Oct. 2, 1951 E. J. WADE ET AL 2,569,646

ELECTRICAL SIMULATOR

Filed April 26, 1949 3 Sheets-Sheet 1

FIG. 1.

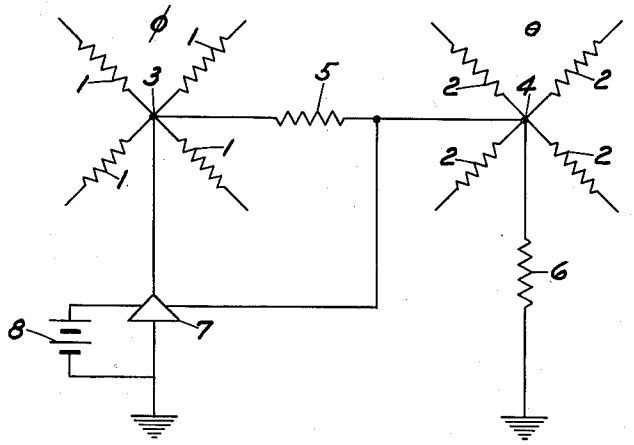
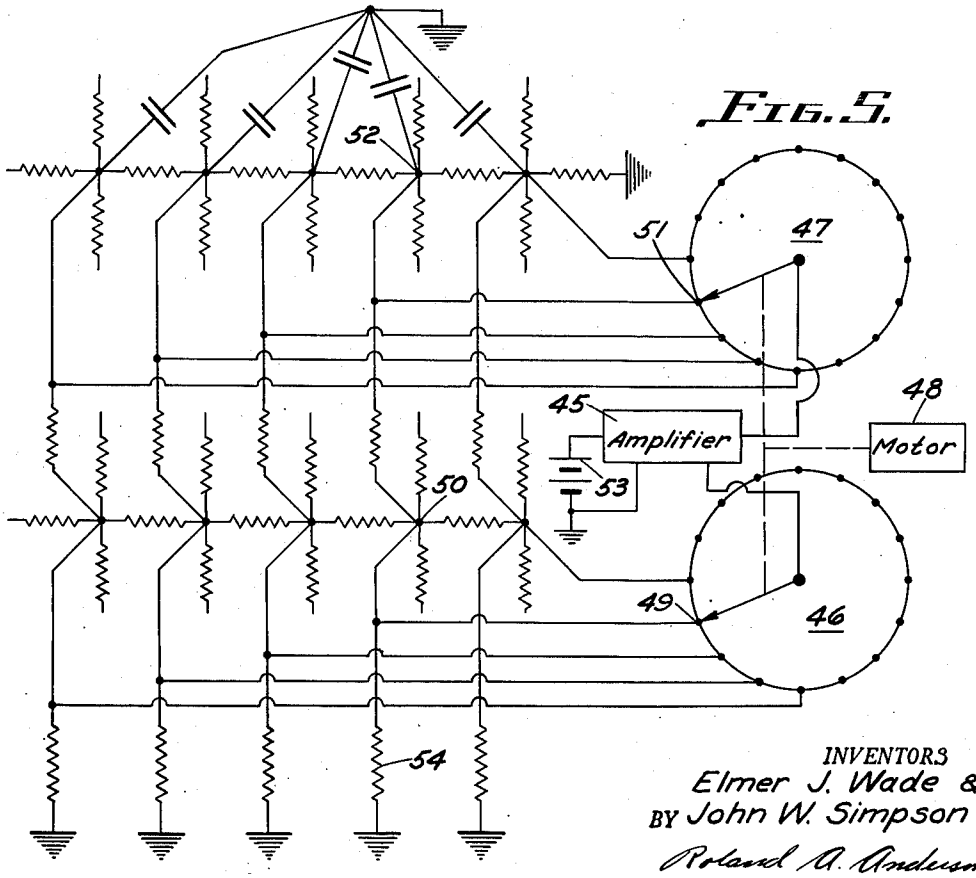


FIG. 5.



