Oct. 14, 1947. J. A. RAJCHMAN 2,428,811

ELECTRONIC COMPUTING DEVICE

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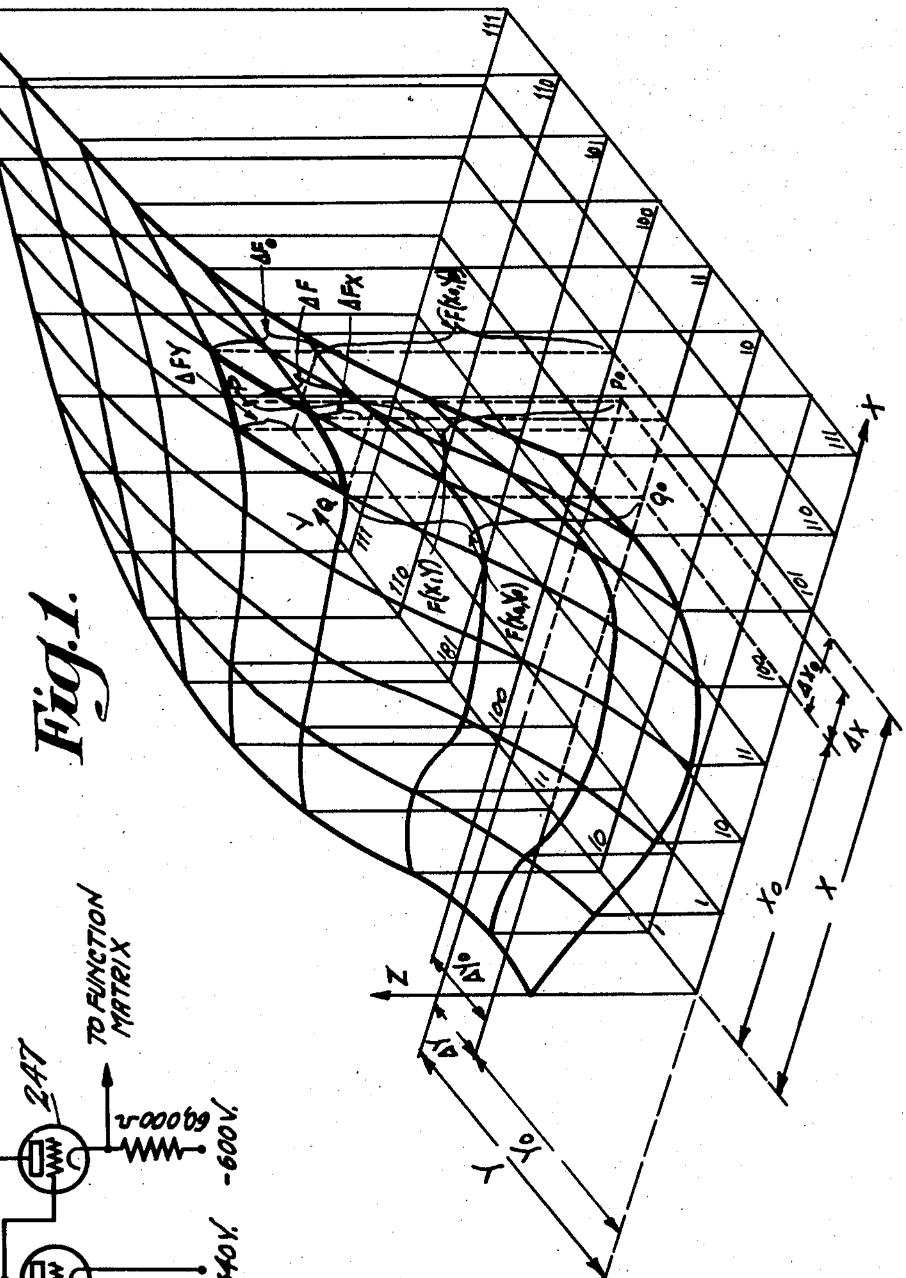

Fig. 1.

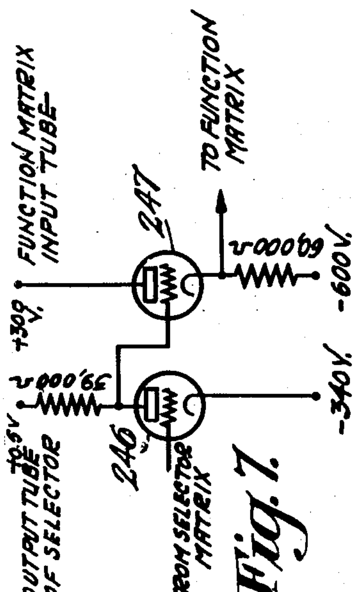

Fig. 7.

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ELECTRONIC COMPUTING DEVICE

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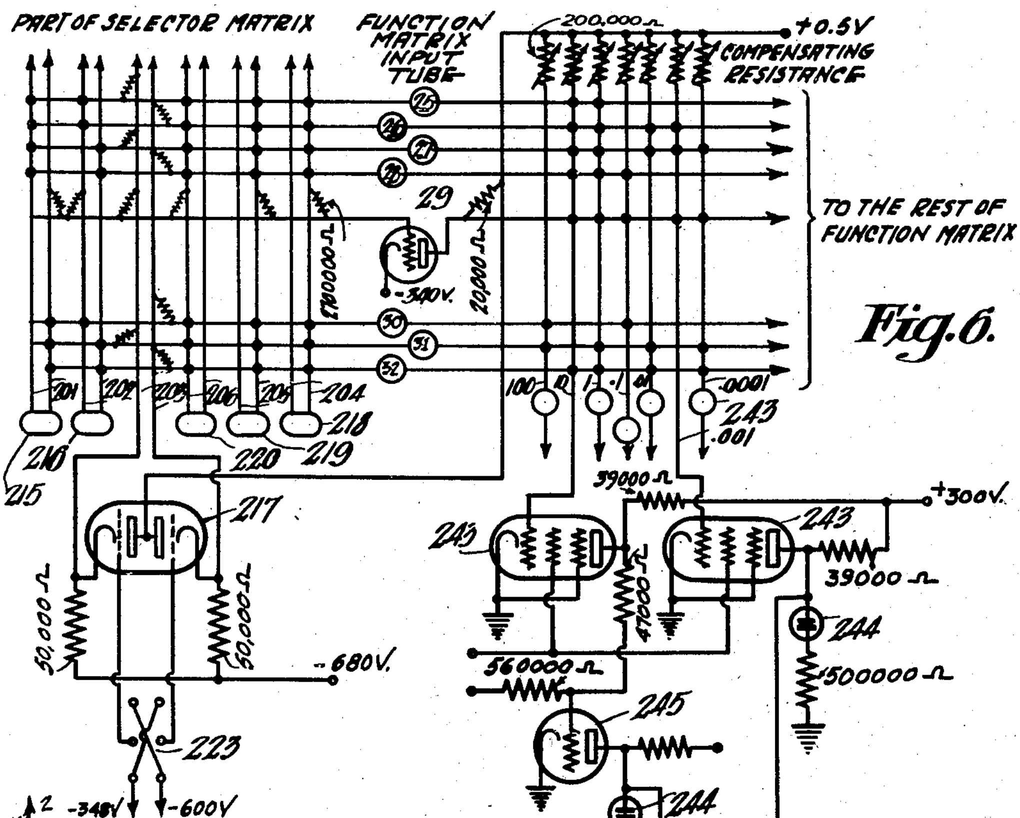

Fig. 6.

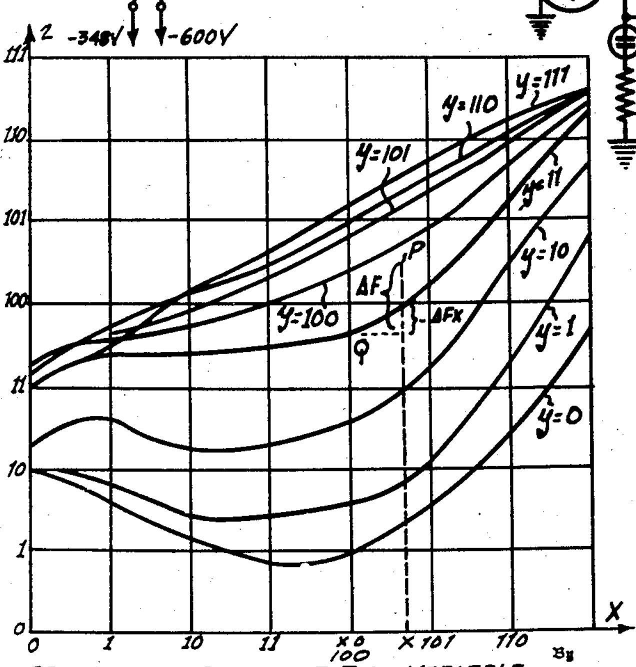

Fig. 2.

REPRESENTATION OF A TWO-VARIABLE FUNCTION BY A FAMILY OF CURVES

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ELECTRONIC COMPUTING DEVICE

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Fig. 3.

Y									
111		57	58	59	60	61	62	63	64
	F	011.0010	011.1011	100.0010	100.1011	101.0100	101.1100	110.0101	110.1010
	ΔFx	+0.1001	+0.0111	+0.1001	+0.0110	+0.1000	+0.1001	+0.0101	
	(ΔFx)'	0.1001	0.0111	0.1001	0.0110	0.1000	0.1001	0.0101	
	ΔFx+S	1.0001	0.1111	1.0001	0.1110	1.0000	1.0001	0.1101	
	ΔFy								
	(ΔFy)'								
	ΔFy+S								
110		49	50	51	52	53	54	55	56
	F	011.0011	011.1100	100.0010	100.1000	101.0001	101.1010	110.0010	110.1010
	ΔFx	+0.1001	+0.0110	+0.0110	+0.1001	+0.1001	+0.1000	+0.1000	
	(ΔFx)'	0.1001	0.0110	0.0110	0.1001	0.1001	0.1000	0.1000	
	ΔFx+S	1.0001	0.1110	0.1110	1.0001	1.0001	1.0000	1.0000	
	ΔFy	-0.0001	-0.0001	+0.0000	+0.0011	+0.0011	+0.0010	+0.0011	0.0000
	(ΔFy)'	1111.1111	1111.1111	0.0000	0.0011	0.0011	0.0010	0.0011	0.0000
	ΔFy+S	0.0111	0.0111	0.1000	0.1011	0.1011	0.1010	0.1011	0.1000
101		41	42	43	44	45	46	47	48
	F	011.0101	011.1011	011.1111	100.0110	100.1110	101.0111	110.0001	110.1010
	ΔFx	+0.0110	+0.0100	+0.0110	+0.1000	+0.1001	+0.1010	+0.1001	
	(ΔFx)'	0.0110	0.0100	0.0110	0.1000	0.1001	0.1010	0.1001	
	ΔFx+S	0.1110	0.1100	0.1110	1.0000	1.0001	1.0010	1.0001	
	ΔFy	-0.0010	+0.0001	+0.0011	+0.0010	+0.0011	+0.0011	+0.0001	0.0000
	(ΔFy)'	1111.1110	0.0001	0.0011	0.0010	0.0011	0.0011	0.0011	0.0000
	ΔFy+S	0.0110	0.1001	0.1011	0.1010	0.1011	0.1011	0.1011	0.1000
100		33	34	35	36	37	38	39	40
	F	011.0101	011.1010	011.1100	100.0000	100.0110	100.1111	101.1100	110.1000
	ΔFx	+0.0001	+0.0010	+0.0100	+0.0110	+0.1001	+0.1101	+0.1100	
	(ΔFx)'	0.0001	0.0010	0.0100	0.0110	0.1001	0.1101	0.1100	
	ΔFx+S	0.1001	0.1001	0.1000	0.1110	1.0001	1.0101	1.0100	
	ΔFy	+0.0000	+0.0001	+0.0011	+0.0110	+0.1000	+0.1000	+0.0101	+0.0010
	(ΔFy)'	0.0000	0.0001	0.0011	0.0110	0.1000	0.1000	0.0101	0.0101
	ΔFy+S	0.1000	0.1001	0.1011	0.1110	1.0000	1.0000	0.1101	0.1101
011		25	26	27	28	29	30	31	32
	F	011.0000	011.0110	011.0111	011.1000	011.1011	100.0100	101.0101	110.0101
	ΔFx	+0.0110	+0.0001	+0.0001	+0.0001	+0.0011	+0.1001	+1.0000	
	(ΔFx)'	0.0110	0.0001	0.0001	0.0001	0.0011	0.1001	1.0000	
	ΔFx+S	0.1110	0.1001	0.1001	0.1001	0.1011	0.1001	1.1000	
	ΔFy	+0.0101	+0.0100	+0.0101	+0.1000	+0.1011	+0.1011	+0.0111	+0.0011
	(ΔFy)'	0.0101	0.0100	0.0101	0.1000	0.1011	0.1011	0.0111	0.0011
	ΔFy+S	0.1101	0.1100	0.1101	1.0000	1.0011	1.0011	0.1111	0.1011
010		17	18	19	20	21	22	23	24
	F	010.0101	010.1010	010.0100	010.0101	011.0100	100.1001	100.1001	101.1100
	ΔFx	-0.0101	-0.0110	+0.0001	+0.0010	+0.1011	+1.0101	+1.0011	
	(ΔFx)'	1111.1011	1111.1010	0.0001	0.0010	0.1011	1.0101	1.0011	
	ΔFx+S	0.0011	0.0010	0.1001	0.1010	1.0011	1.1101	1.1011	
	ΔFy	+0.1001	+0.1100	+1.0011	+1.0011	+1.0010	+1.0000	+0.1100	+0.1001
	(ΔFy)'	0.1001	0.1100	1.0011	1.0011	1.0010	1.0000	0.1100	0.1001
	ΔFy+S	1.0001	1.0100	1.1011	1.1011	1.1010	1.1000	1.0100	1.0001
001		9	10	11	12	13	14	15	16
	F	010.0000	001.1110	001.0111	001.0111	001.1001	010.0010	011.0111	100.1101
	ΔFx	-0.0010	-0.0111	0.0000	+0.0010	+0.1001	+1.0101	+1.0110	
	(ΔFx)'	1111.1110	1111.1001	0.0000	0.0010	0.1001	1.0101	1.0110	
	ΔFx+S	0.0110	0.0001	0.1000	0.1010	1.0001	1.1101	1.1110	
	ΔFy	+0.0101	+0.1100	+0.1101	+0.1110	+1.0000	+1.0010	+1.0010	+0.1111
	(ΔFy)'	0.0101	0.1100	0.1101	0.1110	1.0000	1.0010	1.0010	0.1111
	ΔFy+S	0.1101	1.0100	1.0101	1.0110	1.1000	1.1010	1.1010	1.0111
000		1	2	3	4	5	6	7	8
	F	010.0000	001.1011	001.0010	000.1110	001.0000	001.1001	010.0110	011.1101
	ΔFx	-0.0101	-0.0001	-0.0100	+0.0010	+0.1001	+0.1101	+1.0011	
	(ΔFx)'	1111.1011	1111.1111	1111.1100	0.0010	0.1001	0.1101	1.0011	
	ΔFx+S	0.0011	0.0111	0.0100	0.1010	1.0001	1.0101	1.1011	
	ΔFy	0.0000	+0.0011	+0.0101	+0.1001	+0.1001	+0.1001	+1.0001	+1.0000
	(ΔFy)'	0.0000	0.0011	0.0101	0.1001	0.1001	0.1001	1.0001	1.0000
	ΔFy+S	0.1000	0.1011	0.1101	1.0001	1.0001	1.0001	1.1001	1.1000
	X→	000	001	010	011	100	101	110	111

