350-416 3R

OR 3,302,543

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Feb. 7, 1967 K. F. ROSS 3,302,543

WIDE-ANGLE DIOPTRIC SYSTEM

Filed Oct. 21, 1963 8 Sheets-Sheet 1

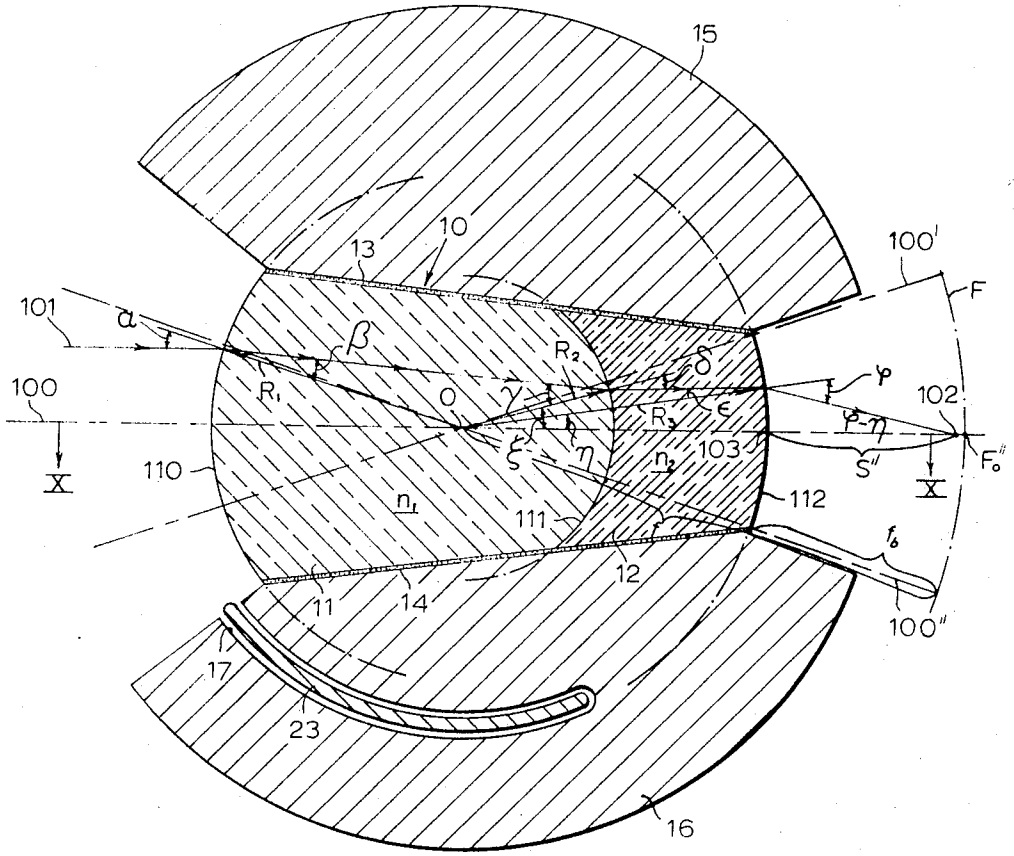


Fig. 1

X733

INVENTOR:
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WIDE-ANGLE DIOPTRIC SYSTEM

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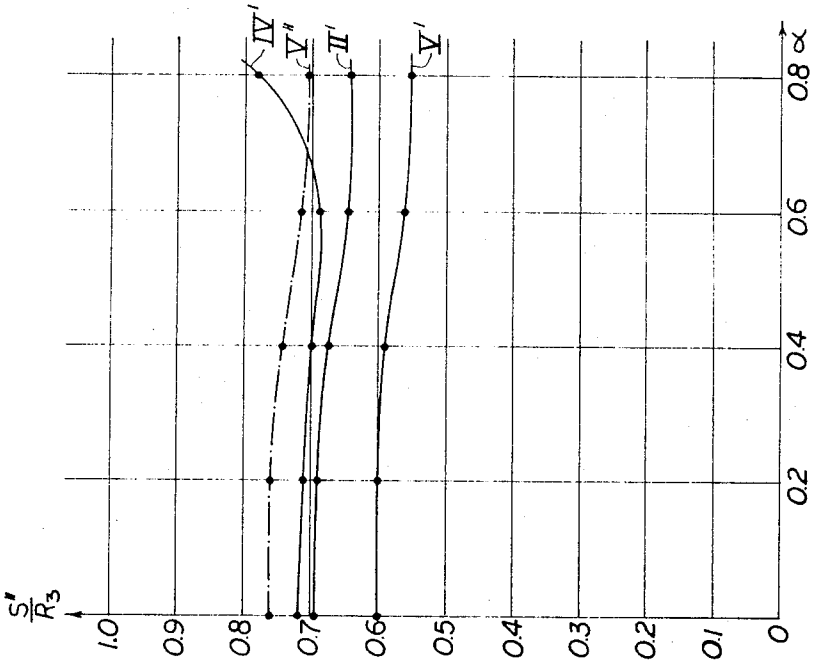


Fig. 5

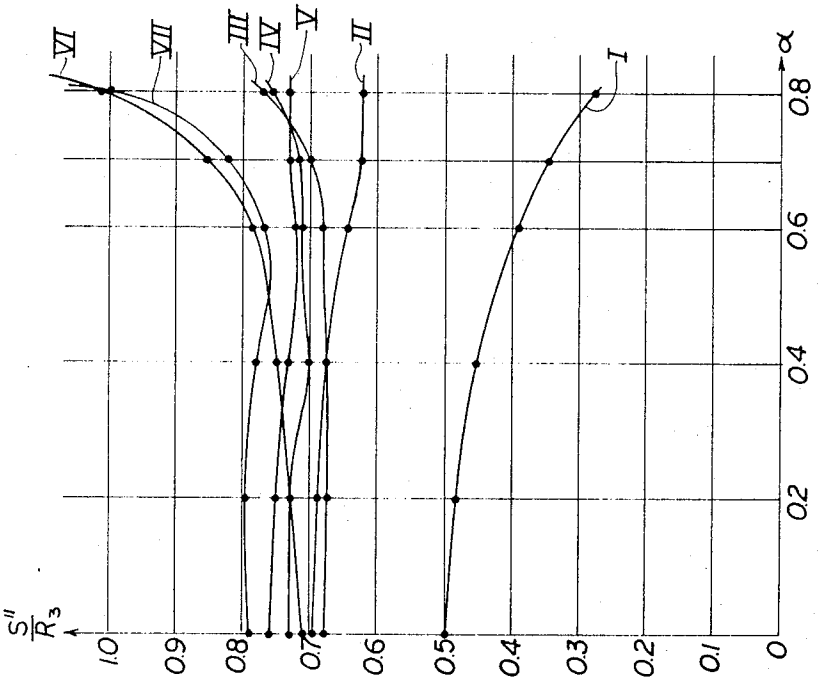


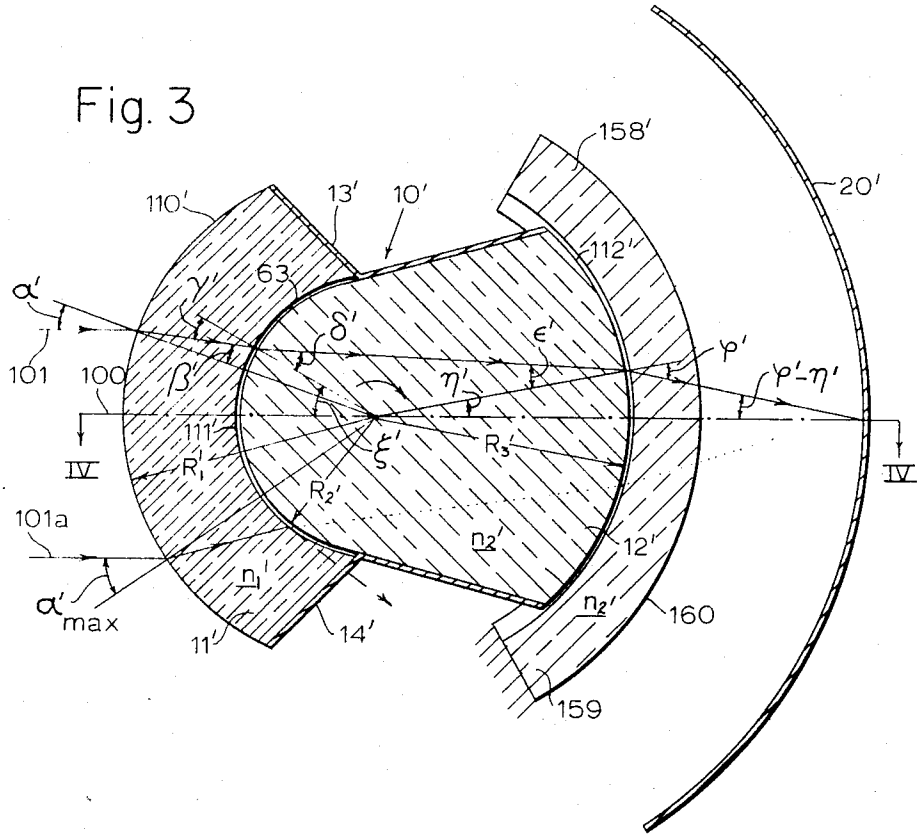
Fig. 2

INVENTOR:
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WIDE-ANGLE DIOPTRIC SYSTEM

