary series inductance ballast which must have a large reactance voltage drop to lower the voltage from the line voltage to the lamp operating voltage and hence has a poor power factor.

In the design of the unit according to the present invention it is desirable that the voltage between the filaments 6 and 7 be high enough to start the lamp on open output circuit, that is, after the filaments are preheated and the starting switch 20 opens. For instance, for a typical 20 watt fluorescent lamp the operating voltage is about 60 volts, but an open output circuit voltage of more than 80 volts is necessary for starting.

Any practical ballast will have some resistance. The equivalent resistances will include copper losses and iron losses. These reduce the power efficiency of the ballast and as such are undesirable even though by adding resistance load they do not improve the line power factor. Another effect of the resistance of a ballast is to provide voltage drop under load in addition to any purely inductive voltage drops. For some fluorescent lamps it is even possible to use only a series resistor as a ballast and obtain unity power factor, but with very poor efficiency. On direct current lines this must, however, be done.

In the design of ballast according to this invention, the losses may be kept low by using large wire for the coils and large cores of low loss material operated at low flux density. The actual designs follow transformer practice except that the coils are arranged for predeterminedly large leakage reactance. This leakage reactance may also be increased by use of air gaps or by saturable core parts, of which magnetic shunts are examples. To achieve the desired results according to the present invention, the leakage reactance should be increased to such a value that it will be small enough not to result in a low power factor, but yet large enough and substantially constant to decreasing in value, when taken with the resistance of the ballast, to drop the ballast output voltage more

with increasing load current than the negative resistance characteristic load voltage drops with equal values of increasing current. Thus, if the leakage reactance is too small the power factor will be high, but as the resistance of the negative resistance characteristic load decreases and thus tends to draw more current, the ballast will not limit current and the load will draw increasing amounts of current until it burns out.

Because of the good regulation of the ballast in this invention, it furnishes a high filament current for quick starting.

A suitable design according to the present invention would, for a typical 20 watt lamp, be constructed as follows: On a shell type core having $\frac{1}{4}$ " wide windows extending $\frac{3}{4}$ " along a $\frac{1}{2}$ " wide center leg of a $\frac{3}{4}$ " stack of transformer laminations there would be two coils. The coils would be 1600 turns of #38 A. W. G. and 4400 turns of #34 A. W. G. enameled copper wire. They would be individually wound as pies and spaced as far apart on the core leg as possible. By this means a large leakage reactance is obtained. Such a ballast connected as in Fig. 1 to a 118 volt 60 cycle line and to a 20 watt fluorescent lamp starts with a standard starter. It has an open circuit output voltage of 90 volts. In operation, this ballast supplies 0.242 ampere at 63 volts to the lamp while drawing 0.195 ampere and 22 watts from the line. Its line power factor is watts/volts $\times$ amperes or $22/118 \times 0.195$ or 0.957. A commercial series reactor ballast for this same lamp has a line power factor of less than 0.65 without a condenser.

The operation of the ballast described in this invention has been theoretically described as being similar to an autotransformer having resistance and large leakage reactance. This has been checked by measuring the constants of the 20 watt ballast described and calculating its performance theoretically. The equivalent circuit of any transformer is an ideal (lossless) transformer with series input and output inductances and resistances corresponding to their respective leakage inductances and equivalent resistances (see any standard text). The input may be shunted by an impedance representing the magnetizing current. The magnetizing current as measured at no load on the 20 watt ballast was less than 0.005 ampere and hence negligible. All the series resistance and reactance as referred to the output side of the 20 watt ballast was measured by shorting the line side and measuring the impedance looking into the ballast from the output side. The values were 98.5 ohms resistance and 122 ohms reactance.

The turns ratio of the ideal transformer in the equivalent circuit is given by the ratio of the line volts to the no-load output voltage of the ballast or 118/90 for the 20 watt unit. The equivalent circuit for the 20 watt ballast then is an ideal transformer of 118/90 ratio and with 98.5 ohms resistance and 122 ohms reactance in series with its output. The lamp which ran at 0.242 ampere at 63 volts is equivalent to 63/0.242 or 260 ohms resistance load. The calculated lamp current for this equivalent circuit is 0.245 ampere which compares with the 0.242 actually measured. The power factor of the equivalent circuit is .930 compared to the .957 of the actual circuit. The calculated watts are 20.6 compared to a measured 22 watts and the calculated current of 0.187 ampere compares with the measured value of 0.195 ampere. These

values all are within the errors of the actual readings.