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ELECTRONIC COMPUTING DEVICE

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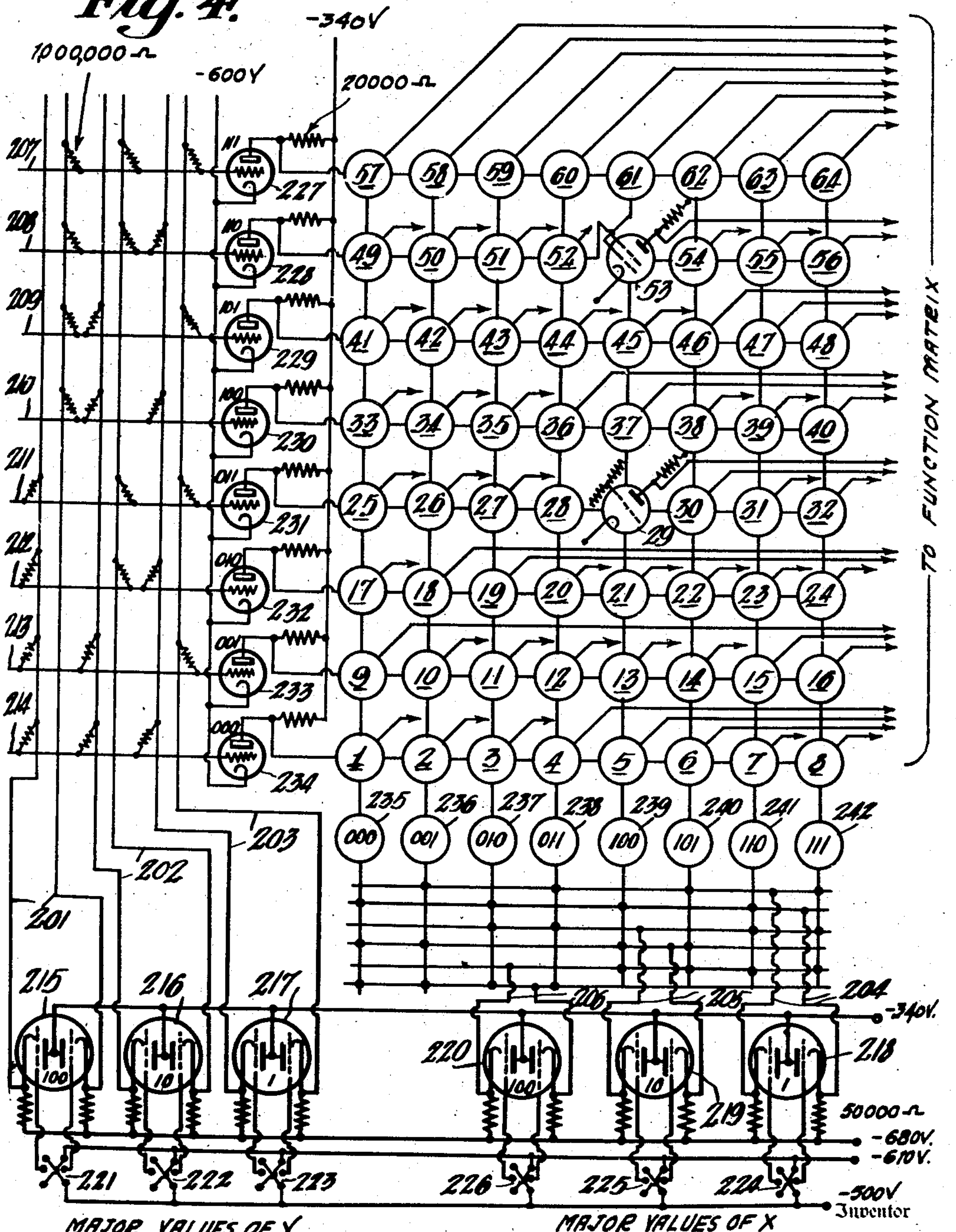
Fig. 4.



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ELECTRONIC COMPUTING DEVICE

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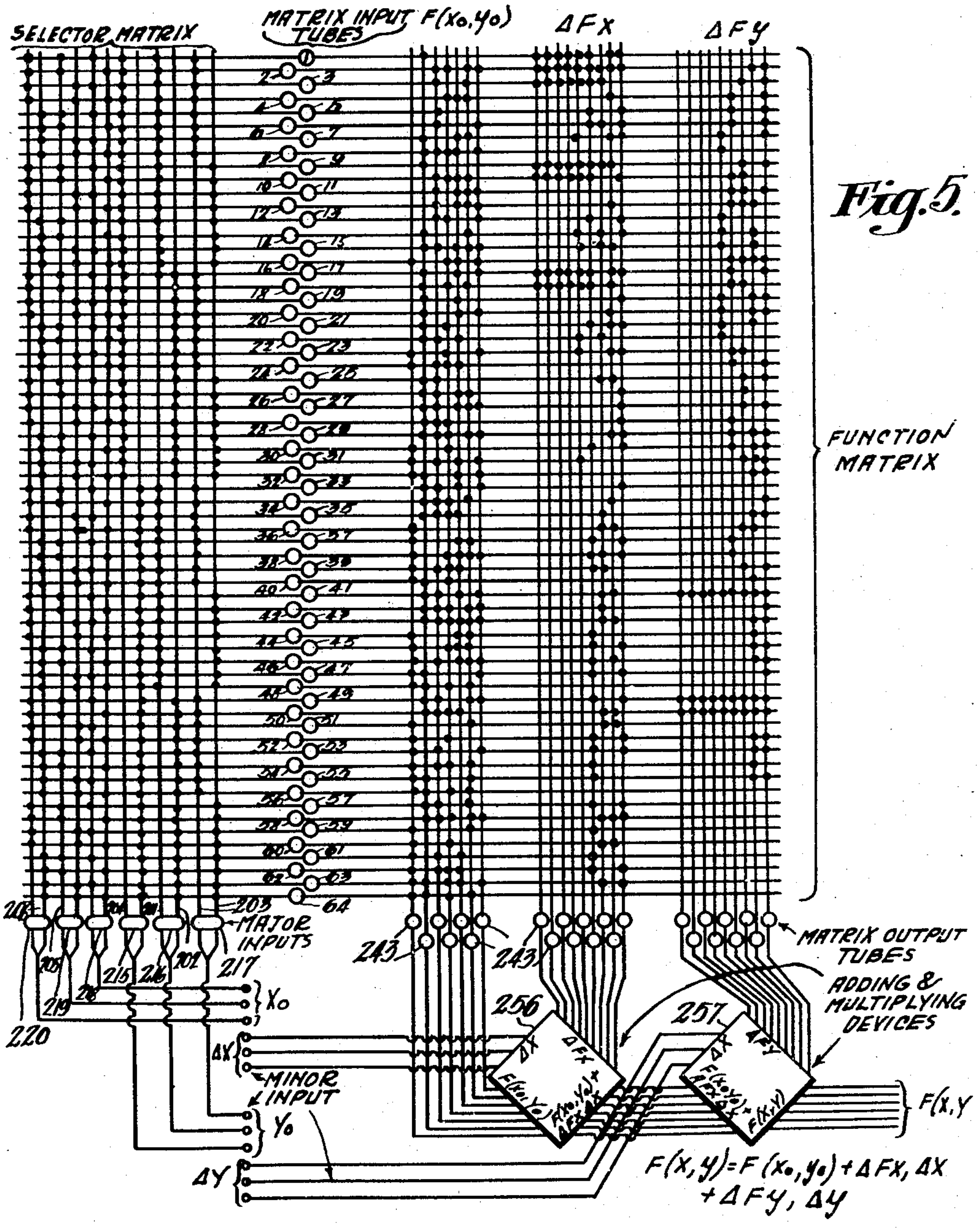

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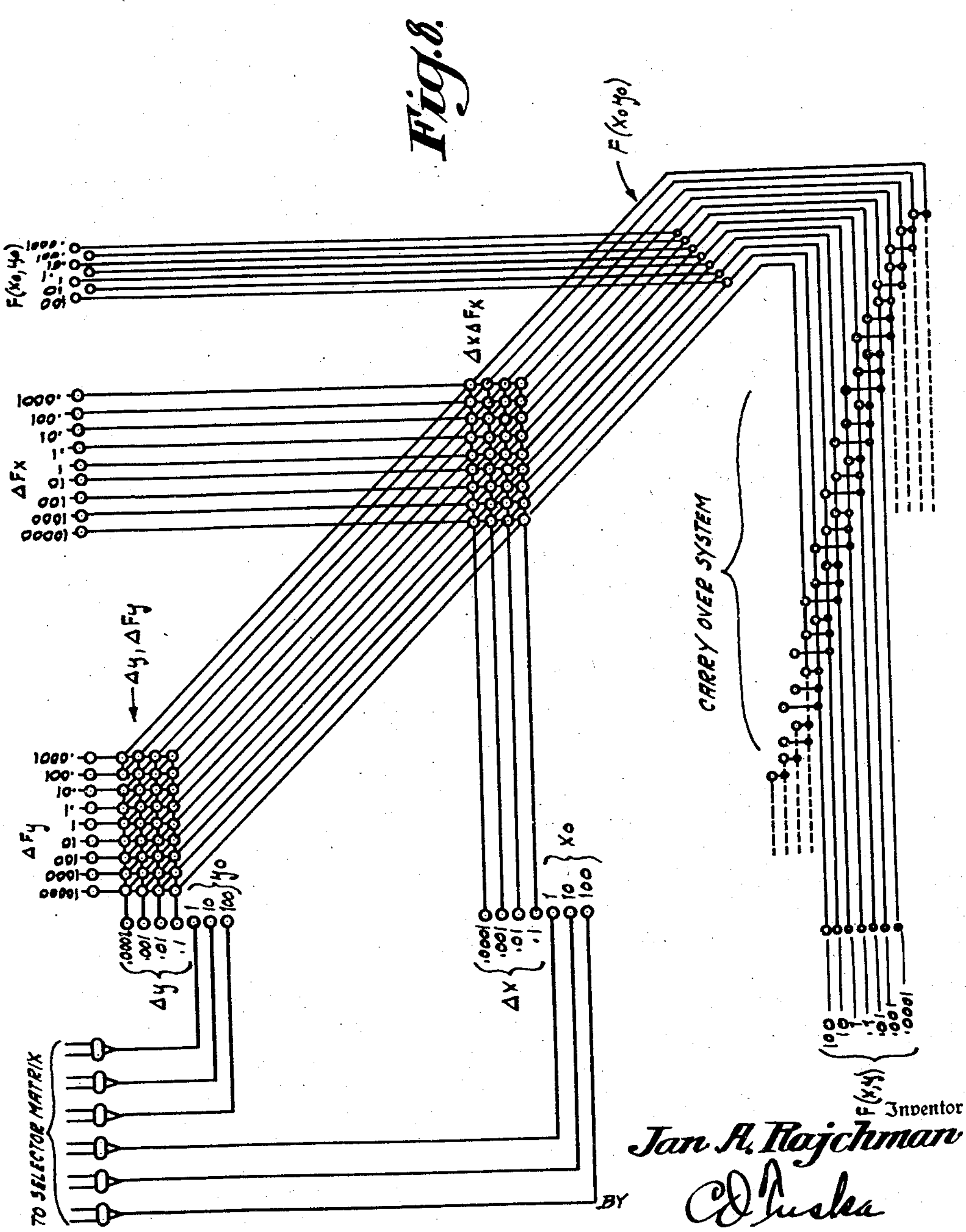

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ELECTRONIC COMPUTING DEVICE

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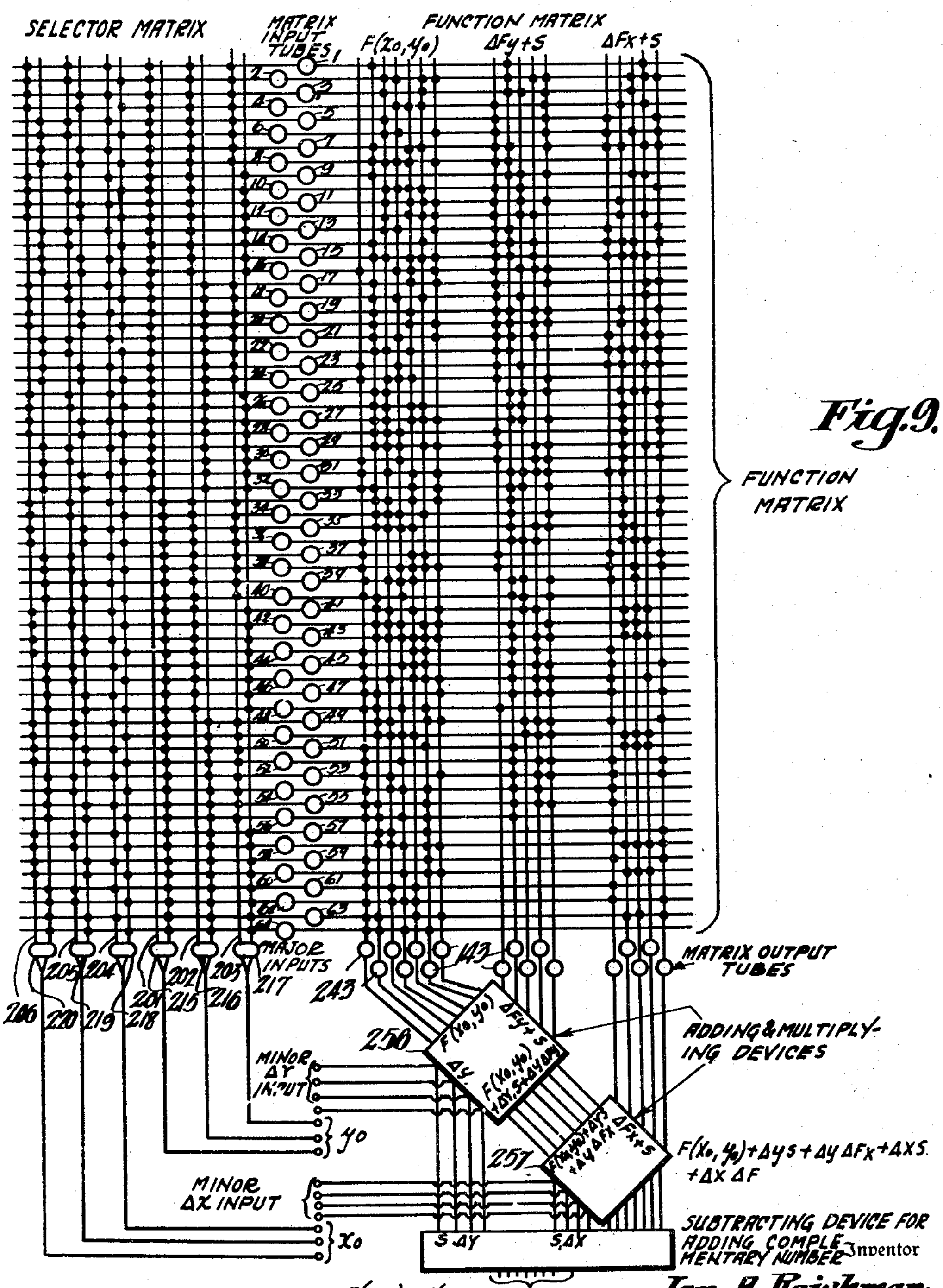