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Fig. 3



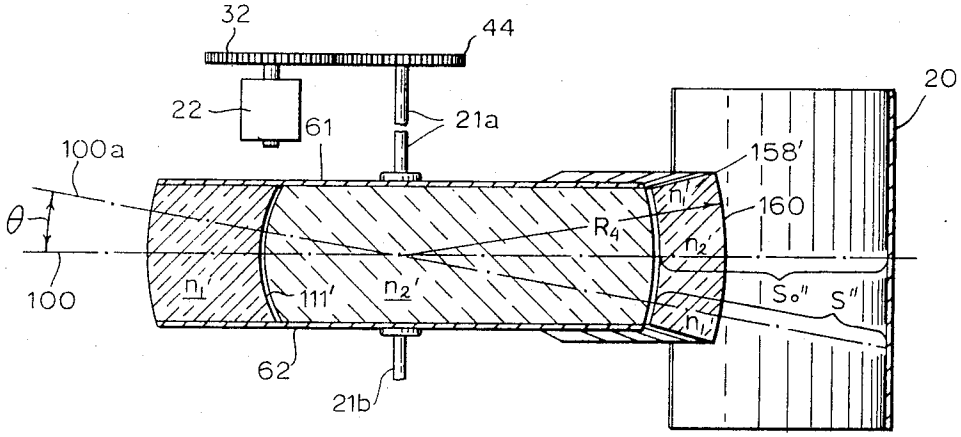


Fig. 4

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WIDE-ANGLE DIOPTRIC SYSTEM

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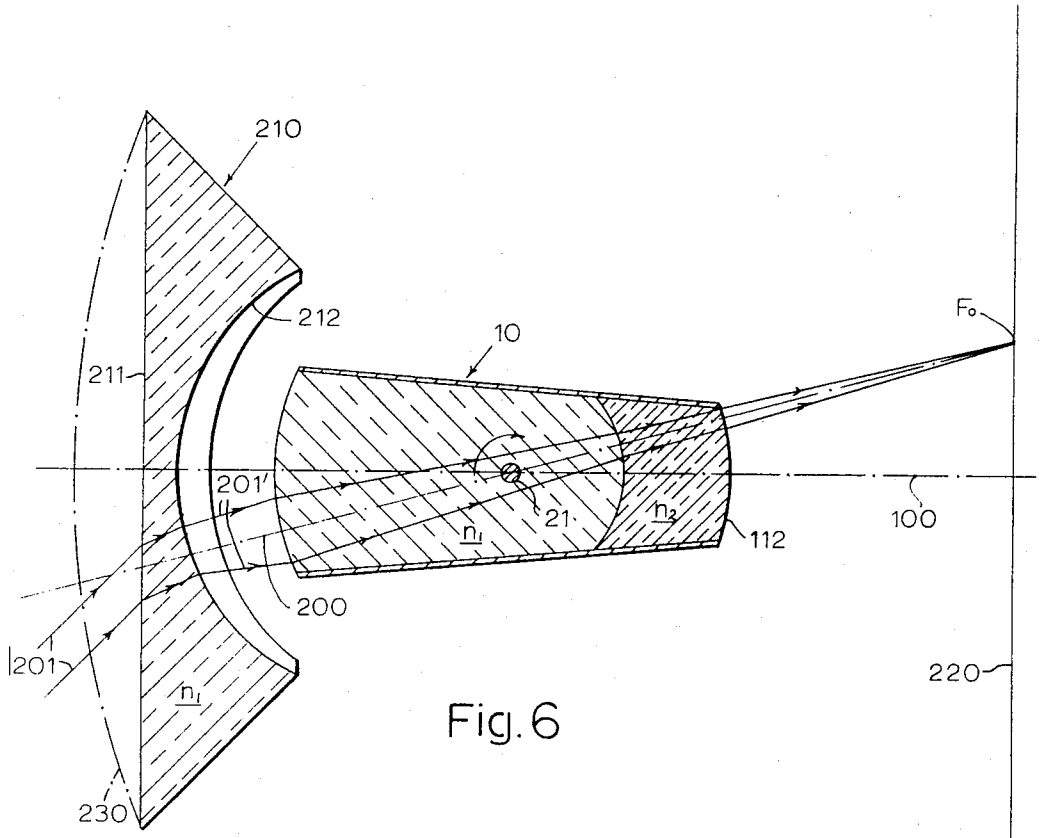


Fig. 6

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WIDE-ANGLE DIOPTRIC SYSTEM

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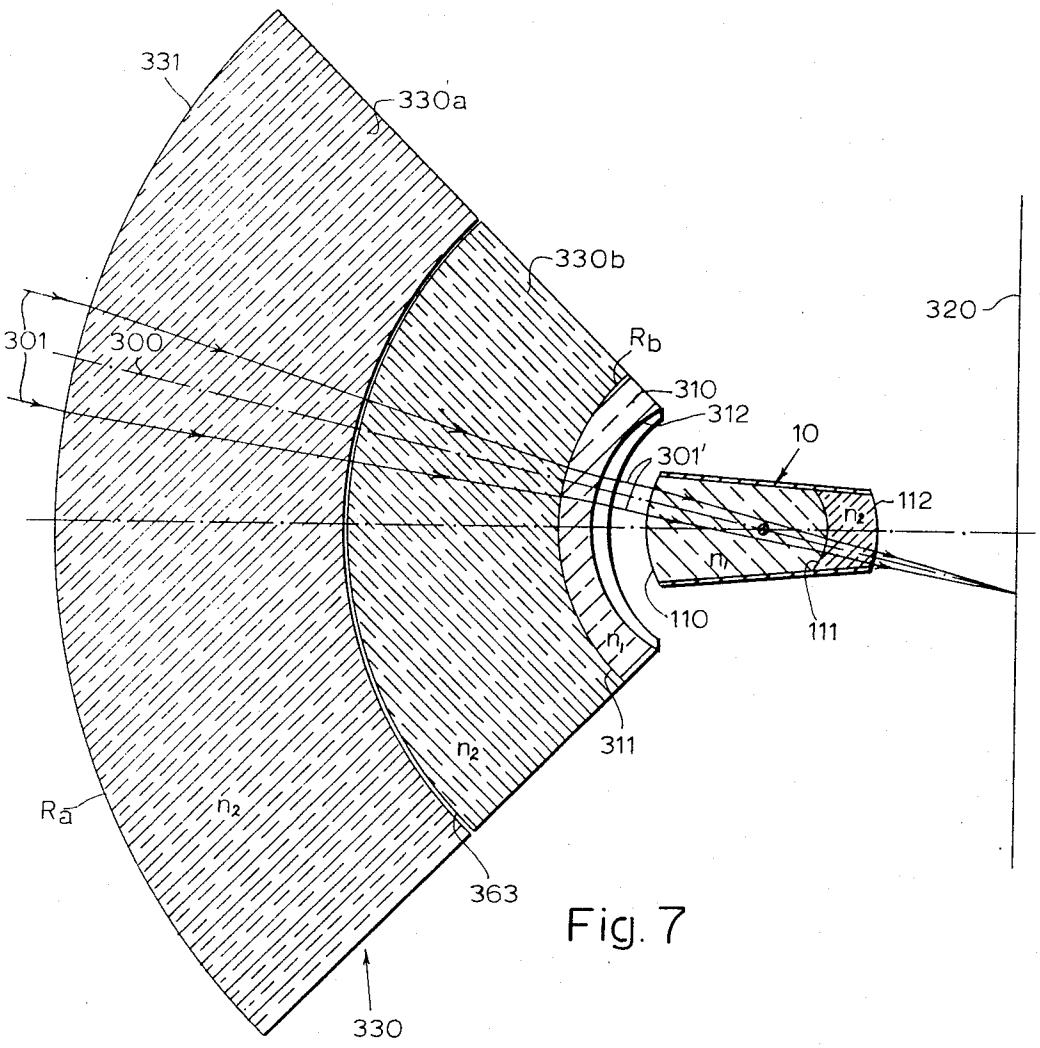


Fig. 7

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WIDE-ANGLE DIOPTRIC SYSTEM

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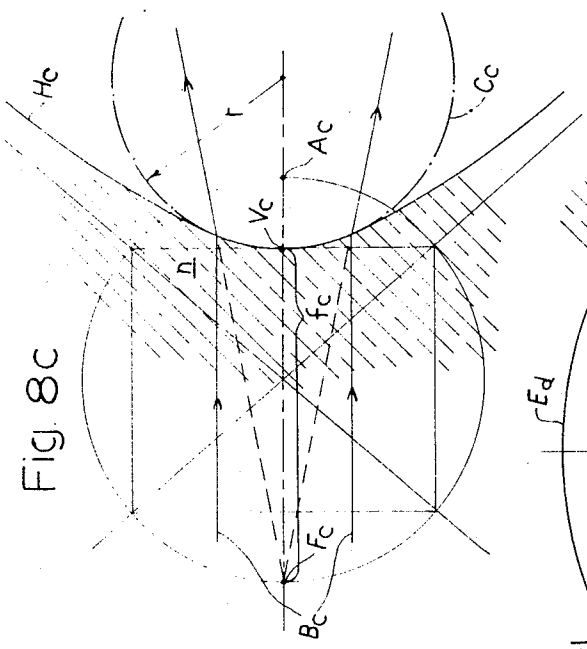


