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(54) VELOCITY DETERMINATION AND RANGE AND VELOCITY DETERMINATION

(71) I, NORMAN SAMSON NEIDELL, a national of the United States of America, of 13054 Taylorcrest, Houston, Texas 77079, United States of America, do hereby declare the invention, for which I pray that a patent may be granted to me, and the method by which it is to be performed, to be particularly described in and by the following statement:—

Echo location systems are designed to identify some subset of individual reflector or target parameters. These parameters include the target position in terms like bearing and elevation angle, the range, target relative velocity and the impedance contrast which causes the echo and is also a measure of target quality. Individual targets distributed within a propagation medium make up a target field. The propagation medium can be attenuative and preferentially cause the loss of the high frequency components of a propagating signal according to some physical law. Additionally, the propagation medium can be dispersive causing the different frequency components of a signal to travel with different velocities hence, introducing distortions of phase and equivalently of form into the propagating signal as a function of its travel.

Such known echo systems emit a signal or signals into the propagation medium; the identification process involves detecting the echo train and performing a variety of appropriate analyses. While this procedure is conceptually straight forward, there are a number of practical difficulties which act to complicate, degrade and make ambiguous such identifications.

First, there is a noise background to consider which is almost always a problem in systems where signals are transmitted and detected. Noise is defined in this instance as any contribution which is not a part of the particular identification process and has as its sources such elements as incoherent scattering by the propagation medium or even the targets themselves. There are a variety of techniques for the detection and enhancement of signals in the presence of noise.

Next, there is the inherent ambiguity between the range and the relative velocity of a target. A moving target can not only stretch or shrink a returning echo signature depending on the sense of its motion, but will also delay or speed up the time of its detection, hence affecting the range calculation. Once again, there are a variety of known techniques which can resolve this ambiguity. It is widely recognized that continuous wave signals, for example, a persistent sinusoid at a single frequency can provide good resolution of the target's relative velocity by means of the Doppler frequency shift. The companion range resolution of such a signal is necessarily poor since its character is indistinguishable from cycle to cycle. Very short duration signals are affected only slightly by target motion and while they provide good resolution in detection time, they convey little or no information about relative velocities. The chirp signal described by Klauder, Price, Darlington and Albersheim in the *Bell System Technical Journal*, Vol. 39, pp 745—808, July 1960, represents a compromise having ambiguity in both velocity and range. Its advantages lie rather in effectiveness of equipment utilization and the noise suppression of its companion correlation detection.

Lastly, there is the problem of resolving target angular parameters such as elevation and/or bearing. Currently, definition of angles is achieved by the use of arrays of broad-beam source or receiver elements, or else by means of narrow-beam source or receiver elements. In both cases, the space in which the target field is distributed must be scanned or viewed only one small part at a time. Scanning is accomplished either electronically by steering array beams or sequencing the operation of large numbers of elements, or even mechanically by rotating operational narrow-beam elements to new positions.

The energy requirements of a scanned system are usually favourable since the entire field of potential targets need not be illuminated at once. On the negative side, however, the individual targets are then not being continuously monitored.

According to the present invention there is provided a method of determining the velocity of a signal-reflecting object positioned in a medium of known signal-propagation characteristics, relative to a signal transmitter and a signal receiver, comprising the steps of:

a) transmitting from the transmitter, at respective instants of time having a predetermined time interval therebetween, two signals, each signal comprising a pair of base signals having a common continuous amplitude spectrum between low and high frequency limits, such base signals having a zero value before and after a finite time interval and such base signals of each pair being in phase quadrature relative to each other, so that the object can reflect the two signals;

b) receiving at the receiver the two reflected signals;

c) measuring the time interval between the two received signals; and

d) determining the relative velocity of the object in dependence upon (1) the measured time interval between the received signals, and (2) the predetermined time interval between the transmitted signals.

Embodiments of this invention may employ encoded signals followed by the correlation of the received echo train with known signal signatures. A relatively long signal train made up of essentially short signals may be used, thereby encompassing both continuous wave and impulse-like properties. The correlation step may achieve a measure of noise suppression. Simultaneous high resolution information about range and relative velocity may be achieved essentially by means of simultaneous solutions involving use of all of the observables embodied in the signal train after detection by correlation.

Both the sources and receivers may operate as broad-beam elements with simultaneous illumination of all targets. Angular resolution may be achieved by appropriately interposing phase distorting lenses between the sources and echo receivers. Information about the angles may be encoded into the phase character of the propagating signal train. Energy requirements may be modest despite the simultaneous illumination of the entire field of targets because the possibly high repetition rate of the system may allow that a rather low echo signal level be tolerated. Also, since range and velocity resolutions are not directly dependent on the use of high frequency signal components, lower frequency band signals may be used with correspondingly less energy loss through attenuation.

Even if the propagating medium does modify the travelling signals by altering their amplitude and phase spectral properties, a preferred system described by the invention may be able to function nevertheless. Some empirical corrections would then have to be made to compensate for the propagation effects, but these could be easily determined by calibration studies using targets of known parameters.

The invention will now be described, by way of example, with reference to the accompanying drawings, in which:—

Figure 1 shows an embodiment with a single transmitter, receiver and target;

Figure 2 shows a processing sequence for the embodiment shown in Figure 1 with a single transmitter, receiver and target;

Figure 3 shows another embodiment in which a single phase lens encodes angular information about targets;

Figure 4 shows a processing sequence for the embodiment of Figure 3;

Figure 5 shows another embodiment utilizing a number of mutually exclusive signal trains from the transmitter, one phase lens being interposed for each signal train;

Figure 6 shows a processing sequence for the embodiment shown in Figure 5;

Figure 7 shows another embodiment having different phase encodings of angular information for each member signal of a single outgoing signal train;

Figure 8 shows a processing sequence for the embodiment of Figure 7;

Figure 9 shows another embodiment where the propagation phase distortion and/or the reflecting phase distortion are treated as an additional lens;

Figure 10 shows another embodiment with a single transmitter, a single receiver and a multiplicity of reflecting targets;

