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(54) Title: MAGNETIC ORBITAL ANGULAR MOMENTUM BEAM ACCELERATION

(57) Abstract: A magnetic orbital angular momentum beam accelerator will accelerate charged particles, electrons or ions, from rest in zero or low magnetic field into a high magnetic field regions with high kinetic energies in the form of magnetic orbital angular momentum. For example, a beam injector that accelerates electrons or ions into IT magnetic fields with tens of keV kinetic energies transverse to the magnetic fields can be used to heat magnetically confined plasmas, to inject an initial energetic plasma component with high magnetic orbital angular momentum and to produce highly transverse particle momenta to the magnetic field for electron or ion beam lithography.



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MAGNETIC ORBITAL ANGULAR MOMENTUM BEAM ACCELERATION

CROSS-REFERENCE TO RELATED APPLICATIONS

The present application claims priority to U.S. Provisional Pat. App. No. 63/228,463,
5 filed August 2, 2021, and is incorporated by reference herein in its entirety.

TECHNICAL FIELD

The present disclosure is drawn to devices, systems, and methods for creating and
using particle beams.

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BACKGROUND

Current particle beam heating techniques for magnetically confined plasmas rely on
neutral beam injectors to allow external, neutral particles to enter the magnetically confined
region. Neutral particles have zero magnetic orbital angular momentum and when they
15 interact in the plasma, a large fraction of these particles after ionization have low magnetic
orbital angular momenta and are not strongly confined.

Additionally, current particle beam lithography techniques use particles with large
linear momentum to cut customized shapes on surfaces, such as nanostructured surfaces, but
such techniques require a large linear momentum normal to the surface, which can negatively
20 impact the structure of several layers below the surface layer.

BRIEF SUMMARY

To avoid these issues, a method and system for particle acceleration may be provided.

In some embodiments, a method for particle acceleration may be provided. The
25 method may include providing particles in a zero or low magnetic field (i.e., a magnetic field
whose strength is sufficiently low to allow for ballistic motion out of the source). The
method may include causing the particles to be in cyclotron motion in a magnetic field that is
strong compared to a momentum of the particles. The particles may have a gyroradius that is
small compared to a transverse dimension of an injection aperture through which the particles
30 will travel, where the magnetic field has a transverse gradient along an average path of the
particles. The method may include utilizing a complementary electric field to balance a
gradient- B drift transverse to the average path of the particles and accelerate the particles
under work of the transverse gradient.

In some embodiments, the particles may include electrons, ions, or a combination thereof. In some embodiments, the method may include directing the particles towards a confined plasma. In some embodiments, the method may include directing the particles towards a substrate. In some embodiments, the substrate may be a semiconductor.

5 In some embodiments, a magnetic orbital angular momentum beam accelerator may be provided, e.g., from a source (such as an electron gun). The accelerator may include a tapered dipole magnet winding configured to have a magnetic field positioned to allow particles to enter the tapered dipole magnet winding, the magnetic field being a low magnetic field configured to cause the particles to begin cyclotron motion. The tapered dipole magnet
10 winding may have a magnetic field gradient that is a transverse gradient along an average path expected of the particles. The accelerator may include a field cage. The field cage may include a plurality of electrodes, configured to form a complementary electric field to balance a gradient-*B* drift transverse to the average path of a beam of the particles and accelerate the particles under work of the magnetic field gradient.

15 In some embodiments, the field cage may be placed within a counter-dipole coil in an upper diagnostic port of a tokamak reactor. In some embodiments, the field cage may include, or be placed within, coils of a solenoid, custom superconducting dipole coils, iron pole-face magnets with shaped pole-faces, configurations of permanent magnets, or a combination thereof. In some embodiments, the accelerator may include an einzel lens
20 configured to accelerate the particles from an initial magnetic field towards the tapered dipole magnet winding, the initial magnetic field being a zero or low magnetic field, the particles initially being low energy charged particles. In some embodiment, the particles may be reflected off a repelling electrode of the einzel lens into the tapered dipole magnet winding. In some embodiments, the particles include electrons, ions, or a combination thereof. In
25 some embodiments, the particles may be accelerated in a low vacuum (i.e., a vacuum sufficiently low to allow unimpeded cyclotron motion). In some embodiments, the tapered dipole magnet winding may include superconducting magnets. In some embodiments, the tapered dipole magnet winding may be symmetrical around a plane extending through a central axis, each half of including a plurality of loops, each loop in the plurality of loops
30 having a contoured rounded rectangular shape, each loop having one side that is substantially located at a first end, and where each loop has a different length.

BRIEF DESCRIPTION OF DRAWINGS

Figure 1 is a flowchart of an embodiment of a method.

Figure 2A is a graph showing a trajectory **200** of a deuterium ion in a balanced drift with an initial kinetic energy of 20 keV at 0.2 T going to 4.7 T region with 1 MeV final
 5 kinetic energy. Dashed lines indicate the gradient of energies, from about 0.2 MeV (210), about 0.4 MeV (220), about 0.6 MeV (230), about 0.8 MeV (240), to about 1.0 MeV (250).

Figure 2B is a graph showing the magnetic field used to generate the trajectory in Figure 2, where the maximum of the B_x field was set to 4.7 T at $Z = 1.6$ m to match a toroidal field near an interface of an upper port of ITER.

10 Figure 2C is a graph showing the electric field generated by a transverse drift filter, where the dimensions are scaled such that the distance between electrodes is 0.3 m.

Figure 3 is an illustration of a tapered dipole magnet winding.

Figure 4 is an illustration of a tapered dipole magnet winding positioned between two adjacent toroidal field coils.

15 Figure 5 is a simplified block diagram of a system.

Figures 6A and 6B are illustrations of magnets.

Figures 7A is an illustration of a three-channel transverse drift filter.

Figure 7B is an illustration of a transverse drift filter in place in a magnet of Figure 6B.

20 Figure 8 is a graph showing net z-velocity along a centerline of the transverse drift filter.

Figure 9 is a graph showing filter electrode voltages and idealized potentials along the center line of the respective wells. Voltages are set for a pitch 60° electron with total kinetic energy 18.6 keV, with $\lambda_{\text{side}} = \lambda_{\text{cent}}$.

25 Figure 10 is a cross-sectional view of a simplified system using a magnet and transverse drift filter.

Figures 11A and 11B are graphs showing the y-position (11a) and transverse kinetic energy (11B) over the length of the filter can be seen for a pitch 90° electron in the “reverse” filter setup that is symmetric about z.

30 Figure 12 is an illustration of a test system using magnets and a transverse drift filter.

DETAILED DESCRIPTION

The present disclosure provides an improvement over existing particle beams, by delivering high energy particle heating through the kinetic energy of magnetically confined particles, or techniques for lithography that avoid the requirement for large linear momentum normal to the surface.

The dream of harnessing energy from controlled nuclear fusion has been proposed for several decades. Intensifying climate change issues increase the desire for a clean and safe energy source. A fusion reactor based on magnetic confinement provides a promising configuration for controlled thermonuclear fusion. To fuse nuclei with large densities for an extended period, it is necessary to heat the plasma to overcome the Coulomb repulsion. The power ratio, Q , of the fusion output power to the input power is proportional to the fusion product $nT\tau_E$, where n and T are the central ion density and temperature. The parameter τ_E is the energy confinement time. In December 2021, the Joint European Torus (JET) achieved a new record and produced 59 MJ of energy with a Q of 0.33 over a τ_E of 5 s. Although remarkable progress has been made to achieve the required n , T , and τ_E , they have not been achieved in the same reactor configuration simultaneously.

To achieve an ignition condition where self-sustaining fusion is possible, additional energy-efficient heating is required. Ohmic heating from the toroidal current wanes at high temperatures. Two external sources are typically used to provide heating power, the resonant absorption of radio frequency electromagnetic waves and the injection of energetic neutral particle beams. The injected beams are neutralized to prevent reflection due to the magnetic field. The neutralization process introduces inefficiency and complicates the instrumentation.

Alternatives to neutral particle beam injection, typically for non-equilibrium fusion reactors, have been explored using different acceleration technologies. The challenges of energy efficiency in particle acceleration are formidable given the high fraction of input power needed to operate relatively low- Q fusion reactors. Radio-frequency acceleration cavities and time-varying electromagnetic fields are, in general, prone to internal ohmic losses and self-heating. Static accelerating fields avoid the bulk of these losses, but are suited primarily for charged particle beams. By construction, the insertion and extraction of charged particles from magnetic confinement systems is thwarted except when necessary, as in the case of divertors. However, non-confining trajectories can be constructed under special conditions through the same processes of cyclotron orbit drift that plague steady-state operation.

In the transverse drift electromagnetic filter developed for the Princeton Tritium Observatory for Light, Early-Universe, Massive- Neutrino Yield (PTOLEMY) experiment, a compact configuration of electromagnetic fields simultaneously transports and decelerates energetic electrons from the tritium β -decay endpoint starting in high magnetic fields of several Tesla to regions where both the kinetic energy and magnetic fields are reduced by several orders of magnitude.

The disclosed approach accelerates low-energy charged particles into a high magnetic field region by, conceptually, operating the PTOLEMY filter in “reverse.”

In some embodiments, a method for particle acceleration may be provided. Referring to FIG. 1, in some embodiments, the method **100** may include providing **110** particles in a zero or low magnetic field. As used herein, the term “low magnetic field” refers to a magnetic field whose strength is sufficiently low to allow for ballistic motion out of the source of the particles. In some embodiments, this may include magnetic fields having a magnetic field strength of 1T or less. In some embodiments, the magnetic field may have a strength of 0.75T or less. In some embodiments, the magnetic field may have a strength of 0.5T or less. In some embodiments, the magnetic field may have a strength of 0.25T or less.

The low magnetic field transition into the accelerator should be a non-adiabatic transition that changes the bending radius of the particle trajectory within a single cyclotron orbit. This allows one to set the initial value of the magnetic moment for the injected particles.

In some embodiments, the particles may include electrons, ions, or a combination thereof. In some embodiments, particles are electrons provided from an electron gun. In some embodiments, the particles are deuterium. In some embodiments, the deuterium is a deuteron beam extracted from a cyclotron.

The method may include causing **120** the particles to be in cyclotron motion in a magnetic field that is strong compared to a momentum of the particles. The particles may have a gyroradius that is small compared to a transverse dimension of an injection aperture through which the particles will travel, where the magnetic field has a transverse gradient along an average path of the particles.

The method may include utilizing **130** a complementary electric field to balance a gradient- B drift transverse to the average path of the particles and accelerate the particles under work of the transverse gradient.

Referring to FIG. 1, In some embodiments, the method may include directing **140** the particles towards a target. In some embodiments, the target is a confined plasma. In some embodiments, the target is a substrate. In some embodiments, the substrate may be a semiconductor.

5

Example

Basics of charged particle beam injection

As described herein, the convention used is that non-bolded symbols of vector quantities refer to the total magnitude unless a component is specified. The equation of motion of a charged particle of mass m and charge q in a magnetic field $\mathbf{B}$ is given by

10

$$\frac{d}{dt} \left(m \frac{d\mathbf{r}}{dt} \right) = q \frac{d\mathbf{r}}{dt} \times \mathbf{B}. \quad (1)$$

The Lorentz force on the right-hand side is perpendicular to the particle's velocity. In a uniform magnetic field, the particle's motion projected on a plane perpendicular to the magnetic field is circular, with a gyroradius given by

15

$$\rho = \frac{mv_{\perp}}{qB} \approx \frac{\sqrt{2mT_{\perp}}}{qB}, \quad (2)$$

For a 1 MeV deuterium ion in a 5 T magnetic field, the gyroradius is about 0.04 m, a small fraction of a typical reactor radius. The ion beam injection energies must be relativistic to be commensurate with the reactor radius.

Relativistic ion beam injection introduces a number of inefficiencies. The plasma does not have the density required to stop energetic ions in a single transit, delivering limited power to the plasma and creating destructive irradiation of the reactor walls. The acceleration methods for relativistic beams involve time-varying fields that have several sources of intrinsic power loss.

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Magnetic orbital angular momentum beam acceleration

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Here, charged particle injection of non-relativistic ions is re-examined as a transport mechanism that drifts charged ions from outside of the reactor volume to the surface of a target (such as the plasma in tokamak reactors, etc.)