Fig. 8c

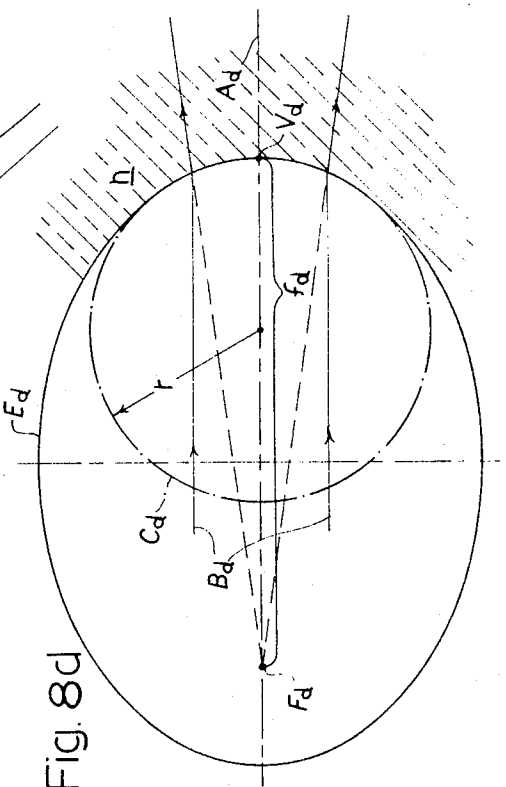


Fig. 8d

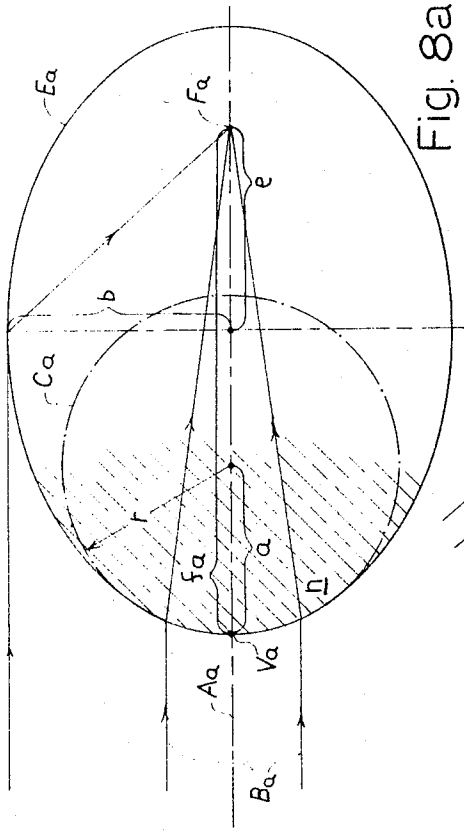


Fig. 8a

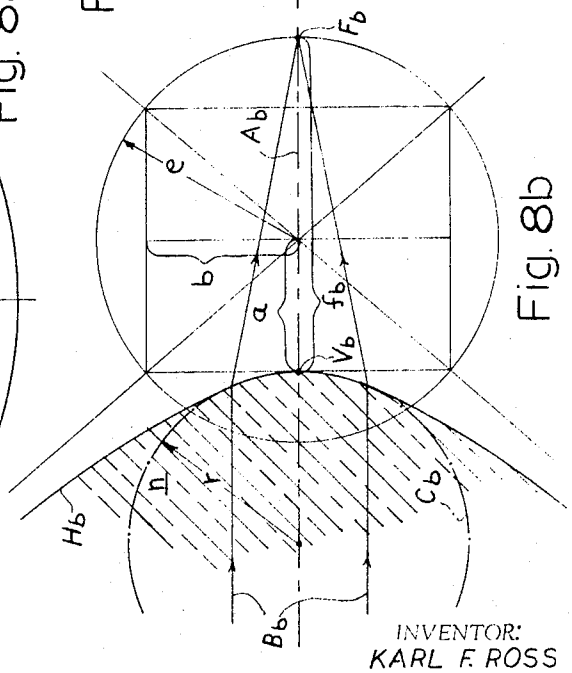


Fig. 8b

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WIDE-ANGLE DIOPTRIC SYSTEM

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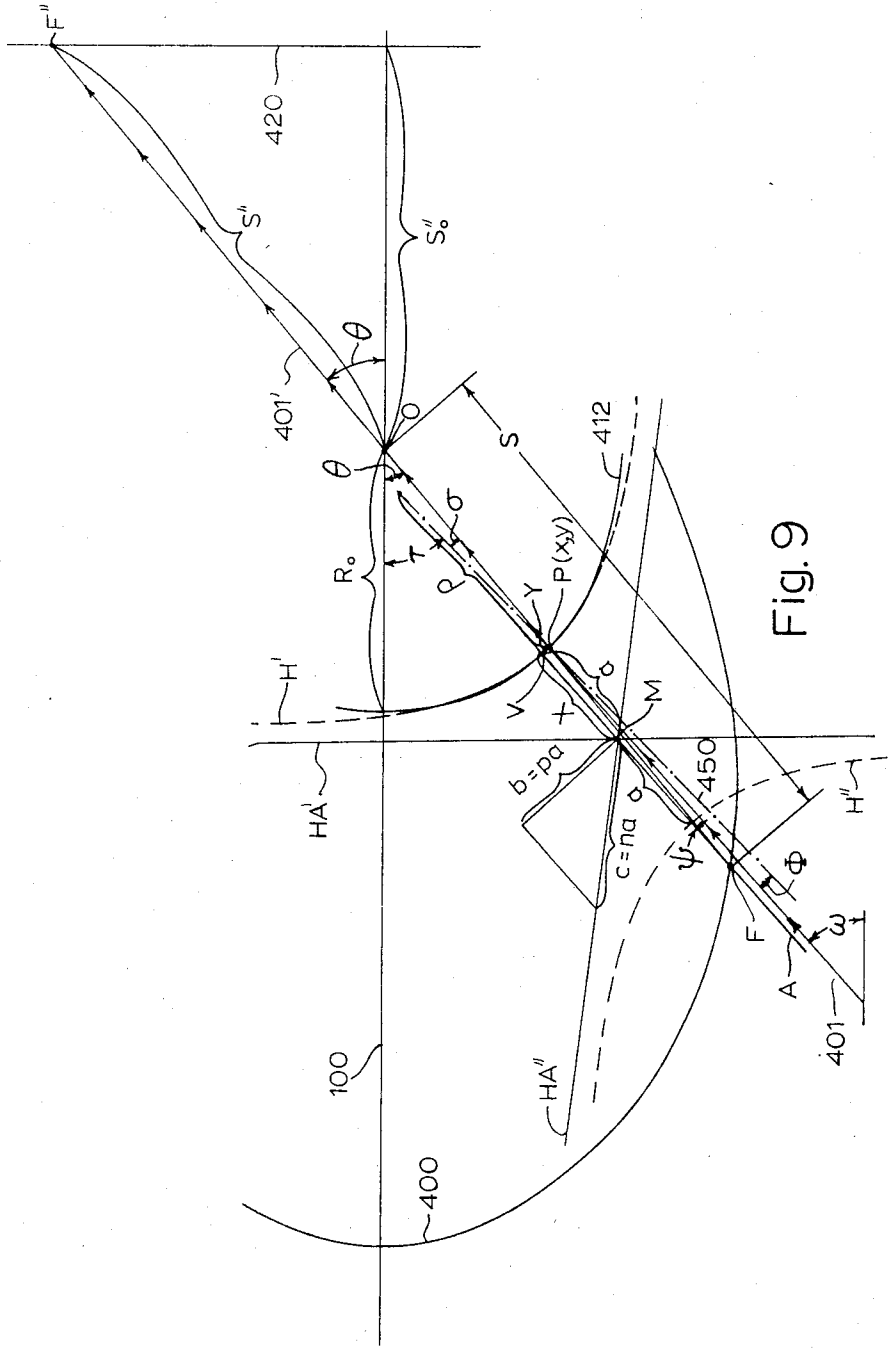


Fig. 9

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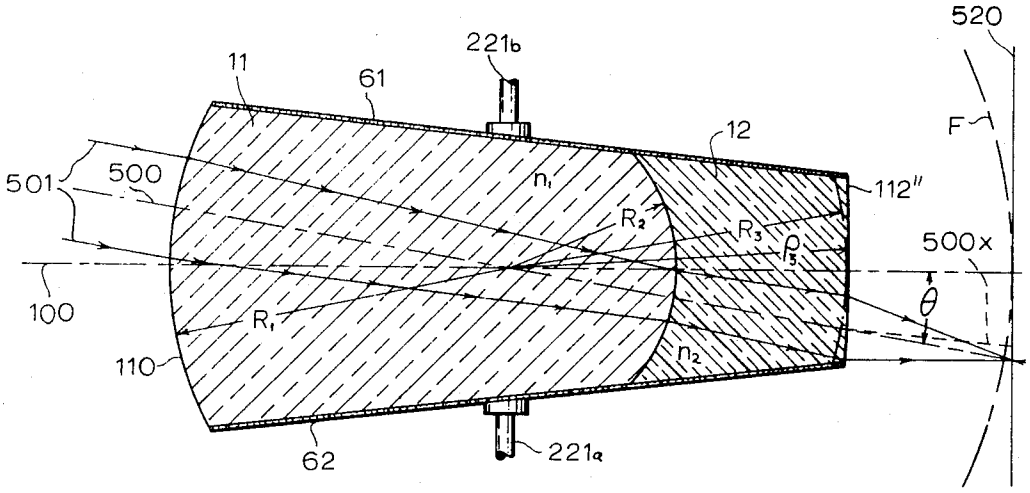


Fig. 10

INVENTOR:
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United States Patent Office

3,302,543

Patented Feb. 7, 1967

3,302,543

WIDE-ANGLE DIOPTRIC SYSTEM

Karl F. Ross, 5121 Post Road, Bronx, N.Y. 10471

Filed Oct. 21, 1963, Ser. No. 317,724
3 Claims. (Cl. 95—16)

My present invention relates to a dioptric system of the type using a lens or lens combination whose optically effective surfaces are all centered on a common point or axis. Lenses of this character have been disclosed in U.S. Patents Nos. 2,923,220 and 3,044,379, issued Feb. 2, 1960 and July 17, 1962 to A. Bouwers.