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ELECTRICAL SIMULATOR

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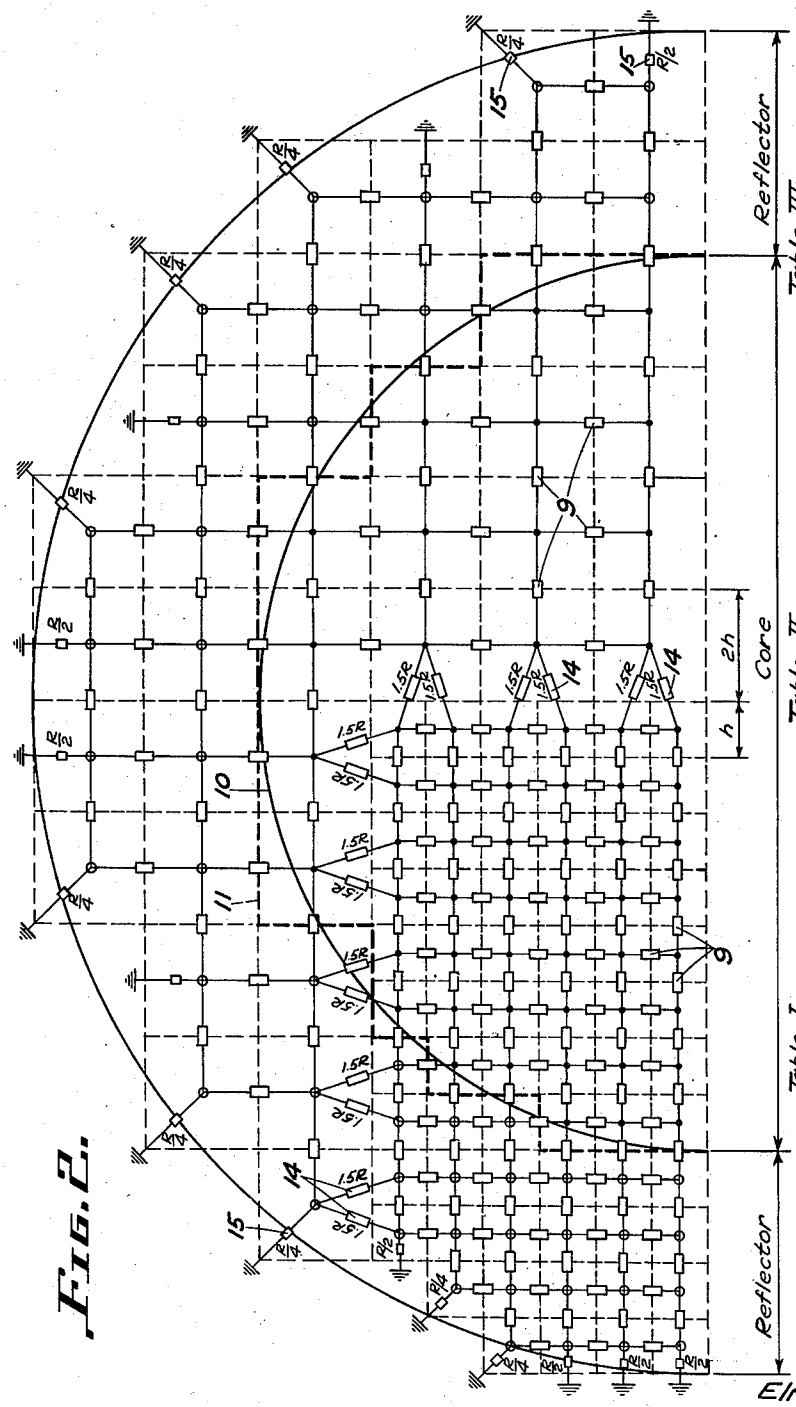


Table III

Resistor Between Node Points of Thermal Network & Ground

	Core	Reflector
Small Lattice	0.1 megohms	1.0 megohms
Large Lattice	24000 ohms	0.24 "

Table II

Resistor Between Node Points of Fast & Thermal Networks

	Core	Reflector
Small Lattice	10 megohms	5.6 megohms
Large Lattice	2.5 "	1.5 "

Table I

	Fast Network-Rf	Thermal Network-Rth
R	2.2 megohms	47000 ohms
R/2	1.2 "	24,000 "
R/4	0.56 "	12,000 "
1.5R	3.3 "	68,000 "

Fig. 2.