30

An alternative method to inject a charged particle beam is to create a beam of particles whose gyroradius is small compared to the transverse dimensions of the injection aperture. The particles are in cyclotron motion in a magnetic field that is relatively strong compared to their momentum. The acceleration mechanism stems from the ability of particles

traveling in cyclotron motion in magnetic field gradients to do work. One, therefore, configures a magnetic geometry such that there is a transverse gradient along the average path of the beam.

A complementary electric field is used to balance the gradient- B drift transverse to the average path of the beam and to accelerate the particles under the work of the magnetic field gradient. The acceleration process will be shown to be adiabatic for relevant injection energies and to maintain the magnetic moment invariance to a good accuracy after an initial stage of zero field ion source injection. The acceleration process does not affect the average linear momentum component of the beam. The increase in the charged particle kinetic energy follows from an increase in the magnetic orbital angular momentum.

Guiding-Center Drifts in adiabatic field conditions

When a charged particle gyrates in a magnetic field with a transverse gradient, the cyclotron-orbit averaged Guiding Center System (GCS) motion can be described in terms of the drift terms of the virtual guiding-center particle if the spatial and temporal field variations within a single cyclotron orbit are taken to be adiabatic, i.e.,

$$\rho_c \ll \left| \frac{B}{\nabla B} \right|, \left| \frac{E}{\nabla E} \right|; \text{ and} \quad (3)$$

$$\tau_c \ll \left| \frac{B}{dB/dt} \right|, \left| \frac{E}{dE/dt} \right|; \quad (4)$$

where ρ_c is the Larmor radius and τ_c the cyclotron period. Under the conditions specified by eqs. (3) and (4), the first adiabatic invariant μ ,

$$\mu = \frac{p_{\perp}^2}{2mB} = \frac{T_{\perp}}{B}, \quad (5)$$

accurately describes an invariant quantity preserved in the motion of the particle and shows that an increase in the magnetic field magnitude is accompanied by a proportional increase in the transverse kinetic energy. Additionally, the deviation of the GCS trajectory from the direction of the magnetic field lines can be described in terms of four fundamental drift terms,

$$\mathbf{V}_D = \mathbf{V}_{\perp} = \left(q\mathbf{E} + \mathbf{F} - \mu \nabla B - m \frac{d\mathbf{V}}{dt} \right) \times \frac{\mathbf{B}}{qB^2}, \quad (6)$$

where $\mathbf{V}_{\perp}$ is the perpendicular component of the GCS velocity with respect to the magnetic field line. The transverse drift velocity, $\mathbf{V}_D$, is composed of individual terms, as appear in equation (6) from left to right, known as (1) the $\mathbf{E} \times \mathbf{B}$ drift; (2) the external force drift; (3) the gradient- B drift; and (4) the inertial drift.

It is possible to configure the electric and magnetic field parameters to manipulate certain drift terms to produce a net linear trajectory in the transverse direction.

Drifts and work

The gradient- B drift is able to drive a charged particle up or down an electrostatic potential. This ability to do work, at first, seems contrary to the notion that magnetic fields do not do work on charged particles, as seen in equation 1, from the cross-product. Similarly, under the motion of $\mathbf{E} \times \mathbf{B}$ drift alone, the cross-product bars work as the electrons will drift on surfaces of constant voltage. This can also be understood by considering that it is always possible to boost into a frame in which the $\mathbf{E} \times \mathbf{B}$ drift is zero.

In contrast, a gradient- B drift due to a spatially varying magnetic field implies a time-varying electric field that cannot be boosted to zero. By itself, i.e., with a magnetic field and no electric field, a gradient- B does no work because there is nothing to do work against. However, when accompanied by an external $\mathbf{E} \times \mathbf{B}$ drift, the external electric potential provides a surface against which the gradient- B drift can do work on. The internal rotational kinetic energy of gyromotion of the virtual guiding-center particle is reduced for a corresponding increase in voltage potential. This is described by inserting terms from equation (6),

$$\frac{dT_{\perp}}{dt} = -q\mathbf{E} \cdot \mathbf{V}_D = -q\mathbf{E} \cdot (q\mathbf{E} - \mu\nabla B) \times \frac{\mathbf{B}}{qB^2} = \frac{\mu}{B^2} \mathbf{E} \cdot (\nabla B \times \mathbf{B}) \quad (7)$$

where $T_{\perp}$ is the internal kinetic energy of gyromotion in the GCS frame.

Balanced drift

To produce a filter or accelerator based on the drift terms in equation (6), the external force and inertial drift terms are first taken to be zero, leaving only the electric and gradient- B drifts to be configured such that the total net drift is along a straight line parallel to the direction of the magnetic field gradient. The gradient- B drift alone is orthogonal to the direction of the magnetic field gradient, so the first step is to create a component of the $\mathbf{E} \times \mathbf{B}$ drift that exactly counters the gradient- B drift. From equation (6) this specifies the requirement,

$$q\mathbf{E}_{\parallel} \times \mathbf{B} = \mu\nabla B \times \mathbf{B}, \quad (8)$$

where $\mathbf{E}_{\parallel}$ is the component of electric field parallel to the magnetic field gradient. In general, the ratio of the parallel electric field to the magnitude of the magnetic field to meet this condition depends on the ratio μ/q times the fractional rate of change of the transverse

component of the magnetic field along the direction of the magnetic field gradient. For an exponentially falling transverse field, the fractional rate of change is $1/\lambda$, the characteristic exponential length scale in units of transverse distance.

To introduce work, the electric field is tilted by adding an additional component, $E_{\perp}$, that is orthogonal to the direction of the magnetic field gradient. The $E_{\perp} \times B$ drift is what moves the charged particle either against or along the magnetic field gradient. As the components of $E_{\parallel}$ and $E_{\perp}$ are in vacuum, the relationship between the components follows from solving Maxwell's equations for a set of voltage plates above and below the direction of balanced drift. Explicit solutions have been found (*see* Betti et al., "A design for an electromagnetic filter for precision energy measurements at the tritium endpoint.", Progress in Particle and Nuclear Physics. 2019;106:120-31, the contents of which are incorporated herein in their entirety).

Given that the magnitudes of $E_{\parallel}$ and $E_{\perp}$ are related, it is not surprising that the net drift velocity along the acceleration direction is constant. There is no linear momentum acceleration present. The acceleration occurs through the increase in the transverse kinetic energy component, the magnetic orbital angular momentum, during a process of constant drift along the magnetic field gradient.

Because the orbital magnetic moment $\mu = T_{\perp}/B$ is invariant, if the B field increases (or decreases) exponentially along the trajectory of the particle, so must its transverse kinetic energy.

Referring to FIG. 2A, one can see the trajectory of a deuterium ion in a balanced drift with an initial kinetic energy of 20 keV at 0.2 T, going to a 4.7 T region with 1 MeV final kinetic energy. In this simulation, using CST studio, the magnetic field (see FIG. 2B) is scaled from the one produced by the PTOLEMY magnet. The maximum of the B_x field is set to 4.7T at $Z = 1.6\text{m}$ to match the toroidal field near the interface of the upper port of ITER. The electric field as in FIG. 2C is generated by a similar electrode structure as in the PTOLEMY transverse drift filter. The dimensions are scaled up such that the distance between the electrodes is 0.3 m.

Injection

The net drift is along the direction of the magnetic field gradient and drives the guiding center of the beam to cross equipotential lines and accelerates the particles. When injecting into a tokamak, it can be seen that as the beam drifts in the direction of ∇B , it

naturally reaches its maximum kinetic energy upon entering the toroidal magnet of a tokamak.

Charged Particle Beam Injection

Via the foregoing mechanism, initial simulations of injecting deuterium ions indicate
5 successful delivery of the beam into a $1/R$ magnetic field in a tokamak using the accelerating structure of Figure 1. Upon exiting the accelerator, the energetic ion continues to drift toward the plasma confinement region under the $1/R$ magnetic field gradient- B drift of the tokamak. Once the particle leaves the injection port, the gradient of the $1/R$ toroidal magnetic field drifts the ions into the center of the plasma. The relatively hot thermal temperature of the 1
10 MeV deuterium ions will thermalize through Coulomb interactions with the plasma. The injection mechanism supports a range of injection energies and ion species. For instance, injection of 4 MeV α -particles through the ITER diagnostic port may be an effective way of studying the effects of fusion final-state ion interactions on the plasma. Charge neutralization can be achieved by instrumenting ion (electron) injection ports on the top (bottom) of the
15 tokamak. The gradient- B drift will drift ions downward (for a given orientation of the azimuthal toroidal magnetic field) and electrons (or negative ions) upward.

Referring to FIG. 3, in some embodiments, the desired injection magnetic field can be produced by a tapered dipole magnet **300** with a superconductor winding **301**, forming a plurality of loops **302**, **303**.

20 Referring to FIG. 4, such a magnet is compact and can be placed within a counter-dipole winding placed in the upper port between two adjacent ITER toroidal field coils **320**, **321**. The counter-dipole reduces the magnetic forces between the injection system and the tokamak field windings. The counter-dipole creates a zero field region for the ion source and reduces Lorentz forces on the primary reactor coils. Note that a field cage **310** with a number
25 of electrodes can be placed inside the magnet to produce the corresponding electric field.

In some embodiments, a magnetic orbital angular momentum beam accelerator may be provided.

Referring to FIG. 5, in some embodiments, the accelerator **500** may include an
30 particles **511** in an initial magnetic field **510**. The initial magnetic field may be a zero or low magnetic field. The particles may initially be low energy charged particles. In some embodiments, the particles include electrons, ions, or a combination thereof.

In some embodiments, the accelerator may include an einzel lens **520** configured to accelerate the particles from an initial magnetic field along a path **550** towards the injection mechanism **530** and eventually a target **540**. In some embodiment, the particles may be reflected off a repelling electrode of the einzel lens into the injection mechanism **530**, which
5 may be, e.g., a tapered dipole magnet winding.

The injection mechanism **530** will generally include a plurality of electrodes **531** configured to form a desired electric field as disclosed herein, and at least one magnet **532** configured to form a desired magnetic field **535** as disclosed herein, where the electrodes **531** are at least partially within the magnetic field **535**. In some embodiments, this may be
10 accomplished with a tapered dipole magnet winding and a field cage, as appropriate.

The at least one magnet **532** (e.g., tapered dipole magnet winding) may be configured to have a magnetic field positioned to allow particles **511** to enter the injection mechanism, the magnetic field being a low magnetic field configured to cause the particles to begin cyclotron motion. The injection mechanism may have a magnetic field gradient that is a
15 transverse gradient along an average path expected of the particles.

In some embodiments, the tapered dipole magnet winding may include superconducting magnets. In some embodiments, the tapered dipole magnet winding may be symmetrical around a plane extending through a central axis **305** (*see* FIG. 3), each half of including a plurality of loops, each loop in the plurality of loops defining a rounded
20 rectangular shape contoured such that an inner surface of the loops define a volume of space having a circular, oval, or rounded rectangular cross-section, each loop having one side that is substantially located at a first end, and where each loop has a different length.

The plurality of electrodes may be configured as a field cage. The field cage may include a plurality of electrodes, configured to form a complementary electric field to balance
25 a gradient-*B* drift transverse to the average path of a beam of the particles and accelerate the particles under work of the magnetic field gradient.

In some embodiments, at plurality of electrodes (e.g., field cage) may include, or be placed within, coils of a solenoid, custom superconducting dipole coils, iron pole-face magnets with shaped pole-faces, configurations of permanent magnets, or a combination
30 thereof.

In some embodiments, the particles may be accelerated in a low vacuum. As used herein, low vacuum refers to a vacuum sufficiently low to allow unimpeded cyclotron motion. In the low vacuum, there will be beam losses from scattering as the vacuum

increases. The tolerated level of vacuum may vary, and is typically set by the amount of acceptable beam loss per cyclotron orbit, where one would want to minimize beam losses. In some embodiments, this may be a pressure of 100 mbar or less. In some embodiments, this may be a pressure of 10 mbar or less. In some embodiments, this may be a pressure of 1 mbar or less.

Example

To better describe how the system operates, it may be helpful to understand how the magnet and transverse drift filter operate to drain a particle's transverse kinetic energy.