Lenses with concentric or coaxial surfaces (for spherical or cylindrical refractivity, respectively) are particularly suitable for wide-angle or panoramic picture taking or projection, owing to the fact that rays incident at a variety of angles can be sharply focused; they also offer the designer the advantage that their principal points coincide with the center of curvature so that the focal length of any such lens, whether simple or compound, is always measured from that center. (In the present context, compound lenses are those whose internal refractive surfaces are concentric or coaxial with the outer lens surfaces.) As is the case with other leneses of circular curvature, however, these lenses provide satisfactory image definition only over a limited field angle for each direction of incidence since only the so-called paraxil rays, i.e., those passing through the lens center or in the immediate vicinity thereof, will focus sharply. It has, therefore, been necessary in the past to provide various types of diaphragms for limiting the effective field angle, with a resultant reduction in the relative aperture of the lens.

An object of my invention is to provide an improved lens of this type in which the field of converging rays is widened so that larger relative apertures may be used.

Another object of my invention is to provide means for increasing the luminous intensity of a concentric or coaxial objective of given design.

A further object of my instant invention is to provide means for effectively flattening the image plane of a spherical or cylindrical objective of the concentric or coaxial type.

It is also an object of the instant invention to provide a high-speed, wide-angle, substantially achromatic dioptric component for photographs and related optical systems.

In the following description I shall generally refer to concentric surfaces, it being understood that—unless otherwise noted—the term is intended to encompass both spherical and cylindrical lens surfaces.

It will be convenient to designated by R_1 the radius of curvature of the front surface of a lens to be discussed (i.e., the surface on the side of the incident or longer light rays), by R_3 and the radius of the opposite or rear surface, and by R_2 the radius of any concentric cemented surface therebetween. Furthermore, let

$$\left|\frac{R_1}{R_2}\right| \equiv K_1 \text{ and } \left|\frac{R_1}{R_3}\right| \equiv K_2$$

it being assumed that R_1 is positive and R_2, R_3 are negative according to conventional notation. Also, the lens portion limited by the surfaces R_1 and R_2 shall have a refractive index n_1 whereas the lens portion limited by the surfaces R_2, R_3 shall have an index $n_2 > n_1$.

It can be shown that the focal length f of a lens so defined is given by the expression

$$f = \frac{R_1}{E-K_2} = R_3\frac{K_2}{E-K_2} \tag{1}$$

where

$$E \equiv \frac{1+K_1}{n_1} - \frac{K_1-K_2}{n_2} - 1 \tag{2}$$

The back-focal length f_b is then given by

$$fb = R_3\frac{E}{E-K_2} \tag{3}$$

and should have a positive value if an object at infinity is to form a real image at a location beyond the lens. Since the internal radius R_2 of the doublet must be smaller than its external radius R_3, it is necessary that $K_1 > K_2$ so that, for $E>0$, $K_1 > n_1 - 1$.

In a conventional spherical or cylindrical lens lacking a cemented surface i.e. a singlet, $|R_1| = |R_3| = R$ and $n_1 = n_2 = n$, R_2 being of indeterminate value owing to the uniformity of the glass. In this case the formula for the back-focal length reduces to

$$fb = R\frac{2-n}{2(n-1)} \tag{3a}$$

With such a lens, in which $K_2 = K = 1$, the back-focal distance s'' for rays with increasing angle of incidence α decreases rather sharply from the value $s_o'' = f_b$ which is valid for $\alpha = 0$. I have found that the relationship $s'' \approx f_b$ may be maintained over a relatively wide range of α if the value of K_2 is close to unity and, preferably, ranges between approximately 0.9 and 1.1. The value of K_1 may be roughly equal to $2K_2$, being advantageously held approximately to the range $2 \geqq K_1 \geqq 1.8$ in order to establish a relatively wide entrance pupil.

A feature of my invention resides in the provision of means for rotating a lens of this description about an axis passing through its center of curvature, at least the front surface of the lens being limited in the plane of rotation to a circular arc subtending an angle not greater than that from which sharp definition of the image is obtainable. As the lens rotates, different proportions thereof focus the same incident rays upon a given point of a concentric receiving surface so that this point is exposed to more light than would be the case if the lens were stationary. Owing to the perfect circularity of each concentric lens surface, the point of convergence of the rays from a given distant object does not change as the lens rotates. In this manner the need for a special diaphragm to restrict the effective field angle is avoided, it being however possible to reduce this field angle selectively by the provision of an adjustable light shield if the luminous intensity of the objective is to be decreased in view of existing light conditions. It will be understood that the rotation of the segmental lens need not be continuous and unidirectional but may consist of alternate sweeps in opposite directions if desired.

The above-described rotatable systems affords a wide angle of view in one plane and, for proper projection, requires that the sensitive film or other receiving surface (e.g. a ground-glass plate used for visual observation) be curved about the axis of rotation of that plane. In some instances, e.g. with panoramic cameras, the dimension of the projected image transverse to the plane of rotation may be so small that the field curvature is negligible; if it is not, a conventional flattening lens (e.g. as known from the aforementioned Bouwers Patent No. 3,044,379) may be fixedly interposed between the film and the concentric lens or may be rotatable with the latter. In accordance with a more specific feature of my invention, however, I may provide a circular flattening lens adapted to be placed in front of the (rotating or stationary) concentric lens, the parameters of the flattening lens being so chosen that the value of f_b changes proportionally to the secant of the angle of incidence so that the resulting image surface will be a plane.

The invention will be described in greater detail with reference to the accompanying drawing in which:

FIG. 1 is a cross-sectional view of a concentric lens according to the invention;

FIG. 2 is a set of graphs illustrating the changes in relative image distance s''/R_3 with increasing angles of incidence α as plotted for difference parameters of the lens shown in FIG. 1;

FIG. 3 is a view similar to FIG. 1, illustrating an inversion;

FIG. 4 is a cross-sectional view taken on the line IV—IV of FIG. 3;

FIG. 5 is a set of graphs similar to FIG. 2, pertaining to the lens shown in FIGS. 3 and 4;

FIG. 6 shows the combination of a lens similar to that of FIG. 1 with a special flattening front lens therefor;

FIG. 7 is a view similar to FIG. 6, showing a modified form of flattening front lens;

FIGS. 8*a*–8*d* and 9 are graphs used in explaining the construction of the flattening lenses of FIGS. 6 and 7; and

FIG. 10 is a cross-sectional view of the lens of FIG. 1 in modified form, taken on the line X—X of that figure.

Reference will first be made to FIG. 1 for a discussion of the properties of a lens adapted to be used in a wide-angle exposure system according to this invention, it being understood that the lens could also be used in a projector by simple inversion of the direction of the light rays as is well known.

The lens **10** shown in FIG. 1 is a doublet composed of a positively refracting first member **11** and a negatively refracting second member **12**. Member **11** has a refractive index n_1 and is bounded by a front surface of radius R_1 and an internal surface of radius R_2 along which it is cemented onto member **12**; the latter has as its front boundary the cemented surface of radius R_2 and as its rear boundary an outer surface of radius R_3. All the radii are measured from a point **0** serving as the common center of the surfaces referred to.

For the present, only tne cross-section of the lens visible in FIG. 1 is to be considered. In its transverse plane the lens **10** may have the cross-section illistrated in FIG. 10 where its front and internal surfaces are shown to have the same radii R_1 and R_2 (being thus segments of concentric spheres) whereas the rear surface has an aspherical configuration more fully described hereinafter. In some instances, however, all the lens surfaces may be spherical, e.g. in the event that the lens **10** is used together with one of the flattening lenses described hereinbelow.

As shown in FIG. 1, R_1 is positive, R_2 is negative and smaller than R_1 so that

$$K_1 \equiv \left|\frac{R_1}{R_2}\right| > 1$$

and R_3 is negative and larger than R_1 so that

$$K_2 \equiv \left|\frac{R_1}{R_3}\right| < 1$$

It is also assumed that n_1 is smaller than n_2 in order that lens member **12** may be of dispersive character. The axial thicknesses of the members **11** and **12** are $|R_1|+|R_2|$ and $|R_3|-|R_2|$, respectively.

The axis **100** passes through the center O and lies at the intersection of two mutually perpendicular planes of symmetry of the lens. At **101** a light ray parallel to axis **100** impinges on the front surface **110** of lens member **11**, including an angle α with the radius R_1 thereof. Upon refraction, the ray continues inside lens member **11** at an angle β with reference to radius R_1 and strikes the internal surface **111** between lens members **11** and **12** at an angle γ with reference to its radius R_2. This internal surface, being negatively refracting, further deflects the ray so that its angle with radius R_2 within member **12** will have the value δ. As the ray strikes the rear surface **112** of member **12**, it includes with the radius R_3 of that surface an angle ϵ which changes to φ as the ray emerges after further refraction. If ξ and η are the angles which the radii R_2, R_3 respectively include with axis **100**, the emerging ray approaches this axis at an angle φ—η and intersects it at a point **102** which, for small values of α, is in the vicinity of point F_o'' representing the focal point of the lens on axis **100**. The distance s'' between point **102** and the rear vertex **103** of the lens along axis **100** is thus substantially equal to the back-focal length f_b of the lens measured between this vertex and focal point F_o''. It will be apparent that, owing to the circularity of the lens surfaces, other focal points will exist which have the same distance f_b from the rear lens surface **112** and which lie on other axes passing in different directions through the center O, such as the axes **100'** and **100''**. The locus of all these focal points is the curve F which is a circular arc of radius f centered on O, f being the focal length of the system.

