

Tri-Cubic and Hybrid Interpolation in a 3D Texture Shader

Background

[0001] As computers render three-dimensional (3D) volumetric seismic data, they often only use tri-linear or tri-cubic interpolation methods to render the data. Tri-linear is fast but may not produce smooth-looking results. Tri-cubic produces smooth results, but generates large computational loads even when using a powerful computer. It is challenging to perform 3D texture rendering that is both smooth and fast.

Brief Description of the Drawings

[0002] Fig. 1 is a view of a tri-linear rendering showing processing artifacts.

[0003] Fig. 2 is a view of a tri-cubic rendering showing the elimination of the processing artifacts.

[0004] Fig. 3 is a view of a tri-linear rendering showing processing artifacts.

[0005] Fig. 4 is a view of a tri-cubic rendering showing the elimination of the processing artifacts..

[0006] Fig. 5A is a perspective view of sixty-four points used in interpolation.

[0007] Fig. 5B is a view of sixteen data points resulting from analysis of one axis of the sixty-four points illustrated in Fig. 5A.

[0008] Fig. 5C is a view of four data points resulting from analysis of the sixteen data points illustrated in Fig. 5B.

[0009] Fig. 6 is an illustration of a hybrid Bezier spline with G1 continuity at each data point.

[0010] Fig. 7A is a view of a set of data points to be used in interpolation.

[0011] Fig. 7B is an illustration of the building of a Bezier curve.

[0012] Fig. 7C is an illustration of the building of a linear Bezier curve.

[0013] Fig. 7D is an illustration of the building of a cubic Bezier curve.

[0014] Fig. 7E is an illustration of the building of a quadratic Bezier curve.

[0015] Fig. 7F is an illustration of the building of a cubic Bezier curve.

[0016] Fig. 8 is a portion of a flow chart for building the hybrid Bezier curve.

5 [0017] Fig. 9 is a portion of a flow chart for building the hybrid Bezier curve.

[0018] Fig. 10 is a portion of a flow chart for building the hybrid Bezier curve.

Detailed Description

[0019] The following detailed description illustrates embodiments of the present disclosure. These embodiments are described in sufficient detail to enable a person of ordinary skill in the
10 art to practice these embodiments without undue experimentation. It should be understood, however, that the embodiments and examples described herein are given by way of illustration only, and not by way of limitation. Various substitutions, modifications, additions, and rearrangements may be made that remain potential applications of the disclosed techniques. Therefore, the description that follows is not to be taken as limiting on the scope of the appended
15 claims. In particular, an element associated with a particular embodiment should not be limited to association with that particular embodiment but should be assumed to be capable of association with any embodiment discussed herein.

[0020] A conventional three-dimensional (3D) texture shader renders raw volumetric data into a two-dimensional (2D) or 3D image using interpolation, in which pixel or voxel values are
20 derived from neighboring data values. As illustrated in Fig. 1, images rendered using tri-linear interpolation include artifacts, such as lines, that are not accurate representations of the data. A user who relies on such an artifact in the image may interpret the data incorrectly. For example, one of the lines may be misinterpreted as a void or as a feature, and relying on that misinterpretation may have costly consequences. For example, in the oil field, incorrectly
25 analyzing raw seismic data may result in incorrect placement of an oil well. In the medical field, incorrectly analyzing magnetic resonance imaging (MRI) data may result in a misdiagnosis.

[0021] Fig. 2 shows the data from Fig. 1 analyzed using a tri-cubic interpolation, which reduces the number of artifacts.

[0022] Fig. 3 shows a tri-linear interpolation, which includes artifacts.

[0023] Fig. 4 shows the data from Fig. 3 analyzed using a tri-cubic interpolation, which reduces the number of artifacts.

[0024] A hybrid shader (which is part of the 3D texture shader) uses cubic, quadratic, or linear interpolation, as discussed below, on each of three physical dimensions (x, y, and z). The hybrid shader may also use a fourth dimension (time-axis). Any four consecutive vertices uniquely generate a curve segment, and each pair of curve segments is smoothly connected.

10 [0025] For example, if A0, A1, A2, A3, and A4 are five consecutive vertices (which have associated scalar values of a0, a1, a2, a3, and a4, respectively), then the A0, A1, A2, and A3 vertices build the first curve segment, and the A1, A2, A3, A4 vertices build the second curve segment.

[0026] The formula to build the first curve segment F1(t), t=[0,1] is:

15
$$A = (a_0 + a_2 - 2 * a_1) / 2;$$

$$B = (a_2 - a_0) / 2;$$

$$D = (a_1 + a_3 - 2 * a_2) / 2;$$

$$E = (4 * a_2 - 3 * a_1 - a_3) / 2;$$

$$t\_1 = (1 - t);$$

20
$$tt2 = t * t;$$

$$ampl1 = A * tt2 + B * t + a1;$$

$$ampl2 = D * tt2 + E * t + a1;$$

$$F1(t) = (ampl1 * t\_1 + ampl2 * t); \quad (1)$$

[0027] Replacing (a0, a1, a2, a3) with (a1, a2, a3, a4) in the above formula will produce:

$$F2(t), t \in [0,1] \quad (2)$$

[0028] F2 is the second curve segment, which is smoothly connected to the first curve segment, as shown by verifying:

$$5 \quad F1'(1) = F2'(0) \quad (3)$$

[0029] As shown in Figs. 5A – 5C, a single curve segment, such as curve segment C20, is built for each fragment (i.e. pixel, voxel or physical dot on a graphics display) that is itself a one dimensional (1D) texture (e.g. wiggle display) computation in the 3D texture shader.

10 [0030] For a 2D texture, each fragment requires creation of five curve segments. Forming a 4X4 2D vertex array 505 requires access to sixteen vertices. This is due to the requirement that four curve segments be built in any one of the two dimensions. Then, the four curves are evaluated at the same parameter “t” (on each of the four curves) to form four vertices 510, which can be interpolated to produce a final curve segment C20.

15 [0031] For 3D textures, each fragment (e.g. C20) is built using twenty-one curve segments, built from sixty-four vertices in a 4X4X4 3D vertex array, such as the array 515 illustrated in Fig. 5A. This is due to the requirement to build sixteen curve segments in any single one of the three dimensions, and the need to evaluate these sixteen curve segments at the same parameter “t” (on each of the 16 curves) to form a 4X4 array 505.

20 [0032] Any of the coordinate axes (x, y or z) can be chosen as the first dimension to build the hybrid Bezier curve. The majority of the computational load is associated with analysis of the first dimension. Sixteen of the twenty-one computations are performed in the first dimension.

25 [0033] After performing computations in the first dimension, the procedure is the same as 2D texture procedure detailed above. That is, four curve segments (C16, C17, C18, and C19) are built in the second dimension, as illustrated in Fig. 5B, and the same value is evaluated on these four curves to form the four vertices shown in Fig. 5C. The final curve segment C20 is built from these four vertices, as illustrated in Fig. 5C. These four vertices are evaluated to get final

fragment value. Thus, sixty-four vertices are used to build twenty-one curve segments in order to compute one 3D fragment.

[0034] It is challenging to use hybrid curve computations in the second and third dimensions, because the curve does not pass through any data points, thus it cannot be predefined. Input data points for the second dimension are the dynamic output from computation of the first dimension (detailed above). Thus, the first dimension curves can be pre-defined based on fixed 3D data points, which takes minimal time to build as compared to the total computation. This method can reduce the computation load significantly.

[0035] For each fragment, the 3D texture shader needs to create $(16 + 4 + 1) = 21$ curve segments, from sixty-four 3D data points (for example, located in a 3D texture buffer).

[0036] If a curve segment is represented by a cubic Bezier curve, it needs six weighted average computations to evaluate a point on the curve. If the curve segment is a quadratic Bezier curve, it will need three weighted average computations. If the curve segment is a linear Bezier curve, it will need one weighted average computation. The technique described herein uses lower degree curves whenever possible and higher degree curves when necessary.

[0037] In most texturing applications, the tri-dimensional shader interpolations are either linear (often using the texture defaults in the graphics card), or cubic. Quadratic interpolation is rarely used, because in general, it cannot handle inflection points in one curve segment, thus not guaranteeing C1 continuity of the combined curve and therefore, rarely used. C1 is defined herein as the first derivative at the junction of consecutive curve segments; both segments' first derivatives are required to be equal.

[0038] The technique described herein uses hybrid degree curve segments during curve construction. That is, some curve segments will be created using linear interpolation, some will be created using quadratic interpolation, and some will be created using cubic interpolation. Using non-cubic (i.e., linear or quadratic) interpolation for at least some of the curve segments significantly reduces the computation load with a reduction in continuity reflected only in the difference between C1 continuity (defined above) and G1 continuity. G1 continuity between two curves requires that they meet at a junction point and that their tangents at the junction point

be the same. The difference between C1 and G1 is that in G1 the derivative magnitude can be different. The human eye can hardly tell the difference between C1 and G1 continuity.

[0039] The major reason to use cubic interpolation, instead of quadratic interpolation, is that cubic interpolation can handle inflection points while quadratic interpolation cannot. The method described herein uses quadratic wherever there is no inflection point. The method also uses linear where the data values are almost homogeneous.

[0040] A hybrid Bezier spline curve 600, as illustrated in Fig. 6, is built from curve segments including cubic curve segments (labeled C_C), quadratic curve segments (labeled C_Q), and linear curve segments (labeled C_L). The dashed line along the hybrid Bezier curve 600 (which is the solid curved line) represents the individual extended tangent vectors 605 (only two are labeled) used to construct the hybrid Bezier spline curve 600. The point of intersection (Q) that connects two extended tangent vectors 605 is used to determine the degree of the curve segment (described in detail below). As described above, a quadratic curve segment C_Q cannot handle inflection points (Ip). Three extended tangent vectors 605 (between data points P2 and P3, P3 and P4, and P4 and P5) are analyzed using cubic interpolation to construct a cubic curve segment C_C along the inflection point (Ip).