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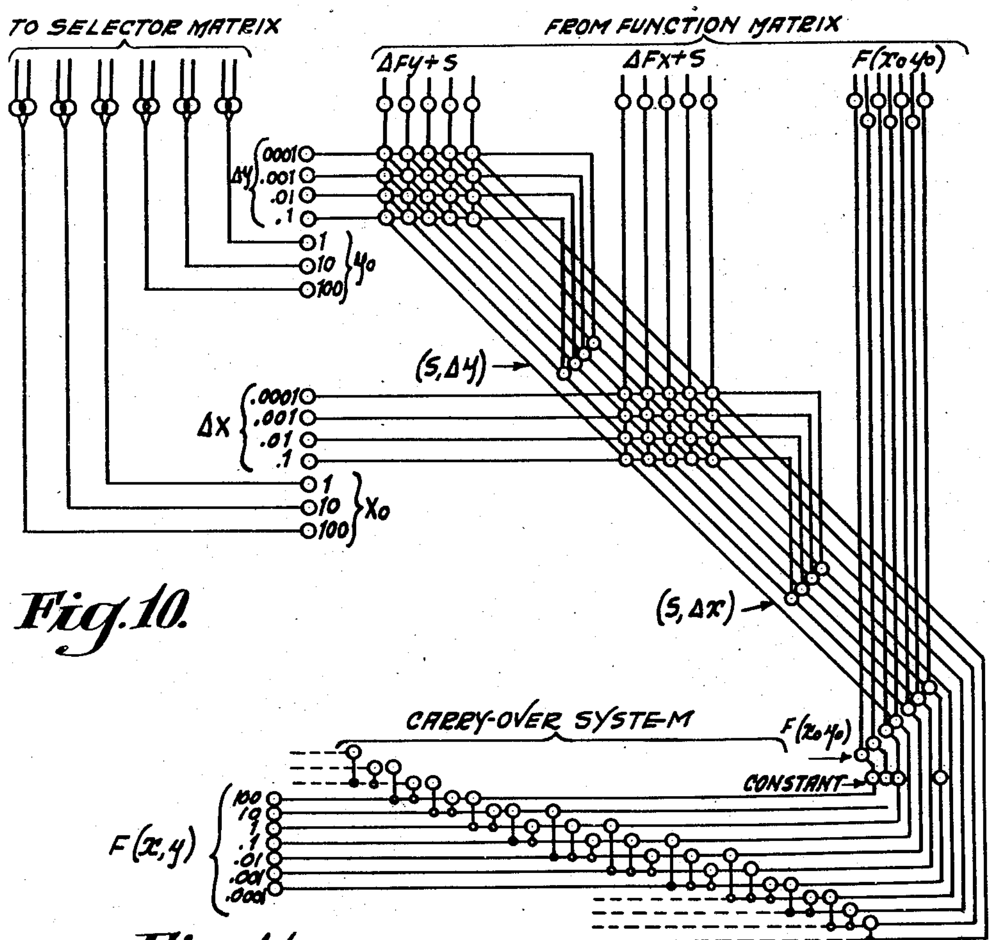

Fig. 10.

Fig. 11.

EXPANDED REGION (36A – 38B)

ORDER NUMBER OF SELECTED SET.	33	34	35	36A	36B	37A	37B	38A	38B	39	40
$F(x_0, y_0)$ TRUE	011.0101	011.1010	011.1100	100.0000	100.0010	100.0110	100,1010	100.1111	101.0101	101.1100	110.1000
DIFFERENCE	.0101	.0010	.0100	.0010	.0100	.0100	.0101	.0110	.0111	0.1100	
$AFx/\Delta x_0$	.0101	.0010	.0100	.0100	.1000	.1000	.1010	.1100	.1110	0.1100	
$F(x_0, y_0)$ CORRECTED	011.0101	011.1010	011.1100	100.0000	100.0010	100.0110	100.0101	100.1111	100.1110	101.1100	110.1000
$\frac{\Delta Fx}{\Delta x_0}+S$	.1101	.1010	.1100	.1100	1.0000	1.0000	1.0010	1.0100	1.0110	1.0100	
$\Delta Fy+S$	.1000	.1001	.1011	.1110	.1111	1.0000	1.0000	1.0000	1.0000	.1101	.1010
x	000	001	010	011.0	011.1	100.0	100.1	101.0	101.1	110	111

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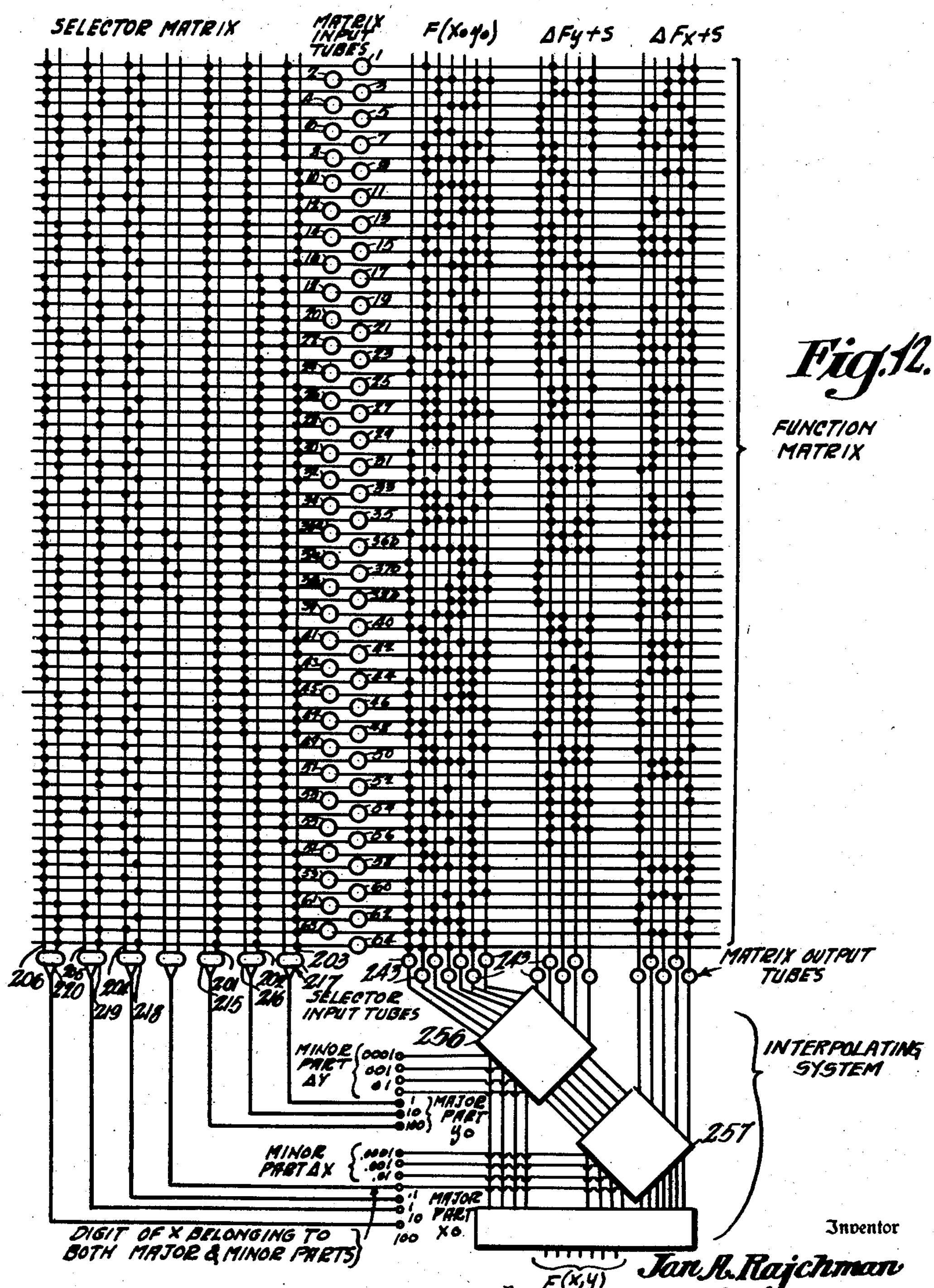

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Fig. 13.

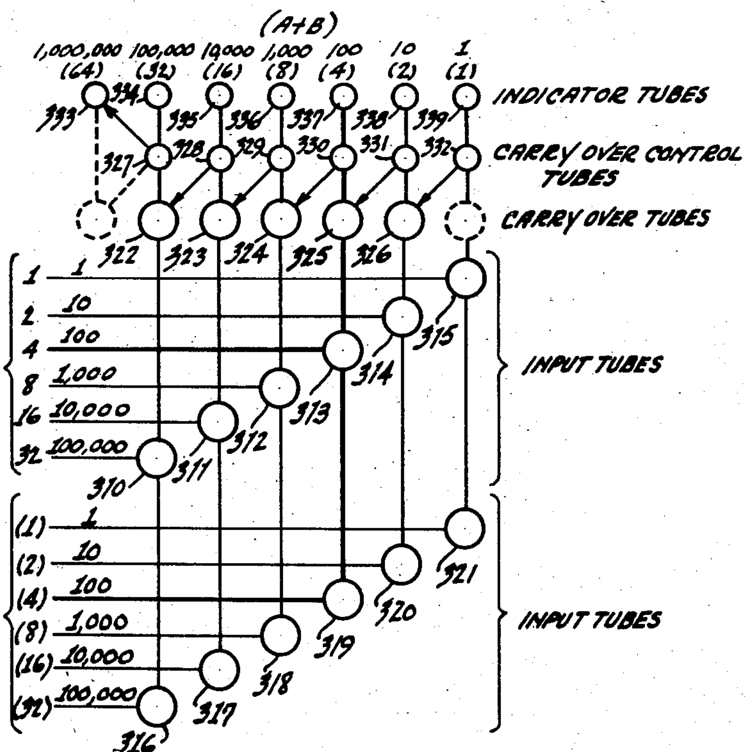
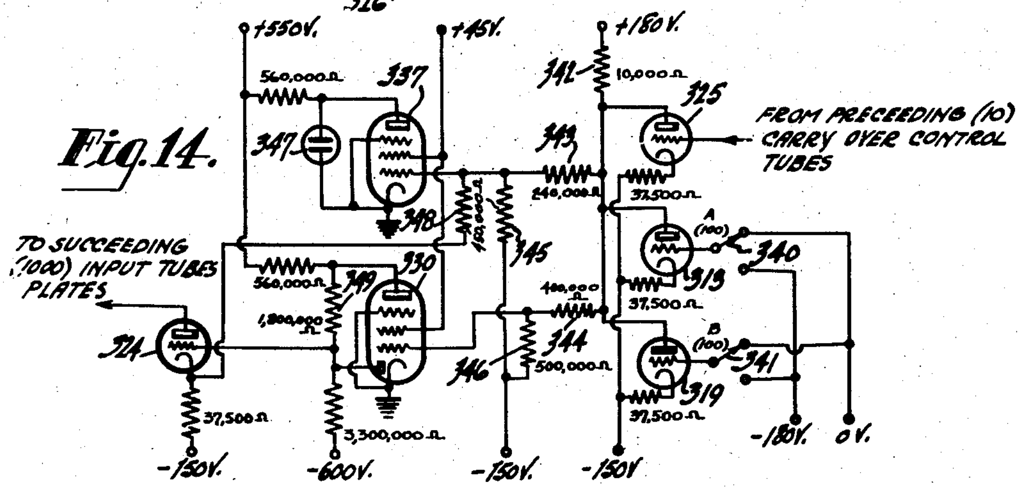


Fig. 14.



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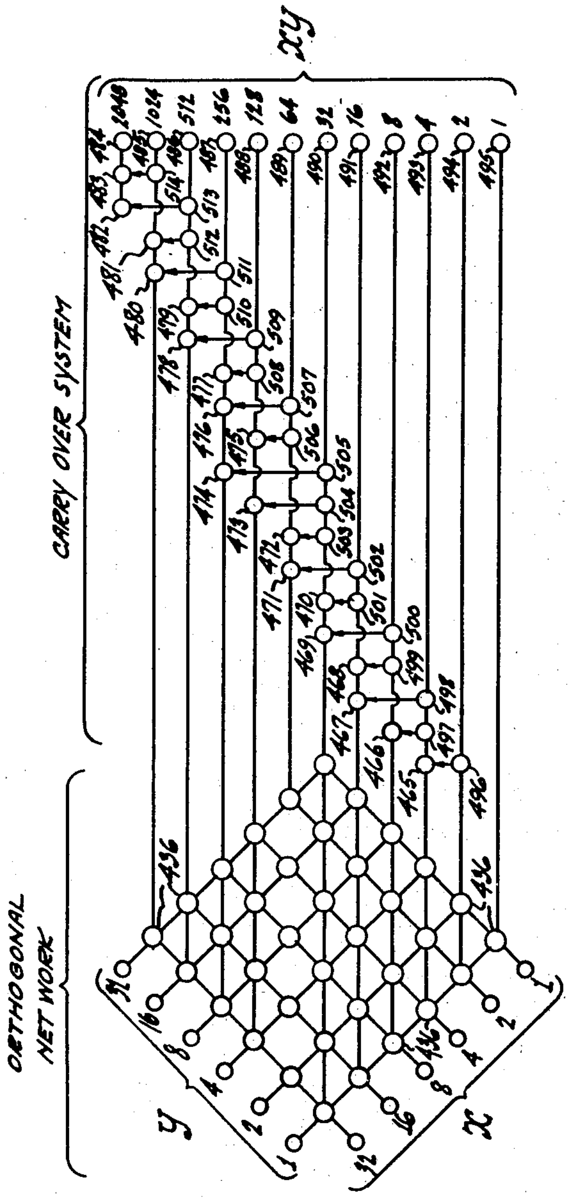

Fig. 15.

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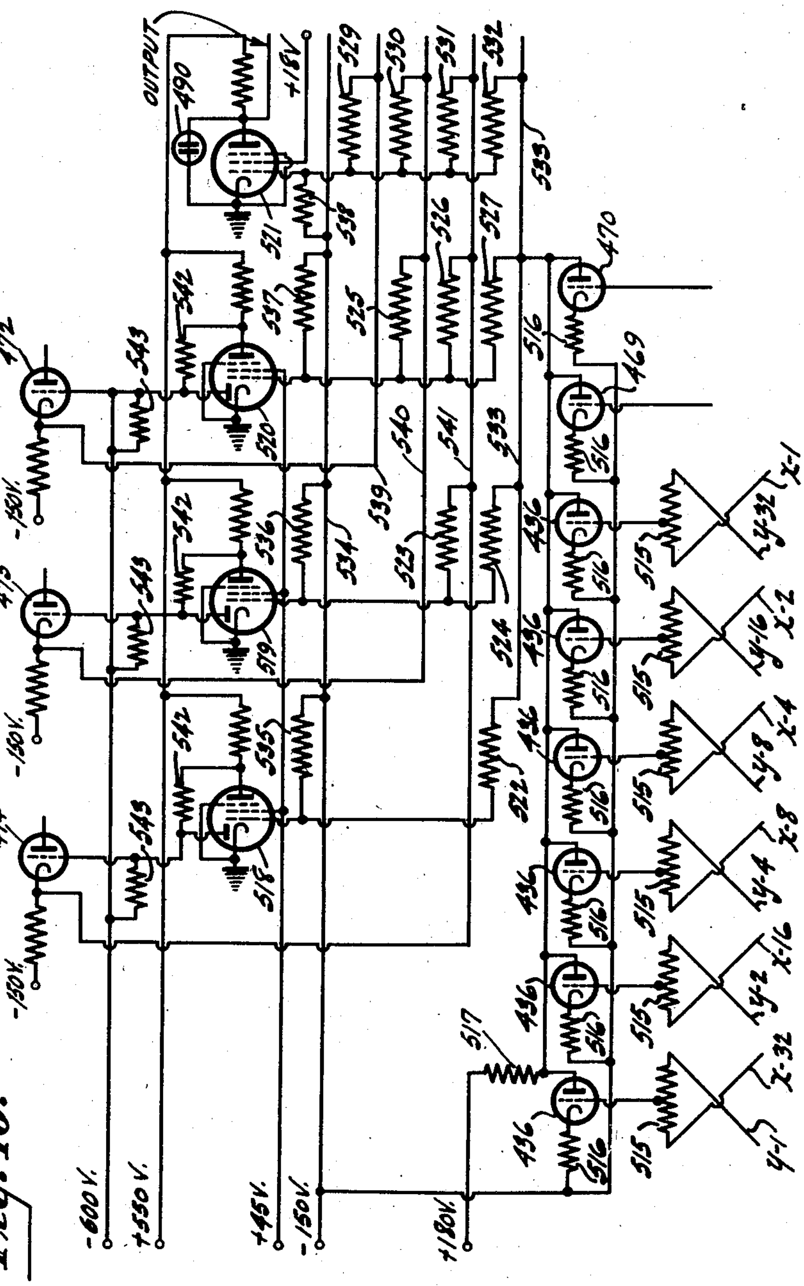

Fig. 16.