In accordance with the laws of geometry and optical refraction it can be shown that the following relationships obtain:

$$\sin\beta = \frac{\sin\alpha}{n_1} \tag{4}$$

$$\sin\gamma = \frac{R_1\sin\alpha}{R_2 n_1} = K_1\sin\beta \tag{5}$$

$$\sin\delta = \frac{R_1\sin\alpha}{R_2 n_2} = \frac{n_1\sin\gamma}{n_2} \tag{6}$$

$$\sin\epsilon = \frac{R_1\sin\alpha}{R_3 n_2} = \frac{K_2\sin\delta}{K_1} \tag{7}$$

$$\sin\varphi = \frac{R_1\sin\alpha}{R_3} = n_2\sin\epsilon \tag{8}$$

$$\xi = \beta+\gamma-\alpha \tag{9}$$

$$\eta = \xi+\epsilon-\delta = \beta+\gamma-\alpha-\delta+\epsilon \tag{10}$$

As will be readily apparent from FIG. 1, point **102** is separated from center O by a distance

$$R_3+s'' = R_3\cos\eta + R_3\frac{\sin\eta}{\tan(\varphi=\eta)} \tag{11}$$

For small values of α we can write:

$$\varphi \approx \sin\varphi = K_2\sin\alpha \approx K_2\alpha \tag{8a}$$

and

$$\eta \approx \frac{\alpha}{n_1}+\frac{K_1\alpha}{n_1}-\alpha-\frac{K_1\alpha}{n_2}+\frac{K_2\alpha}{n_2} = E\alpha \tag{10a}$$

where

$$E \equiv \frac{1+K_1}{n_1}-\frac{K_1-K_2}{n_2}-1 \tag{2}$$

Thus, with $\sin\eta\approx\eta$ and $\tan(\varphi-\eta)\approx\varphi-\eta$, we can simplify Equation 11 for paraxial rays to obtain

$$R_3+s'' \approx R_3\left(1+\frac{\varphi-\eta}{\eta}\right) \approx R_3\left(1+\frac{E}{K_2-E}\right) \tag{11a}$$

whence, for $\alpha=0$,

$$s_o'' \equiv f_b = R_3\frac{E}{K_2-E}$$

The lens **10** is shown bounded along its sides by rearwardly converging opaque strips **13**, **14** of nonreflecting character. The separation of these strips, i.e. the thickness of the lens in the plane seen in FIG. 1, is so selected that the arcs intercepted by them on the entrance and exit surfaces **110**, **112** of the lens are limited to regions within which $s''\approx f_b$. When the lens is rotated about its center O, its axis **100** together with the field defined by the limiting rays **100'**, **100''** ($\alpha=\pm\alpha_{max}$) will sweep acros the surface F through an angle which may be much larger than $2\alpha_{max}$ and in practice may approach 180°. In specific cases, e.g. with slanted incident rays as shown in the two Bouwers patents referred to, the angle of exposure and projection may be extended to a full 360°.

Two opaque segmental bodies **15** and **16** are secured to the lens **10** outside its surfaces **13** and **14** to block the passage of stray light rays when the lens is rotated

about a transverse axis through point O as described hereinafter with reference to FIG. 4. Segment **16** is formed with a slot **17** adapted to receive an arcuate light shield **23** which is adjustable with reference to the lens.

Since, from equations 4 and 5,

$$\sin \lambda = \frac{K_1 \sin \alpha}{n_1}$$

the value of α_{max} is also determined from the relationship

$$\sin \alpha_{max} = \frac{n_1}{K_1} \qquad (12)$$

inasmuch as sin γ cannot be larger than unity. This is further borne out by inspection of FIG. 1 which shows that a considerable reduction of R_2 below the magnitude illustrated would require a sharper convergence of the lens sides **13** and **14** with the result that some of the rays incident upon the front surface of the lens would strike these sides before reaching the cemented surface. With $n_1=1.5$, for example, α_{max} could not be substantially greater than 45° if $K_2=2$, regardless of any improvement in the deviation of s'' from f_b that may be effected by suitable selection of the other parameters of the system.

In FIG. 2 I have shown a series of graphs II to VII in which s''/R_3 has been plotted against α for different values of K_1 and K_2. Curve I, included for purposes of comparison, relates to a homogeneous spherical lens wherein $R_1=R_3=R$ (there being no internal refractive surface of radius R_2) and $n_1=n_2=1.5$, the ratio f_b/R being computed as 0.5. It will be noted that the value of s''/R_3 drops at a steadily increasing rate with rising values of α. Curves II to VII, on the other hand, apply to lenses of the type illustrated in FIG. 1 having the following parameters:

TABLE A

Curve	n_1	n_2	K_1	K_2	f_b/R_3	α_{max}
II	1.5	1.75	1.8	1	0.696	0.99
III	1.5	1.75	1.9	0.9	0.68	0.9
IV	1.5	1.75	1.9	1	0.725	0.9
V	1.5	1.75	1.9	1.1	0.76	0.9
VI	1.5	1.75	2	0.925	0.715	0.84
VII	1.5	1.75	2	1.1	0.795	0.84

It will be seen from FIG. 2 that, for values of K_1 and K_2 in the vicinity of 2 and 1, respectively, the value of s''/R_3 remains substantially constant for angles of incidence α up to approximately 0.6 and above. It is interesting to note that, for higher values of α, the curve tends to droop when K_1 is small and/or K_2/K_1 is large (curves II and V) but tends to rise when the situation is reversed (curves III, VI, VII). Particularly good results are observed with $K_1 \approx 1.9$ and K_2 ranging within about ±10% of unity, as is true of the systems represented by curves III, IV and V; the last-mentioned curve has exceptional flatness up to and beyond $\alpha=0.8$.

The indicated values for n_1 and n_2 are not critical but have been chosen near the lower and upper limits of the range of available refractive indices; if the difference between n_1 and n_2 is materially reduced, the lens begins to approximate a homogeneous one with the characteristic illustrated by graph I in FIG. 2.

In accordance with Equation 12 the field of view is limited by the value of K_1 to an angle α_{max} as given in the foregoing table. Thus, K_1 cannot be substantially greater than 2 if a field close to 90° is desired.

With the listed values for n_1 and n_2, E as derived from Equation 2 assumes the value

$$\frac{2K_1-7+12K_2}{21}$$

whence, according to Equation 3,

$$\frac{fb}{R_3} = \frac{12K_2+2K_1-7}{9K_2-2K_1+7}$$

in order that both E and f_b/R_3 be positive, K_2 (which of course must be smaller than K_1) has to be greater than 1/4 for $K_1=2$ and greater than 1/3 for $K_1=1.5$. Actually, the lower limit of K_2 is also determined by the requirement that ξ and η, as given in Equations 9 and 10, be greater than zero for all angles of incidence up to α_{max}; under the assumed conditions of refractivity it then becomes necessary that, e.g. for a field of view encompassed by $\alpha_{max}=\pm 1$, $K_2>0.47$ *if* $K_1=1.5$. In practice it will usually be desirable to make f_b equal to a major fraction of R_3, preferably upwards of about $R_3/2$, which imposes further limitations upon the range within which K_2 can be varied; thus, with $K_1=1.8$ and a field of view as stated above, K_2 must be chosen greater than 0.72 if the back-focal length is to exceed half the radius of the rear surface **112** of the lens.

The Equation 3 for the back-focal length may also be written in the form

$$\frac{fb}{R_3} = \frac{K_2 n_1 + K_1(n_2-n_1) - n_2(n_1-1)}{n_1 K_2(n_1-1) - K_1(n_2-n_1) + (n_2-1)} \qquad (3b)$$

which is more convenient for calculating the dispersion factors ν_1, ν_2 of the two cemented lens members **11**, **12** that may be used in order to make the lens **10** substantially achromatic. For this purpose it will be assumed that the value of

$$\nu = \frac{n_d-1}{n_f-n_c}$$

is proportional to

$$\frac{n_d-1}{n_d-n_c}$$

for the optical glasses considered, with $n_d-n_c \equiv \Delta >> n_d$, so that

$$\frac{\nu_1}{\nu_2} = \frac{\Delta_2(n_1-1)}{\Delta_1(n_2-1)} \qquad (13)$$

If, now, the refractive power of the lens **10** is to be the same for light rays of the D line and of the C line of the spectrum, then Equation 3*b* must be valid not only for $n_1 \equiv n_{d1}$ and $n_2 \equiv n_{d2}$ but also for $n_1-\Delta_1=n_{c1}$ and $n_2-\Delta_2=n_{c2}$. Hence,

$$\frac{K_2 n_1 + K_1(n_2-n_1) - n_2(n_1-1)}{n_1 K_2(n_2-1) - K_1(n_2-n_1) + n_2(n_1-1)} = \frac{\begin{array}{c} K_2(n_1-\Delta_1) + K_1(n_2-n_1-\Delta_2+\Delta_1) \\ -(n_2-\Delta_2)(n_1-\Delta_1- \end{array}}{\begin{array}{c} 1)(n_1-\Delta_1)K_2(n_2-\Delta_2-1) - K_1(n_2-n_1-\Delta_2-\Delta_1) \\ +(n_2-\Delta_2)(n_1-\Delta_1-1) \end{array}} \qquad (14)$$

which, if one neglects the second-order terms Δ_1^2, Δ_2^2 and $\Delta_1.\Delta_2$, can be rewritten as

$$\frac{K_2\Delta_1 - K_1(\Delta_2-\Delta_1) - \Delta_2(n_1-1) - \Delta_1 n_2}{K_2[\Delta_1(n_2-1)+\Delta_2 n_1] - K_1(\Delta_2-\Delta_1) + \Delta_2(n_1-1) + \Delta_1 n_2} = \frac{K_2 n_1 + K_1(n_2-n_1) - n_2(n_1-1)}{n_1 K_2(n_2-1) - K_1(n_2-n_1) + n_2(n_1-1)} \equiv \frac{N}{D} \qquad (15)$$

reducing to

$$\Delta_1\{(K_2-K_1-n_2)D - [K_2(n_2-1)+K_1+n_2]N\} = \Delta_2[(K_2 n_1 - K_1 + n_1 - 1)N - (K_1-n_1+1)D] \qquad (16)$$