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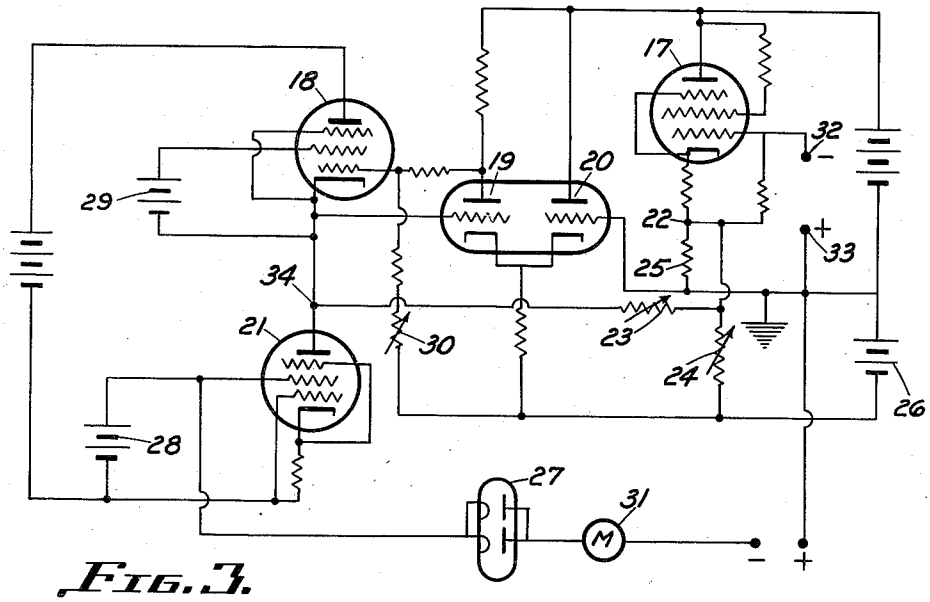


FIG. 3.

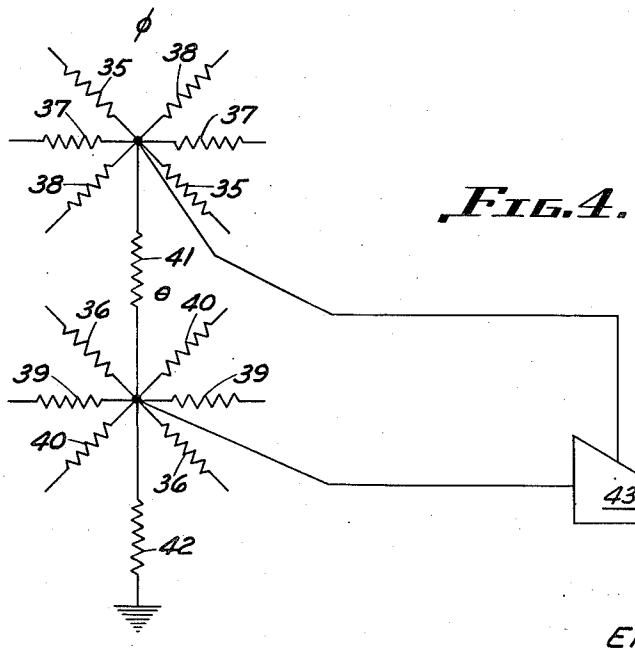


FIG. 4.

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ELECTRICAL SIMULATOR

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Application April 26, 1949, Serial No. 89,628

6 Claims. (Cl. 235—61)

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Our invention relates to a method of and apparatus for solving mathematical equations and, in particular, to a method of and apparatus for solving simultaneous partial differential equations.

The use of finite difference meshes for the solution of various partial differential equations has been described. See, for example, Relaxation Methods in Engineering Science, R. V. Southwell, Oxford Press, 1940, and "The Numerical Solution of Partial Differential Equations," Quarterly of Applied Mathematics, October 1944. An electrical network may be constructed whose characteristic equations defining the network currents and voltages in terms of the network impedances, are the same as the finite difference form of the equations to be solved. Then the parameters of the mathematical expression and the value of any variable in the equation may be found by measuring the appropriate electrical quantity. Many physical phenomena cannot be expressed satisfactorily by one equation, but the relation between two or more variable elements must be stated in at least two simultaneous equations. Since no single network would represent these equations, their solution was formerly done mathematically and required much time and effort, especially in the more complex equations.