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[signature]

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Patented Oct. 14, 1947 **2,428,811**

UNITED STATES PATENT OFFICE

2,428,811

ELECTRONIC COMPUTING DEVICE

Jan A. Rajchman, Princeton, N. J., assignor to Radio Corporation of America, a corporation of Delaware

Application October 30, 1943, Serial No. 508,343

19 Claims. (Cl. 235—61)

This invention relates to computing devices such as are utilized to generate a desired function of one or more variables. The function and the variables are represented by systems of electric potentials. It has for its principal object the provision of an improved computing device and method of operation whereby a function of one or more variables may be derived continuously and without appreciable delay as the different variables change from one value to another.

This improved computing device includes, among other elements, a selector matrix which operates to select a set of values of the different variables, a function matrix which generates certain component functions of this selected set of values, and an interpolator which so combines the various components as to present at its output terminals potentials which are representative of the different digits of a number by which the value of the function is expressed. As will appear, each of these three elements may assume different forms, depending on the conditions under which the device is operated.

All the computations are performed in terms of numbers. The present computing device is therefore of the numerical type, as contrasted with devices using continuously variable physical quantities, such as voltage, current or phase, as the variable of computation. The whole computation is made in the binary system of numeration so that any number is expressed as a sum of powers of two in which the coefficients of the terms are zero or one. These are the only two digits of the binary system.

In this system, a number is expressed thus:

$$A=a_n^{2n}+a_{n-1}^{2n-1}+ \dots a_k^{2k}+ \dots a_0$$

where the coefficients a_k are either one or zero. The numbers can be written in the usual digital representation as shown for the first seventeen numbers in the following table:

0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000

For any number the first digit from the right, or first "digital position," signifies whether there is a $1=2^0$ in the number or not, the second digital place whether there is a $2=2^1$ or not, the third whether there is a $4=2^2$ or not, the fourth whether there is an $8=2^3$ or not, etc.

It is obvious that fractions and fractional numbers can be expressed in the binary system in a manner similar to the decimal fractions by using a "binal point" analogous to the "decimal point." A table of a few fractions would be:

0	.0000
1/16	.0001
1/8	.0010
3/16	.0011
1/4	.0100
5/16	.0101
3/8	.0110
7/16	.0111
1/2	.1000
9/16	.1001
5/8	.1010
11/16	.1011
3/4	.1100
13/16	.1101
7/8	.1110
15/16	.1111
1	1.0000
1 1/16	1.0001
1 1/8	1.0010
13 27/64	1101.011011

For any number the first digit from the right of the decimal point signifies whether there is a $\frac{1}{2}=2^{-1}$ in the number or not, the second whether there is a $\frac{1}{4}=2^{-2}$, the third whether there is a $\frac{1}{8}=2^{-3}$ or not, etc.

This system of numeration was chosen because most electronic computations are more easily performed in it than in any other system. This unusual method of expressing numbers does not involve any practical difficulty so long as the input and output of the computing device are converted automatically to control some physical apparatus, such as an anti-aircraft fire control system. Under such conditions, no ciphering or deciphering from the decimal numeration is involved.

All the operation is made in a direct system in which the binary number is expressed by a system of as many potentials as there are digits in it, each potential having one of two definite values V_1 and V_2 corresponding respectively to the digits zero and one. All these potentials exist simultaneously on a system of conductors each carrying a potential corresponding to one digit of the number. Thus, for example, to express the first seventeen numbers, five conductors would be required. The number 9 would be expressed by the following excitation of the five conductors: $V_1V_2V_1V_1V_2$, since it can be written as 01001.

In a computing device, two or more such systems of potentials are combined and a new system of potentials is derived from them. The result of the computation is the stationary final value of these output potentials. This result depends only on the stationary value of the input potentials, regardless of the manner in which they were reached. A sudden change in one or more input digits will, after short transients, cause the output potentials to reach their correct stationary values, so that the operation of the direct computing device may be considered as "continuous." It does not involve any trigger elements with inherently stable states or any other "holding" devices, nor does it necessitate any definite sequence, timing, or clearing pulses. Therefore, it is not a counter of any sort and does not involve impulses. It is basically the fastest type of numerical device, since no time is wasted in the proper sequencing of operations.

Important objects of the invention are the provision of an improved computing device which is readily adapted to the generation of different functions of one or more variables, the provision of an improved computing device of the direct type as distinguished from types involving sequencing pulses, clearing pulses and the like, and the provision of a computing device which may be provided with interchangeable function matrices whereby different functions of one or more variables may be generated without other modification of the device.

The invention will be better understood from the following description considered in connection with the accompanying drawings and its scope is indicated by the appended claims.

Referring to the drawings:

Figs. 1 and 2 illustrate a function in graphical form, the curved surface representing the function in Fig. 1 and the curves of Fig. 2 representing the function for various values of the y coordinate.

Fig. 3 shows the function in tabular form with the variables in the binary system.

Fig. 4 is a wiring diagram of a selector which operates in response to major values of the variables to select an element corresponding to that particular set of values.

Fig. 5 illustrates a function generator which includes a modified form of selector, a function matrix and an interpolation system.

Figs. 6 and 7 illustrate details in the connections of Fig. 5.

Fig. 8 illustrates an interpolation system which differs from that of Fig. 5 in that it involves a single carryover system.

Fig. 9 illustrates a function generator adapted to the use of augmented interpolating coefficients.

Fig. 10 illustrates the details of the interpolation system forming a part of the generator of Fig. 9,

Fig. 11 illustrates an expanded scale section of the function table of Fig. 3,

Fig. 12 is a function generator adapted for use of the scale expansion illustrated by Fig. 11.

Figure 13 is a diagrammatic representation of an adding circuit,

Figure 14 is a wiring diagram of the circuit of Figure 13,

Figure 15 is a diagrammatic representation of a multiplying circuit, and

Figure 16 is a wiring diagram of the circuit of Figure 15.

The purpose of the present invention is to generate arbitrary functions by the direct method of computation operative in the binary system. The device can operate to generate functions of one, two or any number of independent variables. It will be explained for the case of two variables x and y as this case includes the complications due to several variables without being unduly involved. The function to be generated is assumed to be continuous in the mathematical sense. Such functions are usually encountered when they relate to physical phenomena, as for example the ballistic functions of a gun. The function may be in an explicit mathematical form such as $F(x, y)=\sqrt{x^2+y^2}$ or $F(x, y)=xy(x+y)$ or $F(x, y)=x^2+y^2+xy$. In this case it may be simpler to perform the mathematical operations defining the function rather than to use the present device. However, most empirically found functions are not susceptible of being expressed by simple mathematical formulas. They are given in general in the terms of tables or graphs. An example of an "arbitrary" function determined in that manner is given here in graphical form (Figs. 1 and 2) and in tabular form (Fig. 3).

Fig. 1 shows the function $F(x, y)$ plotted along the coordinate z, as a function of the coordinates x and y in a rectangular system of coordinates x, y, z. Thus the surface $z=F(x, y)$ may be considered to represent the function. Two families of plane curves can be obtained by intersecting that surface by a series of $(x-z)$ and $(y-z)$ planes. These curves are shown in perspective in Fig. 1. The curves $(x-z)$ for different values of y are also plotted in Fig. 2. The surface $Z=F(x, y)$ or the curves $Z=F(x)y=$constant, represent the function completely for all points.

Any table, however large, cannot represent the function for all values of the variables since there is an infinite number of such values. It must therefore give the function for certain values only. The table of Fig. 3 gives the value of $F(x, y)$ for 64 sets of values of x and y. These particular values will be called the "major" values and referred to as x_0 and y_0. (The values of the function for these points were obtained by measuring the Z values in Fig. 1.) The table is in the binary numeration. The "major" values of the variable are chosen to correspond to exact values of two, in this case simply the integers 1, 10, 11, 100, 101, etc. In each square of the table, the first number given is the value of the function. Thus, for the point $X_0=100$ and $Y_0=11$, the value of the function is found in square No. 29 to be 011.1011. This is also represented by point Q of Fig. 1.

To find the function for values of the variables other than the major values X_0 and Y_0, interpolation must be used on the basis of the knowledge that the function $F(x, y)$ has been assumed to be continuous. The simplest kind of interpolation is the so-called "linear" interpolation which is used in the first form of the invention. It consists in replacing the actual surface $Z=F(x, y)$ by little planes for each major area corresponding to the interval between consecutive major values of the variables, as shown, for example, by the heavily drawn area surrounding the point P on Fig. 1, the region defined by x_0 from 100 to 101 and y_0 from 11 to 100. It is easy to see, then, that the value of the function for any set of values x, y (represented by the point P) is given approximately by:

$$F(x, y)=F(x_0+\Delta x, y_0+\Delta y)=F(x_0, y_0)+\frac{\Delta Fx(x_0, y_0)}{\Delta x_0}\Delta x+\frac{\Delta Fy(x_0, y_0)}{\Delta y_0}\Delta y$$

This relation is similar to the so-called Taylor series, in which only the first order terms are taken into account. The values Δx and Δy, by which the actual values of the variable differ from the major values, will be called the "minor" values. For the sake of simplicity, in the present example they are the fractional parts of the variables, although in general it is not necessary to make the separation between major and minor parts just where the binal point is. The unit intervals Δx_0 and Δy_0, differences between consecutive major values, are thus equal to one. Therefore the ratios $\Delta Fx/\Delta x_0$ and $\Delta Fy/\Delta y_0$, which are the rates of change of the function with respect to x and y, respectively, are simply numerically equal to ΔFx and ΔFy. These values ΔFx and ΔFy are the differences of the function between consecutive major values of the variables and the corresponding geometrical intervals are shown on Fig. 1. It will be noted, incidentally, that, with this simple linear interpolation, the increment of the function ΔF for a given change Δx_0 and Δy_0 of the variables of the sum of the separate increments ΔFx_0 and ΔFy_0

(2) $$\Delta F = \Delta Fx_0 + \Delta Fy_0$$

as can be seen on Fig. 1.

If the values of ΔFx and ΔFy are known in addition to the values of $F(x_0, y_0)$ for all major points, the value of the function for any other point can be calculated by the relation (1). This is precisely what is done in the first forms of the invention. The table of Fig. 3 gives the values $F(x_0, y_0)$, ΔFx and ΔFy for all the major values.

As an example, consider the point P of coordinates:

$$x=100.1101 \quad x_0=100 \quad \Delta x=.1101$$
$$y=11.1110 \quad y_0=11 \quad \Delta y=.1110$$

From the table, square No. 29, we find:

$$F(x_0, y_0)=011.1011$$
$$\Delta Fx=.0011$$
$$\Delta Fy=.1011$$

Therefore, the value of the function $F(x, y)$ at P is:

$$F(x, y)=F(100.1101;\ 11.1110)=011.1011+(.0011)\ (.1101)+(.1011)\ (.1110)=011.1011+.00101+.10011=100.0111 \text{ since } (.0011)\ (.1101)\sim .00101 \text{ and } (.1011)\ (.1110)\sim .10011.$$

As heretofore indicated, the function generator includes three principal elements or units, namely, the selector which selects a particular point neighboring that at which the function is to be evaluated, the function matrix which makes available values of the function and desired functions at that selected point, and the interpolator which combines these values with the differences of the variables at the desired and selected points, to produce the function in the form of electric potentials representing its various digits in the binary numeration.

The selector unit is disclosed in two different forms (Figs. 4 and 5). In the first of these forms (that of Fig. 4), the major values x_0 and y_0, which are given in terms of binary numbers, are transformed separately to control an orthogonal network of function matrix input tubes. In the second of these forms (that of Fig. 5), the intermediary step of separate transformation of the major values x_0 and y_0 is avoided. It is to be understood that the network is represented as orthogonal only for convenience of explanation and that the two sets of conductors may have any other convenient arrangement.

The selector of Fig. 4 is shown as adapted for eight possible major values of x_0 and the same number of major values y_0. There are therefore 64 possible sets of values, as indicated by the orthogonal network of tubes 1 to 64 which are connected in the input leads of the function matrix hereinafter described in connection with Fig. 5.

These values are established through three pairs of conductors 201, 202 and 203 for the y_0 input and a similar number of pairs 204, 205 and 206 for the x_0 input. Each pair of these conductors is connected at one end to some of the horizontal conductors (207 to 214 for the y_0 input and others for the x_0 input) of the selector and at the other end to the cathodes of the selector matrix input tubes 215 to 217 for the y_0 input and 218 to 220 for the x_0 input. Potential is applied to the grids of the tubes 215 to 220 from leads maintained at −610 volts and −500 volts, as indicated in Fig. 4. The application of these potentials to the grids may be controlled by switches 221 to 226 as illustrated, or by another computing device, or any other means which will maintain these grids at potentials corresponding to the binary numbers of the major inputs. The selector output tubes 227 to 242 have been interposed between the selector input tubes 215 to 220 and the matrix input tubes 1 to 64.

It will be apparent that the switches 215 to 220 control the grid potential of the selector matrix input tubes, i. e., set them at $V_1=-610$ volts or $V_2=-500$ volts. These tubes are merely amplifier tubes so that no appreciable power is drawn in the input circuit, and they are operated as "cathode followers." It will be apparent that the pairs of "y" vertical conductors (left, Fig. 4) will be excited according to the binary input potentials, i. e., the left conductor of any pair will be at −610 volts and the right one at −500 volts for the digit 0 and vice versa for digit one. The horizontal conductors 207 to 14 are connected to the grids of the selector output tubes 227 to 234 and are coupled through high resistances (1,000,000 ohms) to the certain vertical conductors of the converter.

The pattern of these couplings is as follows: Each horizontal conductor is coupled to the vertical wire which is at the most negative potential (−610 volts) of every pair for the particular combination of excitation of these vertical wires corresponding to its order number. For example, the grid of tube 231 (binary 011 or 3 in decimal numeration) is connected to the left wire of the pair corresponding to the digit 2^2 (100), since it is at −610 volts when "0" is set on the switch 100, to the right wire of the pair corresponding to the digit 2^1 (10), since it is at −610 volts when "1" is set on the switch 10, and finally to the right wire of the pair corresponding to the digit 2^0 (1) since it is at −610 volts when "1" is set on switch 1.

It is apparent that the potential of the grid of any one of these selector output tubes will be at −610 volts for the particular combination of excitations of the inputs which correspond to it, and will be somewhat more positive (at least by 33 volts) for any other combination of excitation since at least one of the vertical conductors to which it is connected is at −500 volts. Therefore, one and only one of the selector output tubes will be cut off, since the cathodes are maintained at −600 volts, and it will be the one corresponding to the particular combination of potential excitation corresponding to its order number. All other tubes will be conducting.

It may be noted that in the resistive network of the selector every conductor is connected to every other one through many "parasitic" circuits. However, this network behaves substantially as stated above because the driving circuit is at the low impedance of 50,000ω while the network is at relatively high impedance since it is composed of 1,000,000ω impedances. Furthermore, the cathode follower driving tends to keep the cathode at approximately the grid potential independently of the load. The selector for the x_0 variable is shown only diagrammatically on Fig. 4; the coupling megohm resistances are heavy dots and the converter output tubes 235 to 242 are shown as circles. This mode of representation will be used subsequently in order to simplify the drawings.