Given the values of n_1, n_2, K_1 and K_2, we can compute Δ_2/Δ_1 from Equation 16 and, from Equation 13, the ratio ν_1/ν_2 can be calculated. Following are several representative values:

TABLE B

n_1	n_2	K_1	K_2	Δ_2/Δ_1	ν_1/ν_2
1.5	1.75	1.9	1.1	5	3.33
1.5	1.75	1.9	0.9	4.04	2.69
1.5	1.75	2	1	4.1	2.74
1.5	1.75	2	0.925	3.72	2.48

The first, second and fourth lines of the foregoing Table B correspond to curves V, III and VI of Table A, respectively. It will be seen that the dispersion ratio of the two glasses decreases generally inversely with the ratio K_1/K_2, the requirement for achromatism being thus more easily satisfied with parameters which do not afford optimum focusing over the largest field of view. Curve III of FIG. 2 may be regarded as a suitable compromise if, for example, $\nu_1=25$ and $\nu_2=67$.

The lenses heretofore described are reversible, i.e. their optical behavior is similar when the direction of the light rays passing through them is inverted. Thus, I have shown in FIGS. 3 and 4 a lens **10'** which is substantially an inversion of the lens **10** of FIG. 1, its front and rear portions **11'**, **12'** being separated by a forwardly convex internal surface **111'** while the refractive index n_1' of front member **11'** is greater than the index n_2' of rear member **12'**. An incident ray **101**, parallel to axis **100**, includes with the radius R_1' of front surface **110'** an angle α' which changes to β' within the member **11'**: at the internal surface **111'** the ray includes with the radius R_2' of that surface an angle γ' within member **11'** and an angle δ' within member **12'**. The angle of incidence at the rear surface **112'**, with reference to its radius R_3', is ϵ' ahead of that surface and φ' beyond it; by analogy with FIG. 1, the angles included by the radii R_2', R_3' with the axis **100** have been designated ξ' and η', respectively.

With this inverted system, in which

$$\left|\frac{R_1'}{R_3'}\right| \equiv K_1'$$

and

$$\left|\frac{R_1'}{R_3'}\right| \equiv K_2'$$

the magnitude of E' in the expression

$$f_b = R_3' \frac{E'}{K_2' - E'} \tag{3'}$$

is given by

$$E' = \frac{K_1' + K_2'}{n_2'} - \frac{K_1' - 1}{n_1'} - 1 \tag{2'}$$

The following table lists, by way of example, the parameters of three lenses constituting inversions of the systems represented by the curves II, IV and V of FIG. 2 (see also Table A); the deviation curves of these three inverted lenses have been shown at II', IV' and V' in FIG. 5.

TABLE C

Curve	n_1'	n_2'	K_1'	K_2'	f_b/R_3'	$\frac{f_b+R_3'-R_1'}{R_1'}$
II'------	1.75	1.5	1.8	1	0.696	0.696
IV'------	1.75	1.5	1.9	1	0.725	0.725
V'------	1.75	1.5	1.73	0.91	0.6	0.76

The final column of Table C lists the value

$$\frac{f_b + R_3' - R_1'}{R_1'} = \frac{f_b}{R_1'} + \frac{1}{K_2'} - 1$$

which must be identical with the value f_b/R_3 given for the corresponding system in Table A and which is, of course, equal to f_b/R_3' whenever, as with the first two systems, $K_2'=1$. The curve V'' has been included in FIG. 5 to show, for purposes of better comparison with the curve of FIG. 2, the parameter

$$\frac{s''}{R_3'} + \frac{1}{K_2'} - 1$$

plotted against α'. It will be seen that the graphs II', IV' and V' of FIG. 5 are generally similar to the graphs II, IV and V, respectively, of FIG. 2.

The lens **10'**, as also the lens **10** of FIG. 1 (cf. FIG. 10), is armored along its sides with opaque and nonreflecting shielding **61**, **62** (FIG. 4) in addition to the light shields **13'**, **14'** visible in FIG. 3.

The means for rotating the sectoral lens **10** about its axis have been shown diagrammatically in FIG. 4 as comprising a drive motor **22** coupled with a two-part lens shaft **21***a*, **21***b* via gears **32**, **44**.

Furthermore, in contradistinction to lens **10**, the lens **10'** is formed with a narrow air gap **63** along its internal surface **111'**, this air gap separating the two portions **11'**, **12'** from each other. The effect of the air gap **63** is to cut off the light rays incident at an angle greater than the limiting angle α'_{max}, as illustrated for the ray **101**α, as these rays undergo internal reflection on reaching the surface **111'**. The value of α_{max} is given by the relationship

$$n_1'\gamma'_{max} = K_1'\alpha'_{max} = 1 \tag{12a}$$

whence

$$\alpha'_{max} = 1/K_1' \tag{12b}$$

Thus, the gap **63** acts as a virtual diaphragm limiting the field of incident rays for each beam direction. If this gap is sufficiently narrow, the rays that are not internally reflected will have substantially the path they would have if the lens members **11'**, **12'** were cemented together.

It will be apparent that the gap **63** need not be provided at the junction of the two differently refracting lens members, though this is most convenient, but could also be located within either of these members forwardly or rearwardly of surface **111'**. In the first case, i.e. if member **11'** is split concentrically, the value of α'_{max} will be greater than that given by Equation 12*b*; in the second case, i.e. if the gap is within member **12'**, it will be less. Naturally, such a gap could also be provided within the member **11** of lens **10** (FIG. 1), in order to reduce the maximum field angle to less than the value α_{max} given in Table A.

The use of a beam-limiting concentric gap in a spherical or cylindrical system is, of course, not limited to rotating sectoral lenses; in a stationary lens the gap could extend, for example, over an arc of 180° or even 360°.

Finally, I have shown in FIGS. 3 and 4 a stationary dispersive flattening lens **158'**, to the read of lens **10'**, whose front and back surfaces **159**, **160** are concentric with those of the rotating lens **10'** and whose optical effect derives from its varying refractive index n_v. The radius of surface **159** is here shown but slightly greater than R_3' (this being, of course, possible only if R_3' is at least equal to R_1' in order that lens **158'** should not interfere with the rotation of lens **10'**), the radius of surface **160** having been designated R_4. Within the central plane of rotation visible in FIG. 3, which includes the axis **100**, the value of n_v is relatively small and may substantially equal n_2'; in a direction transverse to that plane, as seen in FIG. 4, this index increases progressively to a relatively large value which may be substantially equal to n_1'. In order to afford the desired flattening of the image as projected upon the film **20'**, the back-focal distance s'' of the overall system should vary according to the relationship

$$s'' = \frac{R_4(1-\cos\theta) + s_o''}{\cos\theta} \tag{17}$$

θ being the angle of incidence of a ray **100***a* with reference to axis **100** in the plane of FIG. 4; the relationship between s'' and n_v may be expressed as

$$s'' = \frac{f_b R_3' R_4 n_v}{R_3' R_4 n_v - f_b(R_4 - R_3')(n_v - 1)} \tag{18}$$

whence, upon substitution of n_2' for n_v,

$$s_o'' = \frac{f_b R_3' R_4 n_2'}{R_3' R_4 - n_2' - f_b(R_4 - R_3')(n_2' - 1)} \tag{18a}$$

From these equations the law of n_v as a function of θ may be derived, yet this law need not be strictly adhered to because, in practice, the small spread of refractive in-

dices limits the field angle to relatively small values $\pm\theta_{max}$ if the lens **158'** is to be of moderate width. Thus,

$$\text{if } \frac{R_4}{R_3'}=1.2,\ \theta_{max.}\approx 5°;\ \text{if } \frac{R_4}{R_3'}=2,\ \theta_{max.}\approx 8°$$

FIG. 6 illustrates the combination of lens **10** with a special flattening lens **210** fixedly positioned in front thereof. Lens **210** has a planar forward face **211** in a concave rear face **212** of noncircular cross-section, described more fully hereinafter with reference to FIG. 9, which is so shaped that the rays emerging from the rear surface **112** of lens **10** are focused on a flat surface **220** transverse to axis **100**. To this end it is necessary that the incident rays **201**, taken to originate at an infinitely distant object and therefore to be parallel, are so refracted by the surface **212** as to diverge to a different extent for different angles of incidence so that the secondary focus F'' will always fall on the plane **220**. It is assumed here that both lenses **210** and **10** have identical curvatures in all planes which include their common axis **100**, yet the extent of lens **210** in the plane of rotation of lens **10** about axis **21** may be considerably greater than in a plane transverse thereto. With this arrangement it is, of course, no longer necessary to provide a flattening lens behind the lens **10**, e.g. as shown at **158'** in FIG. 4.