In previous work, it was found that it is possible for certain configurations of static, non-uniform electromagnetic fields to drive an electron up a potential hill while maintaining an overall trajectory that is straight on average in the transverse plane, and an electromagnetic filter was designed around this effect. The relevant drift terms are the $\mathbf{E} \times \mathbf{B}$ drift, which drives transport, and the non-electric gradient- B drift, which does work against the increasing potential along the trajectory. The gradient- B drift is proportional to $\nabla \perp \mathbf{B} / B = \lambda$; this term is the radius of curvature of the field lines. It is constant if B is an exponential in the transverse direction z with the decay parameter λ , i.e. $B \sim e^{-z/\lambda}$. The magnitude of $\mathbf{E} \times \mathbf{B}$ drift is proportional to E/B , so if E is also an exponential with the same λ as B , then exact canceling is achieved between one of the $\mathbf{E} \times \mathbf{B}$ components and the gradient- B drift, and the other $\mathbf{E} \times \mathbf{B}$ components are also constant. The drift terms are calculated in the so-called Guiding Center System (GCS) frame of reference, i.e. the cyclotron orbit-averaged trajectory of the electron. The field changes are adiabatic relative to the motion of the trajectory.

The potential hill and E are produced by electrodes lined up along the trajectory. The precise geometry of the filter electrodes is described below. For B , it was found in that solutions to Maxwell's laws in the vacuum regions between flat coils of current-carrying wire, so-called pancake coils, could satisfy the field conditions above.

Here, an iron-core magnet design is introduced using high-permeability soft iron that is more practical than the pancake coils for an initial field magnitude of approximately 1 T. Following the equivalence of λ and the radius of curvature, one can look for patterns of magnetic field lines that have an apparently constant radius of curvature along one dimension. If such a pattern is observed, it follows from the uniqueness theorem that the

field magnitude in that region must be decreasing exponential with a decay parameter λ equal to the radius of curvature of the field lines.

An iron pole-face gap magnet, such as a pair of counterposed 'E'-shaped magnet cores, typically produces a region of uniform field in the air gap between the two poles.

5 Extending transversely out from the air gap, the field decays roughly dipole in character, i.e. as $1/z^3$. This relation can be observed visually through the increasing radius of curvature of the field lines away from the gap, as all of the flux exiting one pole face must eventually return to the opposite pole face.

10 By introducing symmetric iron extensions to the side walls of such a magnet above and below the air gap, it is possible to turn the dipole-like field into a region of field with a constant radius of curvature.

In FIGS. 6A and 6B, perspective views of a standard dual E-magnet (6A) and one with 32 cm extensions (6B) can be seen. The sections with diagonal slashes as current loops
610.

15 The effect of the extensions, modeled in FIG. 6B as rectangular bars, is to divert some of the flux away from the return yokes and back into the vacuum above and below the transverse plane to be recycled into the air gap. The resulting flux pressure constrains the expansion of the original flux radius, and with the right extension length, a channel of field lines with apparently constant radius of curvature can be created. The effect is minimal if the
20 horns are too short; if the extensions are too long or approach each other too closely, a flux loop between them is closed and a quadrupole point is formed in the center region where the field goes to zero and switches direction. When viewing the field lines from magnets with different length extensions (0, 25 cm, 32 cm, and 40 cm), it is clear that only 32 cm shows the desired region of approximately constant radius of curvature.

25 The extensions need not be rectangular and fine adjustments to the location of the quadrupole point and the variance of λ can be made by varying the shape. Such methods can be used in principle to achieve an arbitrary level of precision in λ ; however, in practice, this is unnecessary as what is important is not that the field has a specific or exactly constant value of λ , but that the opposing drift components cancel out at each point in z along the trajectory
30 of the electron. This can be accomplished regardless of small deviations in λ if the $e^{-z/\lambda}$ term of the drift components, which is used to set the voltages on the filter electrodes, is replaced by the sampled values from a precision magnetic field map of the magnet in use. Concretely,

the B_x -component of the field is sampled along z then normalized to the nearly constant B_x at the air gap $z = 0$:

$$B_x(z)/B_x(z=0) \approx e^{-z/\lambda} . \quad (9)$$

Filter Geometry Parameters

5 The filter geometry and coordinate system are shown in simplified FIG. 7A. The magnetic field and motion of the electron parallel to it is in x ; the transverse plane is $y - z$. The electron enters the filter through the region of uniform field in the air gap and the overall transverse drift of the trajectory is in the $+z$ -direction. The electrodes responsible for the $E \times B$ drift are referred to as the filter electrodes and are placed a distance $\pm y_0$ (y_0 is distance **751**),
10 from the center line ($y = 0$) and span a length $2x_0$ in x (x_0 is distance **750**). Placed at $\pm x_0$ are two long electrodes that extend the full length of the filter in z . These are referred to as the bounce electrodes; they close off the volume enclosed by the filter electrodes and contain the electron within the filter by reflecting the parallel momentum at each end.

Referring to FIG. 7A, a simplified illustration of a transverse drift filter can be seen.
15 In FIG. 7A, the filter **700** can be seen to include electrodes and wires forming a hollow rectangular shell through which particles may travel. The centerline **740** is the nominal GCS trajectory of the particles passing through the shell (here, a path aligned with the z -axis in the $+z$ direction).

The filter may include a plurality of filter electrodes **710**, **711**, **712**. The filter
20 electrodes may be aligned in three groups – a central group of electrodes **711**, and two side groups **710**, **712**. Each group may form a “well” or “channel”. The filter electrodes will generally form two sides of the rectangular shell; in FIG. 7A, they are aligned parallel to the x - z plane, at distances of $\pm y$. In some embodiments, the negative- y electrodes may all be at a constant voltage, while the positive- y electrodes may vary in voltage.

25 The filter may include a plurality of bounce electrodes **720**. The bounce electrodes will generally form the other two sides of the rectangular shell; in FIG. 7A, they are aligned parallel to the y - z plane, at distances of $\pm x$.

The filter may include a plurality of field wires **730**.

The parameters x_0 (distance **750**) and y_0 (distance **751**) are largely determined by the
30 dimensions of the air gap of the magnet. A small aspect ratio y_0/x_0 is favorable to maintain field uniformity inside the filter. Nonetheless, y_0 should be large enough to accommodate the final cyclotron radius $\rho_c(z)$ of the electron as it grows with decreasing $B(z)$. The value of x_0 is

best if maximized as close to the size of the air gap as possible; since the arch of the field lines is bookended by the two pole faces, it turns out $x_0 \approx \lambda$. In some embodiments, a λ of 5 cm was used. In some embodiments, the air gap is 12cm with $\lambda \approx 6$ cm. In some embodiments, $x_0 = 5$ cm.

5 To minimize y_0 , one needs the cyclotron radius ρ_c as a function of z ,

$$\rho_c(z) = \frac{\sqrt{2m(T_{\perp}(z))}}{|q|B(z)} \quad (10)$$

where q is the charge of the electron, $T_{\perp}(z) = \mu B(z)$ the transverse kinetic energy of the electron in the GCS frame, and μ the orbital magnetic moment of the electron. In terms of B_0 , the initial field magnitude in the uniform region, and the initial cyclotron radius, $\rho_0 =$

10 $\rho_c(z=0)$,

$$\rho_c(z) = \rho_0 \sqrt{B_0/B(z)} \quad (11)$$

$$= \rho_0 \sqrt{e^{z/\lambda}} \quad (12)$$

where in the last equality we use the approximation $B(z) = B_0 e^{-z/\lambda}$. For an electron with initial $T_{\perp} = 18.6$ keV and $B_0 = 1$ T, the initial radius is $\rho_0 \approx 0.45$ mm. With $\lambda = 6$ cm this becomes approximately 1.25 cm at $z = 40$ cm, corresponding to a final kinetic energy of ≈ 20 eV or three orders of magnitude reduction (when configured to reduce kinetic energy). To accommodate a final radius of 1.25cm one can choose $y_0 = 1.5$ cm, which yields an aspect ratio $y_0/x_0 = 0.3$. At ratios larger than this, the field uniformity inside the filter degrades rapidly.

20 The ability to accommodate the growth of the cyclotron radius while maintaining field-uniformity for drift balancing is the biggest challenge against further reduction in $T_{\perp}$. One possibility to extend filter performance is to introduce a more elaborate filter geometry in which the aspect ratio y_0/x_0 is kept small while the overall dimensions increase with the radius. In general, though, the additional field gradients that come with an expanding or
25 varying geometry make this procedure complex. A more efficient way to increase filter performance is to leave the geometry intact and increase the initial B .

The filter power increases as B^2 in that a factor of three increase in B results in a factor of nine decrease in the final kinetic energy. The cyclotron radius goes as $1/B$ while the energy, for a fixed radius, goes as ρ_c^2 . If the filter dimensions are unchanged and the starting
30 field is increased to 3T, the same radius 1.25cm is achieved at $z \approx 53$ cm, corresponding to a final kinetic energy of ≈ 2 eV or four orders of magnitude reduction. The same principle of

using iron extensions to redirect flux can be implemented with superconducting coils in place of an iron core to produce the necessary field.

Voltage Setting Optimization

5 A net y-drift of zero along the central line is achieved if the potential along the central line ($x = y = 0$) satisfies

$$\begin{aligned}\phi(z)|_{x,y=0} &= \phi_0 - \frac{\mu B_0}{|q|} (1 - e^{-z/\lambda}) \\ &= \phi_0 - T_{\perp} + T_{\perp} e^{-z/\lambda}\end{aligned}\quad (13, 14)$$

where ϕ_0 is the initial potential at $z = 0$ and $T_{\perp}$ the initial transverse kinetic energy in eV. To turn this into voltages for the filter electrodes, we begin with the simplifying assumption that the potential at z is just the average between the two filter electrode voltages at that z , i.e., $\phi(z)|_{x,y=0} = [V_{y+}(z) + V_{y-}(z)]/2$, where $V_{y+}(z)$, $V_{y-}(z)$ are the voltages on the positive- and negative-y side electrodes. The accuracy of this approximation increases as the aspect ratio y_0/x_0 decreases.

All of the negative-y electrodes are set at a constant voltage which determines the total kinetic energy drained and only the positive-y electrode voltages vary along z ,

$$\begin{aligned}V_{y-}(z) &= \phi_0 - \frac{\mu B_0}{|q|} \\ &= \phi_0 - T_{\perp} \\ V_{y+}(z) &= \phi_0 + \frac{\mu B_0}{|q|} (2e^{-z/\lambda} - 1) \\ &= \phi_0 - T_{\perp} + 2T_{\perp} e^{-z/\lambda}\end{aligned}\quad (15, 16, 17, 18)$$

This is a schematic equation for the filter electrode voltages; as noted in the previous section, the $e^{-z/\lambda}$ term is substituted for by the normalized sampled B_x -component along the center line when the filter voltages are actually set.

20 Unlike previous analytical conditions, the non-zero aspect ratio of the filter and the transition region from uniform to decaying field may require corrections to the above voltages until precise drift balancing is achieved. Drift balancing can be calculated explicitly with the precision magnetic field map. The gradient- B drift is

$$V_{\nabla B}(z)|_{x,y=0} = -\frac{\mu \times \nabla_{\perp} B(z)}{qB(z)} \quad (19)$$

where μ is invariant under the adiabatic field conditions of the filter. Along the center line, the B_y and B_z components are nearly zero and therefore the magnitude $B(z) \approx B_x(z)$, and the transverse gradient $\nabla_{\perp} B(z) \approx dB_x/dz$, leading to

$$V_{\nabla B}(z)|_{x,y=0} = -\frac{\mu}{qB_x} \frac{dB_x}{dz} \hat{y} \quad (20)$$

The y-component of $\mathbf{E} \times \mathbf{B}$ drift that counteracts the gradient- B drift is

$$V_{\mathbf{E} \times \mathbf{B}}(z)|_{x,y=0} = \frac{\mathbf{E}_z \times \mathbf{B}_x}{B_x^2} = \frac{E_z}{B_x} \hat{y} \quad (21)$$

The sum of the two drifts should be zero; this is the drift balancing condition and yields an expression for E_z that leads to the potential. If the voltages are used without correction, the actual net-drift that results is non-zero.

To correct the electrode voltages, one can take the difference between the observed potential and the calculated potential and add twice this difference to the voltages. Adding twice the difference is used because the adjustment is only made to the positive-y voltages. This procedure is repeated until a desired level of convergence with or desired level of filter performance is achieved. Explicitly, the voltages for the i th iteration, $V_{y+}[i]$, are set by

$$V_{y+}[i] = V_{y+}[i-1] + 2(\phi_{\text{ideal}} - \phi[i-1]) \quad (22)$$

where $V_{y+}[i-1]$ and $\phi[i-1]$ are the voltages and potential from the previous iteration, and ϕ_{ideal} is the solution.