The anodes of the selector output tubes 227 to 234 and 235 to 242 are coupled to the grids of the orthogonal network function matrix input tubes 1 to 64. These tubes can either be triodes or double control grid tubes, and they are arranged in an array corresponding to the squares of the table of Fig. 3. To each tube there corresponds a vertical exciting conductor coming from the x-converter and a horizontal exciting conductor coming from the y-converter. If double grid tubes are used, as shown for tube 53 on Fig. 4, the vertical wire is connected directly to one grid and the horizontal to the other. It will be apparent that only one out of the 64 tubes of the array will conduct, the one for which both grids are at −340 volts, i. e., when the corresponding tube of the x_0 converter is cut off (plate at −340 volts) and the corresponding tube of the y converter is cut off. For all other tubes, one or the other or both of the grids will be more negative than −340 volts, say at −500 volts, so that they will be cut off.

If triodes are used, as shown for tube 29 in Fig. 4, the single grid of the triode is coupled through a 1,000,000 ohms resistance to the horizontal conductor and through another 1,000,000 ohms to the vertical conductor. It is apparent again that only the particular triode lying in the intersection of the excited conductors will be conducting. The grid of that particular triode will be at −340 volts, whereas all other grids will be more negative by at least half the potential variation of the converter output tubes, and will therefore be cut off since the cathodes of all the triodes are at −340 volts. It may be noted here, as it was in the case of the converter resistive networks, that the resistive network of the plate of the converters to the grid of the function matrix input tubes has many parasitic connections producing extraneous excitations. Here again the operation is as stated in spite of these because the driving impedances are low (20,000ω) and the coupling resistances are high (1,000,000ω).

Whether triodes or double grid tubes are used in the orthogonal array of function matrix input tubes, it is apparent from the above description that the selector will operate so as to excite, by making it conduct, one such tube for every set of major values x_0 and y_0. Therefore, it will truly select the point corresponding to the coordinates x_0 and y_0, while it will leave the other tubes non-excited, i. e., non-conducting. Therefore, for every possible combination of excitation of the system of potentials representing x_0 and y_0, one particular horizontal lead of the "function matrix" will be excited. These leads are shown on the right of Fig. 4, or extending above the tubes 1 to 64.

The same result can be obtained with a simple selector built according to the second modification. Such a selector is shown at the left of Fig. 5, while Fig. 6 shows the detail of the circuit. The basic idea is to combine the input potentials corresponding to the major values of x_0 and y_0 together and to consider the resulting combination as one single variable. Therefore, a larger converter from binary to natural order (having 6 inputs and 64 outputs rather than 3 inputs and 8 outputs) can replace both converters and the orthogonal array of tubes of the first modification. It operates quite similarly to the converters of Fig. 4, except that the selected tube is made to conduct rather than to be cut off, as was the case in the converters. To each of the digits (6 in the present example) of this new variable are assigned two potentials carried on two conductors 201 to 206 which bear a "push-pull" or conjugate relation to one another, that is to say, for digit zero one conductor (the left one in Figs. 5 and 6) is at the most positive potential V_1 (−340 volts) and the other (the right one on Figs. 5 and 6) is at the most negative potential V_2 (−600 volts) and for digit one the potentials of the two conductors are interchanged.

The selector matrix is composed of two sets of orthogonal conductors between which high coupling resistances (2,700,000ω in the present example) are connected according to a predetermined pattern. On Fig. 5 the heavy dots on the selector matrix (left) represent such resistances, which are to be understood as connected between the two conductors on the intersection of which they are drawn. The vertical conductors carry the input potentials and the horizontal wires are connected to the grid of the triodes (halves of 6SN7's in the example of Fig. 6) of the matrix input tubes (shown as numbered circles in Figs. 5 and 6). The push-pull signals are obtained from a previous computing device, or they may be set in by a series of switches. The selector input tubes 215 to 220 (lower left, Figs. 5 and 6) are merely amplifiers, so that no appreciable power is drawn in the input circuit and they operate as cathode followers, as did the input tubes to the converters of Fig. 4.

The pattern of resistances is determined as follows: Any one of the horizontal conductors of the selector matrix which is connected to the grid of a triode of the input matrix tubes (numbered 1 to 64 on Fig. 5) is coupled through high (2,700,000ω) resistances to the vertical wires which are at the most positive (V_2=−340 volts) of the two potentials V_1 and V_2 for the particular combination of excitation corresponding to that selected triode. It is thus obvious that this horizontal grid lead will be at the positive potential V_2 (−340 volts) for the combination of excitations corresponding to it and will be at some potential negative with respect to V_2 for any other combination.

This results from the fact that for any other combination at least one of the vertical conductors to which that particular triode-grid is coupled will be at the more negative potential V_2=−600 volts, so that there will be $(\Phi-1)$ [in the present case $\Phi-1=5$] connections at V_2 (=−600 volts) and one at V_1 (=−340 volts) and the potential assumed by the lead will be the average $[V_2(\Phi-1)+V_1]/\Phi$ [in the present case $-(5.340+600)/6\cong-383$ volts] which is negative with respect to V_2. Therefore, with the proper bias (−340 volts in the present case) on the triodes, only the tube corresponding to the "selected" value represented by the input potentials will conduct and all others will be cut off. As an example, consider the connection to tube No. 29 for which x_0=100 and y_0=011; therefore, the new variable has the value 100011, so that the pattern of resistances for that lead is RLLLRR, as can be seen on Figs. 5 and 6.

The remark concerning parasitic connections made in connection with the selectors of Fig. 4 applies as well to the selector matrix of Fig. 5. The possible undesirable extraneous excitations do not interfere with the proper operation because the coupling resistances are high (2,700,000 ohms), the driving resistances low (50,000ω) and the driving circuit is degenerative (cathode follower).

The purpose of the selector described above is to "select" a point x_0-y_0 which corresponds to selecting a certain square on the table of Fig. 3. The purpose of the function matrix is to assign to this square the values of the functions $F(x_0, y_0)$, ΔFx and ΔFy which are indicated by the table.

The function matrix is represented diagrammatically on Fig. 5 (upper right). The details of the connections are shown on Fig. 6. The matrix is illustrated as composed of two systems of orthogonal conductors, the orthogonal relation being a consideration of convenience and not of necessity. The horizontal conductors are connected to the plates of the function matrix input tubes (1 to 64 of Fig. 5) which are both the output tubes of the selector matrix and the input tubes of the function matrix, and the vertical conductors are connected to the grids of the function matrix output tubes 243. The state of excitation of these matrix output tubes represents the three functions to be generated. One such tube is assigned to each binary place of the functions. In the example used here, there are three sets of output tubes, the first for $F(x_0, y_0)$ having seven tubes to take care of the seven places or digits encountered for that function in the table of Fig. 3, the second and third each having nine tubes to take care of all significant places encountered in the functions ΔFx and ΔFy in the table of Fig. 3.

The vertical wires are coupled to the horizontal wires through high coupling resistances (R_c=1,000,000 ohms) according to a predetermined pattern, i. e., certain vertical wires are coupled to certain horizontal ones, the choice of the wires or the pattern determining the functions to be generated. A resistance is connected between a given vertical wire corresponding to a given set of major values x_0-y_0 and a given horizontal wire corresponding to a given binary place of the functions when the digit of that binary place happens to be one. The resistance is omitted if that digit happens to be zero. The function is thus recorded in terms of existing or non-existing resistances. As an example, consider the square 29 of Fig. 3, and the corresponding input matrix triode No. 29 on Figs. 5 and 6. The values of the three functions are

$$F(x_0, y_0)=011.1011$$
$$\Delta Fx=00000.0011$$

and

$$\Delta Fy=00000.1011$$

(The reason for the five zeros on the left of the binal point for ΔFx and ΔFy is explained below). Therefore, the pattern of resistances on the lead 29, Fig. 5, is like the pattern of "ones," i. e., 0RRR0RR, 0000000RR and 00000R0RR.

The operation of the function matrix is as follows: The selected input matrix tube is conducting while the others remain non-conducting.

Therefore, the potential of the selected horizontal wire becomes negative (in the present case about −200 volts, assuming a current of 10 ma. in the tube No. 29, since the +B is at 0.5 volt, see Fig. 6). This causes the potential of the vertical wires to which this selected horizontal wire is coupled to become more negative than the ones to which that wire is not coupled. Therefore, with proper bias (zero in the case of Fig. 6) of the output matrix tubes, the tubes which are coupled will be cut off and the ones which are not coupled will be conducting. Therefore, it follows that the output matrix tubes will be excited according to the predetermined function set in the function matrix of resistances.

The detail of the connections of the matrix output tube is shown on the lower right portion of Fig. 6. Consider a typical output tube 243, for example, the one on the lead marked .001 (right). This tube is a pentode 6SJ7. A neon lamp 244 is inserted in the plate circuit to provide a visible indication of the state of conduction of the tube. This, of course, is not essential since the functions $F(x, y_0)$, ΔFx and ΔFy are to be combined according to the relation of Equation 1 before the final result $F(x, y)$ is obtained. The reason for the different circuit shown for the tube connected to the lead 10 is explained below.

In the function matrix of resistances every vertical conductor is connected to all horizontal conductors and vice-versa. In fact, there is a connection between any two conductors. Therefore, when the input conductors are excited they produce not only the desired excitation on the desired output leads, but also parasitic excitations on other leads. The matrix is therefore designed in such a manner as to reduce these parasitic effects to such an extent as to make the ratio of useful to parasitic excitation sufficiently large so that the output tubes can discriminate between a true and a false signal. This is achieved by using coupling resistances Rc of high impedance (1,000,000 ohms) with respect to the plate resistances Rb of the driving tubes (R_b=20,000 ohms). The problem of parasitic signals has been analyzed quantitatively. The main results may be summarized as follows: Let N be the number of input horizontal conductors (64 in the present example), p the number of vertical conductors (9+9+7=25 in the present case); i the current through the driving triodes 1 to 64 (10 ma. in the present case); Rb their plate resistance (20,000 ohms), Rc the coupling resistance (1,000,000 ohms in the present case), Es the true signal and Ep the parasitic signal on the grids of the output tubes. Then assuming the worst possible conditions, i. e., the weakest true and largest false signal, the following relations hold:

$$(3) \qquad Es=i\frac{Rb}{N}$$

$$(4) \qquad \frac{Es}{Ep}=\frac{(p-1)+\frac{Rc}{Rb}}{(p-1)}$$

Therefore the attenuation, i. e., the ratio of driving signal on triodes to useful signal, depends only on the number N of input elements. In this case N=64 so that the weakest useful signal on the grid of the output pentodes is about

$$200/64=3.1$$

volts which is sufficient to cut off the tubes 243. The ratio of true to false signals depends only on the ratio of plates to coupling resistances and the number of output leads p. In the present case this ratio is

$$\left(24+\frac{1,000,000}{20,000}\right)/24=3$$

Therefore, the strongest possible parasitic signal is one volt, which is not sufficient to cut off the tubes, since the bias has been adjusted to be +0.5 volt. It follows that the matrix as designed is operative.

In connection with the strength of the useful and parasitic signals, two features of the invention are of interest as they have considerable practical importance.

The attenuation of the useful signal by the factor N shown by relation (3) arose from the assumed worst case when all the horizontal leads are connected to a given vertical lead. This case may well occur. In spite of this the attenuation factor may be reduced to N/2 by the following expedient. The number of coupling resistances for any one vertical lead may be smaller or greater than N/2. If it is smaller, the attenuation of the signal for that particular signal will be less than N/2. If it is larger, that means that there are more digits "one" than digits "zero" for that particular vertical conductor. The expedient consists in that case in using a coupling resistance at every place which corresponds to the digit zero (instead of one) and omitting it for places corresponding to the digit one (instead of zero). In this manner the number of resistances on every vertical wire is less (or equal to) than N/2 (32 in the present case), so that the maximum attenuation is N/2. Of course, this expedient requires that the polarity of the signal on the wires where the resistances have been permuted be reversed. This is shown for the line marked **10** in the lower right part of Fig. 6. An additional "phase inverting" tube **245** (6SN7) is used for the purpose. It can be seen that the true signal on the matrix output tubes is now 200/32≈6 volts (instead of 3 volts).

The relations (3) and (4) mentioned above have been established for the case of maximum attenuation, i. e., one vertical lead is connected to all horizontal leads (and the corresponding digit is always one) and maximum parasitic signal, when all the matrix is full of resistances except a particular one. In practice, the matrix will be filled in some random manner so that the useful and parasitic signals will be different for each output conductor and different for every excitation. It could happen therefore that the ratio of the weakest useful signal at some particular conductor to the strongest parasitic signal at some other conductor would not be as great as indicated by relation (4). To bring all conductors to a standard condition of attenuation equal to N/2 the vertical leads are coupled to the plate supply of the input tubes (+0.5 volt in our example) through compensating resistances R_d which are adjusted for each conductor so as to make the loading uniform. (See Fig. 6.) For example, the value of the resistance for lead marked **100** is 200,000 ohms. This value was obtained by considering that the maximum number of coupling resistances on any one vertical lead is 32, and that the number of resistances on that lead is 27; therefore, a loading corresponding to five coupling resistances in parallel is necessary to bring that lead to the "32" loading, standard condition. Therefore the corresponding compensating resistance is 1,000,000/5=200,000 ohms. Of course, the compensating resistances are different for every conductor.

In many applications it is desirable to generate many functions of the same variables, as is the case, for example, in anti-aircraft fire computers. It is also the case for a single function if a method of interpolation more refined than linear is used. Any number of functions of the same variables can obviously be generated by the same device. It suffices merely to connect their "function" matrices to the first function matrix. It amounts to increasing the number p of vertical conductors. This can be done without further changes in the device as long as the parasitic signal does not become unduly large. However, it is seen from relation (4) that as p increases, the ratio of true to false signal tends toward unity. By using a large R_c/R_b ratio the number of usable places can be fairly large. There is a method by which there is no limit to the number of functions which can be generated. It consists of using a degenerative drive of the matrix, using two tubes instead of one for every input place of the matrix.

Fig. 7 shows one typical driving arrangement for any input (horizontal) conductor. The output of the selector and input of one function matrix are two separate tubes (rather than a single one). The output of the selector is connected as before. The input of the function matrix is derived from the cathode rather than the plate. The tube **247** is connected as a cathode follower so that the potential of the cathode is approximately equal to that of the grid regardless of the load. This means, therefore, that the parasitic signals will be suppressed all together since the potential of all the non-excited horizontal leads of the function matrix will be forcibly maintained at +0.5 volt, in spite of parasitic effects while only the excited lead will become negative.

The function table as described has two matrices of resistance, the selector matrix, which determines which values of the major parts of the variables are assigned to which values of the function, and the function matrix, which determines the nature of the function. Both of these are "arbitrary," that is to say, the pattern of resistances can be chosen to express any desired function.

A particularly convenient method of mounting the large number of resistances consists in holding them in holes drilled in a Bakelite board. The board is drilled according to the desired pattern and the resistances are inserted and soldered in place with the two sets of conductors on opposite sides of the board. If the generator is installed in some computing device where it is desirable to change frequently the nature of the function, as may be the case in computers for fire control when types of guns or shells are changed, the board of resistances may be provided with jacks and may be plugged in and out without disturbing any permanent connections.

When the function $F(x_0, y_0)$ and the interpolating coefficients ΔFx and ΔFy are known, it suffices to make the two multiplications and the two additions indicated in relation (1) to obtain the desired function $F(x, y)$. Means for doing this are shown diagrammatically on Fig. 5. The squares **256** and **257** symbolize multiplying and adding devices operating by the direct method. The upper sides are the terms of the product, the lower left side the number to be added and the lower right side the result.