At **230** I have shown, in dot-dash lines, an auxiliary lens of conventional sperical curvature (not necessarily concentric with lens **10**) which may be used in addition to lens **210**, or as integral part thereof, to focus the system upon objects at finite distances, or to allow it to project images upon a screen or the like from, say, a film passing along plane **220**. Lens **230** may, of course, be designed in keeping with the usual technique as a singlet, a doublet or any suitable combination of members with the desired focal length; its refractive index and/or Abbé number ν may or may not be equal to those of lens **210**. The sole requirement, insofar as the present invention is concerned, is its ability to let the light rays of a given beam pass parallel between the surfaces **211** and **212** of lens **210**.

It will be noted that the optically effective surface **212** of flattening lens **210** extends, in the plane of rotation about axis **21**, over an angle which is substantially greater than the sectional field angle of the circularly curved focal surface F (FIG. 1) of lens **10**, i.e. the angle defined by lines **100'** and **100''** in FIG. 1.

In FIG. 7 I have shown a modified system similar to that of FIG. 6 wherein, however, a field of parallel rays between surfaces **311**, **312** of a lens member **310** is produced (for a primary focus at infinity) with the aid of an auxiliary lens member **330** bounded by spherical surfaces **311**, **331** concentric with the surfaces **110**, **111** of lens **10**. This is accomplished by choosing the radii R_a and R_b of surfaces **331** and **311**, respectively, so that the power of member **330** is zero, it being assumed that the refractive index of member **310** is less than that of member **330**. If, for convenience, we again assign the values $n_1=1.5$ and $n_2=1.75$ to these two members, we find that the power of member **330** is given as

$$\frac{0.75}{R_a}-\frac{0.25}{R_b}+\frac{0.75(R_a-R_b)}{7R_aR_b}$$

whence, for zero power,

$$R_a=\frac{9}{2}R_b$$

Naturally, for focusing upon finite distances a smaller ratio of R_a/R_b may be used. The front lens **331** of FIG. 12, while bulky, may be useful in situations where large pressure differences are to be sustained, e.g. in an aircraft or missile. As the rear surface of lens **310** is identical with that of lens **210**, the image is again projected upon a planar surface **320** (or, conversely, a film on that plane may be reproduced by projection onto a screen through the lens system **10**, **310**, **330**). Lens **330** is shown composed of two portions **330***a*, **330***b* separated by a narrow, forwardly convex spherical gap **363** which acts as a virtual diaphragm, limiting the field of incident rays, in the manner described in conjunction with FIGS. 3 and 4.

The curvature of the concave rear surfaces **212** and **312** of lens members **210** and **310** in FIGS. 6 and 7 will now be described with reference to FIGS. 8*a*–8*d* and 9.

According to the classical theory of optics, a planar beam of parallel light rays traversing a circular boundary between two media of different optical densities is caused to converge upon a real focus beyond the boundary if the latter is concave toward the denser medium but is caused to diverge, seemingly originating at a virtual focus before the boundary, if the latter is of the opposite curvature. If the refractive indices of the two media are 1 and n, respectively, the distance between the focus and the boundary is given as $r/n-1$ if the beam comes from the side of the more permeable medium (e.g. air) and as $nr/n-1$ if it originates within the denser medium. The first situation is illustrated in FIGS. 8*a* and 8*d*, the second in FIGS. 8*b* and 8*c*.

FIG. 8*a* shows a beam B_a, composed of rays parallel to an axis A_a, impinging upon a positively refracting boundary between an optically thinner and an optically denser medium, the latter having the refractive index n. The boundary may be considered in first approximation, for rays close to the axis, a circle C_a illustrated in dot-dash lines. More exactly, however, this boundary is constituted by an ellipse E_a, the focal length f_a constituting the distance between the vertex V_a and the distal focus F_a of the ellipse. The eccentricity e of the ellipse, whose half-axes bear the usual designations a and b, is given by the relationship

$$e=\sqrt{a^2-b^2}=\frac{a}{n} \tag{19}$$

Curve C_a is, of course, the osculatory circle of the ellipse at its vertex V_a, the radius a of this circle being equal to a^2/b. With $n=1.5$, the focal length $f_a=a+e=3r$.

FIG. 8*b* shows a similar positively refracting surface represented in first approximation, for paraxial rays, by a circle C_b. The beam B_b parallel to axis A_b originates within the denser medium to the left of the boundary and is focused on a point F_b whose distance from vertex V_b is given (for the previously assumed value of n) as $f_b=2r$, r being again the radius of the circle. It can be shown that F_b is the distal focus of a hyperbola H_b which osculates the circle C_b and whose half-axes a, b and eccentricity e satisfy the relationship

$$e^2=\sqrt{a^2+b^2}=na \tag{20}$$

FIG. 8 shows the case of a negatively refracting boundary traversed by a beam parallel to axis A_c, the boundary being defined by a circle C_c osculating a hyperbola H_c. The virtual focus F_c is again the distal focal point of hyperbola H_c, its distance f_c from the vertex V_c being given by Equation 20. In this figure, as in FIG. 8*b*, the beam impinges upon the boundary from the side of the denser medium.

A reversal of the situation shown in FIG. 8*c* has been illustrated in FIG. 8*d* wherein a beam B_d, centered on an axis A_d, enters the denser medium by way of a negatively refracting boundary defined by a circle C_d osculating an ellipse E_d. The parameters of the ellipse E_d are the same as those of ellipse E_a in FIG. 8*a*, point F_d being one of the foci of the ellipse whose distance f_d from the distal vertex V_d is defined by Equation 19.

It should be noted that the ellipses and hyperbolas described in connection with FIGS. 8*a*–8*d* apply only to parallel rays; the more general case, in which rays originating at one point are focused upon another, cannot be solved with any second-order curve. Moreover, these conic sections are of but limited utility in practical optical applications since their focusing action extends only to beams incident in a certain direction, i.e. parallel to the axis.

In my copending application Ser. No. 850,628, filed November 3, 1959, now Patent No. 3,112,355, issued November 26, 1963. I have shown in connection with a catoptric system that it is possible to construct a curve which osculates in each point a different conic (specifically a hyperbola), the locus of the foci of all these conics being a predetermined curve which is the conjugate of a straight line. In the present instance, as will be more fully described with reference to FIG. 9, I propose to utilize the same technique in plotting the curvature of lens surface **212** or **312** (FIGS. 6 and 7) to insure that the rays projected through lens **10** converge upon the plane **220** or **320** at all angles of incidence.

In FIG. 9 the spherical lens **10**, whose center is shown at O, has not been illustrated since its only parameter of interest for the subsequent analysis is its focal length f. In fact, for purposes of this discussion the lens **10** could even be a homogeneous spherical segment having a characteristic as shown in graph I of FIG. 2, provided of course that its field of view were limited to a solid angle small enough to reduce the defocusing effect for slanting rays to a tolerable value.

The curve **412** in FIG. 9 represents the profile of the surfaces **212** and **312** of FIGS. 6 and 7, it being understood that this curve could also be the outline of a cylindrical surface if focusing in only one plane were desired. The line **420**, perpendicular to axis **100**, is representative of the projection surfaces **220** and **320** of the two preceding figures.

If a lens centered on point O, having a focal length f, is to concentrate incident parallel rays upon the line **420** at all angles of incidence, these rays must reach the lens as divergent beams originating from point F on a curve **400** having a distance s from O, the distance s'' of O from the conjugate point f'' on line **420** being given by the known relationship

$$\frac{1}{s}+\frac{1}{s''}=\frac{1}{f} \tag{21}$$

If the angle of incidence is designated θ, and if the distance between point O and line **420** is named $s_o''=kf$, then

$$s''=\frac{kf}{\cos\theta} \text{ and } s=\frac{kf}{k-\cos\theta} \tag{22}$$

The curve **412**, representing the dispersive boundary between two media of refractive indices n and 1, respectively, is so designed as to osculate in each point P(x, y) a respective hyperbola H', H'' whose axis A is parallel to the direction of a ray **410** incident at P which is refracted by the boundary **412** toward the point O; axis A intersects the curve **400** in a point F whose distance from the vertex V proximal to point P satisfies the relationship established with reference to focus F_c and vertex V_c of hyperbola H_c in FIG. 8c as given by Equation 20; thus, the distance $\overline{FV}$ equals $a+e=a(n+1)$, a and $b\equiv pa$ being the half-axes of the hyperbola whose asymptotes are shown at HA', HA'' and which is centered on a point M. In a co-ordinate system based upon the axes a and b of the hyperbola, with M as the origin, point P has the abscissa X and the ordinate Y bearing the well-known relationship

$$1=\frac{X^2}{a^2}-\frac{Y^2}{b^2}=\frac{X^2}{a^2}-\frac{Y^2}{p^2a^2} \tag{23}$$

From Equations 20 and 23 we obtain

$$p^2=n^2-1 \tag{24}$$

whence

$$\frac{X^2}{a^2}-\frac{Y^2}{(n^2-1)a^2}=1 \tag{25}$$

and

$$Y=\sqrt{(n^2-1)(X^2-a^2)} \tag{26}$$

At **450** I have indicated the perpendicular to curve **412** in point P. This perpendicular includes an angle Φ with the direction of the axis A and that of the incident ray **401** parallel thereto. The refracted ray **401'** lies on a straight line between points F and O, the angle included between this line and the axis A being designated ψ. The radius of curvature ρ, which of course coincides with the perpendicular **450**, includes with the principal axis **100** an angle τ and with ray **401'** an angle σ. The angle of incidence between ray **401** and axis **100** has been designated ω.