Figure 11 shows the derivation of angles resolution by triangulation for an embodiment with a single transmitter, two receivers and a multiplicity of reflecting targets;

Figure 12 shows the time and Fourier frequency domain properties of a design base signal pair having respectively odd and even symmetry;

Figure 13 shows a typical outgoing signal connection of two members;

Figure 14 shows the graphical evaluation of the amplitude spectrum of a cross-correlation in the Fourier frequency domain between a detection signal pair and a propagating signal which has been Doppler distorted;

Figure 15 shows a computer simulation of the phase encoding of angular information in a two member signal train;

Figure 16 shows a simulated development of a correlation amplitude function for the signal train with phase encoded angular information;

Figure 17 shows a sequence of cross-correlations of a sine chirp with Doppler distortions of its original forms;

Figure 18 shows the computation of phase and amplitude spectra of member signals carrying phase-encoded angular information; and

Figure 19 shows a computer simulation of an embodiment of the invention utilizing three moving targets in the presence of random noise.

Figures 1 and 2 show an elementary echo location system.

A suitable transmitter 1 emits a signal train 1A having at least two members into a propagation medium 2 in which is embedded a reflecting target 3. An echo 3A from the target propagates to a receiver 4 where it is detected and sent on to the processing sequencer 5. The outputs of the processing sequencer are directly interpretable as the target parameter estimator 6, which gives the relative velocities between the source or transmitter 1, receiver 4, and reflecting target 3, and the range to the reflecting target 3 as a sum of the distances to the transmitter 1 and the receiver 4. It should be understood that transmitter 1 and receiver 4 may be coincident.

The system shown in Figure 1 embodies components which are essentially standard for the particular application. For example, in the case of a sonar system, transmitter 1 might be a transducer capable of introducing pressure waves into the water of a form and sequence prescribed by a control signal. Receiver 4 might be a hydrophone while a submarine can serve as a reflecting target 3. In this instance the sea water would be the propagation medium 2 and processing sequencer 5 could be simple electronic network or alternatively digital logic accomplishing equivalent operations.

At least two signals are needed in the outgoing train to provide a calibration standard to separate the contribution of the relative velocity to the echo arrival time from the effect of target range. By such means, the echo location system described is capable of simultaneously making quite simple, highly resolved estimates of both the target relative velocity and range.

Each member signal of the outgoing train has like polarization characteristics if these are applicable and a common amplitude spectrum as defined by the modulus of the exponential Fourier transform. Further, the common amplitude spectrum F(w) is essentially flat or smoothly unimodal over the ranges of positive and negative angular frequencies $w_o \leq |w| \leq w_f$ and zero for practical purposes outside the band as defined by these constant limiting frequencies w_o, w_f.

Additionally, each member signal will be a linear combination of a pair of base signals $f_o(t)$, $f_i(t)$ which have the following properties to the practical approximation required by the system:

I. $f_o(t)$, $f_i(t)$ share the common amplitude spectrum F(w) as described.

II. There is a finite time interval of duration α, before and after which both $f_o(t)$ and $f_i(t)$ may be considered to be zero.

III. $f_o(t)$ and $f_i(t)$ are in quadrature. Every Fourier frequency component necessary for the description of $f_o(t)$ is displaced in phase as measured from any origin in time by 90° from its counterpart in the description of $f_i(t)$. Property I above implies that all Fourier frequency components are present in approximately equal amounts in $f_o(t)$ and $f_i(t)$.

As a mathematical formalism, Property III states that $f_o(t)$ and $f_i(t)$ are a Hilbert transform pair at least to the accuracy of the system and

$$f_1(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f_0(u)}{t-u}\, du \qquad 1$$

By way of illustration, several pairs of signals which satisfy the above specified properties are:

I. "Chirp" signals s(t), c(t)

$$s(t)=A\sin\left(w_o t+\frac{w_f-w_o}{2\alpha}t^2\right) \qquad \text{2A}$$

$$c(t)=A\cos\left(w_o t+\frac{w_f-w_o}{2\alpha}t^2\right)$$

$$A=\text{constant},\ s(t)=c(t)=0 \text{ for } \begin{cases} t\geq\alpha \\ \text{or} \\ t\leq o \end{cases}$$

II. "Klauder" Signals $k_o(t)$, $k_1(t)$

$$k_o(t)=A\frac{\cos w_f t-\cos w_o t}{(w_f-w_o)t} \qquad \text{2B}$$

$$k_1(t)=A\frac{\sin w_f t=\sin w_o t}{(w_f-w_o)t}$$

A=constant; $k_o(t)=k_1(t)=0$ for

$$t\geq\frac{|\alpha|}{2}$$

III. "Gabor" Signals $g_o(t)$, $g_1(t)$

$$g_o(t)=A\frac{(t/t_o)^2}{1+(t/t_o)^2} \qquad \text{2C}$$

$$g_1(t)=A\frac{1}{1+(t/t_o)^2}$$

A=constant; $g_o(t)=g_1(t)=0$ for

$$t\geq\frac{|\alpha|}{2}$$

Figure 12 shows the time and frequency domain properties of the Klauder Signals depicted as $f_0(t)$, $f_1(t)$, the base signal pair.

If $f_k(t)$ is a member signal of the outgoing train and there are no distortions introduced by the propagation medium 2, then the received echo 3A for a moving reflecting target 3 will have the mathematical form

$$f_k(st-T) \qquad 3$$

T is a time delay associated with the target range at the time of detection while s is a scale factor which arises from the target relative velocity. For a reflecting target fleeing detection, s>1 and a stretched signal signature will result. We have assumed a positive impedance contrast between the propagation medium 2 and the reflecting target 3. A "soft" target having a smaller impedance than the medium for signal travel would cause a sign reversal (or equivalently a 180° phase shift) in the echo signal 3A.

The stretching or compression of the propagating signal is the familiar Doppler effect and gives rise to the ambiguity in resolution between target relative velocity and target range. Suppose one could recognize some signal characteristic which occured at $t=t_o$. In the echo 3A it would occur at $t=st_o+T$. If it were assumed, nevertheless, that it occurred at $t=t_o+T$, one would make an error in determining the echo arrival time of $(l-s)t_o$ which would correspondingly cause an error in the estimation of target range.