In a first form, the adding and multiplying device is simply the one tube device such as is disclosed in the aforesaid copending application Serial No. 496,746. The inputs to the electronic calculating tube are direct without any coupling impedances, as explained in that application. Of course, it must be kept in mind that the different inputs must be on the same D.-C. level, which means that the minor parts of the variables Δx and Δy must be brought to the same level as the interpolating coefficients ΔFx and ΔFy.

In a second form, the adding and multiplying devices are the "direct multiplier and adder" described in my copending application Serial No. 511,729, filed Nov. 25, 1943. As previously mentioned, the various multiplications and additions required to combine the major values x_0 and y_0, the minor values Δx and Δy and the interpolating coefficients ΔFx and ΔFy, as indicated by the squares and legends at the lower right-hand corners of Figures 5, 9 and 12, may be performed by devices of the type disclosed by my copending application Ser. No. 496,746. Thus in the case of the square **256** of Figure 5, for example, the ΔFx leads are connected to the "multiplicand" leads of Figure 5 of the copending application, the Δx and $F(x_0, y_0)$ leads are connected respectively to the "multiplier" and the "input B" leads and potentials representative of the value of Δx,

$$\Delta Fx + F(x_0, y_0)$$

are produced at the output leads bearing the numerals 2^5, 2^6, 2^7 and 2^8. The square of Figure 5 indicated the numeral **257** is a second device which is similar to that of the copending application and functions to derive the product Δy, ΔFy and to add this product to the output Δx, $\Delta Fx + F(x_0, y_0)$ of the device **256**.

The various additions involved in deriving the value of the function also may be performed by the adding circuit which is disclosed in my copending application Ser. No. 519,299 (Patent No. 2,404,250) and was made prior to the filing date of the present application.

This adding circuit is disclosed in Figures 12 and 13 of the present application.

Figure 13 is a diagrammatic representation of a computing circuit arranged in accordance with the invention for adding two numbers (A and B) of six digital positions, circles being used to indicate the electron discharge devices involved in the various connections.

The circuit of Figure 13 includes one group of input tubes **310** to **315** to which are applied potentials representative of the various digits of a number A and another group of input tubes **316** to **321** to which are applied potentials representative of the various digits of a number B. In each of these groups, the lowest digital position is at the top and highest digital position is at the bottom. This is indicated by the binary numbers placed above the various input leads. When −180 volts are applied to an input lead, the digital position which it represents contains a zero. When zero voltage is applied to one of these input leads, the digital position which it represents contains a one. Switches **340** and **341** (Fig. 14), another computing circuit or any other suitable means may be utilized to apply these digital representative potentials to the input leads.

The condition of the two groups of input tubes **310** to **315** and **316** to **321** is determined by the digits of the two numbers to be added. Each of these tubes is conducting a standard amount of current of about 4 ma. when a potential (0 v.) representative of the digit one is applied to its control grid and is biased off when a potential (−180 v.) representative of the digit zero is applied to its control grid. These standard unit currents are supplied through a common resistor **342** and are determined by the fact that each of the tubes is connected to operate as a cathode follower. Under these conditions, the anode potential of all the input tubes decreases by a standard amount for each tube that is made to conduct the standard units of 4 ma. and each of the tubes may be considered as representing a digit one or a digit zero.

For converting these various digits into a binary number which is the sum of the two numbers, a group of carry over tubes **322** to **326** and a group of carry over control tubes **327** to **332** are provided. The resulting sum is indicated by a group of indicators **333** to **339** which may include a neon lamp or the like. The manner in which these results are accomplished will be more easily understood in connection with Figure 14.

Figure 14 shows the details of that part of the circuit which appears in the heavy lines of Figure 13. It will be noted that the input tubes **313** and **319** are connected to the same terminal of the resistor **342** as the carry over tube **325** which has its control grid connected to the carry over control tube **332** for applying a positive potential when a one is to be transferred from the second digital position to the third digital position which is represented by the input tubes **313** and **319**. All the carry over tubes **322** to **326**, like the input tubes **310** to **321**, are of the cathode follower type so connected as to conduct a standard unit (4 ma.) of current.

It is apparent that the potential at the lower terminal of the resistor **342** is reduced by a predetermined amount when one of the tubes **325**, **313** or **319** takes current, by twice this amount when two of these tubes take current and by three times this amount when all three of these tubes take current. These different voltages are applied through the resistors **343** and **344** to the first or control grids of the indicator tube **337** and the carry over control tube **330**. Potential is applied also to these grids from a −150 v. lead through resistors **345** and **346**. To the second or screen grids of the tubes **330** and **337**, potential is applied from a +45 v. lead.

Connected in shunt to the tube **337** is a neon tube **347** for indicating when this tube is not conducting (a condition existing when a digit zero is in the third digital position of the sum of the two numbers being added).

The carry over tube **324** of the fourth digital position has the upper end of its cathode resistor connected through a resistor **348** to the first or control grid of the indicator tube **337**. The control grid of the tube **324** is connected to the diode element of the tube **330** and through a resistor **349** to the anode of the tube **330** so that the tube **324** conducts current only when the tube **330** is biased off. The purpose of the diode element of the tube **330** is to establish at the grid of the carry over tube **324** a predetermined potential which is intermediate those of the +550 v. and −600 v. leads when the tube **330** becomes nonconducting and no plate current is drawn through its anode resistor by the tube.

The manner in which the circuit operates to convert the digits established by the tubes **325**, 313 and 319 into a binary number is indicated by the following tabulation:

Input Tubes Conducting	Tube 330	Tube 337	Tube 324	Sum
None	on	on	off	0
1	on	off	off	100
2	off	on	on	1,000
3	off	off	on	1,100

From this tabulation, it is evident that the tubes 330 and 337 are conducting and the tube 324 is biased off when none of the tubes 325, 313 and 319 is conducting. When one of the tubes 325, 313 or 319 is conducting, the potential at the lower end of the resistor 342 is reduced sufficiently to bias off the tube 337 thereby lighting the lamp 347 and indicating a digit one in the third digital position of the binary number. When two of the tubes 325, 313 or 319 are conducting, the potential at the lower end of the resistor 342 is reduced sufficiently to bias off the tube 330. This has two results. It makes the tube 324 conducting so that a digit one is carried over to the fourth digital position. When the tube 324 conducts, a positive potential is applied to the control grid of the tube 337 so that this tube takes current and the lamp 347 is extinguished. When all of the tubes 325, 313 and 319 conduct the potential at the lower end of the resistor is sufficiently negative to bias off both tubes 330 and 337 so that the carry over tube 324 remains conducting and the lamp 347 is lighted. Under these conditions, a binary number of 1100 is established in the part of the circuit detailed in Figure 14. How the complete sum of two numbers represented by potential applied to all the input leads is established is readily understood from the foregoing explanation. How the carry over system of Figures 13 and 14 is extended to produce the sum of numbers having a higher number of digits will be understood readily from consideration of the multiplying circuit of Figures 15 and 16, since this multiplying circuit employs the same carry over and indicating system as that of the adding circuit.

The multiplying circuit of Figures 15 and 16 may be utilized to perform the various multiplications required to produce the selected value of the function. By this multiplying circuit, the product of two binary numbers is obtained in the form of potentials representative of the digits of such product.

The product of two binary numbers x and y

$$(1)\quad x=A_p2^p+A_{p-1}2^{p-1}+\ \ldots\ +A_0=\sum_{i=0}^{i=p}A_i2^i$$

$$(2)\quad y=B_p2^p+B_{p-1}2^{p-1}+\ \ldots\ +B_0=\sum_{k=0}^{k=p}B_k2^k$$

where A_i and B_k are equal to one or zero, can be written as:

$$(3)\quad x.y=\sum_i^p\sum_k^p A_iB_k2^{i+k}=\sum_{j=0}^{j=2p+2}C_j2^j$$

In the product the coefficients A_iB_k are also equal to zero or one, but are equal to one only if both A_i and B_k are equal to one. This property of the binary system of numeration in which the multiplication table of the basic digits reduces itself to a pure coincidence effect, is unique. In all other scales the number of answers in the basic multiplication table of digits is always greater than the radex. In accordance with this invention, that coincidence effect is utilized to produce the terms A_iB_k. However, additions of intensities are utilized for the summations, although this could be accomplished in other ways.

The two systems of potentials representing x and y are carried by two systems of wires arranged, as shown in Figure 15, in an orthogonal network whose intersections constitute the elements of a matrix corresponding to the terms A_iB_k. A vacuum tube 436 on each of these intersections is made to conduct only if there is coincidence of excitation of the two wires at such intersection, and therefore give a value to A_iB_k only when A_i and B_k are both one. All the coefficients A_iB_k corresponding to the power 2^{i+k} are located on diagonal lines, vertical on the Figure 15. Therefore, if all the plate currents of the tubes on these lines are added, the total current will be proportional to the coefficient of the 2^{i+k} term, provided that each tube, when conducting, contributes a standard amount of current. The system of potentials assumed by the plates is already a representation of the product, but it is not in the binary system, since the coefficient of each power of 2 may be larger than one. To obtain the answer in the binary system, the S_j current steps appearing in the j row of each $2p+2$ rows must be revalued into binary number places to excite the proper carry-over and indicating tubes. This can be done in several different ways.

It has been found that the most convenient manner is as follows: Let

$$(4)\quad S_j=C_j+2D_{j+1}\ \ldots\ 2^mD_{j+m}$$

It is apparent that if there are m carry-over tubes located on the rows $(j+1)$,

$$(J+2)\ \ldots\ (J+m)$$

which are excited when the corresponding coefficient D is one, the proper carry over's will be obtained provided that the circuit of each carry-over tube, added to the circuits of the proper row, will contribute the same standard amount of current as the tubes of the matrix. This is so because any one of the coefficients $D_{j+\lambda}$ multiplies the power $2^{j+\lambda}$ and is added precisely to the row corresponding to the $(j+\lambda)^{th}$ power, as hereinafter explained in detail. To obtain the result, it suffices merely to provide an indicator on each of the $(2p+2)$ rows which will be excited when the corresponding coefficient C_j is equal to one. The carry-over tubes 465 and 483 and indicator tubes 484 to 495 for the case of $p=5$ are shown on Figure 15. The thirty-six tubes 436 of the matrix and the carry-over tubes 465 to 483 all contribute a standard current when conducting. The tubes 436 contribute no current when there is no excitation from one of the corresponding leads. Auxiliary amplifying tubes 496 to 513 are used in one modification of the invention.

The basic part of the multiplier is the circuit which will produce the signals to excite the indicator and carry-over tubes, according to Equation 4. This circuit is repeated on each one of the $(2p+2)$ rows, with various degrees of complication because the maximum values which $S_{j\,max}$ can obtain are different for each row.

The range of numbers covered by the arrangement of Figure 15 is five binary places in the multiplicand and five binary places in the multiplier, as indicated by the numerals under the "x" and "y" braces. The general mode of operation will be readily understood from a few examples. There can, of course, be no carry-over from the tube 436 at the lower corner (diagonal No. 1) of the orthogonal network because the current output from this tube can only represent one or zero. If it is one, the indicator **495**, which may be a neon lamp or the like, so indicates. If it is zero, this lamp is not lit.

The second diagonal includes two tubes **436** and may, therefore contribute none, one or two units of current, depending on whether none, one or both tubes are simultaneously excited from the $X-1$, $X-2$ $Y-1$ and $Y-2$ leads. One carry-over tube **465** is therefore required to carry one to the next digital position when both tubes are excited and the total is 4 (100 in the binary system since each position in this row represents 2).

The third diagonal includes three tubes **436** which may contribute as many as three units of current, depending on whether one or more of the tubes are excited from the leads $x-1$, $x-2$, $x-4$, $y-1$, $y-2$ and $y-4$. To this may be added one unit of current from the carry-over tube **465**, making a possible total of four current units. One carry-over tube **466** is required to carry one to the next highest digital position when two or three tubes are excited and the total is 8 or 12 (1000 or 1100 in the binary system). Another carry-over tube **467** is required to carry one to the "16" digital position when four tubes are excited and the total is 16 (10,000 in the binary system).

The sixth diagonal includes six tubes **436** which may contribute as many as six units of current depending on whether one or more of the tubes are excited from the x and y leads. To this may be added the two units of the carry-over tubes **469** and **470**, making a total of eight units of current. One carry-over tube **472** is required to carry one to the "64" digital position when two or three of the tubes are excited and the total is 64 or 96 (1,000,000 or 1,100,000 in the binary system). A second carry-over tube **473** is required to carry one to the "128" digital position when four, five, six or seven tubes are excited and the total is 128, 160, 192 or 224 (10,000,000, 10,100,000, 11,000,000 or 11,100,000 in the binary system). A third carry-over tube **474** is required to carry one to the "256" digital position when all eight tubes are excited and the total is 256 (100,000,000 in the binary system).

How the various matrix and carry-over tube are made to deliver the units of currents discussed above will be understood upon consideration of Figure 16, which shows a detailed wiring diagram of part of a modification which is similar to Figure 15 as indicated by the use of the same reference numerals to indicate corresponding parts.

It will be observed that the tubes **436** of Figure 2 are those of the sixth or longest diagonal of the matrix of Figure 15, that at the intersection of the x and y leads on this diagonal they are interconnected through resistors **515** (1,000,000Ω), and that the grids of the tubes **436** are connected to the midpoints of these resistors. The cathodes of the matrix tubes **436** and carry-over tubes **69** are connected through resistors **516** (37,500Ω) to a source of −150 volts. The unexcited potential of the x and y leads of the matrix is −350 volts and their excited potential is zero. It is apparent that any triode will conduct only when both driving wires are zero, and in that condition the current through the triode will be essentially determined by the 150 volts applied to the cathode resistance returns, and the value of this resistance, and will depend only slightly on the actual characteristic of the triode. The currents of all the S triodes located on the same row, that is corresponding to the same power of 2, are caused to flow through a common resistance **517** (3,200Ω). To each row of triodes corresponds a number M of amplifying pentodes **518**, **519** and **520** and an indicator tube **521** equal to the number $(m+1)$ of terms in Equation 4, expressing S as a binary number.

The method used to excite these tubes **518**, **519** and **520** consists of coupling to each control grid, in addition to the potential of the common anodes, several additional potentials through an appropriate network of resistances **522** to **527** and **529** to **532**. These additional potentials are those of cathodes of the carry-over tubes **472**, **473** and **474**, each of which is controlled by a corresponding amplifying tube. The values of the resistors are 450,000Ω for **522**, 562,500Ω for **523**, 1,275,000Ω for **524**, 1,275,000Ω for **525**, 562,500Ω for **526**, 450,000Ω for **527**, 2,662,000Ω for **529**, 562,500Ω for **530**, 562,500Ω for **531** and 450,000Ω for **532**.

The grids of the indicator tube **521** and of the three amplifying tubes **518**, **519** and **520** are all coupled to the bus **533** at the common potential of the plates of the intensity tubes **436**, and are, in general, all coupled through resistors **535** to **538** to the bus **534** maintained at a convenient negative potential. The values of these coupling resistors are 773Ω for **535**, 1,785,000Ω for **536**, 5,560,000Ω for **537** and ∞ for **538**. In addition, each grid is connected to buses **539**, **540** and **541**, but only to the buses corresponding to the carry over of a power of two greater than that corresponding to that particular amplifier tube. Thus tube **521** is coupled to **534**, **539** to **541** and **533**; **520** to **534**, **540**, **541** and **533**; **519** to **534**, **540** and **533**, and **518** to **534** and **533**. The values of the coupling resistances between these buses and the grids of the amplifying tubes are so chosen that when the first carry-over tube **472** is turned on (bus **539**) it tends to vary the potentials of the grids to which it is coupled by an amount twice that which tends to be established by the standard variation of potential of the bus **533** or main driving potential, when the second carry-over tube **473** is turned on (bus **540**) it tends to vary the potentials of the grids to which it is coupled by an amount four times that of the tending of bus **533**, and finally when the last carry-over tube **474** is turned on (bus **541**) it tends to produce a potential change eight times as great. The variations due to buses **541**, **540** and **539** tend to oppose those of bus **533**. Couplings to bus **534** establishes only constant biases in addition to these variations.