Fig. 3 shows another form of this invention wherein the center leg 26 of the core 27 is made with an air gap 28 tending to decrease the mutual flux in the coils and hence to increase the proportion of leakage flux and thus leakage reactance. The circuit is the same as that of Figs. 1 and 2.

Fig. 4 shows another form of this invention wherein the leakage flux between the coils 8 and 11 is increased by inserting magnetic shunts 30 and 31 between them. For high power factor operation these shunts should be of a design or material, such as a nickel-iron alloy, that saturates at a lower field strength than the balance of the core material which usually will be silicon steel. These shunts will then tend to saturate when load current is drawn from the ballast. Their saturation in the vicinity of full load will limit the leakage flux and reduce the leakage reactance thus improving the power factor and regulation of the ballast. However, the leakage reactance of the design is still greater than it would be without the shunts 30 and 31, to provide the desired operating voltage drop. By this method a higher starting voltage can be obtained without sacrifice of power factor.

Fig. 5 shows the form of the invention for discharge devices which require starting voltages near or above line voltage. This consists of two coils 32 and 33 in series across the lines 35 and a third coil 34 connected to the junction of the first two and which steps the voltage up to, near or above line voltage while providing large leakage reactance.

Fig. 6 shows the circuit of this form of the invention. In order to increase the leakage reactance of the form of the invention disclosed in Figs. 5 and 6 wherein an additional voltage coil 34 is employed, core gaps or saturable core parts as for example magnetic shunts may be employed, but it has been found most satisfactory to merely space any two of the coils. However, any combination of gaps, shunts or coil spacing may be used. As shown, coils 32 and 33 are spaced, but if desired, the spacing could be between the coils 32 and 34. However, it is to be understood that a slight spacing of each of the coils might equally well be used to obtain the amount of leakage reactance desired.

Another form of the invention is disclosed in Fig. 7 wherein two lamps 37 and 38 are operated. The system of Fig. 7 is essentially two of the same circuits as disclosed in Fig. 6 except that one is inverted and superimposed on the other. Employing the terminology applied to the coils of Fig. 1 wherein the coil 11 has been designated as a shunt coil and the coil 8 was designated as a series coil, it will be clear that in Fig. 7 for the lamp 37 the shunt coil equivalent is the combination of the coils 39 and 40 and the series coil equivalent is the coil 41. The equivalent of the voltage coil 34 of Figs. 5 and 6 is the voltage coil 42. For the lamp 38 the shunt coil equivalent is coils 40 and 41, the series coil equivalent is the coil 39 and the voltage coil is the coil 43. It is to be understood as hereinabove explained that voltage coils such as the voltage coils 42 and 43 are employed where an operating voltage at about or above the line voltage is required. Such an instance is for the operation of 100 watt fluorescent lamps from a 120 volt line. In the case of a 20 watt fluorescent lamp, the operating voltage is about 63 volts as hereinabove explained.

A voltage coil is not required when a 120 volt line is used as a source of power for this lamp.

In some applications of the invention it may be desirable to omit the center coil 40 of Fig. 7. Such an arrangement is shown in Fig. 8. The operation of the arrangement of Fig. 8 is identical to that shown in Fig. 7 except that the shunt coil for the lamp 37 consists of only the coil 39 and the shunt coil for the lamp 38 consists of only the coil 41.

The arrangement of Fig. 7, since it employs voltage coils 42 and 43, is, as explained above, for use with lamps operating at a voltage at or above line voltage. If the lamps may be operated at a voltage less than line voltage, the voltage coils 42 and 43 may be omitted as disclosed in Fig. 9.

Still another form of the invention is disclosed in Fig. 10 which is a variation of the circuits disclosed in Figs. 8 and 6 for the operation of two lamps 45 and 46. In this form of the invention each lamp has a common shunt coil 47 and series coil 48, but separate voltage coils, the lamp 45 having a voltage coil 49 and the lamp 46 having a voltage coil 50.