[0041] In operation, a plurality of a data points are received from a source (not shown). The data points may be seismic data, MRI data, or any other similar data. The data points may be uniformly distributed or non-uniformly distributed. As illustrated in Fig. 7A, four consecutive data points (P1, P2, P3, and P4) are selected from the plurality of data points. The data points P1, P2, P3, and P4 are consecutive in that an x-axis 700 coordinate of P1 is less than the x-axis 700 coordinate of P2. Note that the coordinate system labeled AB applies to Figs. 7A and 7B, and the coordinate system labeled CDEF (which is rotated relative to coordinate system AB to be the same angle that a line through data points P2 and P3 is rotated from Figs. 7A and 7B to Figs. 7C, 7D, 7E and 7F) applies to Figs. 7C, 7D, 7E and 7F. The x-axis 700 coordinate of P2 is greater than the x-axis 700 coordinate of P1, but less than the x-axis 700 coordinate of P3. The x-axis 700 coordinate of P3 is greater than the x-axis 700 coordinate of P2, and less than the x-axis 700 coordinate of P4. The x-axis 700 coordinate of P4 is greater than the x-axis 700 coordinate of P3.

[0042] A first line 705 is created through data points P1 and P3. A first tangent vector 710 is created through P2. The first tangent vector 710 originates at P2. The first tangent vector 710 is parallel to the first line 705. The first tangent vector 710 has a direction such that the projection of the first tangent vector 710 onto the x-axis 700 in coordinate system AB is in the direction of the projection of P3 onto the x-axis 700 in coordinate system AB. The first tangent vector 710 may have a unit magnitude. For parts of the analysis, the first tangent vector 710 has an infinite magnitude, in which case it is referred as the “extended first tangent vector.”

[0043] A second line 720 is created through the data points P2 and P4. A second tangent vector 725 is created through P3. The second tangent vector 725 originates at P3. The second tangent vector 725 is parallel to the second line 720. The second tangent vector 725 has a direction such that the projection of the second tangent vector 725 onto the x-axis 700 in coordinate system AB is in the direction of a projection of P2 onto the x-axis 700 in coordinate system AB. For parts of the analysis, the second tangent vector 725 has an infinite magnitude, in which case it is referred as the “extended second tangent vector.”

[0044] A baseline 735 is created through P2 and P3. The Bezier curve is created, where the degree of the Bezier curve is based on a comparison of the first tangent vector 710, the second tangent vector 725, and the baseline 735.

[0045] The degree of the Bezier curve is determined by defining a plane (represented by dashed box 740). The plane 740 contains the first tangent vector 710 and the second tangent vector 725. The baseline 735 divides the plane 740 into a first half plane 745 and a second half plane 750.

[0046] A linear curve segment C_L (where the degree of the Bezier curve is one) is built, as illustrated in Fig. 7C, when the extended first tangent vector 710 is substantially parallel to the baseline 735, and the extended second tangent vector 725 is substantially parallel to the baseline 735. “Substantially parallel” is defined herein to mean parallel within one degree in one embodiment, within five degrees in another embodiment, and within ten degrees in another embodiment.

[0047] A cubic curve segment C_C (where the degree of the Bezier curve is three) is built, as illustrated in Fig. 7D, when the extended first tangent vector 710 and extended second tangent vector 725 are on different half planes.

[0048] When the extended first tangent vector 710 and the extended second tangent vector 725 are on the same side of the baseline 735, the determination of whether to build a cubic curve segment or a quadratic curve segment requires further analysis. A point of intersection (Q), illustrated in Figs. 7E and 7F, of the extended first tangent vector 710 and the extended second tangent vector 725 is defined. The intersection (Q) is projected onto the baseline 735 as point Q'. A distance is defined between P2 and P3 as "len." A distance between P2 and Q' is defined as "d." A quadratic curve is built, as illustrated in Fig. 7E, when:

$$(t1 * len \leq d \leq t2 * len), \quad (4)$$

where,

$$t1 = 1 - t2 = 0.1,$$

[0049] A cubic curve is built, as illustrated in Fig. 7F, when:

$$(d < t1 * len \text{ OR } d > t2 * len), \quad (5)$$

where,

$$t1 = 1 - t2 = 0.1.$$

[0050] A curve inflection point can be modeled by cubic curve construction, and not with a lower degree curve.

[0051] In use, as illustrated in Fig. 8, a plurality of data points (such as data points P1, P2, P3 and P4) is received whose coordinates are established in a coordinate system comprising an x-axis (such as x-axis 700) along a first dimension; a y-axis along a second dimension orthogonal to the first dimension; a z-axis along a third dimension orthogonal to the first dimension and the second dimension; a time-axis along a fourth dimension orthogonal to the first dimension, the second dimension and the third dimension (block 805). A consecutive set of data points (such as

data points P1, P2, P3 and P4) is selected, wherein an x-axis (such as x-axis 700) coordinate of P1 < an x-axis (such as x-axis 700) coordinate of P2, the x-axis (such as x-axis 700) coordinate of P2 < an x-axis (such as x-axis 700) coordinate of P3, and the x-axis (such as x-axis 700) coordinate of P3 < an x-axis (such as x-axis 700) coordinate of P4 (block 810). A first line (such as first line 705) through P1 and P3 is created (block 815). A first tangent vector (such as first tangent vector 710), originating at P2, parallel to the first line (such as first line 705), and having a direction such that the projection of the first tangent vector (such as first tangent vector 710) onto the x-axis (such as x-axis 700) is in the direction of a projection of P3 onto the x-axis (such as x-axis 700) is created (block 820). A second line (such as second line 720) through P2 and P4 is created (block 825). A second tangent vector (such as second tangent vector 725) through P3 parallel to the second line (such as second line 720), originating at P3, parallel to the second line (such as second line 720), and having a direction such that the projection of the second tangent vector (such as second tangent vector 725) onto the x-axis (such as x-axis 700) is in the direction of a projection of P2 onto the x-axis (such as x-axis 700) is created (block 830). A baseline (such as baseline 735) through P2 and P3 (block 905) is created, as illustrated in Fig. 9. A Bezier curve between P2 and P3 is created, wherein the Bezier curve has a degree (block 910). A degree of the Bezier curve is determined based on a comparison of the first tangent vector (such as first tangent vector 710), the second tangent vector (such as second tangent vector 725), and the baseline (such as baseline 735) (block 915).

[0052] Determining the degree of the Bezier curve includes defining an extended first tangent vector (such as extended first tangent vector 710) to be the first tangent vector (such as first tangent vector 710) with an infinite magnitude (block 920). An extended second tangent vector (such as extended second tangent vector 725) is defined to be the second tangent vector (such as second tangent vector 725) with an infinite magnitude (block 925). If both the extended first tangent vector (such as extended first tangent vector 710) and the extended second tangent vector (such as extended second tangent vector 725) are determined not to be substantially parallel to the baseline (such as baseline 735) (block 930) analysis proceeds to Fig. 10 as indicated by bubble "B". Otherwise, a linear curve is built (block 935).

[0053] Turning to Fig. 10, analysis continues to determine whether a quadratic curve or a cubic curve should be built. A plane (such as plane 740) is defined that contains the first tangent

vector (such as first tangent vector 710) and the second tangent vector (such as second tangent vector 725) (block 1005). The baseline (such as baseline 735) divides the plane (such as plane 740) into a first half plane (such as first half plane 745) and a second half plane (such as second half plane 750) (block 1005). If it is determined that the extended first tangent vector (such as extended first tangent vector 710) and extended second tangent vector (such as extended second tangent vector 725) are not on the same side of the baseline (such as baseline 735) (block 1010), a cubic curve is built (block 1015). If it is determined that the extended first tangent vector (such as extended first tangent vector 710) and extended second tangent vector (such as extended second tangent vector 725) are on the same side of the baseline (such as baseline 735) (block 1010) analysis proceeds to block 1020.

[0054] The intersection (Q) between the extended first tangent vector (such as extended first tangent vector 710) and the extended second tangent vector (such as extended second tangent vector 725) is defined (block 1020). Intersection (Q) is projected onto the baseline (such as baseline 735) as point Q' (block 1025). The distance between P2 and P3 is defined as "len" (block 1030). The distance between P2 and Q' is defined as "d" (block 1035).

[0055] If it is determined that $(t1 * len \leq d \leq t2 * len)$, as shown in Fig. 7E (block 1040), a quadratic curve C_Q is built (block 1050). If it is determined that $(d < t1 * len \text{ OR } d > t2 * len)$, as shown in Fig. 7F (block 1045), a cubic curve C_C is built (block 1055). In one or more embodiments, $t1 = 1 - t2 = 0.1$. In one or more embodiments, $t1$ and $t2$ have other values.

[0056] In one aspect, a method features receiving a plurality of data points whose coordinates are established in a coordinate system. The coordinate system includes an x-axis along a first dimension. The coordinate system includes a y-axis along a second dimension orthogonal to the first dimension. The coordinate system includes a z-axis along a third dimension orthogonal to the first dimension and the second dimension. The coordinate system includes a time-axis along a fourth dimension orthogonal to the first dimension, the second dimension and the third dimension. The method includes selecting a consecutive set of data points including P1, P2, P3, and P4. An x-axis coordinate of P1 < an x-axis coordinate of P2. The x-axis coordinate of P2 < an x-axis coordinate of P3. The x-axis coordinate of P3 < an x-axis coordinate of P4. The method includes creating a first line through P1 and P3. The method includes creating a first

tangent vector originating at P2, parallel to the first line, and having a direction such that the projection of the first tangent vector onto the x-axis is in the direction of a projection of P3 onto the x-axis. The method includes creating a second line through P2 and P4. The method includes creating a second tangent vector through P3, wherein the second tangent vector is parallel to the second line, originates at P3, is parallel to the second line, and has a direction such that the projection of the second tangent vector onto the x-axis is in the direction of a projection of P2 onto the x-axis. The method includes creating a baseline through P2 and P3. The method includes creating a Bezier curve between P2 and P3, wherein the Bezier curve has a degree. The method includes determining the degree of the Bezier curve based on a comparison of the first tangent vector, the second tangent vector, and the baseline.