It is, therefore, a primary object of our invention to provide electrical networks capable of representing two or more partial differential equations and a method of rapid accurate solution of the equations simulated for any unknown quantity.

An example of a physical phenomenon which can be expressed in two simultaneous equations is the neutron diffusion in a chain reacting nuclear pile. According to best diffusion theory, the neutrons in such reactor or pile may be divided for convenience into two energy groups; fast and thermal. The behavior of the two energy groups is appreciably different with respect to materials which make up the pile. Any mathematical expression for the spatial distribution of the total neutron flux must, for accuracy, be expressed in two equations. Because fast neutrons lose their energy in the pile and become thermal, and because thermal neutrons are absorbed and cause fission of a nucleus with emission of fast neutrons, the fast and thermal neutron fluxes are closely interrelated and cannot be solved for independently, but require that the above equations be solved simultaneously.

Therefore, it is a further object of our invention to provide a method of and apparatus for

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solving the neutron diffusion equations of a nuclear reactor.

It is an additional object of our invention to simulate the distribution of both fast and thermal neutron fluxes in a nuclear reactor so as to provide a visual indication of the exact conditions at any desired point inside the reactor after any change in parameters which it may be desired to introduce.

The novel features of our invention are set forth with particularity in the appended claims. The invention itself, however, will be best understood from the following description, when read in connection with the accompanying drawings, in which

Fig. 1 is a schematic diagram illustrating the principle of operation of our invention.

Fig. 2 is a schematic diagram of the resistor network employed in simulating a particular neutronic reactor.

Fig. 3 is a schematic diagram of one form of electronic amplifier which may be employed in conjunction with the resistor network of Figures 1 and 2.

Fig. 4 is a schematic diagram illustrating a single lattice unit of another form of our invention when three dimensional simulation of the neutron flux in a reactor is desired.

Fig. 5 is a schematic representation of a switching arrangement which may be used with a single amplifier, such as that of figure 3, in place of a separate amplifier for each lattice unit.

Figure 1 illustrates schematically the principle of operation of our invention in the use of finite difference meshes in the solution and simulation of two simultaneous partial differential equations of the form

$$\Delta^2\phi - a\phi + b\theta = 0 \qquad (1)$$

$$\Delta^2\theta - c\theta + d\phi = 0 \qquad (2)$$

where

ϕ, θ are independent variables

a, b, c, d, are constants, and

Δ^2 is the Laplacian operator, having the Cartesian form

$$\Delta^2 = i\frac{\delta^2}{\delta x^2} + i\frac{\delta^2}{\delta y^2} + i\frac{\delta^2}{\delta z^2}$$

Equations 1, 2, may be converted to finite difference form by use of Taylor's expansion, adding, and collecting terms. They then take the form

$$\Sigma_4\phi_1 - (4 + ah^2)\phi_0 + bh^2\theta_0 = 0 \qquad (3)$$

$$\Sigma_4\theta_1 - (4 + ch^2)\theta_0 + dh^2\phi_0 = 0 \qquad (4)$$

where h is the finite mesh interval, ϕ_0, θ_0 are

the values of the variables ϕ, θ, at any given point and ϕ_1, θ_1, are the values of ϕ, θ, at a neighboring point in the mesh.

Analysis of the neutronic reactor in accordance with diffusion theory, considering all neutrons in the pile as either fast or thermal neutrons, results in the so-called two-group theory pile equations

$$\Delta^2\phi-\frac{1}{\tau}\phi+\frac{3K\Sigma ath}{\lambda tf}\theta=0 \qquad (5)$$

$$\Delta^2\theta-\frac{1}{L^2}\theta+\frac{3\Sigma af}{\lambda tth}\phi=0 \qquad (6)$$

where ϕ represents fast neutron flux, which may be considered the product of the fast neutron density at a particular point in the reactor and the average velocity of these neutrons,

θ represents thermal neutron flux, the product of neutron density and average velocity for the thermal neutrons at a point,

τ is the Fermi constant,

K is the multiplication constant, the number of new thermal neutrons produced for each thermal neutron absorbed, assuming a reactor infinite in extent; that is, having no leakage loss,

L is the diffusion length for thermal neutrons, a quantity proportional to the net distance a neutron travels away from its origin before being absorbed,

Σath is the macroscopic absorption cross section for thermal neutrons,

Σaf is the apparent cross section for fast neutrons becoming thermal,

λtf is the transport mean free path for fast neutrons.