Another form of the invention is disclosed in Fig. 11 which employs the same components as disclosed in Fig. 7, but the connections are such as to provide less output voltage. As explained above, in Fig. 7 the shunt coils for the lamp 37 consist of the two coils 39 and 40 and the series coil consists of the coil 41. In Fig. 11, the shunt coil for the lamp 37 consists of the single coil 39 and the series coil consists of the two coils 40 and 41. Thus the step-down turns ratio of the transformer is more for the Fig. 7 arrangement than it is for the Fig. 11 arrangement. Likewise, the connections for the lamp 38 are such that its shunt coil consists of the single coil 41 and its series coil consists of the two coils 39 and 40.

In each of the forms of the invention, Figs. 5 through 11, the desired leakage reactance as explained for the arrangement of Fig. 5 could be obtained by the use of core gaps, saturable core parts as for examples magnetic shunts, or by spacing of coils, but preferably it is obtained by the appropriate spacing of the respective coils. Thus, for a two lamp arrangement there should be the equivalent of spacing between at least two of the coils for each lamp so as to provide leakage reactance components effective for each lamp.

A specific example of a ballast constructed according to the teachings of Fig. 8 is one having the coils 42, 43, 39 and 41 wound on the center leg of a core as shown in Fig. 5, with the coils 42 and 43 physically located between the coils 39 and 41. The stack comprising the core is 1½ inches high. The center leg is 1" wide and all outside legs are ½" wide. Coils 39 and 41 are each 545 turns of Number 22 enameled copper wire and coils 42 and 43 are each 600 turns of Number 23 enameled copper wire. Such a ballast operates two 100 watt fluorescent lamps at a power factor of .85 with a 45 watt loss. It will be apparent that the step-down autotransformer turns ratio for each lamp is 545+545 or 1090 to 545, and that the step-up turns ratio from line voltage is

$$\frac{600+545}{545+545} \text{ or } \frac{1145}{1090}$$

It will be understood that the voltage under load drops due to the leakage reactance of the transformer.

Another specific example of the invention is as follows and is applied to the form of the invention shown in Fig. 8. The core is the same as is shown in Fig. 5 and consists of a stack of laminations 1" deep with a 1" wide center leg and one-half inch wide other legs and ends. The windows are seven-sixteenths wide. All turns are on the center leg. The coils 42 and 43 are 705 turns of Number 23 copper wire and the coils 39 and 41 are 545 turns of Number 23 copper wire. Each lamp is 100 watts. A power factor of .80 is obtained and an open circuit voltage of 135 volts from a 118 volt line.

Still another form of the invention is illustrated in Figs. 12 and 13. In this form of the invention there is an irregular core structure, the irregularity of which provides a leakage reactance obtained in other forms of the invention by core gaps, magnetic shunts, or by spacing of the coils. The specific embodiment disclosed in Figs. 12 and 13 is designed for use with two 40 watt fluorescent lamps.

The small core portion 50 consists of a stack of laminations ½ inch wide and ½ inch deep which is mortised in the larger core portion 51 which is ⅝" wide at the end, 1 inch wide at the legs and 1¼ inches deep. The coils 52 and 53 consist of 1800 turns each of #26 copper wire, the coils 54 and 55 consist of 350 turns each of #26 copper wire, and the coils 56 and 57 consist of 880 turns each of #28 copper wire.

The operating statistics for the ballast of Figs. 12 and 13 are as follows: Power factor .84, and open circuit voltage 176 when connected to a 118 volt line. The difference between a theoretical turns ratio of turns across the lamp to turns across the line, times the line voltage, which is approximately 188 volts, and the actual open circuit voltage of 176 volts, is accounted for by losses.

The voltage coils in each of the forms of the invention where the operating voltage of the load is about or above line voltage should have a turns ratio with its associated series coil to provide the desired operating voltage. This will be clear when it is observed that the basic circuit in each case, without a voltage coil, is a step-down autotransformer. Thus, with a voltage coil there is provided what might be termed a "step-down-step-up" transformer, wherein the step-up turns ratio is the ratio of turns across the lamp to turns across the line.

