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RIGHT TRIANGLE SOLVER USING FEEDBACK

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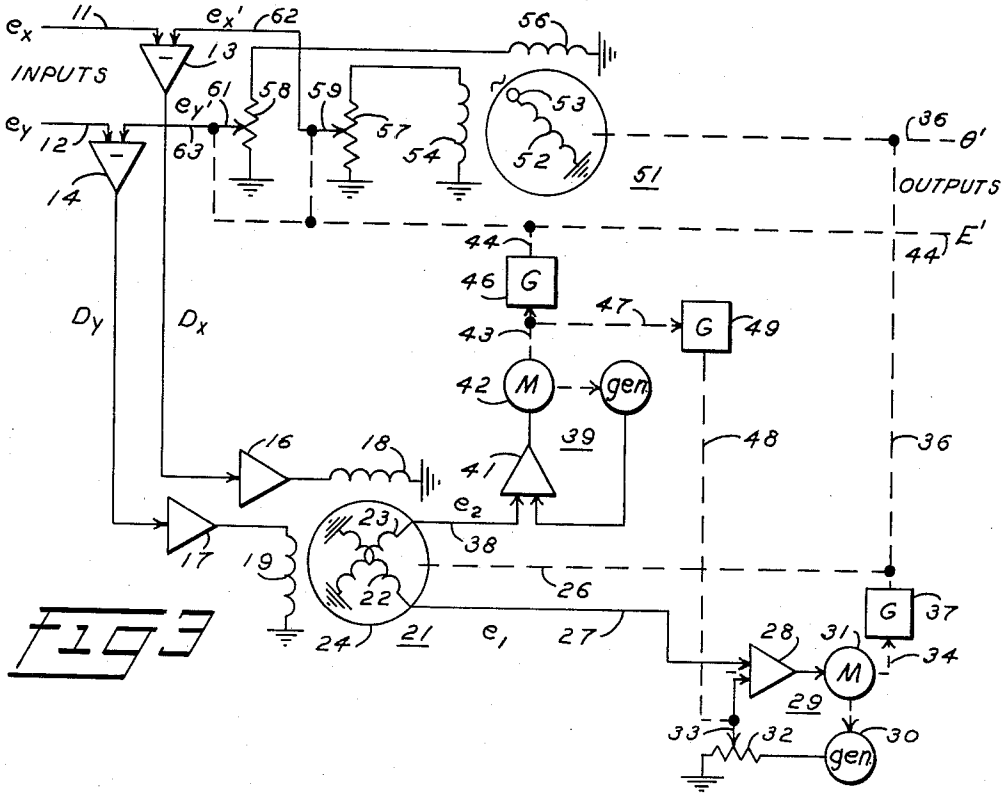

Fig 3

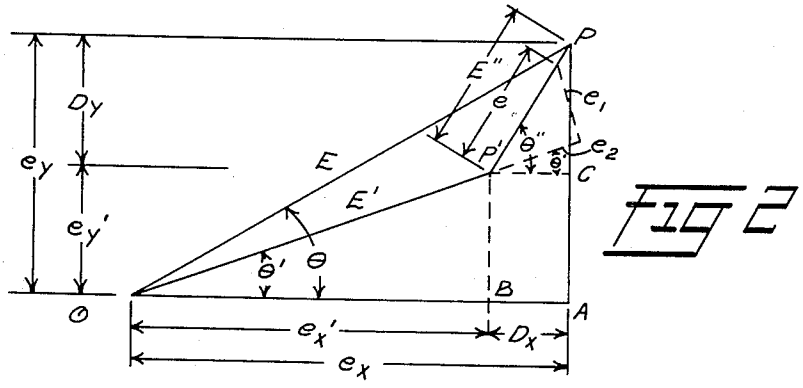

Fig 2

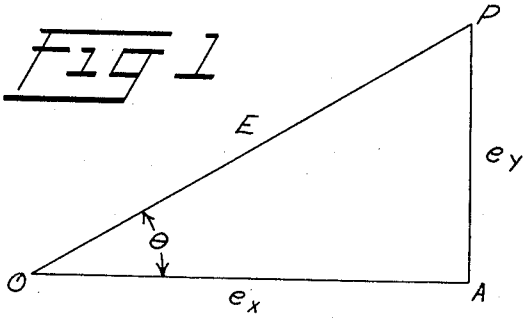

Fig 1

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RIGHT TRIANGLE SOLVER USING FEEDBACK