[0057] Implementations may include one or more of the following. Determining the degree of the Bezier curve may include defining an extended first tangent vector to be the first tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining an extended second tangent vector to be the second tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include determining that the extended first tangent vector is not substantially parallel to the extended second tangent vector. Determining the degree of the Bezier curve may include defining a plane that contains the first tangent vector and the second tangent vector. The baseline may divided the plane into a first half plane and a second half plane. Determining the degree of the Bezier curve may include determining that the extended first tangent vector and extended second tangent vector are on the same side of the baseline. Determining the degree of the Bezier curve may include finding the intersection (Q) between the extended first tangent vector and the extended second tangent vector. Determining the degree of the Bezier curve may include projecting Q onto the baseline as point Q'. Determining the degree of the Bezier curve may include defining the distance between P2 and P3 as len. Determining the degree of the Bezier curve may include defining the distance between P2 and Q' as d. Determining the degree of the Bezier curve may include determining that $(t1 * len \leq d \leq t2 * len)$, and, in response, setting the degree of the Bezier curve to quadratic. Determining the degree of the Bezier curve may include $t1 = 1 - t2 = 0.1$.

[0058] Determining the degree of the Bezier curve may include defining an extended first tangent vector to be the first tangent vector with an infinite magnitude. Determining the degree

of the Bezier curve may include defining an extended second tangent vector to be the second tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include determining that the extended first tangent vector is not substantially parallel to the extended second tangent vector. Determining the degree of the Bezier curve may include

5 defining a plane that contains the first tangent vector and the second tangent vector. The baseline may divide the plane into a first half plane and a second half plane. Determining the degree of the Bezier curve may include determining that the extended first tangent vector and extended second tangent vector are on the same side of the baseline. Determining the degree of the Bezier curve may include finding the intersection (Q) between the extended first tangent vector and the

10 extended second tangent vector. Determining the degree of the Bezier curve may include projecting Q onto the baseline as point Q'. Determining the degree of the Bezier curve may include defining the distance between P2 and P3 as len. Determining the degree of the Bezier curve may include defining the distance between P2 and Q' as d. Determining the degree of the Bezier curve may include determining that ($d < t1 * len$ OR $d > t2 * len$), and, in response,

15 setting the degree of the Bezier curve to cubic. Determining the degree of the Bezier curve may include $t1 = 1 - t2 = 0.1$.

[0059] Determining the degree of the Bezier curve may include defining an extended first tangent vector to be the first tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining an extended second tangent vector to be the second

20 tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining a plane that contains the first tangent vector and the second tangent vector. The baseline may divide the plane into a first half plane and a second half plane. Determining the degree of the Bezier curve may include determining that the extended first tangent vector and extended tangent vector are in different half planes, and, in response, setting the degree of the

25 Bezier curve to cubic.

[0060] Determining the degree of the Bezier curve may include defining an extended first tangent vector to be the first tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining an extended second tangent vector to be the second tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may

30 include defining a plane that contains the first tangent vector and the second tangent vector. The

baseline may divide the plane into a first half plane and a second half plane. Determining the degree of the Bezier curve may include determining that both extended first tangent vector and extended second tangent vector are substantially parallel to the baseline, and, in response, setting the degree of the Bezier curve to linear.

5 [0061] The method may include selecting a consecutive set of non-uniformly distributed data points. The method may include selecting a consecutive set of uniformly distributed data points. The method may include an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the y-axis. The method may include an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the z-axis. The method may
10 include an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the time-axis.

[0062] In one aspect, a non-transitory computer-readable medium, on which is recorded a computer program that, when executed, performs a method. The method includes receiving a plurality of data points whose coordinates are established in a coordinate system comprising. The
15 coordinate system includes an x-axis along a first dimension. The coordinate system includes a y-axis along a second dimension orthogonal to the first dimension. The coordinate system includes a z-axis along a third dimension orthogonal to the first dimension and the second dimension. The coordinate system includes a time-axis along a fourth dimension orthogonal to the first dimension, the second dimension and the third dimension. The method includes
20 selecting a consecutive set of data points including a P1, a P2, a P3, and a P4. An x-axis coordinate of P1 < an x-axis coordinate of P2. The x-axis coordinate of P2 < an x-axis coordinate of P3. The x-axis coordinate of P3 < an x-axis coordinate of P4. The method includes creating a first line through P1 and P3. The method includes creating a first tangent vector originating at P2, parallel to the first line, and having a direction such that the projection
25 of the first tangent vector onto the x-axis is in the direction of a projection of P3 on the x-axis. The method includes creating a second line through P2 and P4. The method includes creating a second tangent vector through P3 parallel to the second line originating at P3, parallel to the second line, and having a direction such that the projection of the second tangent vector onto the x-axis is in the direction of a projection of the P2 on the x-axis. The method includes creating a
30 baseline through P2 and P3. The method includes creating a Bezier curve between P2 and P3,

wherein the Bezier curve has a degree. The method includes determining the degree of the Bezier curve based on a comparison of the first tangent vector, the second tangent vector, and the baseline.

[0063] Implementations may include one or more of the following. Determining the degree of the Bezier curve may include defining an extended first tangent vector to be the first tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining an extended second tangent vector to be the second tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include determining that the extended first tangent vector is not substantially parallel to the extended second tangent vector.

Determining the degree of the Bezier curve may include defining a plane that contains the first tangent vector and the second tangent vector. The baseline may divide the plane into a first half plane and a second half plane. Determining the degree of the Bezier curve may include determining that the extended first tangent vector and extended second tangent vector are on the same side of the baseline. Determining the degree of the Bezier curve may include finding the intersection (Q) between the extended first tangent vector and the extended second tangent vector. Determining the degree of the Bezier curve may include projecting Q onto the baseline as point Q'. Determining the degree of the Bezier curve may include defining the distance between P2 and P3 as len. Determining the degree of the Bezier curve may include defining the distance between P2 and Q' as d. Determining the degree of the Bezier curve may include determining that $(t1 * len \leq d \leq t2 * len)$, and, in response, setting the degree of the Bezier curve to quadratic. Determining the degree of the Bezier curve may include $t1 = 1 - t2 = 0.1$.

[0064] Determining the degree of the Bezier curve may include defining an extended first tangent vector to be the first tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining an extended second tangent vector to be the second tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include determining that the extended first tangent vector is not substantially parallel to the extended second tangent vector. Determining the degree of the Bezier curve may include defining a plane that contains the first tangent vector and the second tangent vector. The baseline may divide the plane into a first half plane and a second half plane. Determining the degree of the Bezier curve may include determining that the extended first tangent vector and extended

second tangent vector are on the same side of the baseline. Determining the degree of the Bezier curve may include finding the intersection (Q) between the extended first tangent vector and the extended second tangent vector. Determining the degree of the Bezier curve may include projecting Q onto the baseline as point Q'. Determining the degree of the Bezier curve may include defining the distance between P2 and P3 as len. Determining the degree of the Bezier curve may include defining the distance between P2 and Q' as d. Determining the degree of the Bezier curve may include determining that ($d < t1 * len$ OR $d > t2 * len$), and, in response, setting the degree of the Bezier curve to cubic. Determining the degree of the Bezier curve may include $t1 = 1 - t2 = 0.1$.

[0065] Determining the degree of the Bezier curve may include defining an extended first tangent vector to be the first tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining an extended second tangent vector to be the second tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining a plane that contains the first tangent vector and the second tangent vector. The baseline may divide the plane into a first half plane and a second half plane. Determining the degree of the Bezier curve may include determining that the extended first tangent vector and extended tangent vector are in different half planes, and, in response, setting the degree of the Bezier curve to cubic.

[0066] Determining the degree of the Bezier curve may include defining an extended first tangent vector to be the first tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining an extended second tangent vector to be the second tangent vector with an infinite magnitude. Determining the degree of the Bezier curve may include defining a plane that contains the first tangent vector and the second tangent vector. The baseline may divide the plane into a first half plane and a second half plane. Determining the degree of the Bezier curve may include determining that both extended first tangent vector and extended second tangent vector are substantially parallel to the baseline, and, in response, setting the degree of the Bezier curve to linear.

[0067] The method may include selecting a consecutive set of non-uniformly distributed data points. The method may include selecting a consecutive set of uniformly distributed data points.

The method may include an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the y-axis. The method may include an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the z-axis. The method may include an interpolation selected from a group consisting of linear, quadratic and cubic
5 interpolations in the time-axis.

[0068] References in the specification to “one or more embodiments”, “one embodiment”, “an embodiment”, “an example embodiment”, etc., indicate that the embodiment described may include a particular feature, structure, or characteristic, but every embodiment may not necessarily include the particular feature, structure, or characteristic. Moreover, such phrases are
10 not necessarily referring to the same embodiment. Further, when a particular feature, structure, or characteristic is described in connection with an embodiment, it is submitted that it is within the knowledge of one skilled in the art to effect such feature, structure, or characteristic in connection with other embodiments whether or not explicitly described.

[0069] The operations of the flow diagrams are described with references to the systems/apparatus shown in the block diagrams. However, it should be understood that the
15 operations of the flow diagrams could be performed by embodiments of systems and apparatus other than those discussed with reference to the block diagrams, and embodiments discussed with reference to the systems/apparatus could perform operations different than those discussed with reference to the flow diagrams.

[0070] The word “coupled” herein means a direct connection or an indirect connection.
20

[0071] The text above describes one or more specific embodiments of a broader invention. The invention also is carried out in a variety of alternate embodiments and thus is not limited to those described here. The foregoing description of an embodiment of the invention has been presented for the purposes of illustration and description. It is not intended to be exhaustive or to limit the
25 invention to the precise form disclosed. Many modifications and variations are possible in light of the above teaching. It is intended that the scope of the invention be limited not by this detailed description, but rather by the claims appended hereto.

Claims

What is claimed is:

1. A method comprising:

receiving a plurality of data points whose coordinates are established in a coordinate
5 system comprising:

an x-axis along a first dimension;

a y-axis along a second dimension orthogonal to the first dimension;

a z-axis along a third dimension orthogonal to the first dimension and the second
dimension;

10 a time-axis along a fourth dimension orthogonal to the first dimension, the second
dimension and the third dimension;

selecting a consecutive set of data points comprising P1, P2, P3, and P4 wherein:

an x-axis coordinate of P1 < an x-axis coordinate of P2,

the x-axis coordinate of P2 < an x-axis coordinate of P3, and

15 the x-axis coordinate of P3 < an x-axis coordinate of P4;

creating a first line through P1 and P3;

creating a first tangent vector originating at P2, parallel to the first line, and having a
direction such that the projection of the first tangent vector onto the x-axis is in
the direction of a projection of P3 onto the x-axis;

20 creating a second line through P2 and P4;

creating a second tangent vector through P3, wherein the second tangent vector is parallel
to the second line, originates at P3, is parallel to the second line, and has a
direction such that the projection of the second tangent vector onto the x-axis is in
the direction of a projection of P2 onto the x-axis;

25 creating a baseline through P2 and P3;

creating a Bezier curve between P2 and P3, wherein the Bezier curve has a degree; and

determining the degree of the Bezier curve based on a comparison of the first tangent
vector, the second tangent vector, and the baseline.