In some embodiments, the difference in filter performance for 1 T vs. 3 T starting magnetic field can be seen. The initial transverse kinetic energy of the electron is 18,600eV. The final GCS transverse KE is 1.2eV for 3T and 9.3eV for 1T. The growth of the cyclotron radius of the electron as B decreases puts a ceiling on filter performance for a given y_0 . The GCS trajectories are calculated from the instantaneous trajectories by averaging values over one cyclotron orbit. The beginning and end of a single cyclotron orbit is defined by intervals

in which the instantaneous y and z velocities of the electron change sign twice in an alternating fashion, indicating circular motion.

For dimensions $y_0/x_0 = 1.5/5\text{cm}$ and a starting field of 3T, the difference in ϕ compared to the ideal and the net y-velocity along the center line for several rounds of iteration can be readily determined. When used to reduce kinetic energy, it is favorable to minimize the potential difference in the transition region $z = 0$ to keep the electron from falling off the center line early on; the net y-drift is not in practice exactly zero but is nearly constant for the majority of the filter and can be counterbalanced by offsets in the starting y-position of the electron.

10 *Boundary Value Method*

An alternate method of solving for the filter electrode voltages based on a boundary-value method is presented here.

From (20) and (21), the voltage optimization is to find the voltage on the filter electrodes such that

$$15 \quad E_z = -\frac{\mu}{q} \frac{dB_z}{dz} \quad (23)$$

along $x, y = 0$. Using the linearity of the Laplace equation, one can generate the E_z^n, E_y^n, ϕ^n for the n th electrode with boundary value $\{V^n\}$ where

$$V^i = \begin{cases} 1 & \text{if } i = n, \\ 0 & \text{if } i \neq n. \end{cases} \quad (24)$$

A least-square fitting was carried out to derive the parameters c_n such that

$$20 \quad \sum_n c_n \cdot E_z^n = -\frac{\mu}{q} \frac{dB_z}{dz}. \quad (25)$$

Transverse Filter Speed

The speed at which the total kinetic energy of the electron is drained by the transverse drift filter depends on the z-component of the $\mathbf{E} \times \mathbf{B}$ drift, which is in the positive z-direction,

$$V_{E \times B}|_{x=0} = -(1/B) \hat{y} \cdot \nabla V(y, z)|_{x=0} = \frac{E_y}{B_z} \hat{z}. \quad (26)$$

It is important that the magnitude of the z drift, which diverges as $1/B$, not exceed the internal transverse velocity of the electron, $|v_{\perp}^*|$, until reaching the end of the filter. The GCS approximation assumes that the transverse drift is a fraction of the cyclotron velocity, giving prolate-shaped cycloid motion in the plane of the cyclotron motion. The transition to curvate-shaped cycloid motion occurs when the transverse drift velocity overtakes $|v_{\perp}^*|$, corresponding to an unraveling of the cyclotron motion in the filter frame of reference.

The divergence in z -velocity as B decreases to zero is mitigated if the E_y component of the field goes to zero faster than B_x does; one way to achieve this is to form a saddle point in the potential just before the quadrupole point, with the local maximum along x and the local minimum along z . The filter electrode voltages in this region can easily be manipulated to produce the saddle point; it also arises naturally in a three-channel filter geometry used to drain the parallel kinetic energy of the electron alongside the transverse.

Referring to FIG. 8, the net z -velocity along the center line can be seen for three different configurations (single channel, 3-channel center, and 3-channel side). The divergence in z -velocity as B approaches zero can be delayed by reducing the E_y component of the field at the end of the filter.

Kinetic Energy Parallel to the Magnetic Field

The transverse drift filter was created with the primary goal of draining the transverse kinetic energy of a charged particle using gradient- B drift, and as disclosed above, the filter electrode voltages were calibrated under the assumption that all of the kinetic energy is transverse to the magnetic field. In practice the electron will also have a parallel component, and it is advantageous for successful operation of the filter for the parallel momentum to be sub-dominant to the transverse.

An electron with non-zero parallel kinetic energy inside the filter will undergo periodic ‘bouncing’ motion in x ; the bounce electrodes reflect electrons back to the center of the filter with a restoring $\mathbf{E} \cdot \mathbf{B}$ term, which, in some embodiments, has been implemented as a harmonic potential.

In general, the motion of an electron inside the filter is continuous cyclotron motion with forward drift in z accompanied by bouncing motion in x . If the parallel component

becomes the dominant part of the total kinetic energy, two important effects begin to manifest. The first is the non-adiabatic nature of the trajectory where less than a single cyclotron orbit is completed before the electron has completed a single bounce, *i.e.*, traversed the full width of the filter. The second is the transverse drift known as curvature drift.

- 5 Curvature drift originates from the centripetal forces that result as a tendency of the cyclotron motion of charged particles to follow magnetic field lines. For an exponentially falling B field, the characteristic length λ and the radius of curvature are equal, $R_c = \lambda$, and

$$\nabla_{\perp} B = -\frac{B}{R_c} \hat{n} \quad (27)$$

- 10 where $\hat{n}$ is the unit vector normal to the magnetic field line curvature. In vacuum, the combined gradient- B and curvature drifts are given by

$$\mathbf{V}_{\nabla B-C} = \frac{1}{2} m (v_{\perp}^2 + 2v_{\parallel}^2) \frac{\mathbf{B} \times \nabla_{\perp} B}{qB^3} = (T_{\perp} + 2T_{\parallel}) \frac{\mathbf{B} \times \nabla_{\perp} B}{qB^3} \quad (28)$$

- in the non-relativistic approximation. For an equal amount of kinetic energy, the curvature drift is apparently a factor of two greater than the gradient- B drift. Unlike the gradient- B drift however, which is constant along the filter owing to the first adiabatic invariant μ , the
- 15 curvature drift depends on the instantaneous value of $v_{\parallel}^2$, which is therefore rapidly averaged over successive bounces in the filter to have an effective bounced-averaged value $\langle v_{\parallel}^2 \rangle$. For a filter geometry symmetric in x the maximum value $|v_{\parallel}^{max}|$ is attained at $x = 0$, and for a harmonic bounce potential along the B field direction, the average value of $\langle v_{\parallel}^2 \rangle = (v_{\parallel}^{max})^2/2$. Plugging this back into (28), the factor of two relative to the gradient- B drift
- 20 disappears.

Therefore, the total gradient- B and curvature transverse drift is proportional to the total kinetic energy $T(z)$ of the electron at $x = 0$ in the filter and points in the negative y -direction,

$$\mathbf{V}_{\nabla B-C} \sim \frac{T(z)}{B(z)R_c} \bigg|_{x=0} \hat{y} . \quad (29)$$

Additionally, the component of transverse drift along the normal of the B field, *i.e.*, along z , introduces an additional contribution to the curvature drift that scales as $|v_{\parallel} V_{E \times B}^z|$ rather than $v_{\parallel}^2$. Therefore, at the end of the filter, the effects of curvature drift can be dramatic if $V_{E \times B}^z$ is a substantial fraction of $|v_{\perp}^*|$. The $V_{E \times B}^z$ drift is mitigated by use of a saddle point, nonetheless it is advantageous to also drain the parallel energy faster relative to the transverse to avoid this runaway drift.

Bounce Electrodes, Field Wires and Side-Well Potentials

The parallel kinetic energy of an electron inside the filter can be drained by modifying the design into a three-channel geometry as follows. Consider an electron undergoing bounce motion in the filter. At the turning points of the motion, the parallel kinetic energy goes to zero while the transverse kinetic energy is largely unchanged. When the electron is reflected back towards the center of the filter by the bounce electrodes, the parallel kinetic energy returns to its maximum value at $x = 0$. However, because the electron is also moving forward in z as the bounce occurs, the potential at $x = 0$ upon return to the center is not the same as it was when it left; it has increased. In the GCS frame, since this increase in potential is along the direction of the bounce motion, and not in the transverse, the parallel kinetic energy is the one that is drained, in an amount nominally equal to the potential difference at $x = 0$ before and after the bounce.

The same effect is observed in the original single-channel filter, but the parallel drain is small since the forward displacement in z between bounces is small. The amount of parallel drain can be increased if the electron is made to linger in the side region of the filter before returning to the center, increasing the forward displacement in z .

This can be achieved to good precision if the initial parallel kinetic energy of the electron is known. Referring to FIG. 7A, the interior of the filter may be split into three discrete potential wells; which can be referred to as center-wells (e.g., under “center” electrodes **711**) and side-wells (e.g., under “side” electrodes **710** and **712**). To the center potential, previously configured only for the transverse kinetic energy drain, is now added the parallel draining term so that the end of the filter remains the total energy drained. The side potential at z is increased (decreased, as it were, for an electron) by an amount equal to the parallel kinetic energy of the electron at z . Since the potential step between side-center-side is in the direction of parallel motion, when the electron enters the side-well from the center its

parallel kinetic energy is reduced to nearly zero, thus causing it to linger there while undergoing transverse drain forward in z , before eventually bouncing back to the center as before, achieving the desired mechanism.

Explicitly, with $T_{\perp}$ and $T_{\parallel}$ the initial transverse and parallel kinetic energies,
5 respectively, the idealized potentials are now, setting the reference offset $\phi_0 = T_{total}$,

$$\begin{aligned}\phi_{cent}(z)|_{y=0} &= T_{\perp} e^{-z/\lambda_{cent}} + T_{\parallel} e^{-z/\lambda_{side}} \\ \phi_{side}(z)|_{y=0} &= \phi_{cent}(z) - T_{\parallel} e^{-z/\lambda_{side}} \\ &= T_{\perp} e^{-z/\lambda_{cent}},\end{aligned}\quad (30, 31)$$

where λ_{side} , which sets the rate of parallel energy drain, is not required to be the same as λ_{cent} .

If $\lambda_{side} = \lambda_{cent}$,

$$\begin{aligned}\phi_{cent}(z)|_{y=0} &= T_{total} e^{-z/\lambda} \\ \phi_{side}(z)|_{y=0} &= T_{\perp} e^{-z/\lambda},\end{aligned}\quad (32, 33)$$

10 which makes clear that the center potential is the total energy drained and the side potential is just the potential for an electron with the same transverse kinetic energy but zero parallel component, i.e. transverse-only draining. The corresponding electrode voltages are

$$\begin{aligned}V_{cent, y_{-}}(z) &= 0 \\ V_{cent, y_{+}}(z) &= 2T_{total} e^{-z/\lambda} \\ &= 2T_{\perp} e^{-z/\lambda_{cent}} + 2T_{\parallel} e^{-z/\lambda_{side}} \\ V_{side, y_{-}}(z) &= 0 - 2T_{\parallel} e^{-z/\lambda_{side}} \\ V_{side, y_{+}}(z) &= 2T_{\perp} e^{-z/\lambda_{cent}} + 2T_{\parallel} e^{-z/\lambda_{side}},\end{aligned}\quad (34, 35, 36, 37)$$

In contrast to the center, the addition of the parallel energy term to the side voltages is
15 split across the top and bottom electrodes rather than applied only to the top electrodes. The total potential difference across the top and bottom in the side well is also increased relative to the center in order to increase the E_y magnitude and therefore the forward drift in z to account for the slight backwards motion in z due to the curvature of the field lines.

Given (34)-(37), the voltages are iterated as before, with the correction term also split
20 across top and bottom for the side voltages.

$$\begin{aligned}
V_{cent, y_{\pm}}[i] &= 0 \\
V_{cent, y_{\pm}}[i] &= V_{cent, y_{\pm}}[i-1] + 2(\phi_{ideal, cent} - \phi_{cent}[i-1]) \\
V_{side, y_{\pm}}[i] &= V_{side, y_{\pm}}[i-1] + (\phi_{ideal, side} - \phi_{side}[i-1])
\end{aligned} \tag{38, 39, 40}$$

Figure 9 shows filter electrode voltages and idealized potentials along the center line of the respective wells. Voltages are set for a pitch 60° electron with total kinetic energy 18.6 keV, with $\lambda_{side} = \lambda_{cent}$.

5 Finally, to increase the sharpness of the transition between the center and side wells, additional field wires are placed along the splits in the filter electrodes. To maintain maximum relative uniformity of the side and center potentials down the entire filter length z , both the field wires and bounce electrodes are segmented in z the same as the filter electrodes. The field wire voltages mirror the opposite side filter electrode voltages and the
10 bounce electrodes, to ensure continuous bouncing but not so much so that the bounce potential contaminates the side and center potentials, are set with a fixed offset plus a decay term proportional to the initial parallel kinetic energy,

$$\begin{aligned}
V_{wire, y_{\pm}}(z) &= V_{cent/side, y_{\pm}}(z) \\
V_{bounce}(z) &= \phi_0 - V_{fixed} - \alpha T_{\parallel} e^{-z/\lambda},
\end{aligned} \tag{41, 42}$$

where $\alpha < 1$.