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2 Claims. (Cl. 235—61)

This invention relates to right triangle solvers and more specifically to right triangle solvers of the electrical analogue type in which the two sides of a right triangle are given and it is required to find the hypotenuse and an angle. Such a solver may also be termed a coordinate converter for, if the magnitudes of the two sides of a right triangle are considered to be the rectangular coordinates of a point, the hypotenuse and an angle constitute the polar coordinates of that point. That portion of the solver which solves for the angle is sometimes termed an arc tangent solver.

This invention provides the instrumentation for such right triangle solution with outputs in the form of shaft angular deflections, and is distinguished by the elimination of the need which conventional solvers have for accurate driver amplifiers. This invention also eliminates any necessity for employing linear synchro resolvers to insure accuracy.

The purpose then of this invention is to provide improved instrumentation for finding the hypotenuse and one angle of a right angle triangle when the two sides are given.

More specifically, the purpose of this invention is to provide a circuit employing components of low individual accuracy to secure a computed result of relatively high accuracy.

A further understanding of this invention may be secured from the following detailed description and the associated drawings, in which:

Figures 1 and 2 illustrate the geometric principles of operation of the invention.

Figure 3 is a schematic electromechanical diagram of the apparatus of the invention.

Referring now to Fig. 1, e_x and e_y represent any two vector quantities having orthogonal relationship. It is desired to find the magnitude E and direction θ of their vector sum. The triangle OPA represents geometrically the relationships of these quantities, and trigonometrically they are expressed by

$$e_x = E \cos \theta \quad (1)$$

$$e_y = E \sin \theta \quad (2)$$

$$\theta = \tan^{-1}\frac{e_y}{e_x} \quad (3)$$

The value of E is given by

$$E = \sqrt{e_x^2 + e_y^2} \quad (4)$$

The problem is, given e_x and e_y, to find θ and E, and is solved by use of the above relations.

The instrumentation of this invention is based on a diagram somewhat more complex than Fig. 1 but which, at null, becomes identical with that figure. This more complex diagram is shown in Fig. 2 and contains the components of Fig. 1 with, in addition, two qunatities E' and E'' whose vector sum equals E. Two right triangles are formed on E' and E'' as hypotenuses, and are lettered OP'B and P'PC. In the right triangle OP'B the

hypotenuse E' is the vector sum of the sides e_x' and e_y', and

$$e_x' = E' \cos \theta' \quad (5)$$

$$e_y' = E' \sin \theta' \quad (6)$$

In the right triangle P'PC the hypotenuse E'' is the vector sum of the sides D_x and D_y, so that

$$E'' = \sqrt{D_x^2 + D_y^2} \quad (7)$$

One instrumentation for performing the mathematical operations depicted in Fig. 2 is indicated in Fig. 3. Two input signals, e_x and e_y, are received at conductors **11** and **12** respectively. These voltages may be either direct or alternating, with appropriate circuit components. However, 400 C. P. S. alternating voltage is employed in this example. These input signals are applied to two similar subtracting circuits **13** and **14** which may be of any type such as, for example, amplifiers with inputs including transformers having subtracting windings or resistors arranged in a subtracting network. Internally generated signals e_x' and e_y' are also applied to the subtracting circuits, so that in circuit **13** the operation

$$e_x - e_x' = D_x \quad (8)$$

is performed, and in circuit **14** the operation

$$e_y - e_y' = D_y \quad (9)$$

is performed. The difference or error signals, D_x and D_y, are amplified by amplifiers **16** and **17**. These amplifiers need not be linear but zero output should correspond to zero input since they amplify only error voltages and not components of the output signal voltages. The amplifier outputs are applied to the orthogonal field windings **18** and **19** of a synchro resolver **21** having two armature windings **22** and **23** on its rotor **24** which set up a composite magnetic field representative of their vector sum E''. If the strength of that magnetic field be termed F and its direction θ'', then