The following relationships obtain between the various angles identified above:

$$\frac{dX}{dX}\equiv Y'=\cot\phi \tag{27}$$

$$\sin\sigma=n\sin\Phi \tag{28}$$

$$\omega=\tau-\Phi \tag{29}$$

$$\psi=\sigma-\Phi \tag{30}$$

Point P can also be defined in polar co-ordinates by its distance R from point O and the angle θ, with $R\cos\theta=X$ and $R\sin\theta=Y$. The distance λ between F and P will be found to equal $nX+a$, with

$$\sin\psi=\frac{(n^2-1)(X^2-a^2)}{nX+a} \tag{31}$$

and

$$\cos\psi=\frac{an+X}{nX+a} \tag{32}$$

Furthermore, as will be clearly apparent from the drawing,

$$\lambda\equiv nX+a=s-R \tag{33}$$

From the foregoing relationships, in conjunction with the well-known formula for the radius of curvature

$$\rho=\frac{(R^2+R'^2)^{\frac{3}{2}}}{R^2+2R'^2-RR''} \tag{34}$$

(where

$$R'\equiv\frac{dR}{d\theta} \text{ and } R''\equiv\frac{d^2R}{d\theta^2}\Big)$$

it can be shown that

$$a=\frac{R^2}{(n^2-1)(R^2+2R'^2-RR'')} \tag{35}$$

and also

$$a=\frac{R(s-R)[\sqrt{n^2(R^2+R'^2)}-R]}{(n^2-1)(R^2+R'^2)} \tag{36}$$

from which

$$s=\frac{R^2(R^2+R'^2)}{(R^2+2R'^2-RR'')[\sqrt{n^2R^2+R'^2(n^2-1)}-R]}+R \tag{37}$$

Let $t\equiv\frac{\tan\theta}{2}$, $\sin\theta=\frac{2t}{1+t^2}$, $\cos\theta=\frac{1-t^2}{1+t^2}$ and $d\theta=\frac{2dt}{1+t^2}$

then, with

$$\dot{R}\equiv\frac{dR}{dt} \text{ and } \ddot{R}\equiv\frac{d^2R}{dt^2}$$

Equation 37 can be rewritten, using the value of s taken from Equation 2, in the following form:

$$\text{Plus}\frac{\frac{kf(1+t^2)}{k-1+t^2(k+1)}=R+2R^2[4R^2+\dot{R}(1+t^2)^2]}{\{4R+2R^2(1+t^2)^2-R(1+t^2)[1+t^2)\ddot{R}+2t\dot{R}]\}[\sqrt{4n^2R^2+\dot{R}^2(1+t^2)(n^2-1)}-2R]} \quad (38)$$

Since curve **412** is to be symmetrical about axis **100**, $\dot{R}_o=0$ for $\theta=0$, $t=0$. If, for convenience, $k=n$ so that s_o has the finite value $nf/n-1'$, the value of $\ddot{R}_o$ is found to be

$$\ddot{R}_o=4nR_o\frac{f-R_o}{nf-R_o(n-1)} \quad (39)$$

It thus follows that, if R_o is selected to equal f, $\ddot{R}_o$ also goes to zero.

The dependent variable R of the polar co-ordinate system R, θ may now be expressed as polynomial

$$R=R_o+\frac{d^2R_o}{2dt^2}t^2+\frac{d^4R_o}{24dt^4}t^4+\frac{d^6R_o}{72dt^6}t_0+\cdots \quad (40)$$

according to Maclaurin's formula, the coefficient of the odd-numbered powers of t being zero. The higher derivatives of R at the point $t=0$ are found by repeated differentiation to have the following values:

$$\frac{d^2R_o}{dt^2}=R_o0$$

$$\frac{d_4R_o}{dt^4}=\frac{-16nR_o}{n-1}$$

$$\frac{d^6R_o}{dt^6}=R_o\frac{46n(n+4)}{n-1}$$

substituting these values in Equation 40, we obtain

$$R=R_o\left[1-\frac{2nt_4}{3(n-1)}+\frac{4n(n+4)t^6}{45(n-1)}-\cdots\right] \quad (41)$$

For the specific value of $n=1.5$, we thus obtain the formula

$$R=R_o\left(1-2t^4+\frac{22}{15}t^6-\cdots\right) \quad (42)$$

which is reasonably accurate for angles up to about $\theta=60°$.

It will thus be apparent that the system illustrated in FIGS. 6, 7 and 9 has an axis of symmetry **100** which passes through the middle of the dispersive stationary lens **210** or **310** and through the center of the rotatable collective lens **10**, the rear face **212** or **312** of the dispersive lens being provided with a noncircular curvature (at least in a plane transverse to axis **21**) which converts incident beams **201, 301** of parallel light rays into diverging beams **201', 301'** with axes **200, 300** trained upon the lens axis **21**, these axes coinciding in FIG. 9 with the ray **401'** which constitutes a principal ray in a bundle of parallel light rays incident upon the boundary **412** in the vicinity of the point P; the intersection between the beam axis or principal ray and the locus **400**, represented by point F in FIG. 9, is one of the foci of a hyperbola (H', H'') which osculates the boundary **412** at the point P but which has different parameters for different such points, except that the relationship expressed by Equation 20 holds true for all these hyperbolas.

In FIG. 10 I have shown, in transverse section, a modification of lens **10** in which the rear surface of lens member **12** is not a spherical segment, as heretofore assumed, but has a noncircular curvature **112''** in a plane including the axis of shafts **221a, 221b**; in a plane transverse thereto, i.e. the one visible in FIGS. 6 and 7, this face still is bounded by a circular arc centered on the point O. The radius ρ_3 of curvature **112''** is so chosen that slanting beams **501** are caused to converge not along the arc F but on a straight line **520** tangent thereto, this line corresponding for example to a generatrix of the transparent cylinder **20** in FIG. 5. For this purpose the radius ρ_3 exceeds the radius R_3 of the circular arc **112** (FIG. 1) by progressive increments with increasing distance from the axis **100**, ρ_3 being equal to R_3 along that axis only.

Equation 3 may be rewritten to give the focal length f of lens **10**, in terms of the radius R_1 of front surface **110**, as

$$f=\frac{R1}{K^2-E} \quad (43)$$

which suggests that, for a focal length varying inversely with cos θ where θ (by analogy with FIG. 9) is the angle between beam axis **500** and the lens axis **100**,

$$\cos\theta=\frac{f_o}{f}=\frac{K^2(\rho)-E(\rho)}{K_2-E} \quad (44)$$

wherein $K_2(\rho)$ and $E(\rho)$ have the same significance as K_2 and E but with substitution of ρ_3 for R_3. For the specific values for n_1 and n_2 heretofore assumed, Equation 44 yields the relationship

$$K_2(\rho)\equiv\frac{R^1}{\rho_3}=K_2\cos\theta+\frac{(2K_1-7)(1-\cos\theta)}{9} \quad (45)$$

whence

$$\rho_3=\frac{9R_1}{(9K_2-2K_1+1)\cos\theta+2K_1-1} \quad (46)$$

The foregoing formula for ρ_3 is, however, not precise because, on the other hand, the noncircular curvature **112''** causes a deflection of the beam axis as exaggeratedly indicated at **500***x*, thereby reducing the effective length of f, and because, on the other hand, the cross-section of the rear surface of lens member **12** in a plane including the axis **500** is, strictly speaking, not a circular arc the the arc of an ellipse with the major half-axis ρ_3 and the minor half-axis R_3, hence the curvature of the rear lens face within that cross-section has a radius smaller than ρ. Since these two aberrations tend to balance each other for small angles θ, the expression for ρ_3 given in Equation 46 or, more generally, derived from Equation 44 may be considered suitable for most cases.

It will be apparent that a flattening lens of the general type shown at **158'** in FIGS. 3 and 4 could also be given an aspherical rear surface similar to the surface **112''** of FIG. 10, in lieu of or in addition to its varying index of refraction, with consequent enlargement of its field angle θ; conversely, the lens **158'**, with or without this modification, could be made integral with lens **10'** for rotation therewith.

My invention is, of course, not limited to the specific embodiments as described and illustrated but may be modified in various respects, including the combination of compatible features from different embodiments, without departing from the spirit and scope of the appended claims.

I claim:

1. In an optical objective system, in combination, supporting means forming a substantially flat receiving surface for projected light rays and dioptric means adapted to focus incident rays upon said receiving surface, said dioptric means including a sectoral lens having two outer surfaces and at least one inner surface all circularly curved around an axis parallel to said receiving surface within a limited sectional sectoral field angle, said lens being ro-

tatable about said axis, and further including image-flattening lens means fixedly positioned with reference to said receiving surface, said lens means having at least one optically effective curved surface extending over an angle substantially larger than said field angle in the plane of rotation; and means for rotating said sectoral lens about said axis.

2. The combination defined in claim 1 wherein said sectoral lens is a doublet.

3. The combination defined in claim 1 wherein said image-flattening lens means comprises a dispersive lens element bounded by surfaces that are substantially concentric with those of said sectoral lens, said lens element being disposed on the side of said lens element opposite said receiving surface.

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