The manner of operation can be followed by considering what happens to the grid of the indicator tube **521** when the tubes **436** of the driving row become conducting one by one. When no tubes are on, bus **533** is at its most positive potential, almost +180 volts, and the bias of the grid of the indicator tube **521** is adjusted so that it is just at zero, and the indicator tube is conducting, a state corresponding to the digit 0. When one intensity tube **436** becomes conducting, the bus **533** becomes negative, the other buses do not change their potential, and the indicator tube **521** is cut off, a state corresponding to the digit 1. When two tubes **436** are turned on, the amplifying tube **520**, which was conducting when there were no or one intensity tube **436** conducting is now cut off by virtue of its proper initial bias. Therefore bus **539** tends to produce a variation of two voltage steps in the grid of the indicator which just compensate the two steps by which the bus **533** has varied, and therefore the indicator tube **521** is brought back to its previous condition and conducts again. For three intensity tubes **436** conducting, the tube **521** is cut off again, since only the bus **533** has varied in potential. For four tubes **436** conducting the amplifying tube **519** is cut off by virtue of its original bias, which causes bus **541** to turn on the indicator of the next highest digital place by compensating exactly the driving potential which reestablishes the previous status, and to turn on the indicator for which the previous status is also reestablished since bus **539** came back to its original potential. It may be seen that the indicator **521** and the amplifying tubes operate similarly for more steps as pointed out above.

The anodes of the amplifying tubes **518**, **519** and **520** are coupled to the grids of the carry-over tubes **474**, **473** and **472** by a voltage divider (resistance **542** of 1,800,000Ω and **543** of 3,300,300Ω) providing the suitable D.-C. bias. When the amplifying tubes are cut off, the grids of the carry-over tubes **474**, **473** and **472** must reach zero potential. In order that this be obtained independently of the accuracy of the coupling resistances **542** and **543**, this grid is connected to a diode which is a part of the amplifying tubes **518**, **519** and **520**. Thus the current through tube **473** depends on the potential of its grid. That potential would be determined only by the ratio of resistances **542** and **543** in the absence of the diodes. Therefore these resistances would have to be very accurate. This requirement is avoided by the diode. When the plate of tube **519** goes positive (i. e. when the tube is cut-off) it tends to bring the plate of the diode to an appreciable positive potential. However, this effect is compensated by the current through the diode which flows through resistance **542** and plate resistance of tube **519**. This current causes a potential drop through resistance **542** so that the potential of the plate of the diode remains only a few volts positive with respect to its cathode, and assumes thus a "standard" potential independent of the accuracy of the resistances **542** and **543**. The complete multiplier is composed of a series of 11 circuits as illustrated on Figure 15, only simpler in that the number S of driving tubes **436** corresponding to the number M of amplifier tubes is smaller. The output carry-over tubes of one circuit are the input carry-overs of the next, as explained above.

The adding circuit of Figures 13 and 14 is provided with a carry-over and indicating system similar to that of Figure 16 with the exception that (1) the number of tubes **436** (corresponding to the input tubes **313** and **319** of Figures 13 and 14) is increased to accommodate any desired increase in the numbers to be added and (2) the conductivity of the tubes **436** is determined by the corresponding digits of these numbers. These devices perform multiplication and addition of binary numbers in the direct method, and utilize a resistive coupling system between ordinary radio receiving tubes. In the operation of this modification, each of the matrix tubes conducts a standard amount of current only when both conductors at the intersection are activated, and the carry-over tubes conduct this standard amount of current between proper output leads of the matrix when a 1 is to be carried over. Also in this case the D.-C. levels of the inputs must be the same.

It is of course apparent that other types of adding and multiplying devices may be utilized to perform the additions and multiplications.

In a third modification, shown by Fig. 8, the multiplying and adding devices made with a circuit comprising resistive couplings between radio tubes as in the second modification, may be simplified. The simplification consists in having only one carry-over system for both multiplication and both additions. As pointed out above, the simultaneous multiplier of the second modification produces the carry over signals on the basis of signals which are the actual sum of standard unit currents corresponding to units of the same powers of two. In this modification all these units, from both products ΔFx, Δx and ΔFy, Δy and from the function $F(x_0, y_0)$ are added together before the carry over procedure is started. This modification reduces somewhat the number of tubes which must be used for the interpolation mechanism. With the values assumed in connection with the previous examples, the function $F(x, y)$ is sufficiently defined by places between 100 and .0001. The additional places introduced by the multiplications do not contribute materially to the accuracy of the result. These places are shown in dotted lines on Figure 8.

It will be noted that the only discrepancy between the example of the function given in Figs. 1, 2 and 3 and the one set in the function matrix of Fig. 5 is that there are twelve binary places necessary for the interpolating coefficients in reality whereas only nine have been shown, for the sake of simplifying drawings, in Figs. 5 and 8.

The function to be generated being "arbitrary" may be increasing for increasing values of the variables for some values of the variables, and for others it may be decreasing. This is the case for the function chosen as an example of Figs. 1, 2 and 3. For example, the slopes

$$\frac{\Delta Fx}{\Delta x_0} \text{ and } \frac{\Delta Fx}{\Delta y_0}$$

are both positive for the point $x_0=100$ and $y_0=11$ (square **29**) while the x slope is negative and the y slope is positive for $x_0=1$ and $y_0=1$ (square **10**); and the reverse, x slope positive, y slope negative, occurs for $x=1$, $y=110$ (square **50**).

Therefore, the second and third terms of the relation (**1**) may be positive or negative. In the latter case, the actual number has to be subtracted rather than added. Therefore, provision must be made for the interpolating devices to take care of the sign.

This may be accomplished quite simply by virtue of a property of numbers in the binary (or in any other scale) system. Let us define as complementary A' of a number A whose highest digit corresponds to the ath power to two, the number

$$A'=2^{a+1}-A \tag{5}$$

This number A' is always positive since $A<2^{a+1}$. It can be calculated by taking A, changing all ones into zeros and vice versa, and adding one. For example, if $A=10110$, $A'=01001+1=01010$. By means of this, if the operation $B-A$ has to be performed, where B has a maximum power of 2 equal to a, it suffices to add the complementary number to B, since:

$$B+A'=B+2^{a+1}-A=2^{a+1}+B-A \tag{6}$$

Now if the value 2^{a+1} is discarded, and only powers of 2 up to a are considered, relation (6) gives the correct result. For example, if $B=11011$ and $A=10110$ as before:

$$B-A=11011-10110=00100$$

or

$$=11011+01010=(1)00100$$

Consider now the case of the product of two numbers A and B. Let the complementary number A' of B be

(7) $A'=2^{\lambda}-A$

where $\lambda=a+b+2$. Then:

(8) $A'B=2^{\lambda}B-AB$

In relation (8) the lowest power of the first term of the right side is λ which is greater than the highest power of two included in the second term. Therefore, if all powers above λ are neglected, the right side of relation (8) is the negative of the product AB.

This property can be utilized to take care of negative slopes, if the complementary numbers [as defined by (7)] are used instead of the numbers themselves for negative interpolating coefficients. For example, the value set in the function matrix for square 10, ΔFx is 111111.1001, which is the complementary of $\Delta Fx=-0.0111$. If $\Delta x=.1100$, for example, the term (.1100)(0.0111)=.0010101≅.0010 would have to be subtracted. Instead the term (111111.1001)(.1100)=1011.1110101100 is added. Both give the same result, as can be verified by calculating the value of $F(x_0, y_0)+\Delta Fx.\Delta x$ both ways. This gives:

$F(x_0, y_0)+\Delta Fx.\Delta x=$
001.1110−.0010=001.1100
001.1110+1011.1110=(1)01.1100

The results are the same if the first digit is discarded. This method of handling negative slopes has been assumed in constructing the table of Fig. 3.

The method of the complementary numbers is quite simple. However, it has a practical drawback in that it increases unduly the number of places which the multipliers performing $\Delta Fx.\Delta x$ and $\Delta Fy.\Delta y$ have to handle. If there were no negative slopes, the maximum number of places in ΔFx or ΔFy in our example is five, while with complementary numbers there are twelve places to be handled (nine places have been shown on the diagram of Fig. 5 for the sake of simplicity).

Another method of handling negative interpolation coefficients which does not have the drawback of an unduly great number of binary places in the multipliers consists in adding to all the interpolating coefficients a constant S equal to the smallest integer power of two which is greater than the greatest absolute value of any negative coefficient. This will make all interpolation coefficients positive. It is then possible to write:

(9) $F(x, y)=F(x_0, y_0)+(\Delta Fx+S)\Delta x+(\Delta Fy+S)\Delta y-S.\Delta x-S.\Delta y$

It is apparent then that if the values of $(\Delta Fx+S)$ and $(\Delta Fy+S)$ are set in the function matrix instead of the values of ΔFx and ΔFy and the interpolation is made as usual, it suffices to subtract the terms $S.\Delta x$ and $S.\Delta y$ from the result to obtain the proper value of $F(x, y)$. This is particularly easy as S is an integer power of two, so that no multiplier is required to perform the multiplications $S.\Delta x$ and $S.\Delta y$, since these products are merely equal to Δx and Δy with the proper position of the binal point. The subtractions are made most conveniently by adding the complementary numbers $(S.\Delta x)'$ and $(S.\Delta y)'$ to the final result. While this involves many digits on account of the larger number of places in complementary numbers, this large number occurs only in additions rather than multiplication so that a saving in the required equipment is realized.

Figs. 9 and 10 illustrate the application of this method in connection with the particular function used as an example throughout this disclosure.

In this case, the number S was chosen to be .1, as the greatest negative coefficient encountered (square No. 10 in the table of Fig. 3) is $\Delta Fx=-0.0111$. This number has been added to all coefficients ΔFx and ΔFy as shown on the fourth and seventh lines of each square in the table of Fig. 3. Thus in square No. 29, $\Delta Fx=0.0011$ so that $\Delta Fx+S=.0011+.1=0.1011$ and $\Delta Fy=0.1011$ so that $\Delta Fy+S=1.0011$.

The pattern of resistances in the function matrix of Fig. 9 has been chosen so as to correspond to these numbers as can be easily observed. Thus for conductor No. 29, the pattern of resistances is 0R0RR for $(\Delta Fx+S)$ and R00RR for $(\Delta Fy+S)$. It will be noted that there are only five binary places per coefficient instead of the twelve which were necessary in the previous form using complementary numbers. The mathematical computations involved in calculating $F(x, y)$ in accordance with the relation of Equation No. 9 can be performed by any of the three interpolating mechanisms mentioned above. The third form of the interpolating mechanisms, shown on Fig. 10, is somewhat similar to that of Fig. 8.

In the interpolation system of Fig. 10, the conductors shown at 45° in the upper part and bent over to the horizontal in the lower part correspond to the binary positions in the final result $F(x, y)$. Through each of these conductors flows the current of (1) all the tubes of the matrix corresponding to the multiplication of Δy by $(\Delta Fy+S)$, (2) all the tubes of the matrix corresponding to the multiplication of Δx by $(\Delta Fx+S)$, (3) one tube corresponding to the proper binary place of Δy and one tube corresponding to the proper binary place of Δx, (4) one tube corresponding to the proper binary place $F(x_0, y_0)$, (5) one tube corresponding to a constant number, and (6) several carry-over tubes.

The total current of any one conductor corresponding to a certain binary place controls the carry over tubes of the next highest binary places depending on (1) whether the number of pairs of conducting tubes each contributing a standard amount of current to the total current is even or odd for the first carry over, (2) whether the number of groups of four of the conducting tubes is even or odd for the second carry over, and (3) whether the number of groups of eight conducting tubes is even or odd for the third carry over. This total current controls also one position of the output, i. e., one digit of $F(x, y)$ according to whether the number of conducting tubes is odd or even.

It is apparent that the proper binary places of the various terms have to be combined together as shown in Fig. 10. The subtraction of the numbers $\Delta x.S$ and $\Delta y.S$ has been handled by the method of complementary numbers. Since $S=.1$ and the maximum value of Δx or Δy is .1111, the maximum value of the product $\Delta x.S$ or $\Delta y.S$ is .01111. Therefore, the complementary number will always be of the form 111.1 $abcd$+.00001 where a, b, c, and d are the inverted digits of Δx or Δy. Therefore, the sum of the complementary numbers $(\Delta x.S)'+(\Delta y.S)'$ will be:

(9a) $(\Delta xS)'+(\Delta yS)'=111.1\,a_xb_xc_xd_x+111.1\,a_yb_yc_yd_y+.00001+.00001=111.00010+.0a_xb_xc_xd_x+.0a_yb_yc_yd_y$

(9b) $(\Delta xS)' + (\Delta yS)' = \text{constant} + .0a_xb_xc_xd_x + .0a_yb_yc_yd_y$

The relation of Equation 9b was used in setting the diagram of Fig. 10, as can be seen by examining the manner in which the binary places of the various numbers have been combined. It will be noted that the lowest places (from .00000001 to .00001) are required to evaluate the higher places but have no significance if the numbers represent physical quantities. Similarly, places higher than 100 do not occur in $F(x, y)$ as can be seen from Figs. 10 or 3, so that carry overs are not necessary to these places. The omitted places are indicated by dotted lines.

Consider, for example, the same set of values as those used above in connection with the explanation of Figs. 1 to 3. These values are as follows:

$$x = 100.1101 \quad x_0 = 100 \quad \Delta x = .1101$$
$$y = 11.1110 \quad y_0 = 11 \quad \Delta y = .1110$$

From function table square No. 29 of Fig. 3,

$$F(x_0, y_0) = 011.1011$$
$$S + \Delta Fx = .0011 + .1 = .1011$$
$$S + \Delta Fy = .1011 + .1 = 1.0011$$

The complemenetary of $(\Delta xS + \Delta yS)'$ is thus, according to (9b):

$$111.00010 + .00010 + .00001$$

The system of Fig. 10 adds as follows:

$F(x_0, y_0)$	011.1011
$\Delta y(S+\Delta Fy) = (.1110)\ (1.0011)$	1.00001010
$\Delta x(S+\Delta Fx) = (.1101)\ (.1011)$	.10001111
Constant	111.0001000
Part of $(\Delta xS)'$	.00010000
Part of $(\Delta yS)'$	.00001000
$F(x, y)$ (1)	100.01110001
	or 100.0111

It will be noted that the result is the same as that obtained above.

The method of generation of the function which consists in generating exact values of the function for certain values of the variable and interpolating between these major values, provides only an approximate value of the function. The degree of approximation desired, i. e., the maximum permissible discrepancy between true and approximated value, will determine the number of major values which will have to be used and will determine thereby the size of the function matrix. In general, the higher the desired accuracy the larger the matrix. The size of the matrix will depend also on the nature of the function. If the function is very "linear," that is to say, the rates at which it changes with respect to its variables do not change themselves very much with respect to the variables, or if the surface representing the function is not very different from plane, i. e., has small curvature, the necessary number of major points is quite small.

The function may be of such a nature that it is fairly linear in some regions (of variations of the variables) and very non-linear in others. In that case, considerable economy in the size of the matrix function can be realized by using small intervals between major values of the variables where the function is non-linear and fairly large ones where it is fairly linear. This "scale expansion" in regions of high curvature of the function is of considerable practical importance. To obtain such a scale expansion, the highest one or few digits of the minor parts of the variables are used not only to multiply the interpolating coefficients, but also as inputs of the selector matrix. These digits may be considered as both major and minor digits.