$$F = kE'' \quad (10)$$

in which k is the constant of proportionality. When the rotor is positioned through its shaft **26** at an angle θ' voltages e_1 and e_2 will be induced in the output windings **22** and **23**, having the values

$$e_1 = mF \sin (\theta'' - \theta') \quad (11)$$

$$e_2 = mF \cos (\theta'' - \theta') \quad (12)$$

in which m is the constant of proportionality. These equations show that the device **21** is a coordinate transformer or coordinate rotator with equal windings, and reproduces the input orthogonal vector quantities in a rotated coordinate system.

Substituting the value of F from (10):

$$e_1 = mkE'' \sin (\theta'' - \theta') \quad (13)$$

$$e_2 = mkE'' \cos (\theta'' - \theta') \quad (14)$$

The winding **22** is connected through conductor **27** to a high gain amplifier **28** which is part of a rate servomechanism **29**. A negative feedback circuit is provided for the amplifier consisting of a motor **31**, generator **30**, voltage divider **32** and its slider **33** connected to the input of the amplifier. The motor shaft **34** has a speed S_1, proportional to e_1/d, in which d is the ratio of divider **32**, or approximately

$$S_1 = \frac{e_1}{d} \quad (15)$$

The motor shaft **34** drives an output shaft **36** through gear **37**; the angular position θ' of this shaft therefore is

$$\theta' = k_1 \int S_1 dt \quad (16)$$

$$= \frac{k_1}{d} \int e_1 dt \quad (17)$$

or

$$e_1=\frac{d}{k_1}\frac{d\theta'}{dt} \tag{18}$$

k_1 being the gear ratio. Shaft **36** is connected to the rotor **24** through shaft **26**, thus positioning the rotor in accordance with the angle θ'.

The winding **23** is connected through conductor **38** to a rate servomechanism **39** comprising an amplifier **41** and motor **42**. The motor output shaft **43** has a speed S_2 proportional to the input voltage e_2, or

$$S_2=Ce_2 \tag{19}$$

in which C is a constant. Motor shaft **43** drives the E' output shaft **44** at a speed proportional to S_2 through the gear box **46** to a final angular displacement representative of the quantity E', or

$$E'=k_2\int S_2dt \tag{20}$$
$$=k_2C\int e_2dt \tag{21}$$

or

$$e_2=\frac{1}{k_2C}\frac{dE'}{dt} \tag{22}$$

k_2 being the gear ratio. Substituting in Equations 13 and 14 for E'' its value taken from Equation 7

$$e_1=mk\sqrt{D_x^2+D_y^2}\sin(\theta''-\theta') \tag{23}$$

$$e_2=mk\sqrt{D_x^2+D_y^2}\cos(\theta''-\theta') \tag{24}$$

Equating Equations 18 and 23,

$$mk\sqrt{D_x^2+D_y^2}\sin(\theta''-\theta')=\frac{d}{k_1}\frac{d\theta'}{dt} \tag{25}$$

Equating Equations 22 and 24,

$$mk\sqrt{D_x^2+D_y^2}\cos(\theta''-\theta')=\frac{1}{k_2C}\frac{dE'}{dt} \tag{26}$$

These equations show that in order for the shafts **36** and **44** to be servoed to stationary or null positions, both D_x and D_y must be made equal to zero individually. When they are so made,

$$\frac{d\theta'}{dt}\text{ and }\frac{dE'}{dt}$$

both become zero, that is, the shafts **36** and **44** become stationary, and from Fig. 2 it is apparent that, when D_x and D_y become zero, e_x' becomes equal to e_x, e_y' becomes equal to e_y, θ' becomes equal to θ and E' becomes equal to E. The shafts **36** and **44** therefore become the output signal shafts.