Summarizing the invention, it will be clear that in order to obtain a limiting of the current of a negative resistance characteristic load, there is used a step-down autotransformer either with or without a voltage coil for increasing the voltage output. The transformer must have a leakage reactance which is large enough to prevent the load from running away, that is, drawing increasingly large amounts of current until it burns up, and is yet small enough to result in a high power factor. To design a ballast according to the present invention, the turns ratio of the autotransformer should be calculated so as to provide a starting voltage equal to that required by the load. Enough leakage reactance then should be introduced until the output voltage at rated load current drops to the rated load voltage. Due to the copper and iron losses in the ballast, it will have an equivalent resistance. This will cause some drop in output voltage with load. It will be understood that the resistance losses in the ballast are power losses and, therefore, should be minimized.

The introduction of leakage reactance and resistance to the theoretical ballast using the above turns ratio will result in a changed no-load starting voltage. Accordingly, the turns ratio should be revised to compensate for the change due to the added leakage reactance and resistance. The resultant ballast may then have to be revised to add or subtract leakage reactance in order to prevent the load from running away. It may still be necessary after this point has been reached to revise the turns ratio. When as many adjustments have been made as necessary, the resultant ballast will have a no-load voltage equal to that required to start the load and will at the same time have sufficient series impedance to prevent the load from running away, that is, the output voltage at rated load current will equal the rated load voltage. As pointed out hereinabove, the leakage reactance may be obtained by spacing the coils, by introducing saturable core parts as for examples magnetic shunts in the core, or by gapping the core. The resistance may be in any of the coils and is adjusted by varying the size of the wire. Resistance could be introduced in series with the load or line, but as this would introduce a power loss, it should not be used. Leakage reactance provided by saturable magnetic shunts decreases with increasing load currents. Leakage reactance provided by spacing of coils or by gapping the core is substantially constant with increasing load current.

The operation of the invention as far as control of power factor is concerned will be clear from an understanding that a voltage drop provided by a leakage reactance is inductive and thus causes a current in the output of the ballast to lag the voltage. Thus the greater the leakage reactance, the greater the current lag and thus the lower the power factor. For this reason a leakage reactance which increases with load current is not desirable. The operation of the invention as far as current limiting is concerned is that the leakage reactance voltage drop limits the flow of current in the ballast and thus in the load.

An analysis of the present invention with the common series reactor ballast and other circuits will probably make the novelty of the invention clear.

Fig. 14 is admittedly a simplified circuit since the coils are necessarily iron cored and have copper ohmic and eddy current losses and the iron has hysteresis and eddy current losses. These losses, however, can be represented accurately by equivalent series resistances in the coils. The saturation of the iron would also be a factor, but if the inductances of the coils were taken at their rated currents, the major effect of the saturation would be to increase the hysteresis series coil resistance.

The lamp can be considered a pure resistance, but not a linear one. That is, its resistance will change with current. However, if the principle can be demonstrated by analysis for any of several fixed resistances, it should apply to one varying in that range.

Therefore, for the sake of simplicity, this first analysis will be made by assuming ideal (lossless) coils and a fixed (linear) resistance. These re-

sults agree with actual measurements hence justifying this simplification.

The circuit analysis which follows is carried out in general terms thus permitting the later substitution of any particular component values.

The circuit of Fig. 15 represents the simplified circuit to be analysed.

The basic circuit equations for the currents and the mesh voltages are given by Kirchhoff's laws. These are vector equations:

$$(1) \quad I = I'' + I'$$

$$(2) \quad E = Ij\omega L + I''j\omega M + I'j\omega L'' + Ij\omega M$$

$$(3) \quad E = Ij\omega L + I''j\omega M + I'R$$

For any given components L, L', M, R and voltage E alternating at ω , only three variables are present. They are $I, I'',$ and I' . Hence the three variables can be found by solution of these three equations.