15 Injection of Charged Particles with a Reversed Filter

One approach is to operate a filter in reverse. Referring to FIG. 10, a system **1000** can be seen. In the system, a source of particles **1010** (here, an electron gun) is configured to provide particles to an injection end of the filter **1020**. Instead of decreasing kinetic energy, the filter is configured to increase kinetic energy. At least a part of the filter **1020** is within a
20 magnetic field generated by the magnet **1030**. In some embodiments, there may be a gap **1025** between the magnet and the filter. The injected electrons are accelerated by gradient- B drift using the same drift conditions as the draining process.

As seen in FIGS. 11A and 11B, the y -position and total kinetic energy (11A), and the transverse kinetic energy (11B) over the length of the filter can be seen. In FIG. 11A, the y -
25 position **1100** can be seen having cyclical motion with ever-decreasing radius of movement as it proceeds down the filter. As that is occurring, the total kinetic energy is increase, from around 1 eV (**1101**) to 10 eV (**1102**), 100 eV (**1103**), 1 keV (**1104**), to over 10 keV (**1105**).

An electron, starting at around 1 eV, is accelerated by gradient-B drift to around 18.6 keV at the end of the filter. As seen in FIG. 11B, the transverse kinetic energy over the length of the filter can be seen increasing to over 10 keV.

As seen in FIG. 12, a system 1200 with a magnet having coils 1201 and iron extensions 1203, and a three-channel transverse drift filter 1202, with a plurality of filter electrodes 1205 can be seen.

The B_x field was mapped out by digital 3-axis hall magnetic sensors. Initial investigation found good agreement between the measured fields and simulations. With new power supplies, the B_x field may start at about 1 T between pole faces and fall exponential with $\lambda \sim 7$ cm.

Tolerance Estimation

The filter electrodes 1205 have 80 individual voltages set over a distance of one meter along the z direction. Each electrode has a width of 6.25mm and a separation gap of 6.25 mm between closest surfaces. According to the simulated B field, the voltage settings are computed using the foregoing boundary-value method.

The foregoing is merely illustrative of the principles of the disclosure, and the apparatuses can be practiced by other than the described embodiments, which are presented for purposes of illustration and not of limitation. It is to be understood that the apparatuses disclosed herein, while shown for use in plasma heating or particle etching, may be applied to apparatuses in other applications.

Variations and modifications will occur to those of skill in the art after reviewing this disclosure. The disclosed features may be implemented, in any combination and subcombination (including multiple dependent combinations and subcombinations), with one or more other features described herein. The various features described or illustrated above, including any components thereof, may be combined or integrated in other systems. Moreover, certain features may be omitted or not implemented.

Examples of changes, substitutions, and alterations are ascertainable by one skilled in the art and could be made without departing from the scope of the information disclosed herein.

* * *

What is claimed:

1. A method for particle acceleration, comprising:
providing particles in a zero or low magnetic field;
causing the particles to be in cyclotron motion in a magnetic field that is strong compared to a momentum of the particles, the particles having a gyroradius that is small compared to a transverse dimension of an injection aperture through which the particles will travel, wherein the magnetic field has a transverse gradient along an average path of the particles; and
utilizing a complementary electric field to balance a gradient- B drift transverse to the average path of the particles and accelerate the particles under work of the transverse gradient.
2. The method according to claim 1, wherein the particles comprise electrons, ions, or a combination thereof.
3. The method according to claim 1 or 2, further comprising directing the particles towards a confined plasma.
4. The method according to claim 1 or 2, further comprising directing the particles towards a substrate.
5. The method according to claim 4, wherein the substrate is a semiconductor.
6. A magnetic orbital angular momentum beam accelerator, comprising:
a tapered dipole magnet winding configured to have a magnetic field positioned to allow particles to enter the tapered dipole magnet winding, the magnetic field being a low magnetic field configured to cause the particles to begin cyclotron motion, and has a magnetic field gradient that is a transverse gradient along an average path expected of the particles; and
a field cage comprising a plurality of electrodes, configured to form a complementary electric field to balance a gradient- B drift transverse to the average path of a beam of the particles and accelerate the particles under work of the magnetic field gradient.

7. The magnetic orbital angular momentum beam accelerator according to claim 6, wherein the field cage is placed within a counter-dipole coil in an upper diagnostic port of a tokamak reactor.
8. The magnetic orbital angular momentum beam accelerator according to claim 6, wherein the field cage includes, or is placed within, coils of a solenoid, custom superconducting dipole coils, iron pole-face magnets with shaped pole-faces, configurations of permanent magnets, or a combination thereof.
9. The magnetic orbital angular momentum beam accelerator according to claim 6, further comprising an einzel lens configured to accelerate the particles from an initial magnetic field towards the tapered dipole magnet winding, the initial magnetic field being a zero or low magnetic field, the particles initially being low energy charged particles.
10. The magnetic orbital angular momentum beam accelerator according to claim 9, wherein the particles are reflected off a repelling electrode of the einzel lens into the tapered dipole magnet winding.
11. The magnetic orbital angular momentum beam accelerator according to claim 6, wherein the particles comprise at least one of electrons and ions.
12. The magnetic orbital angular momentum beam accelerator according to claim 6, wherein the particles are accelerated in a low vacuum.
13. The magnetic orbital angular momentum beam accelerator according to claim 6, wherein the tapered dipole magnet winding comprises superconducting magnets.
14. The magnetic orbital angular momentum beam accelerator according to claim 6, wherein the tapered dipole magnet winding is symmetrical around a plane extending through a central axis, each half of including a plurality of loops, each loop in the plurality of loops having a contoured rounded rectangular shape, each loop having one side that is substantially located at a first end, and where each loop has a different length.

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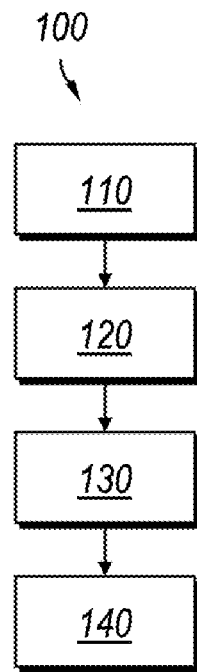


FIG. 1

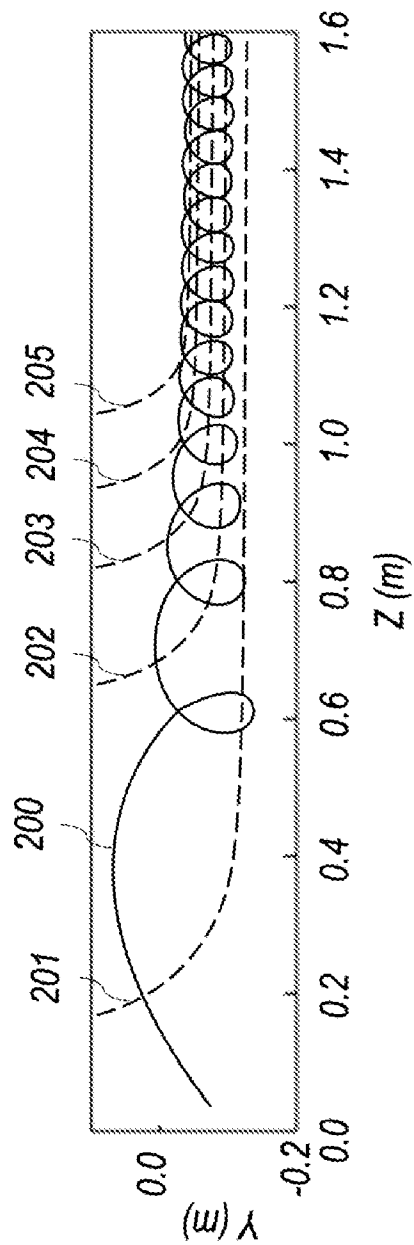


FIG. 2A

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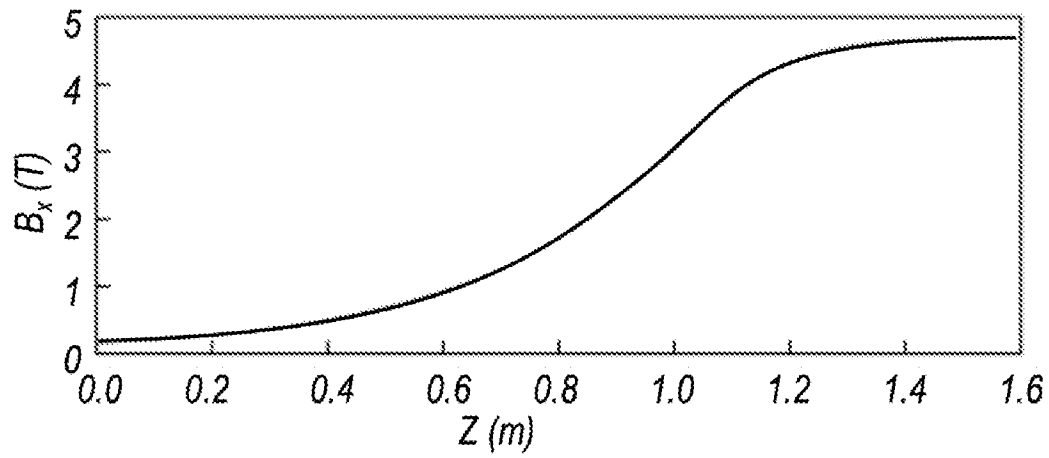