The quantities θ' and E' should approach their final values θ and E smoothly and directly. Described graphically, in Fig. 2 the point P' should approach the point P along the straight path P'P. As so far described this may not occur, but wide excursions of P' from the straight path may be taken because, in Fig. 3, the angle θ' fed back to resolver **21** depends as so far described directly on e_1. In general P' will approach P by a spiral path. If less accuracy is required this is satisfactory, but for greater accuracy the straight path P'P should be followed. This condition is obviously met if the rate of shrinkage of the side e_1 is proportional to the magnitude of e_1, and also the rate of shrinkage of the side e_2 is proportional to the magnitude of e_2. That is, the desirable conditions are that

$$\frac{de_1}{dt}=ke_1 \tag{27}$$

and

$$\frac{de_2}{dt}=ke_2 \tag{28}$$

Let

$$e=\sqrt{e_1^2+e_2^2}$$

Then, in the limit at null, $e=E''$ and $E'=E$, so that approximately

$$e_1=|E'|(\theta-\theta') \tag{29}$$

Differentiating, considering $|E'|$ to be constant,

$$\frac{de_1}{dt}=|E'|\frac{d\theta'}{dt} \tag{30}$$

or

$$\frac{1}{E'}\frac{de_1}{dt}=\frac{d\theta'}{dt} \tag{31}$$

substituting in (27)

$$\frac{d\theta'}{dt}=\frac{Ke_1}{E'} \tag{32}$$

That is, in order to secure the condition indicated by Equation 27, in the process of securing θ' the voltage representing e_1 must be divided by a quantity representing E'.

It will be noted that this Equation 32 is in the same form as Equation 18, and that these two equations become identical when

$$\frac{K}{E'}=\frac{k_1}{d} \tag{33}$$

The voltage divider **32** is provided to perform the operation of division indicated by the right side of Equation 32, dividing e_1 by a function of E' in the servomechanism **29** in order to secure the desired linear approach of the triangle solver to its null position. As is well known, a voltage divider in the negative feedback path of a servomechanism divides the servomechanism input function by the function of the voltage divider. This is what is done by the divider **32**, which is positioned proportionally to E', and which therefore causes the input to amplifier **28**, and therefore its output, to be proportional to

$$\frac{e_1}{E'}$$

This is what is demanded by Equation 32,

$$\frac{d\theta}{dt}$$

being proportional to the output speed of motor **31**.

The shafts **36** and **44** are made to change e_x' and e_y' until they equal e_x and e_y by connecting shaft **36** to a synchro resolver **51** having a single rotor coil **52** connected to the 400 C. P. S. source at the terminal **53**. Two orthogonal stator coils **54** and **56** are provided, each loaded by a voltage divider **57** and **58** respectively. The sliders **59** and **61** thereof are connected to the conductors **62** and **63** respectively, and bear potentials e_x' and e_y' which are applied to the respective subtracting circuits **13** and **14**. The sliders **59** and **61** are connected for actuation by the E' shaft **44**.

The synchro resolver **51** resolves the potential of its rotor coil **52** into the orthogonal potentials e_x'' and e_y'' of its stator coils **54** and **56** so that

$$e_x''\sim\cos\theta' \tag{34}$$

and

$$e_y''\sim\sin\theta'$$

These potentials are multiplied by E' by the voltage dividers **57** and **58** operated by the shaft **44**, so that at their sliders

$$e_x'=E'\cos\theta' \tag{5}$$

and

$$e_y'=E'\sin\theta' \tag{6}$$

and the values of e_x' and e_y' are progressively changed by the shafts **36** and **44** toward the values e_x and e_y, which they attain at the null condition.

To recapitulate, inputs e_x and e_y are subtracted from internally generated e_x' and e_y' to form difference error

signals D_x and D_y which eventually vanish. These error signals are amplified by amplifiers 16 and 17 which need not be accurate since at null, when the computation has been achieved, the amplifier outputs are zero. Their outputs during the automatic computation are converted by a double resolver and servomechanisms to shaft displacements approximating the desired output signals, which are converted by another synchro resolver to signals e_x' and e_y' approximating the input signals, and are balanced against them to secure the null signals.