Signals designed as described, however, have a companion processing sequencer 5 which allows a signal characteristic to be unambiguously identified despite stretching or compression.

One embodiment of this sequencer is shown in Figure 2. The use of a train of signals of known intervals of separation then allows that s be computed from the changes of such intervals in the echo train 3A. Once s is known, the echo arrival times may be compensated for the target relative velocity effects so that accurate estimates of the target range can be obtained.

To illustrate the principle, there is shown in Figure 13 an outgoing signal train of two members, these being respectively $k_o(t)$ and $k_l(t)$, the Kláuder Signals with their origins of definition shifted. In mathematical language, the outgoing train shown is simply given by

$$k_o(t-\alpha/2)+k_1(t-\alpha/2-\tau). \qquad 4$$

Further generality can be added by defining

$$\begin{aligned} f_k(t)&=ck_o(t)+dk_l(t) \\ f_j(t)&=ak_o(t)+bk_l(t) \end{aligned} \qquad 5$$

where a, b, c, d are constants. Let us also introduce a scaling such that

$$\sqrt{c^2+d^2}=\sqrt{a^2+b^2}=1 \qquad 6$$

The outgoing signal train now will be taken to have the more general form

$$f_k(t-\alpha/2)+f_j(t-\alpha/2-\tau) \qquad 7$$

The returning echo 3A will then have the following mathematical form

$$f_k(s(t-\alpha/2)-T)+f_j(s(t-\alpha/2-\tau)-T) \qquad 8$$

The processing sequencer 5 is shown in Figure 2. The received signal is divided into two parts by signal splitter 5B. The correlation in parallel of the two parts with $f_o(t)$ and $f_l(t)$, $k_o(t)$ and $k_l(t)$ in this case is effected by cross-correlators 5C1 and 5C2.

One can evaluate the correlation functions A(t), B(t) and their squared functions $A^2(t)$, $B^2(t)$ very simply by considering only the first member signal of the returning echo as given by equation 8. If we relocate the origin of the first member signal of the echo for convenience, the indicated chain of operations gives the following

$$\begin{aligned} A^2(t)&=1/4[(ck_o(st)+dk_l(st))*k_o(-t)]^2 \\ B(t)&=1/4[(ck_o(st)+dk_l(st))*k_l(-t)]^2 \end{aligned} \qquad 9$$

where * denotes a convolution operation (standard signal processing usage) and $-t$ denotes reversal of the signal in the time variable t.

Owing to the symmetry properties of $k_l(t)$ and anti-symmetry of $k_o(t)$ it is evident that

$$\begin{aligned} k_o(-t)&=-k_o(t) \\ k_l(-t)&=k_l(t) \end{aligned} \qquad 10$$

and one can write

$$\begin{aligned} A^2(t)+B^2(t)&=1/4[-ck_o(st)*k_o(t)-dk_l(st)*k_o(t)]^2 \\ &\quad+1/4[ck_o(st)*k_l(t)+dk_l(st)*k_l(t)]^2 \\ &=1/4\{c^2([k_o(st)*k_o(t)]^2+[k_o(st)*k_l(t)]^2) \\ &\quad+d^2([k_l(st)*k_o(t)]^2+[k_l(st)*k_l(t)]^2)\} \\ &=1/4\{[k_o(st)*k_o(t)]^2+[k_o(st)*k_l(t)]^2\} \end{aligned} \qquad 11$$

or

$$=1/4\{[k_l(st)*k_o(t)]^2+[k_l(st)*k_l(t)]^2\}$$

which follows along the processing sequence through 5E and which has made use of a number of unstated assumptions and extended properties of $k_o(t)$, $k_i(t)$. These latter points will now be briefly discussed.

The result of the convolution of two functions both having either a point of symmetry or antisymmetry is a function which has a point of symmetry. Similarly, if one function has a point of symmetry while the other has a point of antisymmetry, the result will have a point f antisymmetry. If we perform Fourier analyses, using as a coordinate origin these points of symmetry and antisymmetry, the phase spectrum of the resulting transform must either be identically zero, $\pm\pi$ or else equal to $\pi/2$ sgn(w).

From the well-known convolution theorem, three results can be derived. First, if we scale a time variable by a constant s, we scale the frequency domain variable by l/s. In mathematical terms,

$$g(st);\ G(w/s) \qquad 12$$

are time and frequency domain equivalents. Convolution in the time domain is equivalent to multiplication in the frequency domain. Hence,

$$f(t)*g(t);\ F(w)G(w) \qquad 13$$

are time and frequency domain equivalents. In polar form, the product F(w)G(w) is equivalent to a multiplication of the respective amplitude spectra and a simple ordered difference of the phase spectra. Finally, the time and frequency domain equivalents of reversal in time are as follows:

$$g(-t);\ G^{+}(w) \qquad 14$$

where + denotes complex conjugation or simply the reversal in sign of the Phase Spectrum.

With the three results 12, 13 and 14, we return to Equation 11 where we note that they have been used to eliminate cross-product terms having the coefficient cd. Also we recognize that

$$k_o(st)*k_o(t)=k_i(st)*k_i(t) \qquad 15$$

which is in fact yet another Klauder Signal of slightly different frequency content with a point of even symmetry, which we shall call $k_i'(t)$. We can readily appreciate the make-up of $k_i'(t)$ by examining Figure 14 which is a graphical evaluation of the convolution of Equation 15 for an approaching target performed in the Fourier frequency domain.