FIG. 2B

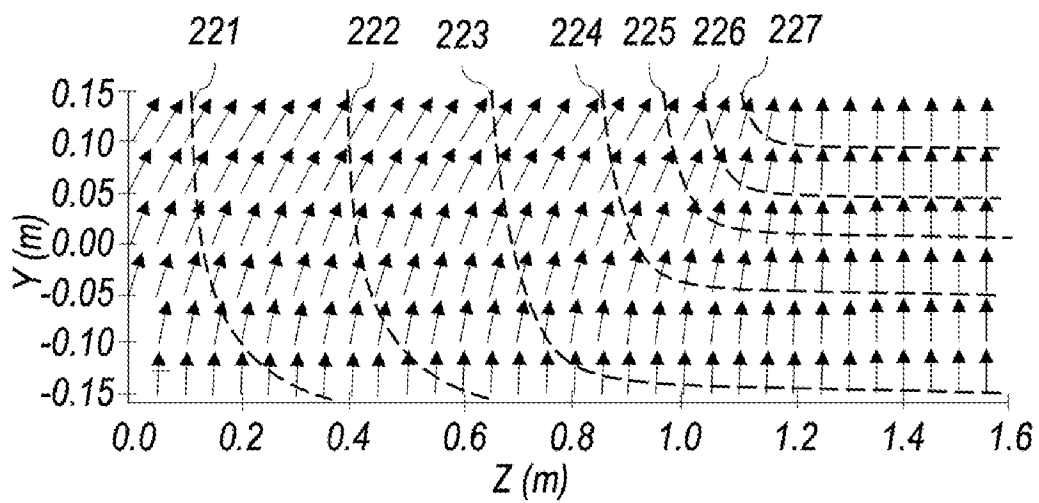


FIG. 2C

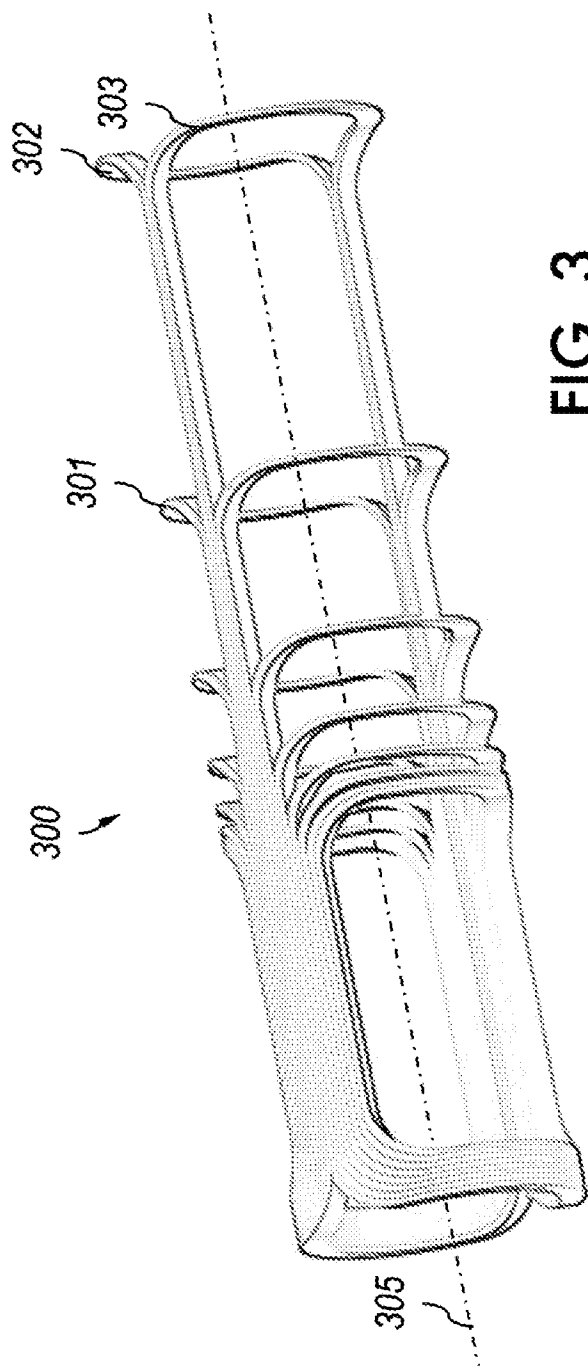


FIG. 3

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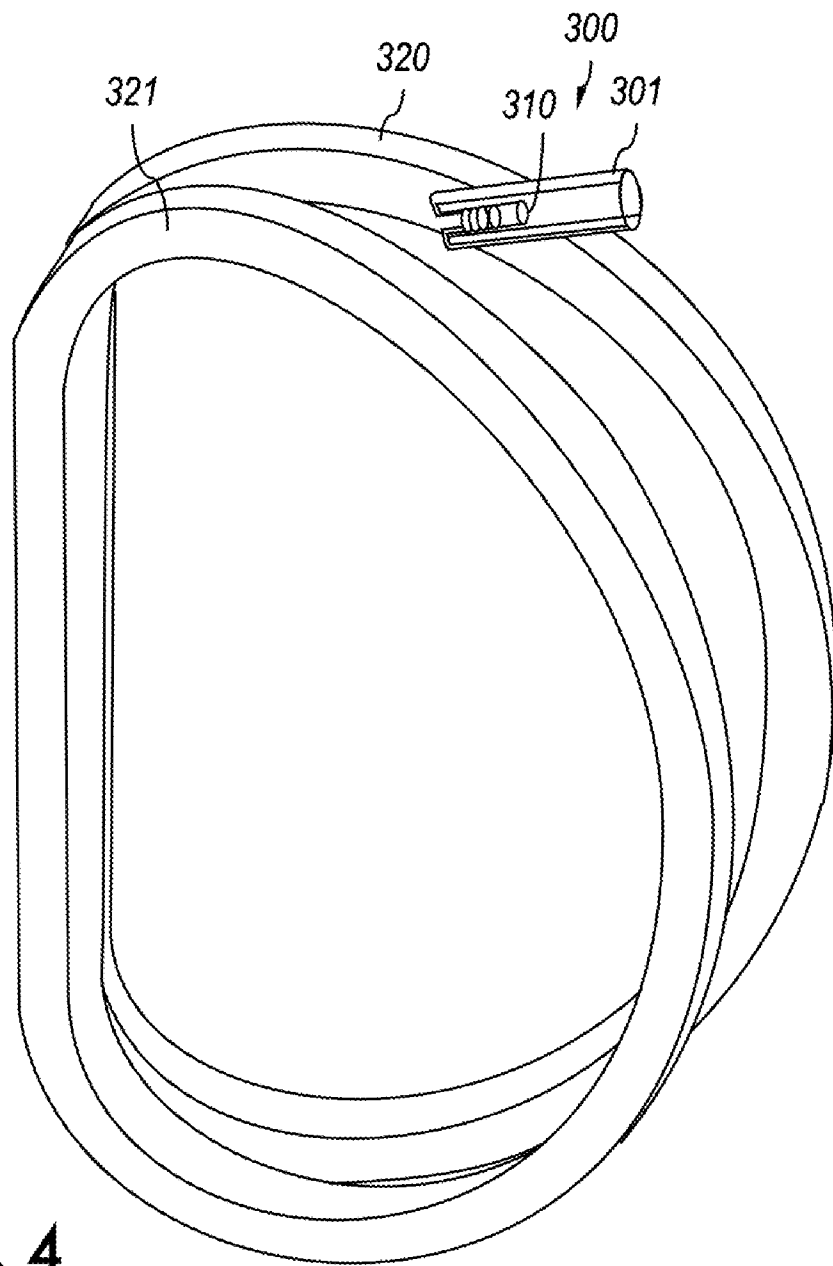


FIG. 4

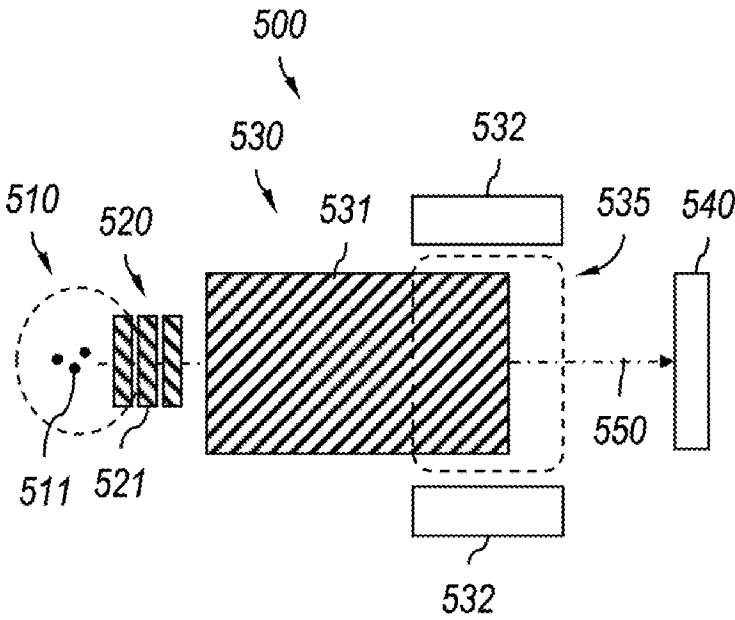


FIG. 5

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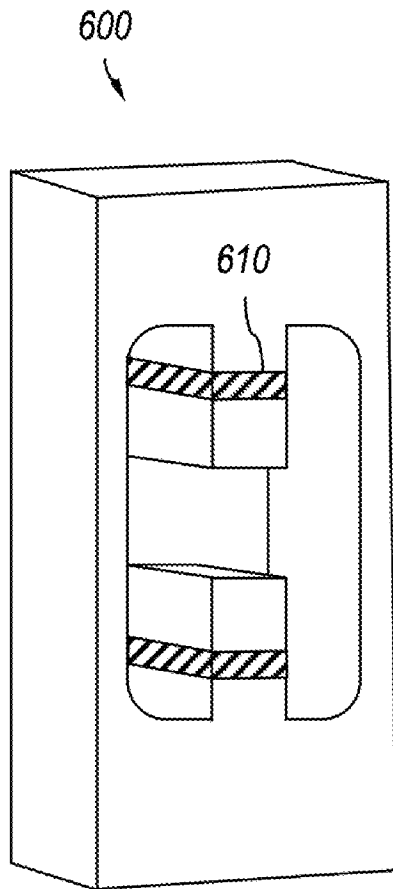


FIG. 6A

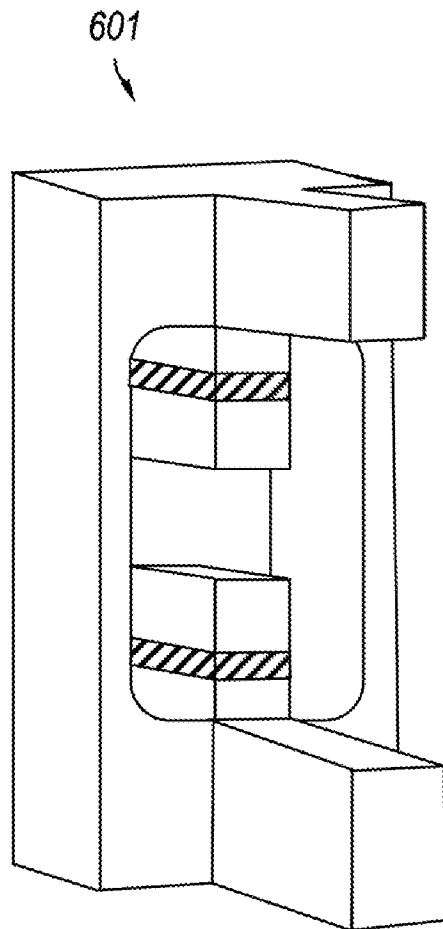
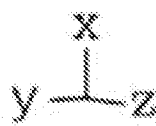


FIG. 6B

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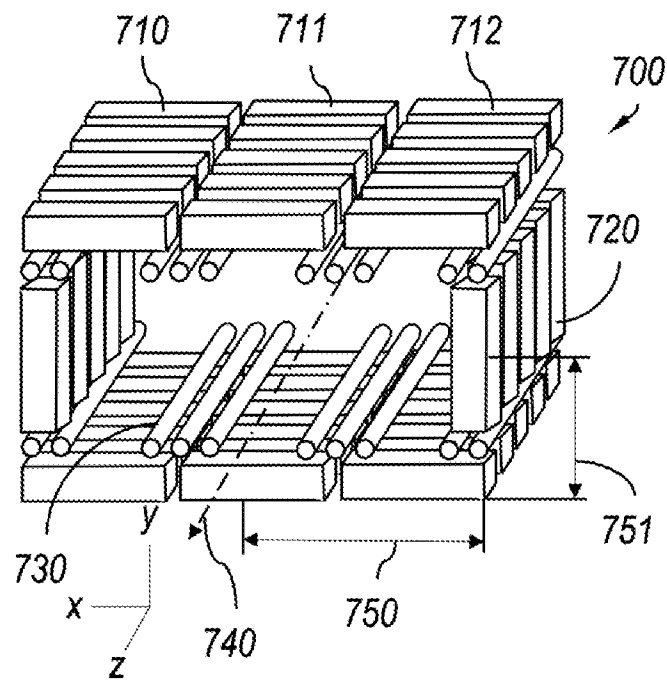