2. The method of claim 1 wherein determining the degree of the Bezier curve comprises:

defining an extended first tangent vector to be the first tangent vector with an infinite magnitude;

defining an extended second tangent vector to be the second tangent vector with an infinite magnitude;

determining that the extended first tangent vector is not substantially parallel to the extended second tangent vector;

defining a plane that contains the first tangent vector and the second tangent vector; wherein the baseline divides the plane into a first half plane and a second half plane;

determining that the extended first tangent vector and extended second tangent vector are on the same side of the baseline;

finding the intersection (Q) between the extended first tangent vector and the extended second tangent vector;

projecting Q onto the baseline as point Q';

defining the distance between P2 and P3 as len;

defining the distance between P2 and Q' as d;

determining that $(t1 * len \leq d \leq t2 * len)$, and, in response, setting the degree of the Bezier curve to quadratic.

3. The method of claim 2 wherein $t1 = 1 - t2 = 0.1$.

4. The method of claim 1 wherein determining the degree of the Bezier curve comprises:

defining an extended first tangent vector to be the first tangent vector with an infinite magnitude;

defining an extended second tangent vector to be the second tangent vector with an infinite magnitude;

determining that the extended first tangent vector is not substantially parallel to the extended second tangent vector;

defining a plane that contains the first tangent vector and the second tangent vector; wherein the baseline divides the plane into a first half plane and a second half plane;

determining that the extended first tangent vector and extended second tangent vector are on the same side of the baseline;

finding the intersection (Q) between the extended first tangent vector and the extended second tangent vector;

5 projecting Q onto the baseline as point Q';

defining the distance between P2 and P3 as len;

defining the distance between P2 and Q' as d;

determining that ($d < t1 * len$ OR $d > t2 * len$), and, in response, setting the degree of the Bezier curve to cubic.

10 5. The method of claim 4 wherein $t1 = 1 - t2 = 0.1$.

6. The method of claim 1 wherein determining the degree of the Bezier curve comprises:

defining an extended first tangent vector to be the first tangent vector with an infinite magnitude;

15 defining an extended second tangent vector to be the second tangent vector with an infinite magnitude;

defining a plane that contains the first tangent vector and the second tangent vector; wherein the baseline divides the plane into a first half plane and a second half plane;

20 determining that the extended first tangent vector and extended tangent vector are in different half planes, and, in response, setting the degree of the Bezier curve to cubic.

7. The method of claim 1 wherein determining the degree of the Bezier curve comprises:

defining an extended first tangent vector to be the first tangent vector with an infinite magnitude;

25 defining an extended second tangent vector to be the second tangent vector with an infinite magnitude;

defining a plane that contains the first tangent vector and the second tangent vector; wherein the baseline divides the plane into a first half plane and a second half plane;

determining that both extended first tangent vector and extended second tangent vector are substantially parallel to the baseline, and, in response, setting the degree of the Bezier curve to linear.

8. The method of claim 1 further comprises selecting a consecutive set of non-uniformly distributed data points.

9. The method of claim 1 further comprises selecting a consecutive set of uniformly distributed data points.

10. A method as in any preceding claim, further comprising performing an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the y-axis.

11. A method as in one of claims 1-9, further comprising performing an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the z-axis.

12. A method as in claims 1-9, further comprising performing an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the time-axis.

13. A non-transitory computer-readable medium, on which is recorded a computer program that, when executed, performs a method comprising:

receiving a plurality of data points whose coordinates are established in a coordinate system comprising:

an x-axis along a first dimension;

a y-axis along a second dimension orthogonal to the first dimension;

a z-axis along a third dimension orthogonal to the first dimension and the second dimension;

a time-axis along a fourth dimension orthogonal to the first dimension, the second dimension and the third dimension

selecting a consecutive set of data points comprising a P1, a P2, a P3, and a P4 point wherein:

an x-axis coordinate of P1 < an x-axis coordinate of P2,

the x-axis coordinate of P2 < an x-axis coordinate of P3, and

the x-axis coordinate of P3 < an x-axis coordinate of P4;
creating a first line through P1 and P3;
creating a first tangent vector originating at P2, parallel to the first line, and having a
direction such that the projection of the first tangent vector onto the x-axis is in
5 the direction of a projection of P3 on the x-axis;
creating a second line through P2 and P4;
creating a second tangent vector through P3 parallel to the second line originating at P3,
parallel to the second line, and having a direction such that the projection of the
second tangent vector onto the x-axis is in the direction of a projection of P2 on
10 the x-axis;
creating a baseline through P2 and P3;
creating a Bezier curve between P2 and P3, wherein the Bezier curve has a degree; and
determining the degree of the Bezier curve based on a comparison of the first tangent
vector, the second tangent vector, and the baseline.

- 15 14. The method of claim 13 wherein determining the degree of the Bezier curve comprises:
defining an extended first tangent vector to be the first tangent vector with an infinite
magnitude;
defining an extended second tangent vector to be the second tangent vector with an
infinite magnitude;
20 determining that the extended first tangent vector is not substantially parallel to the
extended second tangent vector;
defining a plane that contains the first tangent vector and the second tangent vector;
wherein the baseline divides the plane into a first half plane and a second half
plane;
25 determining that the extended first tangent vector and extended second tangent vector are
on the same side of the baseline;
finding the intersection (Q) between the extended first tangent vector and the extended
second tangent vector;
projecting Q onto the baseline as point Q';
30 defining the distance between P2 and P3 as len;

defining the distance between P2 and Q' as d;
 determining that ($t1 * len \leq d \leq t2 * len$), and, in response, setting the degree of the Bezier curve to quadratic.

15. The method of claim 14 wherein $t1 = 1 - t2 = 0.1$.

- 5 16. The method of claim 13 wherein determining the degree of the Bezier curve comprises:
 defining an extended first tangent vector to be the first tangent vector with an infinite magnitude;
 defining an extended second tangent vector to be the second tangent vector with an infinite magnitude;
 10 determining that the extended first tangent vector is not substantially parallel to the extended second tangent vector;
 defining a plane that contains the first tangent vector and the second tangent vector;
 wherein the baseline divides the plane into a first half plane and a second half plane;
 15 determining that the extended first tangent vector and extended second tangent vector are on the same side of the baseline;
 finding the intersection (Q) between the extended first tangent vector and the extended second tangent vector;
 projecting Q onto the baseline as point Q';
 20 defining the distance between P2 and P3 as len;
 defining the distance between P2 and Q' as d;
 determining that ($d < t1 * len$ OR $d > t2 * len$), and, in response, setting the degree of the Bezier curve to cubic.

17. The method of claim 16 wherein $t1 = 1 - t2 = 0.1$.

- 25 18. The method of claim 13 wherein determining the degree of the Bezier curve comprises:
 defining an extended first tangent vector to be the first tangent vector with an infinite magnitude;

defining an extended second tangent vector to be the second tangent vector with an infinite magnitude;

defining a plane that contains the first tangent vector and the second tangent vector; wherein the baseline divides the plane into a first half plane and a second half plane;

determining that the extended first tangent vector and extended tangent vector are in different half planes, and, in response, setting the degree of the Bezier curve to cubic.

19. The method of claim 13 wherein determining the degree of the Bezier curve comprises:

defining an extended first tangent vector to be the first tangent vector with an infinite magnitude;

defining an extended second tangent vector to be the second tangent vector with an infinite magnitude;

defining a plane that contains the first tangent vector and the second tangent vector; wherein the baseline divides the plane into a first half plane and a second half plane;

determining that both extended first tangent vector and extended second tangent vector are substantially parallel to the baseline, and, in response, setting the degree of the Bezier curve to linear.

20. The method of claim 13 further comprises selecting a consecutive set of non-uniformly distributed data points.

21. The method of claim 13 further comprises selecting a consecutive set of uniformly distributed data points.

22. A method as in any preceding claim, further comprising performing an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the y-axis.

23. A method as in one of claims 13-21, further comprising performing an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the z-axis.

5 24. A method as in claims 13-21, further comprising performing an interpolation selected from a group consisting of linear, quadratic and cubic interpolations in the time-axis.