Using analogous reasoning we find also that

$$k_o(st)*k_i(t)=-k_i(st)*k_o(t) \qquad 16$$

which is the antisymmetric counterpart of $k_i'(t)$ and shall herein be called $k_o'(t)$. We shall now state as a general principle that any signal base pair having Properties I, II and III, supra, and further exhibiting respectively antisymmetry and a symmetry characteristic will have a sum of squares which is sharply peaked about the respective point of symmetry and coincident point of antisymmetry. This property holds for all Klauder Signals 2B including

$$k_o(t),\ k_i(t)$$
$$k_o(st),\ k_i(st) \qquad 17$$
$$k_o'(t),\ k_i'(t)$$

and all Gabor Signals 2C. Before presenting some analysis which suggests the validity of this general principle, we should note that equation 11 may be rewritten as

$$A^2(t)+B^2(t)=1/4\{[k_o'(t)]^2+[k_i'(t)]^2\} \qquad 18$$

$k_o'(t)$ and $k_i'(t)$ as functions with coincident points of antisymmetry and symmetry must have Fourier series representations of the form

$$k_o'(t) \simeq \sum_{i=0}^{\infty} h_i \sin w_i t \qquad 19$$

$$k_i'(t) \simeq \sum_{i=0}^{\infty} h_i \cos w_i t$$

The sum of the squares of equation 18 is then simply

$$[k_o'(t)]^2+[k_i'(t)]^2$$

$$= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} h_i h_j \sin w_i t \sin w_j t + \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} h_i h_j \cos w_i \cos w_j t = \sum_{i=o}^{\infty} \sum_{j=o}^{\infty} j_i h_j \cos (w_i - w_j)t \qquad 20$$

Since the common amplitude spectrum F(w) of $k_o'(t)$ and $k_i'(t)$ is smooth and unimodal, we can take all the h_i which are non-zero to be approximately equal. This allows us to recognize that equation 20 has a maximum at t=0 where all the cosine harmonics are in phase. We also recognize that equation 20 is symmetrical about t=0.

An analysis of $f_i(s(t-\alpha/2-\tau)-T)$, the second member signal of the echo train described by equation 8 through the processing sequence of Figure 2 would parallel the considerations of equations 9 through 20. Hence the net effect of the processing sequency of Figure 2 on the echo 3A described mathematically for the particular example by equation 8 is to identify two well-defined correlation peaks of mathematical form of equation 18 which occur at times

$$st=T+s\alpha/2$$
$$st=T+s\alpha/2+s\tau \qquad 21$$

in the correlated variable time scale.

The interval between the peaks is simply $s\tau$ and since τ is known, s can be determined thus providing the estimate of the target relative velocity. Once s is known, the target range may be computed from T and a knowledge of the velocity V of the signal 1A in the propagation medium 2 (Figure 1).

Figures 15 and 16 further illustrate the technique through the results of a digital computer simulation. In Figure 15 we observe seven returning two-member Klauder echo signal trains of the mathematical form of equation 8. The various trains correspond to differing choices of the constants c, d, a, b as they are defined by equations 5 and 6. For the particular illustration

$$w_o=800\,\pi$$
$$s=1.005$$
$$w_f=20{,}000\pi \qquad 22$$
$$\tau=0.0015 \text{ sec.}$$

note that the signal echo train identified by "Return Azimuth" 0° is in fact of the form illustrated by Figure 13. Results of the processing sequencer of Figure 2 are shown in Figure 16. The interval between the peaks is simply $s\tau$ in all cases and it can easily be determined as described.

Application of the technique using signals like the chirps of equation 2A are similar, but require somewhat more subtlety in their explanation. It is well-known that the cross- and auto-correlation signatures of the chirp signals 2A in the absence of any stretching or compression are in fact to a good approximation the Klauder Signals 2B. While this is well-known, we illustrate this point again by computer simulation. The bottom curve of Figure 17 is in fact the auto-correlation of a sinusoidal chirp or sweep where

$$\begin{gathered} w_o=20\pi \\ \alpha=7 \text{ sec.} \\ w_f=80\pi \\ \text{Sample Interval}=0.004 \text{ sec.} \end{gathered} \tag{23}$$

The cross-correlations of the undeformed sweep with stretched versions of itself are also shown.

It is believed that the processing sequence of Figure 2 when applied to the deformed echo chirps will nevertheless produce a result as in Figure 17 with easily identifiable major peaks. As before, the two peaks would have similar form and be separated by an interval $s\tau$. There now will be, however, a correction to be made to the peak arrival times which depends on the computed value of s prior to any estimation of the target range. The nature of the correction and its variation with s may be determined by calibration studies using correlations as illustrated in Figure 17.

As a final note, we emphasize that additional member signals in the outgoing train will provide redundant information about the target relative velocity and range which will enable the echo location technique to function even in noisy environments.

Figure 3 shows another embodiment. The major difference between the embodiments of Figures 1 and 3 in terms of physical elements is the interposition of a phase lens 7 somewhere in the signal path from the transmitter 1 to the reflecting target 3 to the receiver 4. We term lens a "phase lens" since its primary purpose is to introduce into the propagating signal train a phase distortion which varies in a known and single valued manner with certain desired angular information. The outgoing signal 1A will now be designed in such a manner that after the received echo 3A passes through the processing sequencer 5, outlined in some detail in Figure 4, recoverable target parameters will include range and relative velocity as in the embodiment of Figure 1, as well as the angular information which may, for example, relate to the target azimuth or bearing.

Again the transmitter 1 emits a signal train 1A with at least two members. The signal may encounter the phase lens 7 at this time or else intercept it prior to detection by the receiver 4. A reflecting target 3 as before is imbedded in the propagation medium. The principal difference in design of members of the signal train for the application in Figure 3 and the one in Figure 1 is that we now require a fourth property in that $f_o(t)$, $f_1(t)$ must have properties of antisymmetry and symmetry respectively or be "odd" or "even" respectively about the points of antisymmetry or symmetry. Formally stated, the fourth property is

IV. $f_o(t)$ and $f_1(t)$ must be odd (or antisymmetric) and even (symmetric) respectively about the central coordinate value in their interval of definition of duration α.