λtth is the transport mean free path for thermal neutrons.

Converted to finite difference form, as above, the equations become

$$\Sigma_4\phi_1-\left(4+\frac{h^2}{\tau}\right)\phi_0+\frac{3h^2K\Sigma ath}{\lambda tf}\theta_0=0 \qquad (7)$$

$$\Sigma_4\theta_1-\left(4+\frac{h^2}{L^2}\right)\theta_0+\frac{3h^2\Sigma af}{\lambda tth}\phi_0=0 \qquad (8)$$

It is apparent that the Equations 7, 8, are of the same form as the general Equations 3 and 4. The electrical model of Equations 3 and 4, one lattice of which is shown in Figure 1, consists of several admittances 1, 2, which may be resistors arranged in mesh and connected to node points 3, 4. Resistor 5 connects the node points of two networks together and resistor 6 connects node 4 to ground. The voltages in the network consisting of resistors numbered 1 may be represented by ϕ and the voltages in the network consisting of resistors numbered 2 may be represented by θ. The two networks are connected also by the amplifier 7 whose variable transconductance may be represented as A, and which is adapted to deliver an output current i which satisfies the equation $i=A\theta$. A voltage source 8 energizes the amplifier.

Using Kirchhoff's law, $\Sigma i=0$, the characteristic equations of the single lattice unit illustrated, may be written:

$$\frac{1}{R_1}\Sigma_4\phi_1-\phi_0\left(\frac{1}{R_5}+\frac{4}{R_1}\right)+\theta_0\left(A+\frac{1}{R_5}\right)=0 \qquad (9)$$

$$\frac{1}{R_2}\Sigma_4\theta_1-\theta_0\left(\frac{1}{R_5}+\frac{4}{R_2}+\frac{1}{R_6}\right)+\phi_0R_5=0 \qquad (10)$$

where the subscripts 1, 2, 5, 6, represent the correspondingly numbered resistors of Figure 1.

Comparing Equations 7, 8, with Equations 9, 10, it is apparent that they are of the same general form. Therefore, by judicious choice of the constants in the model, its equations can be made equivalent to those of the reactor. A set of values for the several resistors may be determined from the following relations:

$$\frac{R_1}{R_5}=\frac{h^2}{\tau} \qquad \frac{R_2}{R_6}=\frac{h^2}{L^2}\left(1-\frac{L^2}{\tau}\frac{\lambda tf}{\lambda tth}\right) \qquad \frac{R_1}{R_2}=\frac{\lambda tth}{\lambda tf}$$

$$A=K\frac{R_6+R_5}{R_6R_5}-\frac{1}{R_5}=\frac{K-1}{R_5}+\frac{K}{R_6} \qquad (11)$$

If the θ or thermal network is operated at a much lower voltage than the ϕ network, so that the current flowing from point 3 to point 4 is subtantially independent of the voltage at point 4, then current in resistors 1 represents diffusion of fast neutrons into or out of a lattice unit per second; current in resistors 2 represents diffusion of thermal neutrons in or out of a lattice unit per second; current in resistor 6 represents neutrons absorbed per lattice unit per second; and current in resistor 5 represents fast neutrons that become thermal per lattice unit per second.

If the θ voltages be reduced by a factor of C, where $C>>1$, by proper reduction in the values of resistors 2 and 6, then the relation between pile and simulator constants, to a very good approximation, becomes:

$$\frac{R_1}{R_5}=\frac{h^2}{\tau} \qquad \frac{R_1}{R_2}=C\frac{\lambda tth}{\lambda tf} \qquad \frac{R_2}{R_6}=\frac{h^2}{L} \qquad A=\frac{CK}{R_6} \qquad (12)$$

If these values are given to the simulator components and proper boundary conditions are imposed, the voltage distribution of the simulator will give values of ϕ and θ which are solutions of the differential equations of the nuclear reactor. For the reactor which we wish to simulate the neutron flux at the boundary must be 0, so we have grounded the corresponding point in each network.