A solution for I follows:

Equation (1)

$$I'' = I - I'$$

substituting (1) in (2)

$$E = Ij\omega L + (I - I')j\omega M + (I - I')j\omega L'' + Ij\omega M$$

separating I' terms

$$I'j\omega M + I'j\omega L'' = -E + Ij\omega L + Ij\omega M + Ij\omega L'' + Ij\omega M$$

$$(4) \quad I' = \frac{-E + Ij\omega(L + 2M + L'')}{j\omega(M + L'')}$$

substituting (1) in (3)

$$E = Ij\omega L + (I - I')j\omega M + I'R$$

separating I' terms

$$I'j\omega M - I'R = -E + Ij\omega M + Ij\omega L$$

$$I'(j\omega M - R) = -E + Ij\omega L + Ij\omega M$$

$$(5) \quad I' = \frac{-E + Ij\omega(L + M)}{j\omega M - R}$$

equating (4) and (5)

$$\frac{-E + Ij\omega(L + 2M + L'')}{j\omega M + j\omega L''} = \frac{-E + Ij\omega(L + M)}{j\omega M - R}$$

separating I terms

$$\frac{Ij\omega(L + 2M + L'')}{j\omega M + j\omega L''} - \frac{Ij\omega(L + M)}{j\omega M - R} =$$

$$\frac{E}{j\omega M + j\omega L''} - \frac{E}{j\omega M - R}$$

lump I terms and multiply R terms by $-1/-1$

$$I \left\{ \frac{j\omega(L + 2M + L'')}{j\omega M + j\omega L''} + \frac{j\omega(L + M)}{R - j\omega M} \right\} =$$

$$E \left\{ \frac{1}{j\omega M + j\omega L''} + \frac{1}{R - j\omega M} \right\}$$

$$I = \frac{E \left\{ \frac{1}{j\omega M + j\omega L''} + \frac{1}{R - j\omega M} \right\}}{\frac{j\omega(L + 2M + L'')}{j\omega M + j\omega L''} + \frac{j\omega(L + M)}{R - j\omega M}}$$

multiplying by denominators

$$I = \frac{E(R - j\omega M + j\omega M + j\omega L'')}{(j\omega L + 2j\omega M + j\omega L'')(R - j\omega M) + (j\omega L + j\omega M)(j\omega M + j\omega L'')}$$

$$I = \frac{E(R + j\omega L'')}{Rj\omega(L + 2M + L'') - j^2\omega^2ML - 2j^2\omega^2M^2 - j^2\omega^2L''M + j^2\omega^2LM + j^2\omega^2LL'' + j^2\omega^2M^2 + j^2\omega^2ML''}$$

$$I = \frac{E(R + jwL'')}{Rjw(L + 2M + L) - j^2w^2M^2 + j^2w^2LL''}$$

$$(6) \quad I = \frac{E(R + jwL'')}{Rjw(L + 2M + L'') + w^2(M^2 - LL'')}$$

Equation (6) is therefore the general solution for I in terms of E , R , w , L , L'' , and M .

Since the power factor (the cosine of I 's angle) is not readily apparent from (6) we will insert typical values of E , R , w , L , L'' , and M and calculate it. We will then also calculate the power factor for a simple series inductive ballast to give the same lamp current for the same E , R , and w .

Assume: $E=118$ volts 60 cycle A. C.

Therefore $w=2\pi f=2 \times 3.14 \times 60=377$

Also assume $R=288$ ohms, $L=6$ henries, $L''=35$ henries and $M=13.6$ henries. (Approximate values measured on an actual circuit.)

$$(6) \quad I = \frac{118(288 + j377 \times 35)}{j288 \cdot 377(6 + 2 \times 13.6 + 35) + 377^2(13.6^2 - 6 \times 35)}$$

$$I = \frac{.034000 + j1.556000}{-3.550000 + j7.420000} = \frac{1.56/89.9^\circ}{8.22/90^\circ + 25.6^\circ}$$

$$I = .190/25.7^\circ = .171 - j.082$$

Power factor = cosine $25.7^\circ = .90$

$$(4) \quad I' = \frac{-E + Ijw(L + 2M + L'')}{jw(M + L'')} \\ I' = \frac{-118 + (.171 - j.082)j377(6 + 2 \times 13.6 + 35)}{j377(13.6 + 35)}$$

$$I = \frac{-118 + j4400 + 2110}{j18300} = \frac{1992 + j4400}{+j18300}$$

$$I' = \frac{4820/65.6^\circ}{18300/90^\circ} = .263/24.4^\circ$$

Therefore the equivalent lamp voltage $I'R = .263 \times 288 = 75.7$ volts and the watts

$$= I'^2 R = .263^2 \times 288 = 19.9$$

Therefore these typical components will theoretically run a 19.9 watt resistance load at 75.7 volts while maintaining a .90 power factor.