Let us for the moment assume that the phase lens 7 introduces a constant phase shift θ independent of w which varies only as described with the desired angular information. We shall describe the operation of the processing sequencer 5 of Figure 3 for this circumstance and then indicate those modifications which would be required for more complex phase shifts. Recall the signal train 7 and its manner of definition from Klauder signals 2B and the constants c, d, a, b according to equations 5 and 6. We shall use such a signal train or one designed analogously from Gabor signals 2C with the following modification

$$\begin{gathered} a=\sin\theta \\ b=\cos\theta \\ c=\cos\theta \\ d=-\sin\theta \\ \theta=\text{constant} \end{gathered} \tag{24}$$

Note that owing to property IV, the chirp signals 2A are disqualified from direct use in this technique.

Let us consider $f_k(t)$ and $f_j(t)$ (equation 5) individually for the moment and their Fourier frequency domain equivalents. In the Fourier frequency domain:

$$F_k(w) = \int_{-\infty}^{\infty} f_k(t)e^{-iwt}dt = b\int_{-\infty}^{\infty} f_0(t)e^{-iwt}dt - a\int_{-\infty}^{\infty} f_1(t)e^{-iwt}dt$$

$$\begin{aligned} &= bF_o(w) - aF_l(w) = (Fib - a)F(w) \\ &= \mp i(b \mp ai)\ F(w) = F(w)e^{\mp i(\theta+\pi/2)} \\ F_j(w) &= aF_o(w) + bF_l(w) = (\mp ia + b)F(w) \\ &= F(w)e^{\mp i\theta} \end{aligned} \tag{25}$$

In equation 25, F(w) is the common amplitude spectrum (Property I), $\mp$ is a shorthand notation for −sgn(w), and equations 24 have been applied. The properties of $f_k(t)$ and $f_j(t)$ in the complex Fourier frequency domain are much like those of $f_o(t)$, $f_l(t)$ except that a constant phase angle θ has been introduced into each frequency component.

In fact, a phase lens 7 able to introduce phase distortions independent of w would convert the wave train 4 into the form of the wave train 7. If now this phase distortion were diagnostic of the desired angular information, then recovery of it would be tantamount to recovery of the angular information. Assuming such a phase lens 7, we now change our notation for the propagating signal train 7 to

$$f_k(t-\alpha/2\theta)+f_j(t-\alpha/2-\tau\theta) \tag{26}$$

to give expression to the variation of the train with the desired angular information. The received echo (3A and 4) can correspondingly be expressed as

$$f_k(s(t-\alpha/2)-T,\theta)+f_j(s(t-\alpha/2-\tau)-T,\theta) \tag{27}$$

—refer to equation 8. It is important to recognize that θ is unaffected by the Doppler effects owing to the special properties of our signals.

For an understanding of the essentials of processing of Figure 4, it will be more convenient to look at the mathematical analyses entirely in terms of their Fourier frequency domain equivalents. To facilitate comprehension, we refer to the results of equations 12, 13, 14, 25 and Figure 14.

In the Fourier frequency domain, the received signal 5A is simply

$$F(w/s)e^{\mp i\theta}e^{\mp i\ w/s\ T/s}\{e^{\mp i(w/s\ \alpha/2+\pi/2)}+e^{\mp i(w/s[\alpha/2+\tau])}\} \tag{28}$$

The signal splitting operation 5B simply requires division of equation 28 by 2. The cross-correlations 5C1 and 5C2 simply require that the halved components of equation 28 be multiplied respectively by $F^+_o(w)$ and $F^+_l(w)$. Evaluation of the amplitude portion of the convolutions proceeds analogously to the illustration of Figure 14 and we are again led to define the signals $f'_o(t)$ and $f_l(t)$ analogously to $k'_o(t)$ and $k'_l(t)$ as described in the discussions surrounding equations 15 and 16. Recall that $f'_o(t)$, $f'_l(t)$ are much like $f_o(t)$, $f_l(t)$, but span a slightly different frequency range.

In detail, and in the frequency domain, the two convolution products are simply

$$\begin{aligned} A(w) &= F(w)F(w/s)e^{\mp i\theta}e^{\mp i\ w/s\ T/s}\{e^{\mp i(w/s\ \alpha/2)}+e^{\mp i(w/s[\alpha/2+\tau]-\pi/2)}\} \\ B(w) &= F(w)F(w/s)e^{\mp i\theta}e^{\mp i\ w/s\ T/s}\{e^{\mp i(w/s\ \alpha/2+\pi/2)}+e^{\mp i(w/s[\alpha/2+\tau])}\} \end{aligned} \tag{29}$$

Now making use of the signal pair $f_o'(t)$, $f_i'(t)$ we can write explicitly the time domain equivalents of the expressions of equation 29 or

$$\begin{aligned}
A(t) &= 1/2\ T^{-1}\ \{F(w)F(w/s)e^{\mp i\ w/s\ T/s}[\cos\theta\ e^{\mp i(w/s\ \alpha/2)} \\
&\quad +\sin\theta\ e^{\mp i(w/s\ \alpha/2+\pi/2)}+\cos\theta\ e^{\mp i\ (w/s[\alpha/2+\tau]-\pi/2)} \\
&\quad +\sin\theta\ e^{\mp i\ (w/s[\alpha/2+\tau])}]\} \\
&= 1/2(bf_i'(s(t-\alpha/2)-T)+af_o'(s(t-\alpha/2)-T \\
&\quad -bf_o'(s(t-\alpha/2-\tau)-T)+af_i'(s(t-\alpha/2-\tau)-T), \\
&= 1/2(f_j'(s(t-\alpha/2)-T,\theta)-f_k'(s(t-\alpha/2-\tau)-T,\theta)),
\end{aligned} \tag{30}$$

$$\begin{aligned}
B(t) &= 1/2\ T^{-1}\{F(w)F(w/s)e^{\mp i\ w/s\ T/s}[\cos\theta\ e^{\mp i\ (w/s\ \alpha/2+\pi/2)} \\
&\quad -\sin\theta\ e^{\mp i\ (w/s\ \alpha/2)}+\cos\theta\ e^{\mp i\ (w/s[\alpha/2+\tau])} \\
&\quad +\sin\theta\ e^{\mp i(w/s[\alpha/2+\tau]+\pi/2)}]\} \\
&= 1/2(f_k'(s(t-\alpha/2)-T,\theta)+f_j'(s(t-\alpha/2-\tau)-T,\theta)).
\end{aligned}$$

In deriving the results of equation 30, we have defined (and employed) the signal pair $f_k'(t,\theta)$, $f_j'(t,\theta)$ in a manner analogous to the definitions implied by equations 26 and 27. Also, the notation $T^{-1}\{\ \}$ denotes the operation of an inverse Fourier transform.