FIG. 7A

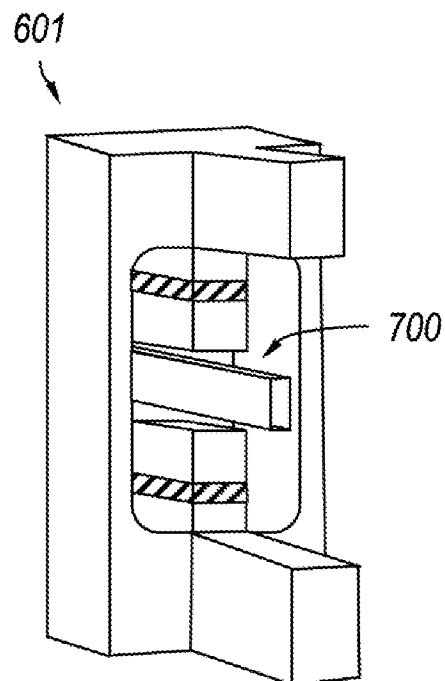


FIG. 7B

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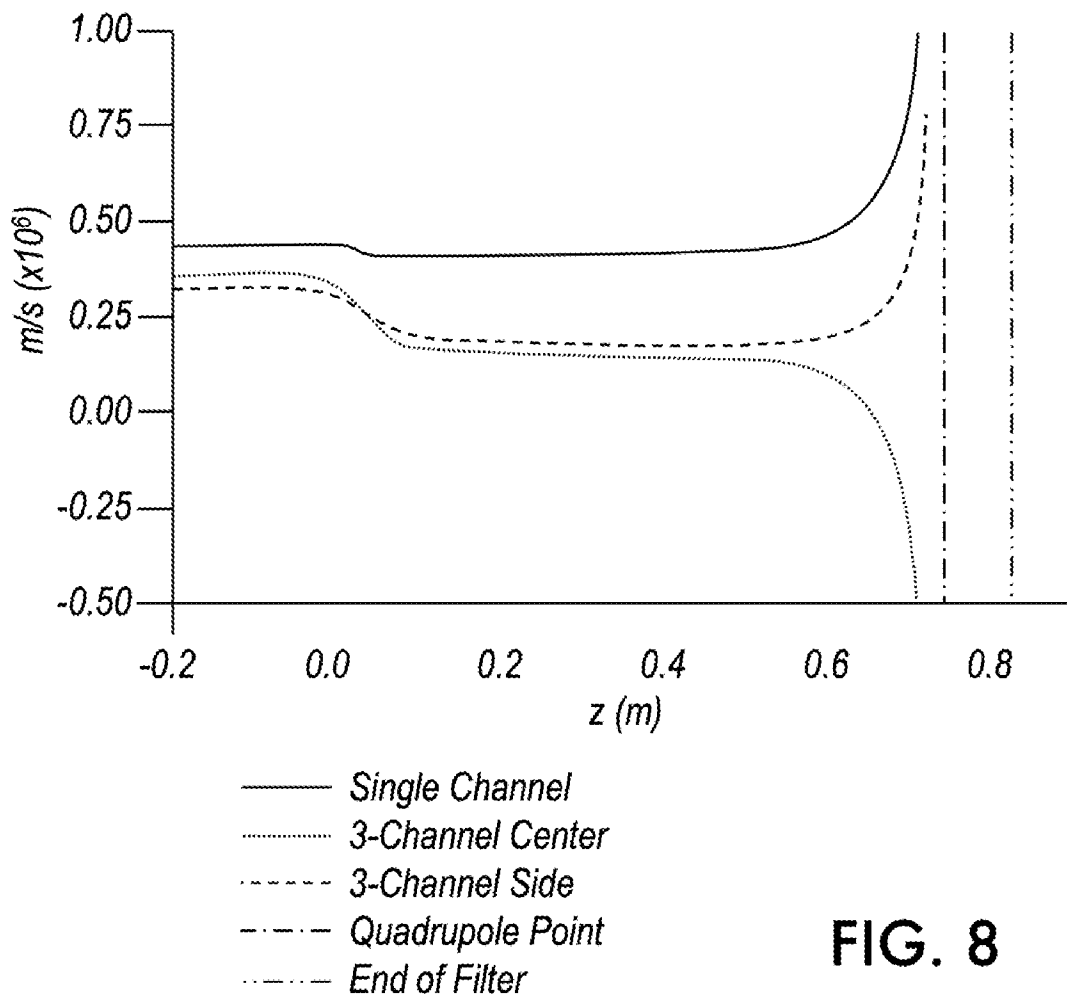
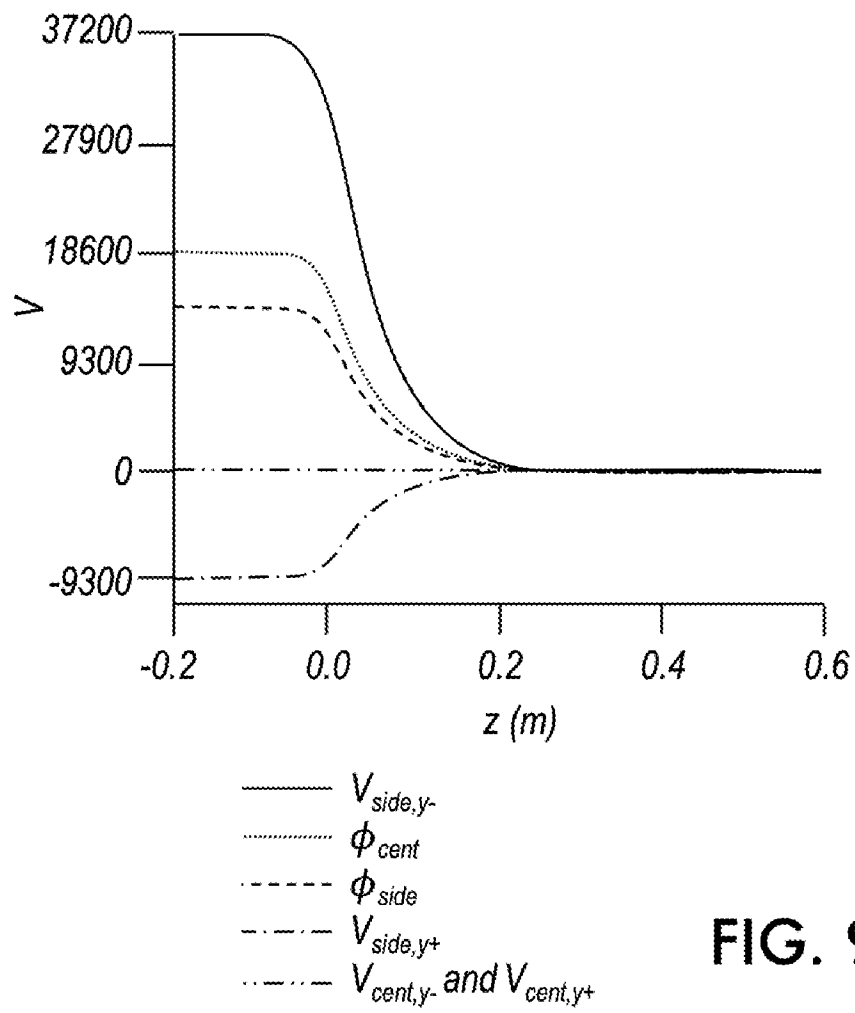


FIG. 8

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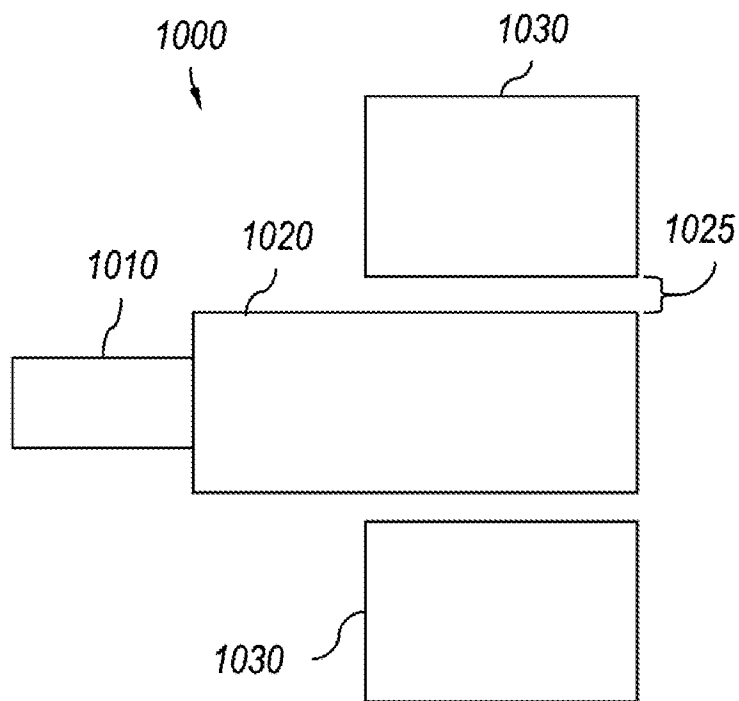


FIG. 10

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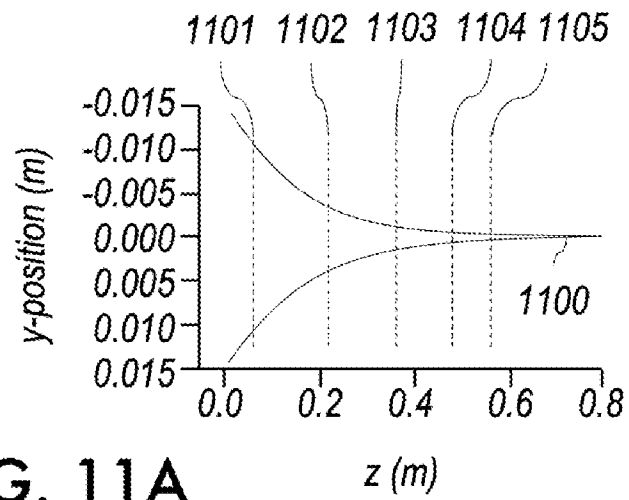


FIG. 11A

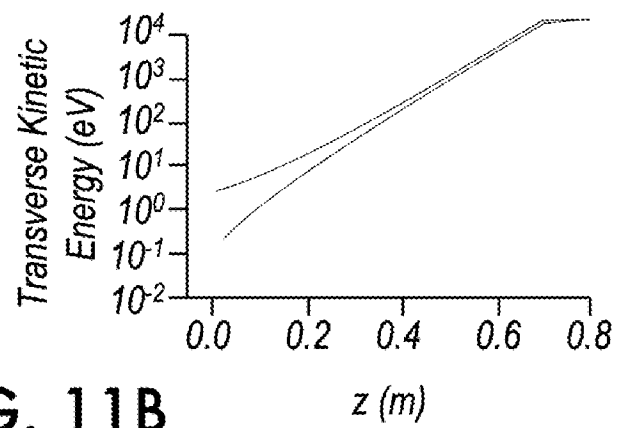


FIG. 11B

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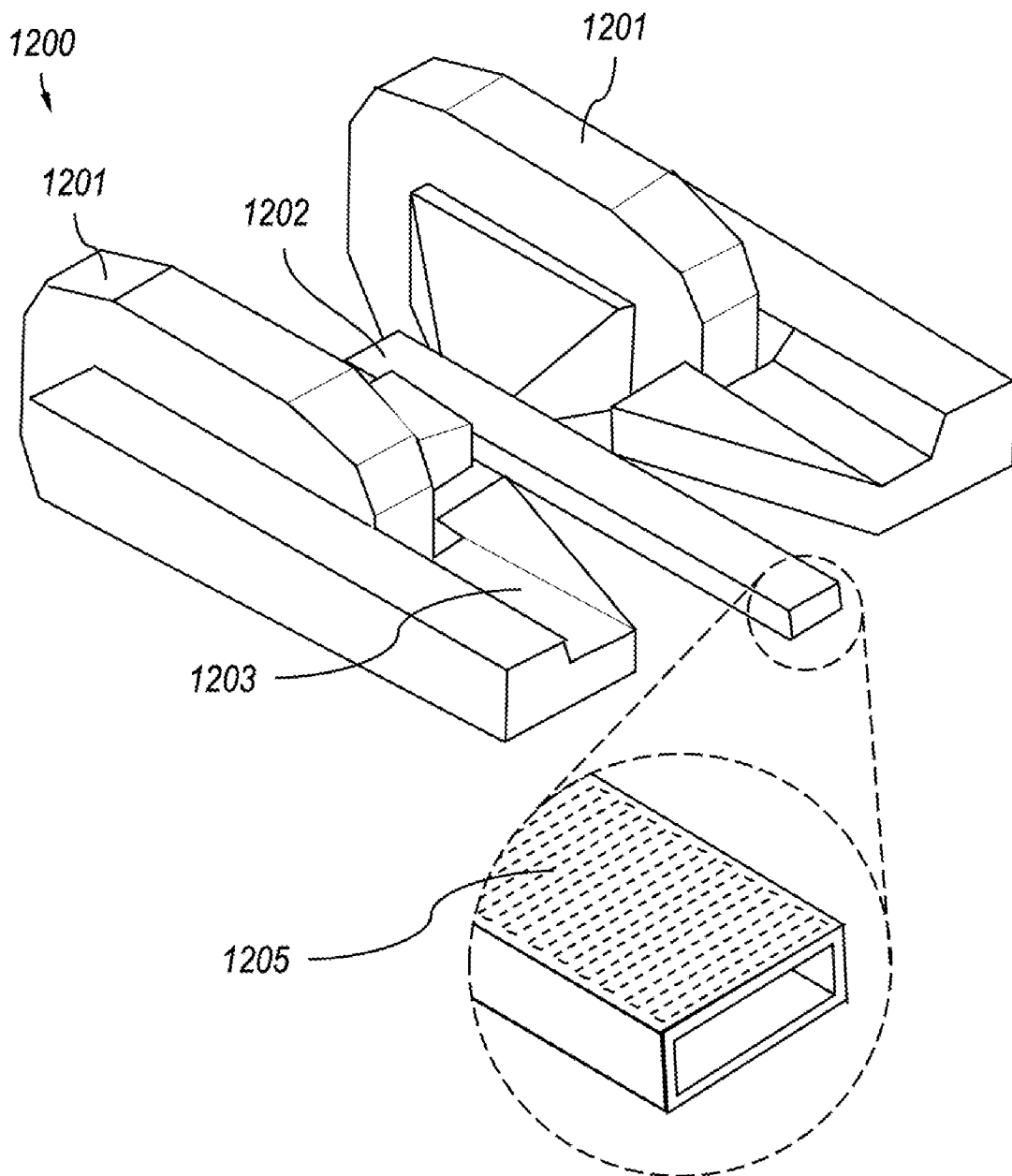


FIG. 12

INTERNATIONAL SEARCH REPORT

International application No.