In further illustration of the simulator, Figure 2 represents an electrical network constructed to simulate an actual reactor. For clarity, only the ϕ network is shown but the θ network takes exactly the same form, differing only in the values of the resistors. Resistor values for both networks are set forth in Table 1. The corresponding node points in the two networks must of course be connected by resistors corresponding to R_5 of Figure 1, and each node point of a θ network is connected to ground by a separate resistor corresponding to R_6 of Figure 1. The values for these resistors are shown in Tables 2 and 3. An amplifier must be connected between each pair of corresponding node points in the networks, as described in connection with Figure 1, to feed a current proportional to the θ voltage into the ϕ network, but for economy we may employ a single amplifier with a switching arrangement which is described hereinafter in connection with Figure 5.

The resistors 9 are connected in a mesh having finite intervals $h=7.62$ centimeters, in parts of the network corresponding to points in the nuclear reactor where a more detailed investigation of the flux is required. Where detailed knowledge of the distribution is not required, the interval may be $2h$. Different values of an inter-connecting and grounding resistor are made necessary by the different mesh intervals as is apparent from Equations 11, 12, and are so shown in Tables 2 and 3. Line 10 represents the actual dividing line between core 12 and reflector 13 in the pile, while line 11 represents the boundary as repre-

sented in the network. Because the slowing down of fast neutrons occurs at a different rate in core and reflector, the values of the resistors corresponding to R_5 and R_6 in Figure 1 are different for the core and reflector section of the simulator, and the different values are shown in the tables. To arrive at the exact values given, the cross-sections and densities of the core and the reflector material must be compared, in accordance with Equations 12.

Resistors 14 connect the points in the large lattice or mesh to points in the smaller mesh. Since there are twice as many node points in the smaller mesh as in the larger mesh, we must provide two resistors from each point in the larger mesh to connect it with the two points adjacent it in the smaller mesh. The resistance is proportional to the distance in the direction of current flow and inversely proportional to the current flow for a given lattice unit. As is illustrated, resistors 14 cover only ¾ of the mesh interval and two resistors join at each large mesh node point. To make the resistance of that parallel connected network equal ¾R, each resistor must be 1.5 times as large as the other lattice resistances. The resistors 15 along the periphery of the simulator may be ½ or ¼ of the value of the other lattice resistance because they represent a distance of only h or $\frac{1}{2}h$ instead of $2h$ in the reactor considered.

The amplifier represented schematically as 7 in Figure 1 must deliver a current output proportional to the voltage at its input, the current representing new neutrons from fission, and the voltage of the thermal neutrons absorbed. The relation between current and voltage must be linear and opposite in sense—a negative transconductance. The operating point of the amplifier must not drift, and the output of the amplifier must not draw current from the network. An amplifier which meets these requirements is illustrated in Figure 3.

A cathode follower is employed for the input stage, the triode-connected tube 17. The output stage may be a "bootstrap" type cathode follower with a preceding amplifier stage. A pentode 21 is utilized as the cathode resistor of the bootstrap, to allow operation of the cathode follower 18 with a reasonable static plate current, thus operating on a more linear portion of the tube characteristics, while retaining the high dynamic impedance in the cathode circuit. The amplifier stage feeding the bootstrap is duo-triode 19, 20 connected as a difference amplifier, the control grid of triode 20 being connected to ground, while the control grid of triode 19 is connected to the cathode of the tube 18 and the anode of triode 19 feeds the control grid of tube 18. The input stage is connected at junction 22 to the transconductance control 23, which may be a Type A Heliopot potentiometer, connected in turn to the cathode of tube 18. A variable resistor 24 is provided to carry the cathode current of the input stage, and is initially adjusted to carry all of the current, such that no voltage appears across resistor 25. Voltage source 26 furnishes 210 volts, and may be a conventional electronically regulated and filtered power supply, but for simplicity is shown as a battery, as are all other sources of potential.

To secure a very high output impedance for the amplifier, output current is taken from the screen grid of tube 21, and fed through a rectifier 27 and a meter 31. The screen supply is represented as battery 28, but may be a rectified and filtered regulated power supply, capable of furnishing up to 10 milliamperes to the load at the output terminals. For greater stability, the screen supply of tube 18 must also be regulated, and a rectified, filtered, voltage-regulator tube stabilized supply may be used in place of battery 29.