At this point, following the sequence of Figure 4, we must take A(t) and B(t), the two expressions of Equation 30, square them (5D1 and 5D2) and add them (5E). Note that owing to a sign difference no "cross-terms" of the form $f_k'(t,\theta)\ f_j'(t,\theta)$ appear. We are left simply with the quantity

$$\begin{aligned}
A^2(t)+B^2(t) &= 1/4\{f_k'^2(s(t-\alpha/2)-T,\theta)+f_j'^2(s(t-\alpha/2) \\
&\quad -T,\theta)+f_k'^2(s(t-\alpha/2-\tau)-T,\theta) \\
&\quad +f_j'^2(s(t-\alpha/2-\tau)-T,\theta)\}
\end{aligned} \tag{31}$$

Before interpreting the significance of equation 31, it is desirable to comment on the nature of the expression which has the form

$$f_k'^2(t,\theta)+f_j'^2(t,\theta) \tag{32}$$

Recalling all of the arguments of similar nature surrounding the understanding of equation 11, and in specific the results developed through equations 18, 19 and 20, we recognize that equation 31 will have well defined maxima at times.

$$\begin{aligned}
st &= T+s\alpha/2 \\
st &= T+s\alpha/2+s\tau
\end{aligned} \tag{33}$$

We can now determine the target relative velocity and range precisely as in the application of Figure 1. Before we consider some alternate methods of recovery of the target angular information 5G1 and 5G2, it is instructive to refer back to Figure 15 where we now recognize a family of two member signal trains with encoded values of θ ranging from 90° to −90° in 30° increments. Note that the computer simulation of the sequence of Figure 4 through 5F (Equation 33) shows that the detected peak interval is for practical purposes independent of the encoded phase information.

We shall now consider some methods of recovery of θ or equivalently the target angular information by the alternate 5G1 of Figure 4. On the stretched time scale of equation 31 consider the arctangent of the ratio of ordinate values of the return signal (equation 27) at the two particular time values corresponding to the correlation maxima. In detail,

$$\text{Phase Angle} = \arctan Z$$

where

$$Z = \frac{\{\cos\theta\, f_o(s(t-\alpha/2)-T) - \sin\theta\, f_l(s(t-\alpha/2)-T)\}_{st=T+s\alpha/2}}{\{\sin\theta\, f_o(s(t-\alpha/2-\tau)-T + \cos\theta\, f_l(s(t-\alpha/2-\tau)-T)\}_{st=T+s\alpha/2+s\tau}} \qquad 34$$

$$\text{Phase Angle} = \arctan\{-\sin\theta/\cos\theta\} = \arctan\{-\tan\theta\} = -\theta$$

To derive the result of equation 34 we have used the definitions of equation 5 and property IV or $f_o(0)=0$ and $f_l(0)=1$. Analogous reasoning allows additional computations of θ using the correlation components A(t), B(t) certain of which are illustrated in Figure 4 as 5G1.

The alternate 5G2 of Figure 4 for determining θ or the target angular information is also readily accomplished. Using the peak positions determined by 5F (equation 33) as coordinate origins, Fourier transforms of the correlated member signals of the trains A(t), B(t), or the returning echo train itself may be computed and the phase spectra computed. Over the band of frequencies encompassed by f(w) (property I), the phase spectrum will be constant and equal to $\pm\theta$ or $\pm\theta$ $\pm\pi/2$. We illustrate this point again by computer simulation.

Figure 18 illustrates a nine member returning signal train in which θ is successively encoded at 10° increments from −90° to 0°. Note that sign convention used here is the opposite of the one used in Figure 15. An amplitude and phase spectrum was computed for each member signal using as the coordinate origin the peak indicated by processing according to Figure 4 up through element 5F. The train is denoted by Sequence B and it is readily observed that the amplitude spectra are identical as they should be. For the meaningful frequency band defined by the amplitude spectrum we see that the phase spectrum is in fact constant and equal in value to θ.

We now address the question of the introduction by the phase lens 7 of Figure 3, phase variations which vary with the target angular information but which are not independent of the frequency. It should be noted, however, that for restricted frequency bands the approximation

$$\theta(w) = \theta_o + \theta_l w \text{ where } \theta_o,\ \theta_l \text{ are constants} \qquad 35$$

may be expected to reasonably represent phase effects. The linear variation of phase with frequency inherent in equation 33 and governed by θ_l necessarily implies that some timing adjustment depending on θ_l will be needed which will change the target range computation. Target relative velocity will be unchanged as all member signals undergo the same phase distortion for any given value of the target angular information hence suffering also the same delays.

To illustrate the previous discussion we note that substituting θ(w) for θ in equation 25 would give in the time domain a place of equation 26

$$f_k(t-\alpha/2-T-\theta_l,\ \theta_o) + f_j(t-\alpha/2-T-\theta_l,\ \theta_o) \qquad 35A$$

The received echo (3A and 4) would be correspondingly modified. If the desired angular information is functionally related to θ(w) then it is similarly related to θ_o, θ_l through equation 35.

Referring to Figure 4 in this circumstance, it is evident that the alternate computation for θ, 5G1 will yield a single characteristic θ_o value which can uniquely be related to the desired target angular information for reasonably behaved phase lenses 7 as described by empirical procedures which make use of reflecting targets of known parameters. The same calibration procedures can be used to relate θ_l to the measured θ_o so that the echo timing correction can be made. Of course, alternate 5G2 will similarly allow a direct estimation of θ_o, and θ_l may then be related to θ_o by empirical calibrations. θ_l is again directly interpretable as the timing correction needed for determining the target range, and θ_o and θ_l are now both available to define the approximation to θ(w) thus allowing as before determination of the target angular information.