Fig. 1

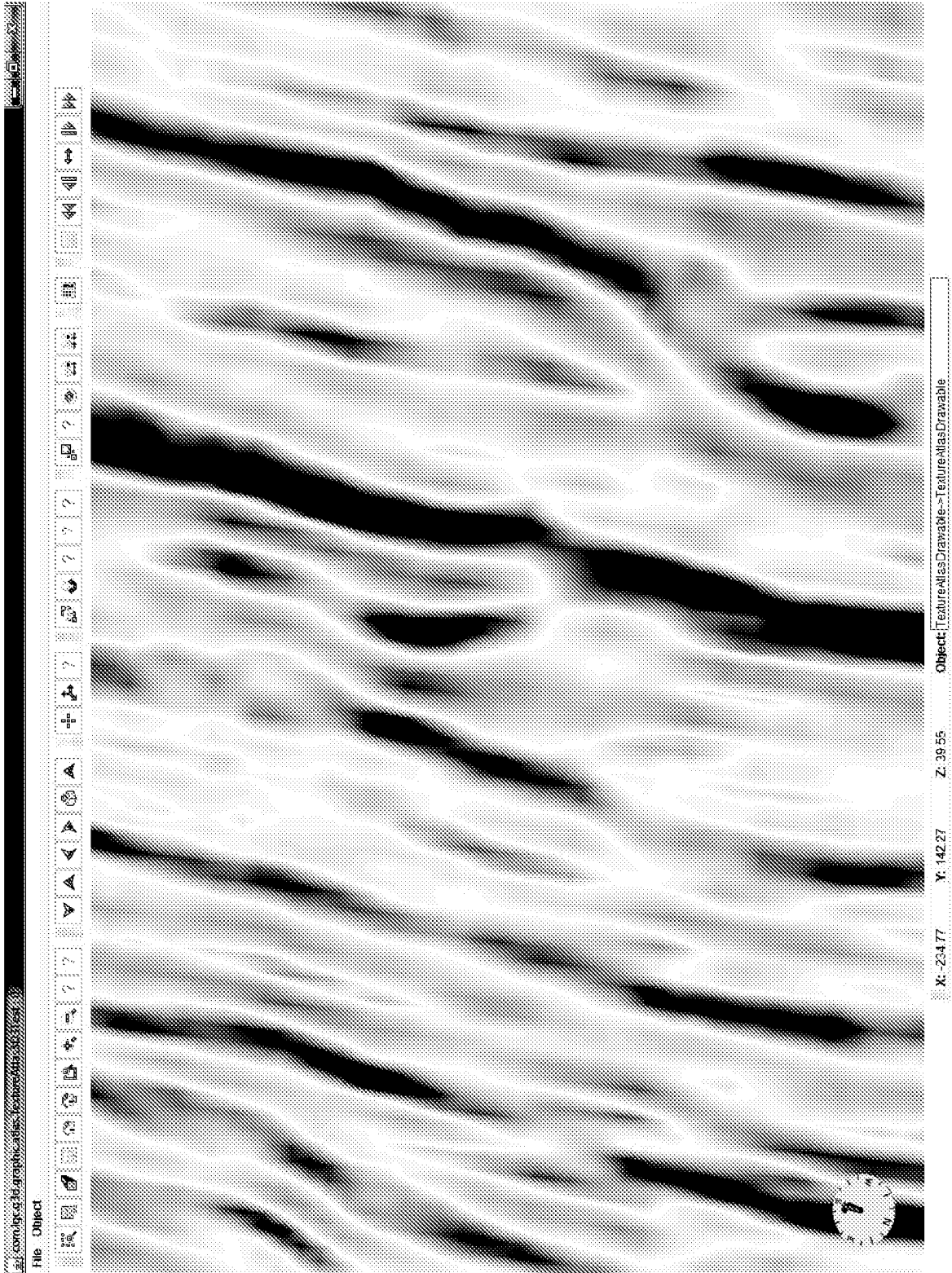


Fig. 2

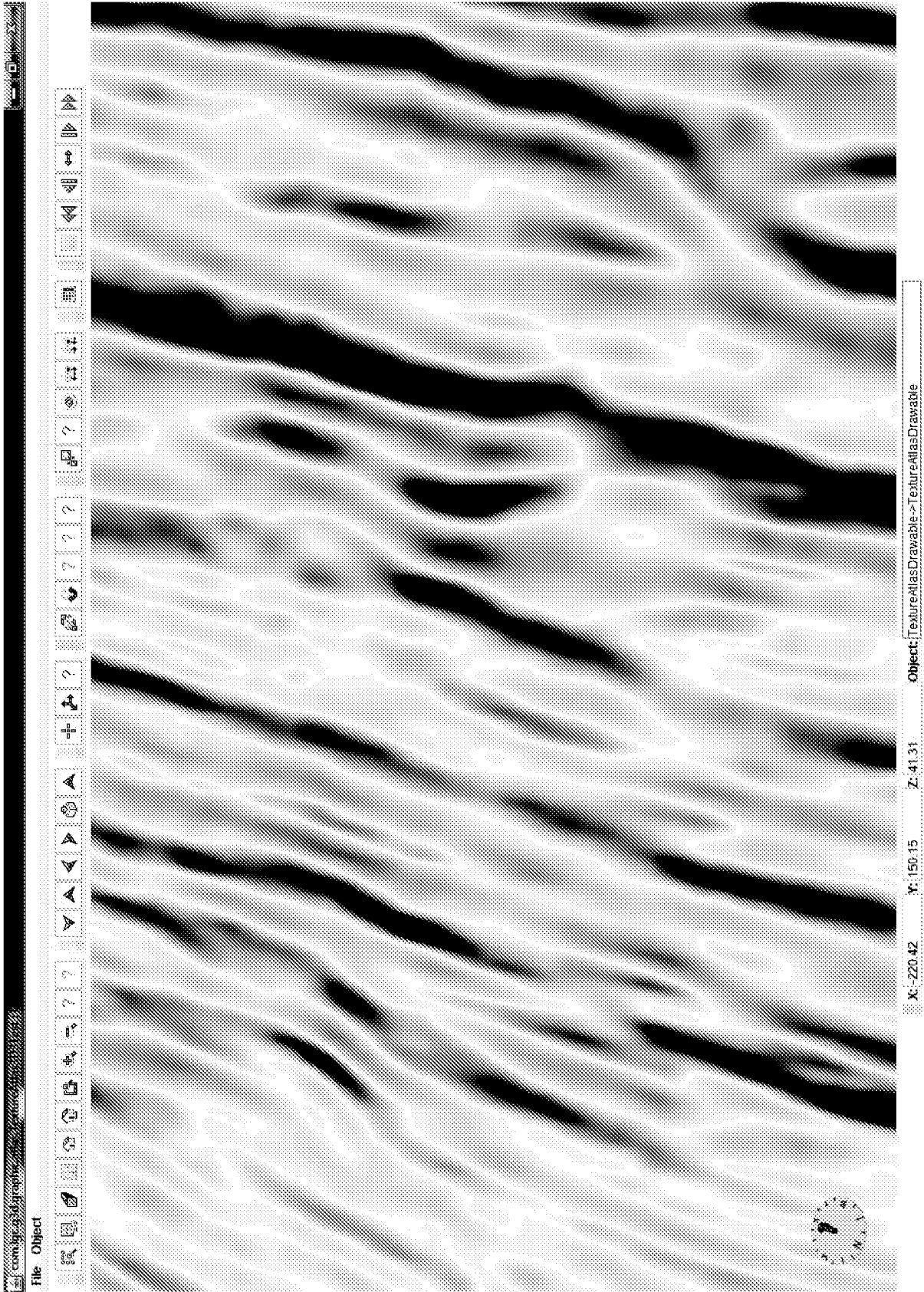


Fig. 3

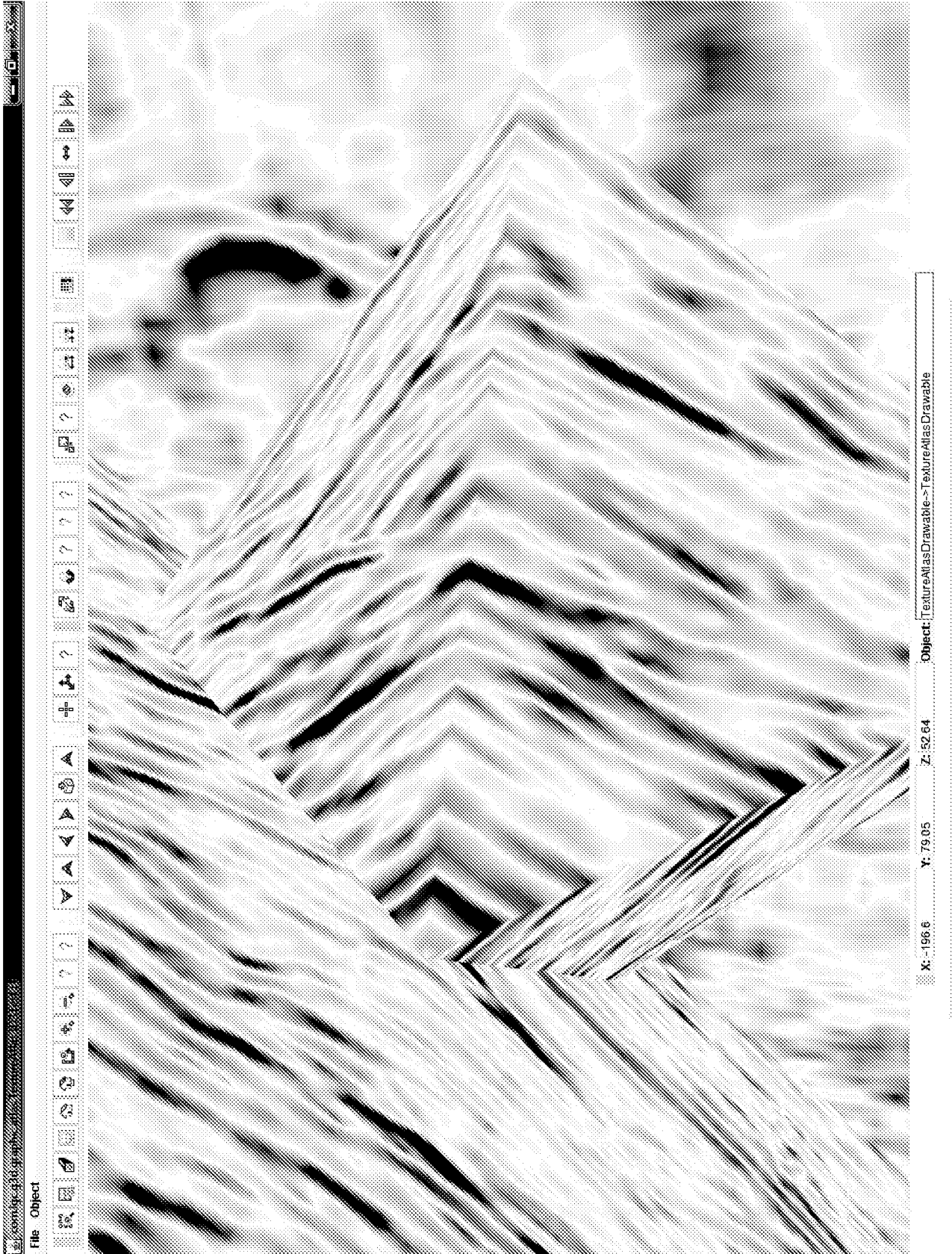
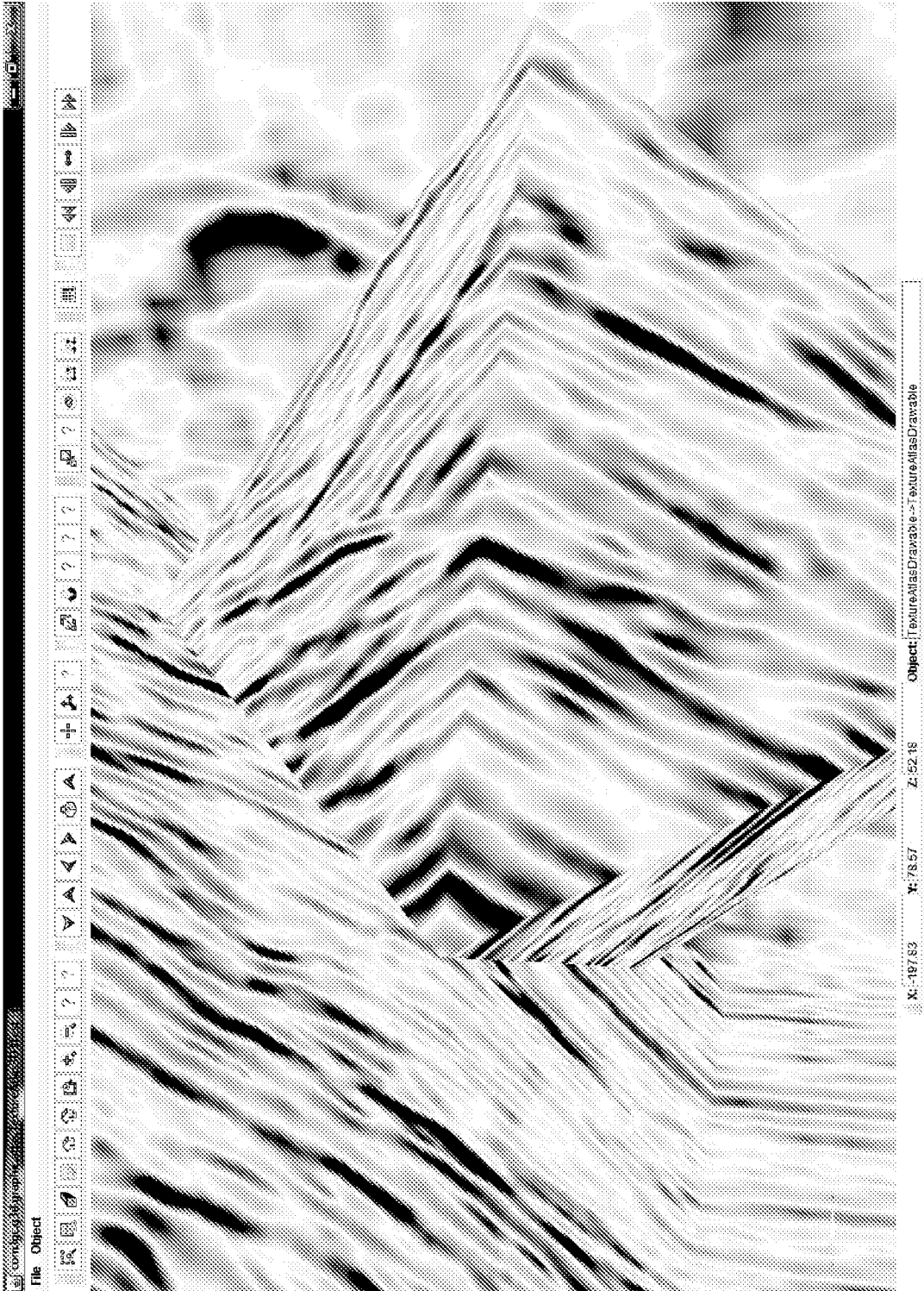