Variable resistor 30 controls the bias on tube 18, and should be so adjusted that no output current flows when the input terminals are shorted and the output terminals are shorted simultaneously, so that no current is delivered to the output when no signal is impressed in the input of the amplifier.

In operation, the negative input terminal 32 is connected to a node point in the θ network, a common ground connection being maintained for the + input terminal 33 and the resistor networks. The terminal 32 may go more negative as the simulator network node is connected thereto, and the potential of junction 22 will rise. Of this potential change, a portion determined by the setting of potentiometer 23 will appear as a rise in potential of junction 34. The control grid of triode 19 will rise in potential, and the anode potential will fall, thus causing the control grid of tube 18 to become more negative, decreasing its cathode current and causing the cathode potential to fall. A constant potential difference is maintained between the anode of tube 18 and the grid of tube 21, a limiting resistor being connected in the grid-cathode circuit of tube 21.

Any resultant change in potential of the junction 34 causes a corresponding change in the current through tube 21 and a change in the resultant screen current. With bias control 24 and zero control 30 adjusted as described hereinbefore, the change in screen current is proportional to the change in potential across the input terminals 32, 33, over a wide range. For the present use voltages up to 3 volts at the input produce currents up to 9 milliamperes at the output in substantially linear relation.

The network described in Figure 2 shows only a two dimensional simulator of the pile. But our invention may be constructed in three dimensional mesh form if desired. A single lattice unit of a three dimensional form is shown in Figure 4. It differs from the lattice of Figure 1 only by the addition of resistors 35, 36. The currents flowing in the added resistors represent the diffusion of neutrons along an imaginary axis perpendicular to the plane of the slice of the pile which the networks including resistors 37, 38, 39, 40, simulate. Resistor 41 inter-connects the two networks and resistor 42 grounds the lower or θ network. An amplifier 43 and voltage source 44 are required, and perform the same function as those described in connection with Figure 1.

Because of the large number of lattice units necessary when a sufficiently small mesh interval is chosen for accurate results, a great many amplifiers are required. To obviate the necessity for building so many amplifiers, we have employed only a single amplifier and by a scanning switch arrangement connect its input and output terminals successively to each pair of corresponding node points in our network. Figure 5 illustrates schematically an arrangement utilizing a single amplifier 45, energized from voltage source 53, and multiple contact switches 46, 47 having their contact arms synchronously rotated by motor 48. Each successive contact on switch 47 is connected to a point on a ϕ network of a simulator while the contacts of switch 46 are successively connected to a node point of a θ network. The switching arm positions are so synchronized that corresponding node points on the two networks are

always energized at the same time. For example, as illustrated, the arm of switch 46 makes contact with terminal 49 which is directly connected to point 50. The voltage at 50 thus appears on the arm of switch 46 and is fed into amplifier 45, causing a current proportional to that voltage to be fed through the arm of switch 47 and contact 51 to the node 52. Large condensers should be connected between each node in the ϕ network and ground to maintain the potential of each point substantially constant while the switch is scanning over the remainder of the points.

It may be desirable to solve the Equations 5, 6 for the value of K required to make a certain pile critical. Utilizing the approximations of Figure 5 with the selector switches set to contact a pair of corresponding points in the θ and ϕ networks such as 50, 52, the motor 48 and the amplifier 45 are energized. The switch arms will rotate synchronously. Then a potential is impressed momentarily upon any point, other than ground, of the simulator network, by means such as a lead from voltage source 53. Current begins to flow in the network because of the applied potential, and develops a voltage drop across resistance 54. The amplifier 45 will magnify this voltage drop, and will produce at its output a current proportional thereto.

The transconductance of the amplifier must be adjusted by the operator until the currents in the network reach a steady, or equilibrium, value. To do so, the variable resistance 23 in the amplifier is varied from one extreme position and the microammeter 31 is watched until no change is apparent. At that point, equilibrium is reached. Resistance 23 may be calibrated, and furnished with a dial and pointer, so that its setting may be read directly in terms of the transconductance A.

From Equation 12, it is apparent that

$$K=\frac{AR_6}{c}$$

Since R_6 is a constant for each particular application, it is apparent that resistance 23 may be calibrated directly in values of K. Or, knowing R_6, the value of K is easily obtained by multiplying R_6 by the value of

$$\frac{A}{C}$$

Thus, we have provided a rapid, accurate method of determining the value of the variable K, which represents the critical multiplication factor, very useful in control of a particular reactor, given the size of the reactor, its composition, and the nuclear properties of the materials.