Yet another embodiment similar to that illustrated in Figure 3 is indicated in Figure 5. Here a reflecting target 3 is identified in terms of its relative velocity, range and angular information in more than one angle as perhaps the bearing and elevation angle. The technique uses a number of the embodiments of the type of Figure 3 in concert, each realization of the former technique occupying different but non-overlapping bands of frequency or polarization (if applicable). The signal bands are selected to retain their exclusion in polarization and/or frequency despite all Doppler effects and effects introduced by the propagation medium 2 or reflecting target 3. Angular information is encoded in each signal band or polarization direction by an appropriate phase lens as in Figure 3 except that the lenses may affect only certain of the signal bands or polarizations, and different angular information may be encoded in the different bands.

Figure 5 illustrates the technique in terms of a number of different frequency bands, but it should be understood that mutual exclusion may still be achieved within a single band if differing directions of polarization are employed. In this case, separation of the signal bands is accomplished by filtering according to polarization first.

The transmitter 1 of Figure 5 emits the coincident signal trains as described which may have any relative alignment in time including simultaneity. Prior to encountering the receiver 4 a series of phase lenses 7A, 7B, 7C, etc. are interposed or some other mechanism employed which encodes desired angular information as a phase distortion in the manner previously described. The lenses 7A, 7B, 7C, etc. or equivalent mechanisms act independently upon the differing signal bands so that differing angular information may be encoded in each of them. Encoding of the same information in differing bands would, of course, provide redundancy and improved recovery of such information in noisy environments.

The propagation medium 2 and reflecting target 3 function analogously to their respective roles in Figure 3. The processing sequence 5 functions much as in Figure 3, but in parallel for each signal band. Figure 6 diagrammatically illustrates the parallel processing for the configuration outlined by Figure 5. The target parameter estimator 6 might encompass certain added features to allow for the effective use of redundant information about target range and relative velocity.

We can readily appreciate that polarization filtering (if applicable) or the cross-correlation operations 5CA, 5CB, 5CC, etc. of Figure 6 do in fact achieve a separation of the signal bands. For example, each such operation develops correlation components analogous to A(t), B(t) of Figure 4 (also Equation 30). Equation 29 shows the counterpart Fourier transforms A(w), B(w) in polar form. If the amplitude spectrum of the received return signal does not overlap that of the base function pair of the cross-correlation 5CA, 5CB, 5CC, etc., no output whatsoever will result. Hence the coincident signal bands do not interfere and may be treated quite independently.

Figure 7 illustrates another embodiment which is like the embodiment of Figure 5 in that the reflecting target 3 is again identified in terms of its relative velocity, range, and angular information in more than one angle as perhaps the bearing and elevation angle. Unlike the embodiment of Figure 5 which utilizes simultaneously many signal bands, the angular information in this approach is phase encoded differently in the various member signals of the outgoing signal train 1A. The transmitter in this instance functions as in Figure 3 utilizing only a single signal band defined as having uniform polarization (if applicable) with impulsive member signals of four properties as described having a frequency content limited to $w_o \leq |w| \leq w_f$.

One possible mechanism for accomplishing the diverse phase encodings is shown in Figure 7 where differing phase lenses 7 are interposed in turn between the transmitter 1 and the reflecting target 3 such that each member signal of the outgoing signal train 1A encounters a different lens. If information is desired about two angles, at least two such lenses are required. Additional angles would require additional lenses and at least an equivalent number of member signals in the outgoing train.

Let each phase encoding be characterized by a single phase value θ_i recognizing that as in preceding discussions the characterization can be generalized to accommodate more realistic encodings of general linear form. We shall now assume that the desired angular information ϕ_j is related to θ_i by the known or empirically determined relation

$$\theta_i = g_i(\phi_j) \qquad 36$$

where i denotes the member signal of the train and j denotes the particular angular information in a plane j containing the transmitter 1 and referred to a line in this plane.

For a more concentric discussion, we shall consider the specific case in which there are only two outgoing member signals, i=1,2 and one value of ϕ_j or ϕ_l. g_1 and g_2 shall be defined as implied by the equations

$$\theta_1=\phi_l \qquad \theta_2=-\phi_l \qquad (37)$$

We may now similarly interpose other phase lenses 8 between echo 3A from reflecting target 3 and receiver 4. One such lens is depicted in Figure 7. As before, the signal value of phase ϕ_i will characterize the encoding, where i again refers to the member signals, but now of the echo train 3A. The ϕ_i are related to the desired angular information referred to a specified line in a plane k through the receiver 4 by the known or empirically determined relation

$$\phi_i=h_i(\gamma_k) \qquad (38)$$

Again, for illustrative purposes and with no loss of generality, we shall assume a single lens 8, a single desired angle γ_l such that for the two returning member signals, the phase encoding by the lens 8 would be

$$\phi_1=\gamma_l \qquad \phi_2=\gamma_l \qquad (39)$$

where h_1, h_2 have implicit definition in equations 39. It is important to add that for each desired angle beyond the first, either $g_i(\phi_j)$ or $h_i(\gamma_k)$ for member signal i must have a single valued form more general than the simple addition of a constant value.

The processing sequence 5 described in greater detail in Figure 8 is analogous to the sequence of Figure 4 through element 5F of that Figure where the timing of peaks in the correlation component square sum leads to estimates of the target relative velocity and range. Figure 8 illustrates alternate 5G1 of Figure 4 being employed to estimate the phase characteristic for each member signal. Referring to equations 36 and 37 these are given generally by

$$\theta_i+\phi_i=g_i(\phi_j)+h_i(\gamma_k) \qquad (40)$$

which are a set of i simulaneous equations for j+k unknowns. The unknown ϕ_j and γ_k constitute the desired angular information and may be determined when $i \geq j+k$.