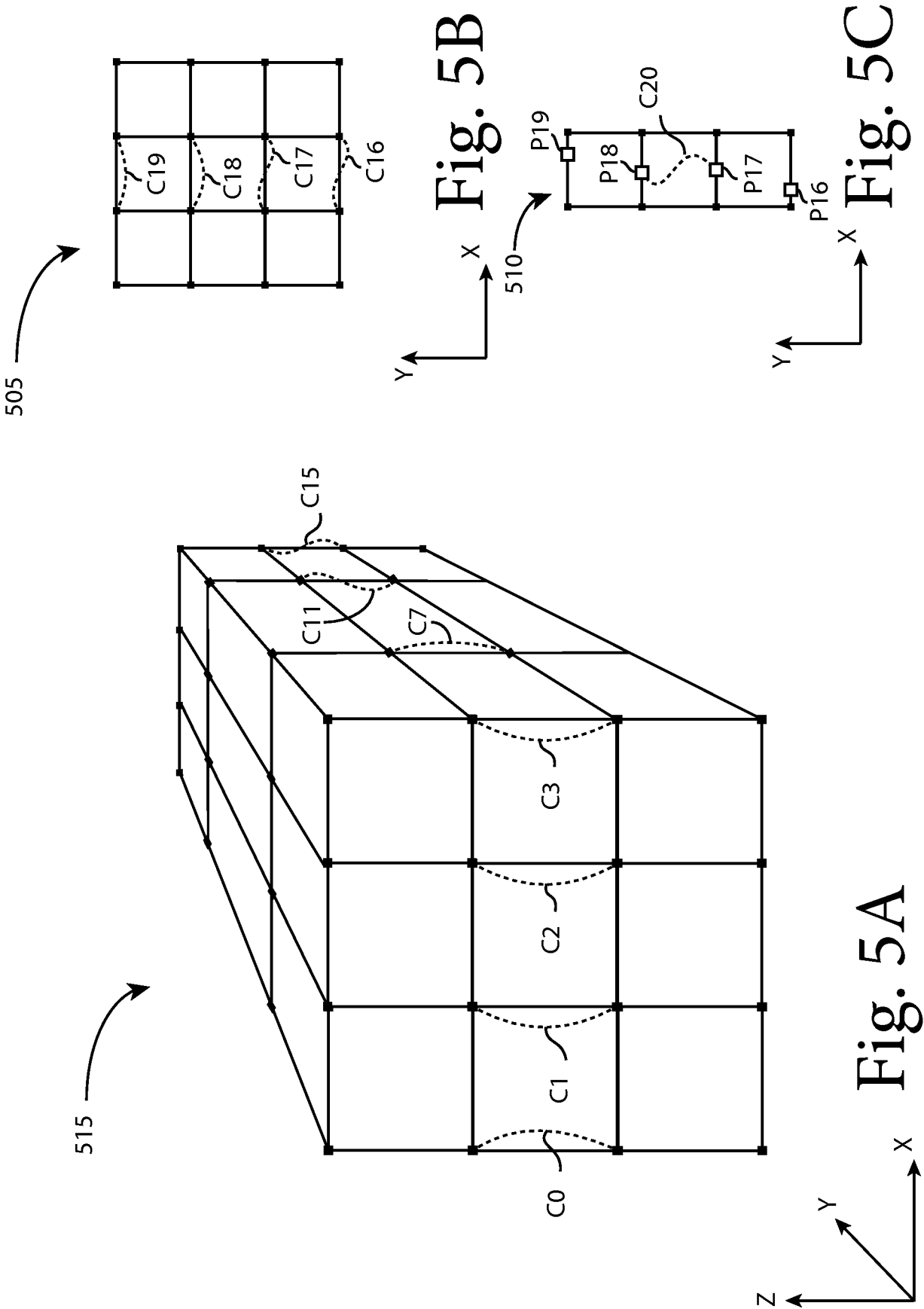


Fig. 4





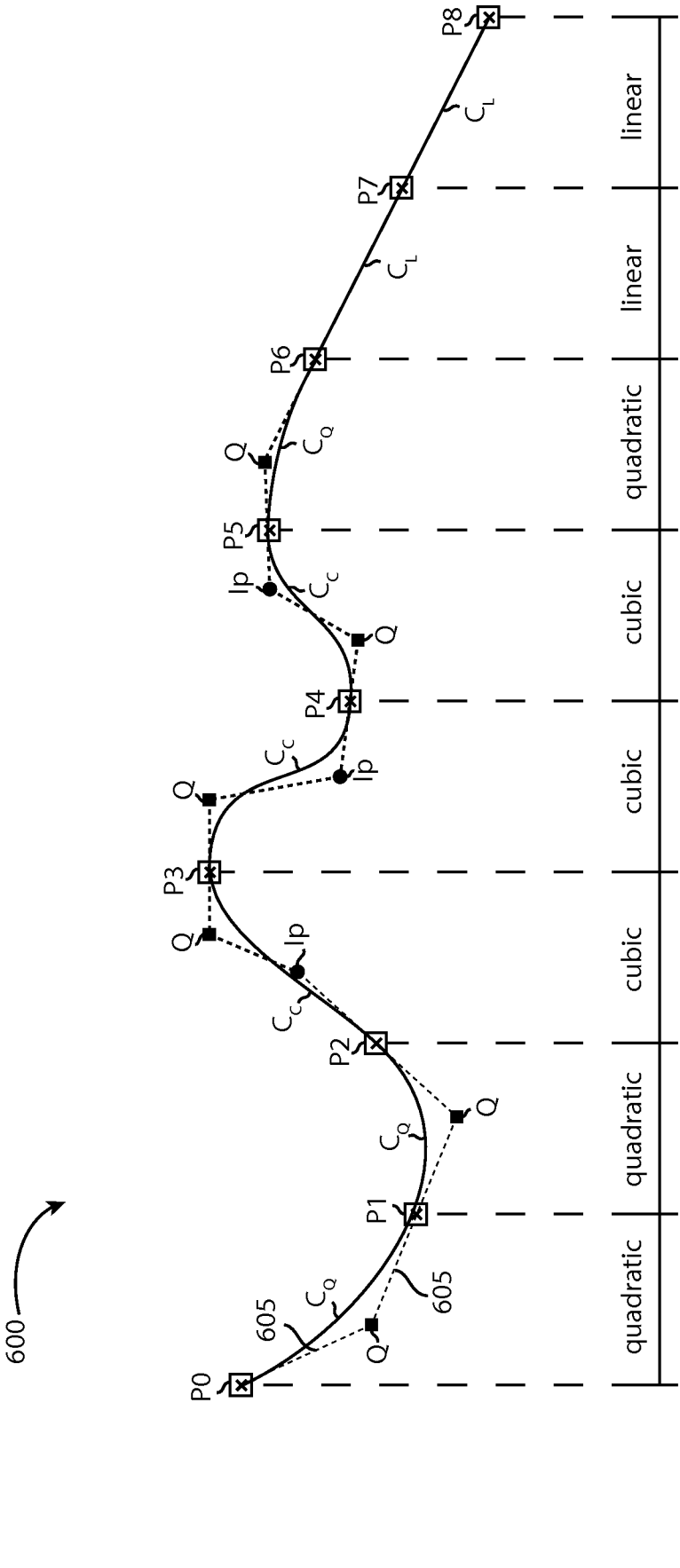


Fig. 6

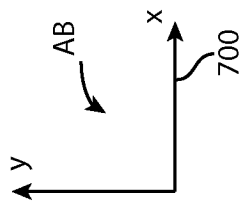
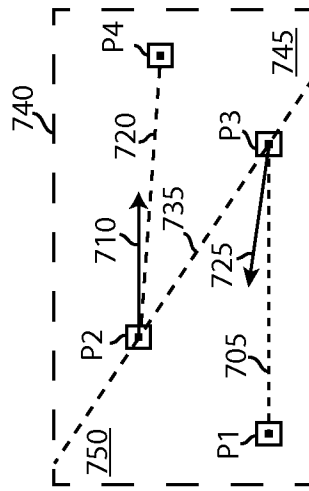