We have also provided a method and apparatus facilitating the determination of the value of the neutron flux at any point in a particular nuclear reactor.

We have, in general, provided a method of and apparatus for solving simultaneous partial differential equations on an electrical model, without resorting to highly complex mathematical treatment.

We claim:

1. A simultaneous differential equation solver including a first network consisting of a plurality of impedance elements connected to a series of junction points, a second network consisting of a plurality of impedance elements connected to a corresponding series of junction points, current source means adapted to feed into each junction point in said first network a current proportional to that potential appearing at the corresponding junction point in said second network, and calibrated means for varying the amplification of said means.

2. A simultaneous equation solver including a first and a second inter-connected electrical networks, each network comprising a plurality of impedance elements connected to a series of junction points, electronic amplifying means having input and output circuits and adapted to furnish at its output a current proportional by a variable factor to the voltage impressed upon said input circuit, switching means adapted to connect said input circuit to each junction point in said second network consecutively and to simultaneously connect said output circuit to each corresponding junction point in said first network consecutively, and capacitance means connected between each junction point and a point of constant potential for maintaining the potential impressed upon said junction point substantially constant during the interval when said amplifying means is disconnected therefrom.

3. A simultaneous equation solver including first and second inter-connected electrical networks, each network comprising a plurality of impedance elements connected to a series of junction points, said impedance elements having values determined by the constants of the two simultaneous equations to be solved, and a plurality of electronic amplifying means each adapted to provide a current at its output proportional by a variable factor to the magnitude of the voltage impressed upon its input, said inputs being connected each to one of said junction points in said second network and said outputs being connected each to the corresponding junction point in said first network.

4. A device for simulating the spatial distribution of neutron flux in a nuclear reactor on an electrical matrix comprising first and second inter-connected electrical networks, the voltages in said first network representing the fast neutron flux in said reactor, and the voltages in said second network representing the thermal neutron flux in said reactor, each network comprising a plurality of impedance elements connected to a series of junction points, each pair of corresponding junction points in said networks representing a discrete point in said reactor, and the voltage at each point representing the magnitude of the appropriate neutron flux, amplifying means, connected to a source of electrical power, switch means adapted to connect consecutively each junction point of said second network to the input of said amplifying means and to simultaneously connect the corresponding junction point in said first network to the output of said amplifying means, and capacitance means connected between each junction point and a point of constant potential in said second network for maintaining the potential periodically impressed thereon by said amplifying means.

5. Apparatus for solving two simultaneous linear differential equations relating to a neutronic reactor comprising first and second interconnected electrical networks, the voltages in said first network representing the fast neutron flux in said reactor, and the voltages in said second network representing the thermal neutron flux in said reactor, each network comprising a plurality of impedance elements connected to a series of junction points, each pair of corresponding junction points in said networks representing a discrete point in said reactor, and the voltage at each point representing the magnitude of

the appropriate neutron flux, amplifying means, connected to a source of electrical power, switch means adapted to connect consecutively each junction point of said second network to the input of said amplifying means and to simultaneously connect the corresponding junction point in said first network to the output of said amplifying means, and capacitance means connected to each junction point in said second network for maintaining the potential periodically impressed thereon by said amplifying means.

6. A device for simulating the spatial distribution of the neutron flux in a nuclear reactor on an electrical matrix comprising first and second inter-connected electrical networks, the voltages in said first network representing the fast neutron flux in said reactor, and the voltages in said second network representing the thermal neutron flux in said reactor, each network comprising a plurality of impedance elements connected to a series of junction points, each pair of corresponding junction joints in said networks representing a discrete point in said reactor, and the voltage at each point representing the magnitude of the appropriate neutron flux, variable transconductance amplifying means connected to a source of electrical power and adapted to furnish at its output a current proportional to the magnitude of the voltage impressed upon its input, switch means adapted to connect consecutively each junction point of said second network to the input of said amplifying means and to simultaneously connect the corresponding junction point in said first network to the output of said amplifying means, and capacitance means connected to each junction point in said second network for maintaining the potential periodically impressed thereon by said amplifying means.

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No references cited.