Of course, this scale expansion will require that the interpolating coefficients and the function be properly corrected to take into account the fact that the intervals Δx_0 and Δy_0 are not the same for all values of the variables. To illustrate this, a simple expansion in the x scale in the region $y=100$ and x from 011 to 110 is shown for the function $F(x, y)$ used throughout this disclosure.

Fig. 11 shows the expanded part of the table of Fig. 3. In this table, the values of $F(x_0, y_0)$ for the squares **36** to **38** corresponding to $y_0=100$ have been expanded. In this expanded region, $\Delta x_0=1$ for the values of squares **33**, **34**, **35**, **39** and **40**, $\Delta x_0=.1$ for the squares **36A**, **36B**, **37A**, **37B**, **38A**, and **38B** and $S=.1$.

The expansion is carried out only for one additional digit so that for each selected set of major values **36**, **37** and **38** of the table of Fig. 3 there are now two sets. The values of the function for these sets are given in the second line. It will be noted that the value of $F(x_0, y_0)$ for $x=011.1$, which is 100.0010, is not the average value between values of 011 and 100 for x, which would be 100.0011 if no expansion scale but simple linear interpolation were used. Differences between consecutive values of the function ΔFx are given in the third line. The values of $\Delta Fx/\Delta x_0$ are given in the fourth line, taking into account the fact that sometimes $\Delta x_0=1$ and sometimes $\Delta x_0=.1$. The reason for the corrected value of $F(x_0, y_0)$ in the fifth line is explained below. The sixth line has the values of $(\Delta Fx/\Delta x_0+S)$ which is used instead of $\Delta Fx/\Delta x_0$ in the function matrix to take care of the negative values of ΔFx as explained above. The values of $\Delta Fy+S$ on the seventh line are the same as in the table of Fig. 3 for the nonexpanded values of x but are different for the expanded values. This gives a better approximation of the function for values of y comprised between 100 and 101 and of x between 011 and 100 because the rate of change of the function with respect to one variable (y) depends in general on the other variable (x). For example, $\Delta Fy+S$ is .1110 for $x=011.0$ and .1111 for $x=011.1$.

Fig. 12 illustrates a function generator adapted for use in connection with scale expansion. It will be observed that only the digit of x corresponding to .1 is a part of both the minor and major parts of x. The selector is the same as before except for the addition of a pair of vertical conductors corresponding to the expanded region. Both the selector and function matrices have additional horizontal conductors corresponding to the new sets of major values. The rest of the device is as illustrated in Fig. 9.

It is apparent that the generator of Fig. 12 operates as previously described for all non-expanded values since the additional vertical pair in the selector has no influence on these values.

For the expanded values, proper selection of the matrix input tubes is obtained due to the additional connections. Thus the proper value of $F(x, y)$ is obtained for values of x for which the major-minor digit of place .1 happens to be zero since the value of $\Delta Fx.\Delta x$ is added to $F(x_0, y_0)$ in the normal manner. However, when the digit happens to be 1, there arises a complication due to the fact that the multiplier of the x variable will add to the value of $F(x_0, y_0)$ given by the matrix for this new major point, a value corresponding to .1 times $\Delta Fx/\Delta x_0$. Therefore, to obtain the true value of the function for that point and the region corresponding to it, it is necessary to set in the function matrix a pattern of resistance corresponding to the difference between the true value of $F(x_0, y_0)$ and the result of the product .1 times $\Delta Fx/\Delta x_0$ which is set in automatically by the interpolating system. These corrected values of $F(x_0, y_0)$ are shown in the fifth line of Fig. 11.

The addition of the pair of vertical leads in the selector makes it possible to have 128 instead of 64 horizontal conductors. Only 67 out of the possible 128 have been used, this additional pair of vertical leads being ignored for 64 horizontal conductors. In general, the number of horizontal conductors or sets of major values of the variables need not be equal to the maximum possible number for the reason that the function may in some cases be non-existant for some sets or may be of no interest.

It is also possible to use a better than linear interpolation, taking for example both first and second order terms in the Taylor expansion.

$$(10)\quad F(x, y)=F(x_0, y_0)+\frac{\Delta Fx\Delta x}{\Delta x_0}+\frac{\Delta Fx\Delta y}{\Delta y_0}+\frac{1}{2}\frac{\Delta^2 fx\Delta x^2}{\Delta x_0^2}+\frac{1}{2}\frac{\Delta^2 Fx\Delta y^2}{\Delta y_0^2}+\frac{\Delta^2 Fxy}{\Delta x_0\Delta y_0}\Delta x\Delta y$$

In that case the six functions

$$F(x_0, y_0), \frac{\Delta Fx}{\Delta x_0}, \frac{\Delta Fy}{\Delta y_0}, \frac{\Delta^2 Fx}{\Delta x_0^2}, \frac{\Delta^2 Fy}{\Delta y_0^2}, \frac{\Delta^2 Fxy}{\Delta x_0 y_0}$$

have to be obtained from the function table and eight multiplications and five additions (counting squaring of Δx_0 and Δy_0 as multiplications) have to be performed. Unless multiplying devices of a particularly simple nature are invented, this particular method, for most practical functions, involves more apparatus than a linear interpolation with sufficiently large function tables.

It has been mentioned that the same function generator can be used to generate many functions of the same independent variables, merely by increasing the size of the function matrix.

It is also possible to make a function generator for function for more than two variables. For n variables $x_1, x_2, x_3 \ldots x_n$, the linear interpolation formula would be:

$$(11)\quad F(x_1x_2x_3 \ldots x_n)=F(x_1^*, x_2^*, x_3^* \ldots x_n^*)+\frac{\Delta Fx_1}{\Delta x_1^*}\Delta x_1+\frac{\Delta Fx_2}{\Delta x_2^*}\Delta x_2+\ldots\frac{\Delta Fx_n}{\Delta x_n^*}\Delta x_n$$

where $x_1^*, x_2^*, x_3^* \ldots x_n^*$ are the major and $\Delta x_1, \Delta x_2 \ldots \Delta x_n$ the minor values. The selector would have as input the single variable obtained by combining all the major values of all the variables. The output of the function matrix would not only include the function for the major values but also the n interpolating coefficients. There would be n multiplications and n additions. A geometrical representation of an n-independent variable function cannot be obtained in the three dimensional space.

The characteristic features of the invention, of course, are the selector herein disclosed in three different forms, the exchangeable matrix which determines the particular components of the function to be developed and which is disclosed in two different forms, and the interpolator which combines these various components to produce the desired function and which is disclosed in four different forms.

I claim as my invention:

1. The combination of means having different parts each representative of a different set of values each including a first value which is that of a major part of a predetermined function of a variable and a second value which includes that of a function derived from said predetermined function, means responsive to potentials representative of a predetermined value of a major part of said variable for selecting one of said sets, and means for combining the value of a minor part of said variable with said selected set of values to obtain a desired function of said variable.

2. The combination of means having different parts each representative of a different set of values each including a first value which is that of a major part function of a variable and a second value which includes that of a function derived in part from said major part function, means responsive to potentials representative of a predetermined value of a major part of said variable for selecting one of said sets, and means for obtaining the product of said second selected value and a minor value of said variable.

3. The combination of means having different parts each representative of a different set of values each including a first value which is that of a major part of a predetermined function of a variable and a second value which includes that of a function derived from said predetermined function, means responsive to potentials representative of a predetermined value of a major part of said variable for selecting one of said sets, and means for obtaining the product of said second selected value and a minor value of said variable, and means for adding said selected first value to said product.

4. The combination of a network having resistors connected between two groups of conductors to provide parts each representative of a different set of values each including a first value which is that of a major part function of a variable and a second value which includes that of a function derived in part from said major part function, means responsive to potentials representative of a predetermined value of a major part of said variable for selecting one of said sets, and means for combining the value of a minor part of said variable with said selected set to derive a desired function of said variable.

5. The combination of a network having resistors connected between two groups of conductors to provide parts each representative of a different set of values each including a first value which is that of a major part of a predetermined function of a variable and a second value which includes that of a function derived from said predetermined function, means responsive to potentials representative of a predetermined value of a major part of said variable for selecting one of said sets, and means for combining the value of a minor part of said variable with said selected set to derive a desired function of said variable, and means for attenuating parasitic signals in said network.

6. The combination of a network having resistors connected between two groups of conductors to provide parts each representative of a different set of values each including a first value which is that of a major part of a predetermined function of a variable and a second value which includes that of a function derived from said predetermined function, means responsive to potentials representative of a predetermined value of a major part of said variable for selecting one of said sets, and means for combining the value of a minor part of said variable with said se-

lected set to derive a desired function of said variable, and means including a source of potential connected through resistors to one of said groups of conductors for attenuating parasitic signals in said network.

7. The combination of a network having resistors connected between two groups of conductors to provide parts each representative of a different set of values each including a first value which is that of a major part of a predetermined function of a variable and a second value which includes that of a function derived from said predetermined function, means responsive to potentials representative of a predetermined value of a major part of said variable for selecting one of said sets, and means for combining the value of a minor part of said variable with said selected set to derive a desired function of said variable, and means including electron discharge tubes connected between said selecting means and said network and operated as cathode followers for suppressing parasitic signals in said network.

8. The combination of a network having resistors connected between two groups of conductors to provide parts each representative of a different set of values each set including a first value which is the value of a major part of a predetermined function of a variable with certain of its digits inverted and a second value which includes the value of a function derived from said predetermined function with certain of its digits inverted, means responsive to potentials representative of a predetermined value of a major part of said variable for selecting one of said sets of values, means for reinverting the inverted digits of said selected set, and means for combining a value of a minor part of said variable with said selected set to obtain a desired function of said variable.

9. The combination of means having different parts each representative of a different set of values, a first group of said sets including the value of a predetermined function of a major value of a variable comprised in an unexpanded scale and the value of a function derived from said predetermined function and a second group of said sets including the value of said function of a major value of said variable expanded by the inclusion of a digit common to the minor value of said variable and the value of a function derived from said function, means responsive to potentials representative of a predetermined unexpanded scale major value of said variable for selecting a set from the first of said groups and to potentials representative of a predetermined expanded scale major value of said variable for selecting a set from the second of said groups, and means for combining said selected set of values with said minor value for producing said predetermined function of said variable.

10. The combination of means having different parts each representative of a different set of values, a first group of said sets including the value of a predetermined function of a major value of a variable comprised in an unexpanded scale and the value of a function derived from said predetermined function and a second group of said sets including the value of said function of an expanded scale major value of a variable and the value of a function derived from said function, means responsive to potentials representative of a predetermined unexpanded scale major value of said variable for selecting a set from the first of said groups and to potentials representative of a predetermined expanded scale major value of said variable for selecting a set from the second of said groups, and means for combining said selected set of values with a minor value of said variable for producing the said predetermined function.

11. The combination of a first network having two sets of conductors interconnected to provide parts each representative of a different set of values each of said sets including a first value which is that of a major value function of a variable and a second value which includes that of a function derived from said major value function, a second network having two sets of conductors interconnected to select one of said sets of values in response to a predetermined major value of said variable, and means connected to pairs of one set of conductors of said second network for applying to the conductors of each pair potentials which represent the digit one or zero, depending on said predetermined value of said variable.

12. The combination of a function network having parts each representative of a different set of values of functions, a selector network including a first group of conductors each connected to a different one of said parts and a second group of conductors arranged in pairs, means connected to each of said pairs for applying thereto potentials representative of the digit one or the digit zero depending on a predetermined value of a variable, and means connecting each conductor of said first group to the conductor of each pair which is the most positive when the part to which such first group conductor is connected is to be selected.

13. The combination of a function network having parts each representative of a different set of values of functions, a selector network including a first group of conductors each connected to a different one of said parts and a second group of conductors arranged in pairs, means connected to each of said pairs for applying thereto potentials representative of the digit one or the digit zero depending on a predetermined value of a variable, and high resistance means connecting each conductor of said first group to the conductor of each pair which is the most positive when the part to which such first group conductor is connected is to be selected.

14. The combination of a function network having parts each representative of a different set of values of functions, a selector network including a first group of conductors and a second group of conductors arranged in pairs, a plurality of electron discharge devices each provided with a grid connected to a different conductor of said first group and with an anode connected to a different one of said parts, means connected to each of said pairs for applying thereto potentials representative of the digit one or the digit zero depending on a predetermined value of a variable, and high resistance means connecting each conductor of said first group to the conductor of each pair which is the most positive when the part to which such first group conductor is connected is to be selected.

15. The combination of a function network having parts each representative of a different set of values of functions, a selector network including a first group of conductors and a second group of conductors arranged in pairs, a plurality of first and second electron discharge element pairs each connecting a different conductor of said first group to a different one of said parts, said first elements having their grids connected

to the conductors of said first group and their anodes connected to the grids of said second elements and said second elements having their cathodes connected to said parts, means connected to each of said pairs for applying thereto potentials representative of the digit one or the digit zero depending on a predetermined value of a variable, and high resistance means connecting each conductor of said first group to one or another conductor of each pair depending on which is the most positive when the part to which such first group conductor is connected is selected by said predetermined value.

16. The combination of a function network having different parts each representative of a different set of values of functions, means including a selector network provided with output terminals, and means including a plurality of electrons discharge elements each provided with a grid connected to a different one of said output terminals and with an anode connected to a different one of said parts.

17. The combination of a plurality of networks each including a number selecting set of conductors, one of said networks including a set of digital position conductors and the other of said networks including a set of digital position pairs of conductors, a plurality of input electron discharge elements each having an output electrode connected to a different conductor of said pairs of conductors, a plurality of coupling electron discharge elements each including a control electrode connected to a different one of the number selecting conductors of one of said networks and an output electrode similarly connected to the number selecting conductors of the other of said networks, a plurality of output electron discharge elements each having a control electrode connected to a different one of said digital position conductors, means interconnecting said digital position conductors with the conductors of one said number selecting sets in accordance with the number to be selected, means connected with said input electron discharge elements for applying to the conductors of each pair higher and lower potentials dependent on numbers to be used in selecting between said interconnected number selecting conductors, and means interconnecting each number selecting conductor of the other of said sets and the conductors of said pairs which have the higher potential for the number by which it is selected.

18. The combination of a plurality of networks each including a number selecting set of conductors, one of said networks including a set of digital position conductors and the other of said networks including a set of digital position pairs of conductors, a plurality of input electron dis-

charge elements each having an output electrode connected to a different conductor of said pairs of conductors, a plurality of coupling electron discharge elements each including a control electrode connected to a different one of the number selecting conductors of one of said networks and an output electrode similarly connected to the number selecting conductors of the other of said networks, a plurality of output electron discharge elements each having a control electrode connected to a different one of said digital position conductors, high resistance elements interconnecting said digital position conductors with the conductors of one of said number selecting sets in accordance with the number to be selected, means connected with said input electron discharge elements for applying to the conductors of each pair higher and lower potentials dependent on numbers to be used in selecting between said interconnected number selecting conductors, and high resistance elements interconnecting each number selecting conductor of the other of said sets and the conductors of said pairs which have the higher potential for the number by which it is selected.

19. The combination of a plurality of networks each including a number selecting set of conductors, one of said networks including a set of digital position conductors and the other of said networks including a set of digital position pairs of conductors, a plurality of input electron discharge elements each having an output electrode connected to a different conductor of said pairs of conductors, a plurality of coupling electron discharge elements each including a control electrode connected to a different one of the number selecting conductors of one of said networks and an output electrode similarly connected to the number selecting conductors of the other of said networks, a plurality of output electron discharge elements each having a control electrode connected to a different one of said digital position conductors, resistance means interconnecting each number selecting conductor of one of said sets with such digital position conductors as corresponds to a predetermined digit in the number selected through it, means connected with said input electron discharge elements for applying to the conductors of each pair higher and lower potentials dependent on numbers to be used in selecting between said interconnected number selecting conductors, and means interconnecting each number selecting conductor of the other of said sets and the conductors of said pairs which have the higher potential for the number by which it is selected.

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