From (2) we see that if no load (R) is connected then I'' must equal I and the starting voltage for the lamp will be

$$\frac{L'' + M}{L + 2M + L''} \times E$$

or for the above units

$$\frac{35 + 13.6}{6 + 2 \times 13.6 + 35} \times 118 = 84.1 \text{ volts}$$

Hence the voltage drops from 84.1 to 75.7 when the load is applied and therefore this device has some load current regulation which is

necessary to limit the current to a fluorescent lamp.

Obviously, an autotransformer could be used to supply 75.7 volts from the 118 volt line but the autotransformer would not limit the load current nor would it provide a higher starting voltage.

A simple series reactor could be used on the 84.1 volt output line of an autotransformer to reduce the voltage to 75.7 for the lamp, limit the current and provide 84.1 starting volts. The power factor of such a unit would be $75.7/84.1$ or .90. Therefore, this method would be equivalent in action to the circuit analysed but the separate reactor required for it would add cost.

A simple series reactor ballast as commonly used would for the same lamp conditions give a power factor of only $75.7/118$ or .641.

A series resistor could also be used and would give a 1.00 power factor, but with too much power loss to be economical.

Therefore the less regulation necessary to limit the lamp current, the smaller the ratio of load to no-load voltage will be. This ratio is equivalent to the power factor hence with less regulation the power factor will be much improved.

The foregoing analysis was derived for lossless coils to show that the operation of this device did not depend upon capacity or resistance for high power factor operation.

In that analysis it was stated that all coil losses were equivalent to resistances in series with these coils. The following analysis derives the basic equations taking into account the practical case with series coil resistances. By substituting typical resistance values, their effect on power factor is shown. Also included is a definition of limits of useful design.

Fig. 16 shows the equivalent circuit with coil resistance r in series with coil L and coil resistance r'' in series with coil L'' . Other conditions are the same as before.

From Equation (6) above in the original analysis we had:

$$(6) \quad I = \frac{E(R + jwL)}{Rjw(L + 2M + L'') + w^2(M^2 - LL'')}$$

Since we are in effect substituting a coil and resistance for an ideal lossless coil we may write:

$$jwL \text{ (ideal)} = r + jwL \text{ (practical)}$$

dividing by jw ;

$$L \text{ (ideal)} = \frac{r}{jw} + L \text{ (practical)}$$

and likewise,

$$L'' = \frac{r'}{jw} + L''$$

substituting the practical cases for the ideal in equation (6),

$$I = \frac{E \left(R + jw \left(\frac{r'}{jw} + L'' \right) \right)}{Rjw \left(\frac{r}{jw} + L + 2M + \frac{r''}{jw} + L'' \right) + w^2 \left(M^2 - \left(\frac{r}{jw} + L \right) \left(\frac{r''}{jw} + L'' \right) \right)}$$

or

$$I = \frac{E(R + r'' + jwL'')}{R(r + jwL + 2M + r'' + jwL'') + w^2 \left(M^2 - \frac{rr''}{jw^2} - \frac{rL''}{jw} - \frac{Lr''}{jw} - LL'' \right)}$$

Multiplying terms with j denominators by

$$\frac{j}{j}$$

and since $j^2 = -1$

(7)

$$I = \frac{E(R + r'' + jwL'')}{R(r + r'' + jw(L + 2M + L'')) + rr'' + rjwL'' + r''jwL + w^2(M^2 - LL'')}$$

Also substituting the practical case for the ideal in equation (4) above we obtain:

$$(8) \quad I' = \frac{-E + I(r + jwL + 2jwM + r'' + jwL'')}{jwM + r'' + jwL''}$$

Using the same inductances, voltage, frequency and equivalent lamp resistance as in the ideal example; $E=118$ volts 60 cycle A. C. ($w=377$), $R=288$ ohms, $L=6$ henries, $M=13.6$ henries and $L''=35$ henries and using typical values of measured coil resistances of $r=120$ ohms and $r''=855$ ohms.