Fig. 7A

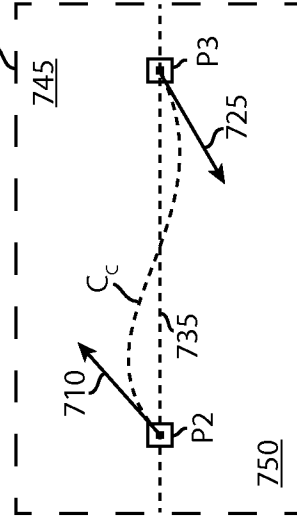
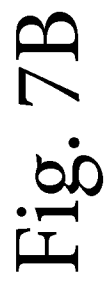


Fig. 7D

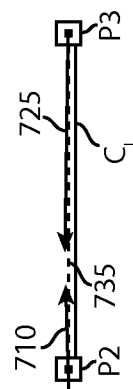


Fig. 7C

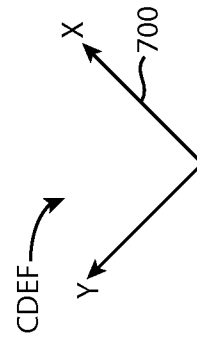
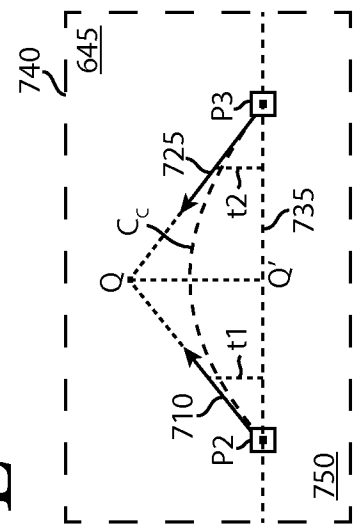
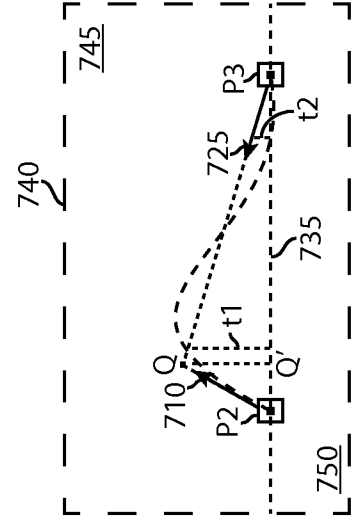