PCT/US 22/39157

A. CLASSIFICATION OF SUBJECT MATTER

IPC - INV. H05H 13/02, H05H 13/08 (2022.01)
ADD. H05H 13/00 (2022.01)

CPC - INV. H05H 13/005, H05H 13/02, H05H 13/08

ADD. H05H 13/00

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

See Search History document

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

See Search History document

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

See Search History document

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US 8,031,824 B2 (Bystriskii et al.) 4 October 2011 (04.10.2011) Entire document, especially Abstract, col 5 in 32-37; col 6 in 8-11; col 12 in 28-29; col 14 in 21-25; col 15 in 44-54; col 17 in 13-20; col 19 in 24-26; col 23 in 39-44; col 33 in 44-46	1-5
Y	US 7,294,969 B2 (Kuninaka) 13 November 2007 (13.11.2007) Entire document, especially Abstract, col 12 in 3-19, 57-60; col 15 in 18-28; col 20 in 66-67; col 21 in 1-8, 32-45	1-5
Y	US 7,488,960 B2 (Zani et al.) 10 February 2009 (10.02.2009) Entire document, especially Abstract, col 2 in 29-42; col 7 in 57-67; col 9 in 16-18; col 17 in 5-19; col 28 in 43-46	4-5
A	Lee, "Pulse shaping of high-current electron beams with gradient B drift transport." United States: N. p., 1986. Web. https://www.osti.gov/biblio/5596355	1
A	Vandas, "Acceleration of Electrons by a Nearly Perpendicular Shock: The Dynamics of Electron Motion." NASA/ADS, 1989, https://ui.adsabs.harvard.edu/abs/1989BAICz..40..189V/abstract .	1
A	US 6,850,011 B2 (Monkhorst et al.) 1 February 2005 (01.02.2005) Entire document	1-5
A	US 7,860,146 B2 (Johnstone) 1 February 2011 (01.02.2011) Entire document	1-5
A	US 7,295,494 B2 (Meier) 13 November 2007 (13.11.2007) Entire document	1-5
A	US 4,246,067 A (Linlor) 20 January 1981 (20.01.1981) Entire document	1-5

☐ Further documents are listed in the continuation of Box C.

☐ See patent family annex.

* Special categories of cited documents:

"A" document defining the general state of the art which is not considered to be of particular relevance

"D" document cited by the applicant in the international application

"E" earlier application or patent but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search

23 September 2022

Date of mailing of the international search report

06 December 2022 (06.12.2022)

Name and mailing address of the ISA/US

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P.O. Box 1450, Alexandria, Virginia 22313-1450
Facsimile No. 571-273-8300

Authorized officer

Kari Rodriguez

Telephone No. PCT Helpdesk: 571-272-4300

INTERNATIONAL SEARCH REPORT

International application No.

PCT/US 22/39157

Box No. II Observations where certain claims were found unsearchable (Continuation of item 2 of first sheet)

This international search report has not been established in respect of certain claims under Article 17(2)(a) for the following reasons:

1. ☐ Claims Nos.:
because they relate to subject matter not required to be searched by this Authority, namely:

2. ☐ Claims Nos.:
because they relate to parts of the international application that do not comply with the prescribed requirements to such an extent that no meaningful international search can be carried out, specifically:

3. ☐ Claims Nos.:
because they are dependent claims and are not drafted in accordance with the second and third sentences of Rule 6.4(a).

Box No. III Observations where unity of invention is lacking (Continuation of item 3 of first sheet)

This International Searching Authority found multiple inventions in this international application, as follows:

-- See Extra Sheet --

1. ☐ As all required additional search fees were timely paid by the applicant, this international search report covers all searchable claims.
2. ☐ As all searchable claims could be searched without effort justifying additional fees, this Authority did not invite payment of additional fees.
3. ☐ As only some of the required additional search fees were timely paid by the applicant, this international search report covers only those claims for which fees were paid, specifically claims Nos.:
4. ☒ No required additional search fees were timely paid by the applicant. Consequently, this international search report is restricted to the invention first mentioned in the claims; it is covered by claims Nos.:
1-5

Remark on Protest

- ☐ The additional search fees were accompanied by the applicant's protest and, where applicable, the payment of a protest fee.
- ☐ The additional search fees were accompanied by the applicant's protest but the applicable protest fee was not paid within the time limit specified in the invitation.
- ☐ No protest accompanied the payment of additional search fees.

INTERNATIONAL SEARCH REPORT

International application No.

PCT/US 22/39157

Continuation of:

Box III Observations where unity of invention is lacking

This application contains the following inventions or groups of inventions which are not so linked as to form a single general inventive concept under PCT Rule 13.1. In order for all inventions to be searched, the appropriate additional search fees must be paid.

Group I: Claims 1-5 directed to a method for particle acceleration, comprising: providing particles in a zero or low magnetic field; causing the particles to be in cyclotron motion in a magnetic field that is strong compared to a momentum of the particles, the particles having a gyroradius that is small compared to a transverse dimension of an injection aperture through which the particles will travel, wherein the magnetic field has a transverse gradient along an average path of the particles; and utilizing a complementary electric field to balance a gradient-B drift transverse to the average path of the particles and accelerate the particles under work of the transverse gradient.

Group II: Claims 6-14 directed to a magnetic orbital angular momentum beam accelerator, comprising: a tapered dipole magnet winding configured to have a magnetic field positioned to allow particles to enter the tapered dipole magnet winding, the magnetic field being a low magnetic field configured to cause the particles to begin cyclotron motion, and has a magnetic field gradient that is a transverse gradient along an average path expected of the particles; and a field cage comprising a plurality of electrodes, configured to form a complementary electric field to balance a gradient-B drift transverse to the average path of a beam of the particles and accelerate the particles under work of the magnetic field gradient.

The inventions listed as Groups I-II do not relate to a single general inventive concept under PCT Rule 13.1 because, under PCT Rule 13.2, they lack the same or corresponding special technical features for the following reasons:

Special Technical Features:

Group I requires a method for particle acceleration, comprising: providing particles having a gyroradius that is small compared to a transverse dimension of an injection aperture through which the particles will travel, not specifically required by Group II.

Group II requires a magnetic orbital angular momentum beam accelerator, comprising: a tapered dipole magnet winding configured to have a magnetic field positioned to allow particles to enter the tapered dipole magnet winding, not specifically required by Group I.

Common Technical Features:

Groups I and II share the technical feature of a method comprising: providing a low magnetic field configured to cause particles to begin cyclotron motion, where the magnetic field has a transverse gradient along an average path of the particles; and providing a complementary electric field to balance a gradient-B drift transverse to the average path of the particles and accelerate the particles under work of the transverse gradient.

However, these shared technical features do not represent a contribution over prior art, because the shared technical feature is being obvious over US 8,031,824 B2 to Bystriskii et al. (hereinafter 'Bystriskii') in view of US 7,294,969 B2 to Kuninaka. Bystriskii discloses a method (col 17 ln 20-24 - '...As explained above, the ion beams are trapped within the chamber 310 in large betatron orbits by this magnetic field. The ion beams may be generated by an ion accelerator, such as an accelerator comprising an ion diode and a Marx generator...'; see instant claim 2) comprising: providing a low magnetic field configured to cause particles to begin cyclotron motion (col 17 ln 17-20 - '...At short intervals, low energy ion beams are injected into the chamber 310 through the injector ports 340 substantially transverse to the externally applied magnetic field within the chamber 310...'; col 19 ln 24-26 - '...The base FRC established by this method exhibits modest magnetic field and particle energies that are typically not at reactor relevant operating parameters...'), where the magnetic field has a transverse gradient along an average path of the particles (col 17 ln 17-20 - '...At short intervals, low energy ion beams are injected into the chamber 310 through the injector ports 340 substantially transverse to the externally applied magnetic field within the chamber 310...'); and providing a complementary electric field with a drift that may be a gradient shift to accelerate the particles under work of the gradient shift (col 12 ln 28-29 - '...The direction of a gradient drift is given by the cross product $B \times \nabla B$, where ∇B is the gradient of the magnetic field...'; col 15 ln 44-54 - '...If there is a strong electrostatic field, the $E \times B$ drift dominates the gradient drift, and the drift orbits rotate in the diamagnetic direction... The electrostatic fields in the colliding beam system may be strong enough, so that the $E \times B$ drift dominates the gradient drift. Thus, the electrostatic field of the system would avoid ion transport by eliminating this ion loss mechanism, which is similar to a loss cone in a mirror device...'), but does not further specifically disclose the method comprising: providing a complementary electric field to balance a gradient-B drift transverse to the average path of the particles and accelerate the particles under work of the transverse gradient.

-- See Extra Sheet --

INTERNATIONAL SEARCH REPORT

International application No.

PCT/US 22/39157

Continuation of:

Box III Observations where unity of invention is lacking

Kuninaka discloses a similar method (Abstract - "...The two-stage Hall-effect plasma accelerator of the present invention can control the ion acceleration and the plasma generation in a highly independent manner..."; col 12 ln 57-60 - "...Each of the above inventions defined by a product claim may be figured out as a method in which respective functions of structural elements of the invention are sequentially executed..."), wherein a magnetic field may be utilized (col 12 ln 3-19 - "...The ion acceleration section includes... magnetic-field generation means for generating a magnetic field having a given intensity distribution in the radial direction from the central axis of the annular acceleration channel..."), along with an electric field (col 15 ln 18-28 - "...Thus, the requirement for the magnetic field distribution is eased as compared to the conventional Hall-effect plasma accelerator 100. This is one advantage of using a high-frequency wave for the plasma generation, as will be described in detail later. The anode 206 is placed at an upstream end of the acceleration channel 202 on the opposite side of an outlet of the acceleration channel 102, and adapted to cooperate with the cathode 207 disposed in the vicinity of the downstream outlet to generate an electric field therebetween, and accelerate ions residing in the acceleration channel 202 by the electric field..."), wherein a gradient-B drift of the fields may be used to control electron trajectory in order to further accelerate the particles with cyclotron motion (col 21 ln 32-45 - "... In FIG. 9, the magnetic field has an intensity gradient in a direction as indicated by the arrow (gradient)B (grad B), and a magnetic field direction in a direction as indicated by the arrow B. In this magnetic field, an electron has a (gradient)B×B drift, and drifts in a direction as indicated by the arrow (gradient)B×B Drift. An electron 355 is rotated around the magnetic line according to a Larmor electron motion 356, and moved along an electron trajectory due to the (gradient)B×B drift and the confinement by the mirror field 353. During this movement, the electron is reflected by a mirror field reflection region 357. Then, when the moving electron passes through the ECR region 354, it is further accelerated by the electron cyclotron resonance to facilitate ionization...). It would have therefore been obvious to one of ordinary skill in the art to combine these references, and to create the method comprising: providing a complementary electric field to balance a gradient-B drift transverse to the average path of the particles and accelerate the particles under work of the transverse gradient, as opposing gradient-B drifts are known in the art to increase particle acceleration and transport plasma within an applied magnetic field (see the document entitled, "Pulse shaping of high-current electron beams with gradient B drift transport" to Lee, see Abstract - "...An intense electron beam can be transported through a plasma via grad B drift in an applied magnetic field..."; see the document entitled, "Acceleration of electrons by a nearly perpendicular shock-The dynamics of electron motion" to Vandas, see Abstract, "Due to the grad-B drift, electrons gain energy..."; page 195 col 1 para 2 - "...Due to the grad-B drift the electrons gain energy, this drift prevails and the electrons are accelerated...").

As the shared technical features were known in the art at the time of the invention, they cannot be considered common technical features that would otherwise unify the groups. Therefore, Groups I-II lack unity under PCT Rule 13.