(7)

$$I = \frac{118(288 + 855 + j377 \times 35)}{228(120 + 855 + j377(6 + 2 \times 13.6 + 35)) + 120 \times 855 + 120j377 \times 35 + 855j377 \times 6 + 377^2(13.6^2 - 6 \times 35)}$$

$$I = \frac{118(1143 + j13400)}{288(975 + j377 \times 68.2) + 10250 + j1580000 + j1930000 + 142000(185 - 210)}$$

$$I = \frac{135000 + j1580000}{281000 + j7400000 + 10250 + j1580000 + j1930000 - 3550000}$$

$$I = \frac{135 + j1580}{-3259 + j10910} = \frac{1581/85.1^\circ}{10400/90^\circ + 16.6^\circ}$$

$$I = .152/21.5^\circ = .141 - j.056 \quad pf. = \cos 21.5^\circ = .93$$

Substituting this for I in equation (8)

$$I' = \frac{-118 + (.141 - j.056)(120 + 855 + j377(6 + 2 \times 13.6 + 35))}{855 + j377(13.6 + 35)}$$

$$I' = \frac{-118 + (.141 - j.056)(975 + j377 \times 68.2)}{855 + j18300}$$

$$I' = \frac{-118 + .141 \times 975 + .141 \times j25700 - j.056 \times 975 - j^2.056 \times 25700}{855 + j18300}$$

$$I' = \frac{-118 + 137 + j3620 - j54.5 + 1440}{855 + j18300}$$

$$I' = \frac{1459 + j3565}{855 + j18300} = \frac{3930/68.2^\circ}{20200/65^\circ} = .191/3.2^\circ$$

The equivalent lamp voltage $= I'R = .191 \times 288 = 55$ volts and the lamp watts $= I'^2 R = .191^2 \times 288 = 10.5$ watts.

Comparison of the results with and without coil resistance shows that the effect of the resistance is:

(1) To improve the power factor from .90 to .93

(2) To reduce the efficiency due to the coil losses

(3) To reduce the current to the equivalent lamp resistance from .263 to .191 ampere and the line current from .190 to .152 ampere while reducing the lamp power from 19.9 to 10.5 watts.

The improvement in power factor is obtained by unduly decreasing the efficiency. Therefore, minimum coil losses are desirable.

While the foregoing invention has been described as applied particularly to fluorescent or

hot cathode lamps operating from 120 volt lines, it is understood that the invention can be used to advantage on other gaseous discharge devices or electrical loads having negative resistance characteristics.

Having thus described my invention, what I desire to secure by Letters Patent and claim is:

1. A ballast for supplying an alternating current voltage to a load having negative resistance characteristics, comprising a plurality of magnetically coupled coils connected in series and constituting a primary winding, a magnetic core on which said coils are arranged so as to have leakage flux between them, the coils being adapted for connection across an alternating current source in such manner that their direction of winding is continuous around the core, and means

adapted to connect said load between one end of said plurality of coils and the junction between two of them, the coil or coils across which the load is connectable constituting a secondary winding, coils of said primary winding having resistance and being spaced to produce leakage re-

actance and resistance of predetermined characteristics in said ballast, said leakage reactance and resistance having a constant to decreasing value with increases in the secondary current, said leakage reactance and resistance also being sufficiently large to decrease the ballast output voltage with increasing secondary current a sufficient amount to limit said secondary current.

2. The combination according to claim 1, further provided with an air gap in said core for producing said predetermined leakage reactance.

3. The combination according to claim 1, there being two magnetically coupled coils constituting said primary winding, said core being further provided with an air gap for producing said predetermined leakage reactance.

4. The combination according to claim 1, further provided with a saturable magnetic shunt for producing said predetermined leakage reactance.

5. The combination according to claim 1, said primary winding constituting two coils, one of said coils constituting said secondary winding.

6. The combination according to claim 1, said primary winding constituting two coils, a third

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coil having one end connected to the junction of said first two coils, said load connecting means serving to connect said load between the other end of said third coil and the side of said primary winding farthest in potential from said other end of the third coil.

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