Fig. 7E



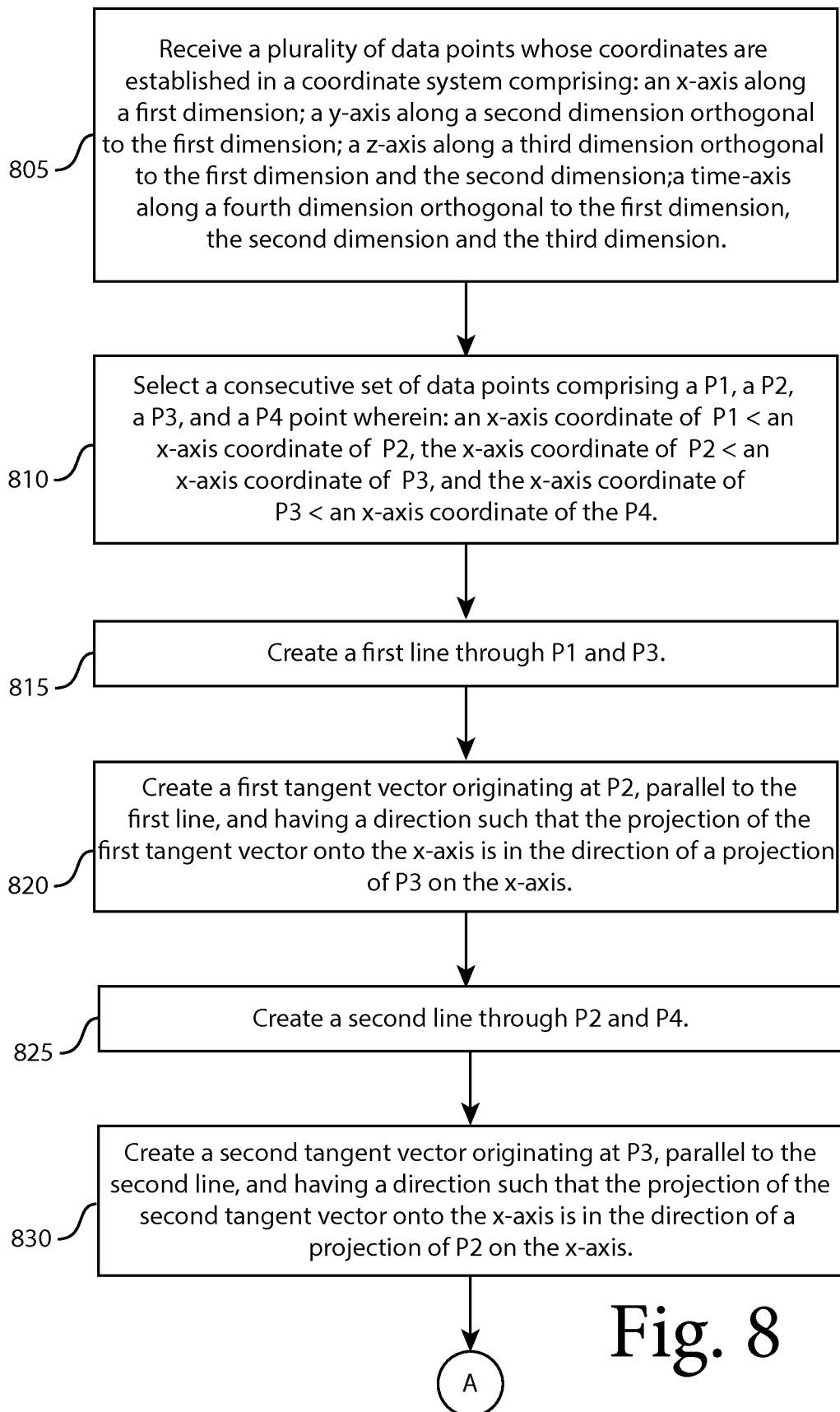


Fig. 8

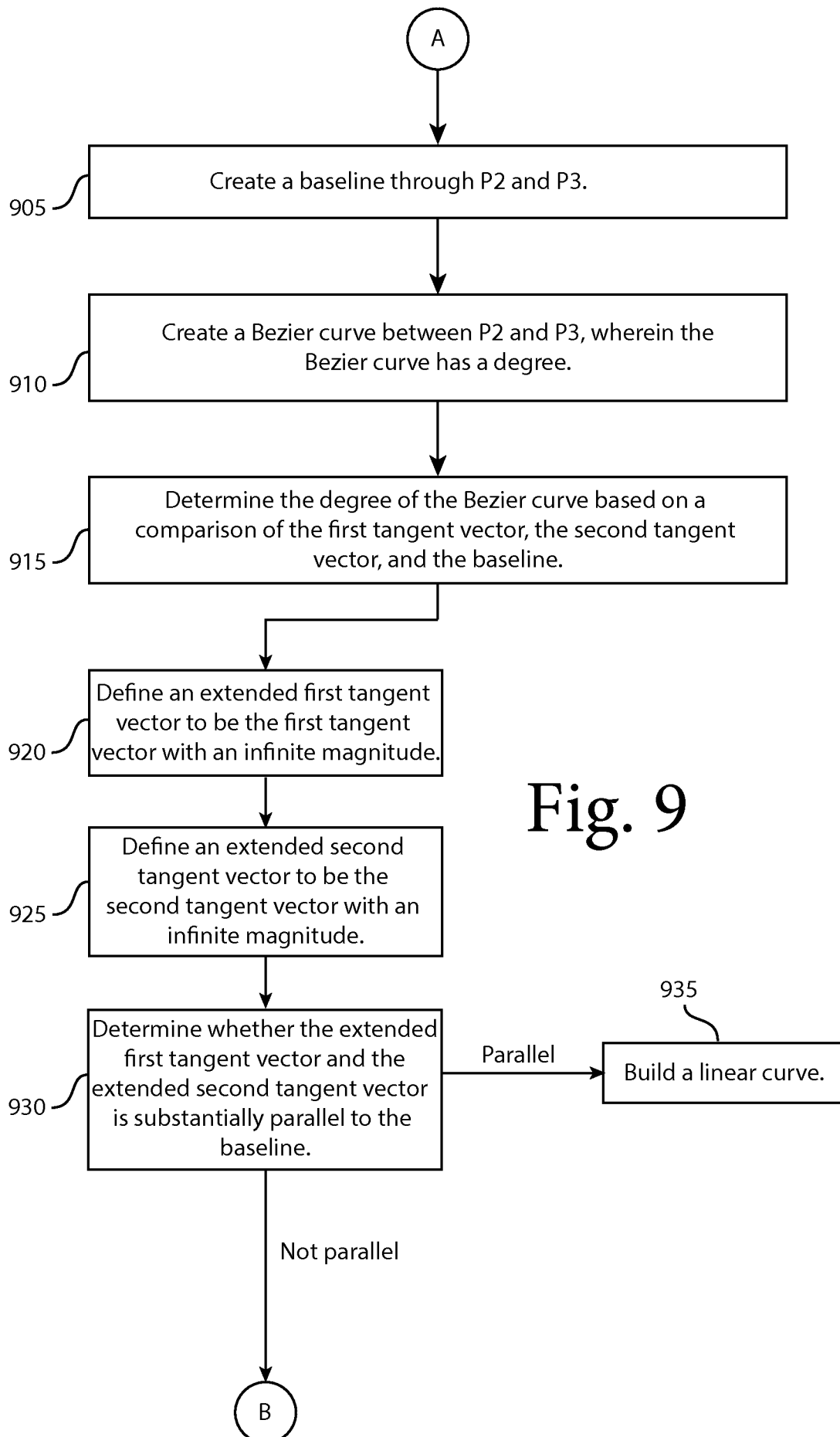


Fig. 9

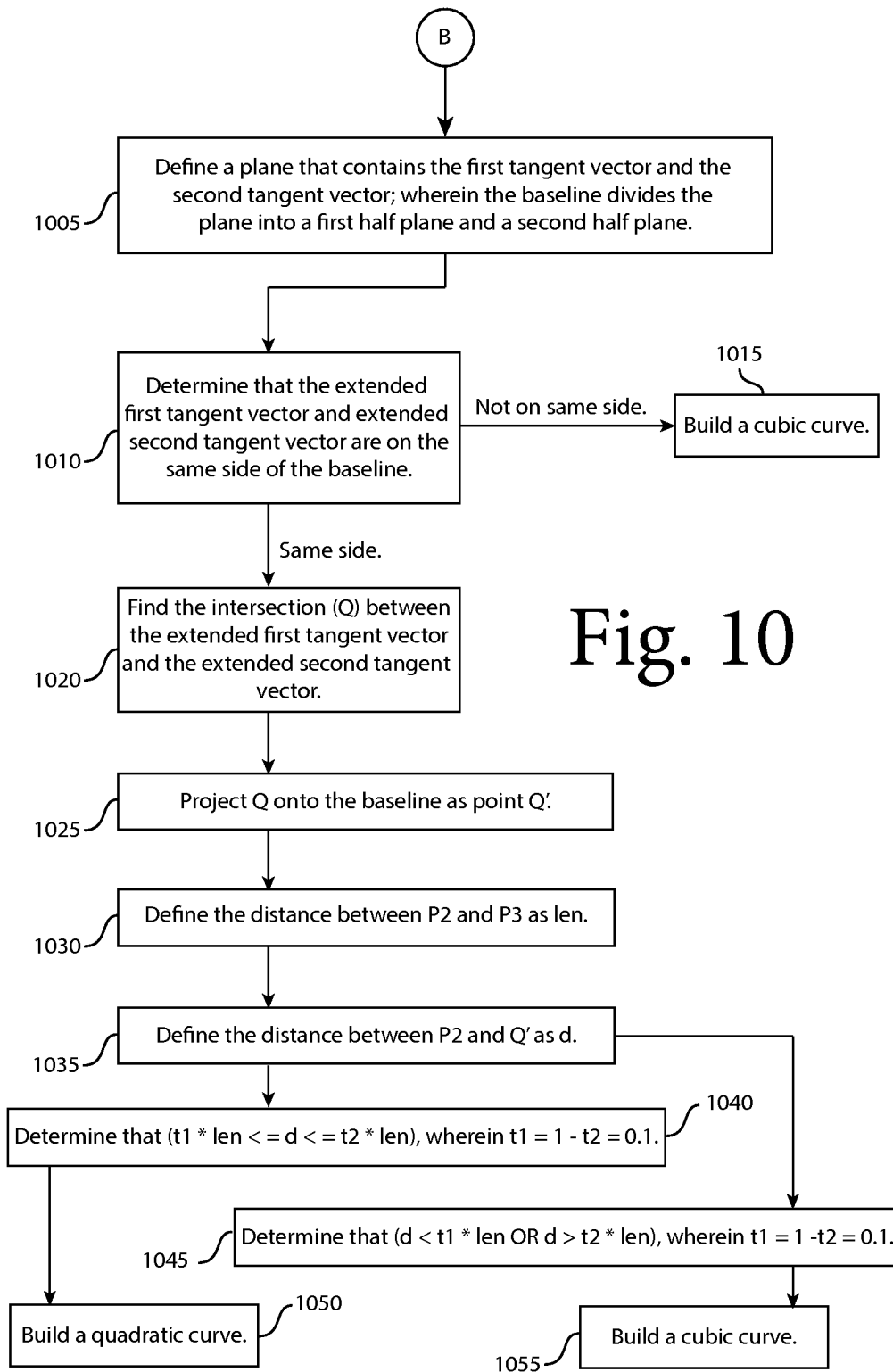


Fig. 10

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US2015/039301**A. CLASSIFICATION OF SUBJECT MATTER****G06T 15/80(2011.01)i, G06T 15/00(2006.01)i**

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHEDMinimum documentation searched (classification system followed by classification symbols)
G06T 15/80; G06T 17/00; G06F 3/033; G06K 11/20; G01C 17/38; G06T 15/00; G09G 5/00Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
Korean utility models and applications for utility models
Japanese utility models and applications for utility modelsElectronic data base consulted during the international search (name of data base and, where practicable, search terms used)
eKOMPASS(KIPO internal) & keywords: data point, coordinate, tangent vector, baseline, Bezier curve, interpolation, texture, shader**C. DOCUMENTS CONSIDERED TO BE RELEVANT**

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	US 2002-0005856 A1 (NOBUO SASAKI) 17 January 2002 See paragraphs [0174]-[0182], [0232]-[0236]; and figures 20, 22A, 34.	1-24
A	US 2009-0033658 A1 (ALYN ROCKWOOD et al.) 05 February 2009 See paragraphs [0192]-[0194]; and figure 26.	1-24
A	EP 0762265 A2 (CANON KABUSHIKI KAISHA) 12 March 1997 See column 6, line 6 - column 10, line 43; and figures 1-7.	1-24
A	US 2005-0246130 A1 (SEAN SPICER et al.) 03 November 2005 See paragraphs [0090]-[0113]; and figures 2-7.	1-24
A	US 6246414 B1 (TOMOYUKI KAWASAKI) 12 June 2001 See column 6, line 60 - column 8, line 5; and figures 4-6F.	1-24



Further documents are listed in the continuation of Box C.



See patent family annex.

* Special categories of cited documents:

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier application or patent but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search

19 May 2016 (19.05.2016)

Date of mailing of the international search report

19 May 2016 (19.05.2016)

Name and mailing address of the ISA/KR

International Application Division

Korean Intellectual Property Office

189 Cheongsa-ro, Seo-gu, Daejeon, 35208, Republic of Korea

Facsimile No. +82-42-481-8578

Authorized officer

LEE, Dong Yun

Telephone No. +82-42-481-8734



INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No.

PCT/US2015/039301

Patent document cited in search report	Publication date	Patent family member(s)	Publication date
US 2002-0005856 A1	17/01/2002	AR 018846 A1	12/12/2001
		AU 1999-31670 A1	01/11/1999
		AU 3167099 A	01/11/1999
		BR 9906309 A	11/07/2000
		CA 2293670 A1	21/10/1999
		CN 100476882 C	08/04/2009
		CN 1272933 A	08/11/2000
		CO 4880830 A1	31/01/2000
		EP 0996094 A1	26/04/2000
		EP 0996094 A4	17/10/2007
		EP 0996094 B1	15/06/2011
		JP 03705923 B2	12/10/2005
		JP 11-296686 A	29/10/1999
		KR 10-0586276 B1	07/06/2006
		US 6373491 B1	16/04/2002
		US 7084883 B2	01/08/2006
		WO 99-53445 A1	21/10/1999
		ZA 9902636 A	11/10/1999
US 2009-0033658 A1	05/02/2009	US 2004-0085311 A1	06/05/2004
		US 2007-0152999 A1	05/07/2007
		US 2007-0176923 A1	02/08/2007
		US 6981695 B1	03/01/2006
		US 7196702 B1	27/03/2007
		US 7236167 B2	26/06/2007
		US 7417635 B2	26/08/2008
		US 7636091 B2	22/12/2009
		US 7755623 B2	13/07/2010
EP 0762265 A2	12/03/1997	CN 1098496 C	08/01/2003
		CN 1151555 A	11/06/1997
		DE 69628380 D1	03/07/2003
		DE 69628380 T2	01/04/2004
		EP 0762265 A3	11/06/1997
		EP 0762265 B1	28/05/2003
		JP 03517496 B2	12/04/2004
		JP 09-069022 A	11/03/1997
		JP 09-091081 A	04/04/1997
		JP 09-106321 A	22/04/1997
		KR 10-0269680 B1	16/10/2000
		US 6278445 B1	21/08/2001
US 2005-0246130 A1	03/11/2005	AU 2005-241463 A1	17/11/2005
		AU 2005-241463 B2	28/01/2010
		AU 2005-241463 C1	29/07/2010
		BR PI0509830 A	16/10/2007
		CA 2564156 A1	17/11/2005
		CA 2564156 C	08/04/2014
		CN 100576250 C	30/12/2009

INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No.

PCT/US2015/039301

Patent document cited in search report	Publication date	Patent family member(s)	Publication date
US 6246414 B1	12/06/2001	CN 101052997 A	10/10/2007
		EA 012579 B1	30/10/2009
		EA 200601977 A2	27/04/2007
		EA 200601977 A3	28/02/2008
		EP 1756517 A2	28/02/2007
		EP 1756517 A4	26/01/2011
		MX PA06012485 A	19/06/2007
		NO 20065397 A	24/01/2007
		US 2008-0129731 A1	05/06/2008
		US 7352369 B2	01/04/2008
		US 7576743 B2	18/08/2009
		WO 2005-108923 A2	17/11/2005
		WO 2005-108923 A3	14/06/2007
		CN 1218940 A	09/06/1999
		EP 0917106 A2	19/05/1999
		JP 03179392 B2	25/06/2001
		JP 11-149567 A	02/06/1999
		KR 10-0349483 B1	18/11/2002