The potential of terminal 53 in resolver 51 should preferably be from the same source as that employed in generating the input signals e_x and e_y, thus eliminating any necessity for strict voltage regulation.

Linearity requirements are not significant in the resolver 21 or in the servomechanisms 29 and 39, since they carry only error signals which are zero at null, and not output signal components. It is therefore possible for each servomechanism to consist merely of a motor driven by a driver amplifier, dispensing with the tachometer generator and feedback loop.

What is claimed is:

1. A right triangle solver comprising, a pair of subtracting circuits each emitting an electrical difference signal, each of said pair of subtracting circuits having applied thereto a respective one of two electrical input minuend signals representing two orthogonal vector quantities, a pair of voltage dividers each having a slider connected to a respective one of said pair of subtracting circuits for applying respective subtrahend signals thereto, a synchro resolver having an input winding energized by a constant potential and having a pair of output windings positioned in space quadrature, said two output windings being respectively connected to said two voltage dividers for applying multiplicant signals thereto, a coordinate transformer connected to said pair of subtracting circuits and actuated by the difference signals thereof to form transformed difference signals, first and second integrators connected to said coordinate transformer and having applied thereto said transformed difference signals producing therefrom respective angle and hypotenuse integral output signals, means for positioning said coordinate transformer and said synchro resolver by said angle integral signal, a dividing circuit associated with said first integrator, said dividing circuit being actuated by the hypotenuse integral output of said second integrator for dividing the value of the transformed difference signal applied to said first integrator by the value of said hypotenuse integral signal, and means for positioning the sliders of said two voltage dividers by said hypotenuse integral signal, whereby said subtrahend signals approach said minuend signals in value and said integral output signals approach the values of the vector sum and of the arc tangent of said two orthogonal vectors.

2. A right triangle solver comprising, a pair of electrical subtracting circuits each emitting an electrical difference signal and each having applied thereto a respective one of two alternating electrical input minuend signals representing by their voltage magnitudes two orthogonally related vector quantities, said two input signals having the same selected frequency and phase, a pair of

linear voltage dividers each having a slider connected to a respective one of said pair of subtracting circuits for applying respective subtrahend signals thereto, a synchro resolver having a single input winding energized by a constant alternating potential having said selected frequency and phase and having a pair of output windings positioned in space quadrature, said two output windings being respectively connected to said two voltage dividers for applying multiplicant signals thereto, a double synchro resolver having a pair of primary windings in space quadrature and a pair of secondary windings in space quadrature relatively rotatable by a shaft, amplifying means respectively connecting said pair of subtracting circuits to said pair of primary windings for actuation thereof by the amplified difference signals to form coordinate-transformed difference signals at said pair of secondary windings, a first integrating servomechanism including an amplifier, motor, generator and negative feedback loop and having a first output shaft, the angular displacement of said output shaft linearly representing the integral of input potential, said servomechanism being connected to one of said pair of secondary windings to form an angle output signal, a second integrating servomechanism including an amplifier, motor, generator and negative feedback loop and having a second output shaft, the angular displacement of said output shaft linearly representing the integral of input potential, said servomechanism being connected to the other of said pair of secondary windings to form a hypotenuse output signal, said first output shaft being connected to said double synchro resolver shaft whereby said resolver shaft is positioned in terms of said angle output signal, said first output shaft being likewise connected to said synchro resolver for the rotational positioning thereof, a voltage divider associated with the negative feedback loop of said first integrating servomechanism, said voltage divider having a slider connected to said second output shaft and positioned in accordance with the angular displacement thereof whereby the value of the transformed difference signal applied from said one of the pair of secondary windings to said first integrating servomechanism is divided by a function of the value of said hypotenuse output signal, and a shaft connecting said second output shaft to the sliders of said pair of linear voltage dividers for positioning them in accordance with said hypotenuse output signal, whereby said subtrahend signals approach said minuend signals in value and said angle output signal and hypotenuse output signal respectively approach the value of the arc tangent and of the vector sum of the orthogonally related vectors represented by said two alternating electrical minuent signals.

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