In the simplified case cited for illustration in equations 37 and 39, the counterparts to equations 40 are

$$\theta_1+\phi_1=\phi_1+\gamma_1 \qquad \theta_2+\phi_2=-\phi_1+\gamma_1 \qquad (41)$$

whose solution can be directly obtained as a simple sum and difference of observed phases or

$$\gamma_1=\frac{\theta_1+\phi_1+\theta_2+\phi_2}{2} \qquad \phi_1=\frac{\theta_1+\phi_1-\theta_2-\phi_2}{2} \qquad (42)$$

Note that the successive phase encodings imposed on the propagating signal by the cascaded phase lenses 7 and 8 are simply additive.

Where the phase distortion imparted by any lens requires a more general description such as a linear approximation as described by equation 35, the procedure is similar except that after determining the desired angular information from member signal i, a timing correction is needed leading once again to a correction to the target range and in this embodiment also to the target relative velocity. Note that in other embodiments where all member signals passed through common phase lenses, no correction was needed for the target relative velocity. The timing correction appertaining to each lens may be determined empirically as a function of phase as before using targets of known parameters. These same calibration techniques would define the relations

$$\theta_i = g_i(\phi_j)$$

$$\phi_i = h_i(\gamma_k)$$

(equations 37 and 38) unless they were determined theoretically or on some other basis.

It should also be recognized that alternate 5G2 of Figure 4 for each member signal would again provide values of $\theta_i + \phi_i$ which could be similarly employed to derive the desired angular information and to correct the target relative velocity and range should such corrections be needed.

Another embodiment which follows from the previous discussions is one which incorporates the elements exemplified by both Figures 5 and 7 and their complementary processing sequences illustrated by Figures 6 and 8. Here again, a reflecting target 3 (of Figure 5 or 7) would be identified in terms of its relative velocity, range and angular information in more than one angle as perhaps the bearing and elevation angle. However, the phase encoding of the desired angular information would not be the same for all member signals of the outgoing train as in Figure 7, and, more than a single band would also be utilized, such bands being mutually exclusive even after Doppler effects, by virtue of polarization and/or frequency differences, with each band having encoded differing angular information.

One practical objective in combining the approaches of Figures 5 and 7 as described is to achieve even greater redundancy of all the reflecting target parameters yet without increasing the period of time which is needed to make such identifications. Details of accomplishing the processing for such identification follows precisely from both Figures 6 and 8.

Figure 9 depicts an embodiment much as described by Figure 7, where again a reflecting target 3 is identified in terms of its parameters as target range, relative velocity and angular information in many angles, but now an accommodation is made also for a phase distortion 2A which is a function of the target range introduced by the propagation medium 2 and/or the reflecting target 3 as a function of the angle of signal incidence. For illustrative purposes in this discussion the phase distortion will be assumed attributable in its entirety to the propagation medium 2, with no loss of generality of the technique implied by such assumption.

For practical materials or propagation media 2, the phase distortion 2A will be smoothly varying and represented well by an analytic expression like equation 35. Such a phase distortion was specifically discussed and treated in all of the previous embodiments of the invention where angular information was to be determined. As a consequence of a distortion of this form a target range correction will result, however, no relative velocity error would be introduced since all of the member signals see the same phase distortion 2A. The constants which approximate the phase distortion in the form of equation 35 will also be estimated.

In Figure 9, the role of the propagation medium 2 with regard to phase is analogous to the interposition of one additional phase lens 7 or 8. Unlike the phase lenses 7 or 8, the phase effects to be introduced are not designed as a part of the embodiment and vary with the target range or better the travel distance of the signal rather than any angular information. Let us designate the phase distortions 2A for a given value of target range R as

$$\lambda(w) = \lambda_o + \lambda_1 w;\ \lambda_o = \lambda(R) \qquad 43$$

Since phase effects are additive, and if the phase distortion introduced by the phase lenses 7 and 8 have form analogous to equation 43, then for the i^{th} member signal of the echo train 3A after being received 4 and processed according to the processing sequence 5 of Figure 8, the detected phase would be

$$\theta_i + \phi_i + \lambda_o = g_i(\phi_j) + h_i(\gamma_k) + \lambda(R) \qquad 44$$

where g_i and h_i are defined as in equations 36 and 38 and where $\lambda(R)$ is a function of the target range.

In this case g_i, h_i and $\lambda(R)$ are all determined theoretically or else by empirical studies using a reflecting target 3 of known parameters. The equations 44 may be solved simultaneously or by least squares for a sufficient number of member signals ($i \geq j+k+1$) to give ϕ_i, γ_k and r which in turn can give values for θ_i, ϕ_i, λ_o as distinct from their sum. For phase characterization of the general form specified it should be recognized that a timing correction would be needed for each member signal which would modify the determinations both of target relative velocity and target range.

The timing correction as described in certain of the previous embodiments can be computed theoretically or else can be determined by some empirical method using a known reflecting target 3 so that for values of θ_i, ϕ_i, λ_o or ϕ_j, γ_k, R an appropriate sum of corrections may be applied. Of course, once λ_o and the timing correction for the propagation medium 2 are known, the phase distortion 2A of the medium is known according to equation 43 as is all other desired information. Note that the timing correction associated with λ_o or R gives rise to no change in the computation for target relative velocity since, as was mentioned, all member signals are subjected to the same timing adjustment.

Figure 9 represents the most general embodiment up to this point in a single signal band as defined by frequency range and polarization direction, if the latter is applicable. Figure 5, on the other hand, depicts an embodiment in which the phase encoding of target parameters and propagation material properties (if applicable) would utilize a number of signal bands. It follows then that a more comprehensive embodiment may yet be envisaged in which a number of signal bands are employed, the technique in each single signal band being represented as in Figure 9.

An embodiment as described would provide the greatest redundancy yet for the identification parameters of the reflecting target and would also allow the characterization of the phase distortion of the propagation medium and/or the reflecting target in a number of frequency bands and polarization directions.

Another embodiment complementing all other variations described up to this point would include the use of amplitude spectral information determined either collectively from the received return signal or else computed from individual member signals as one additional target identification parameter describing its "quality". Referring to Figure 4 in alternate 5G2 we first noted the use of Fourier analysis in the processing sequence for the received return signal 5A.

Throughout preceding discussions, the only modification admitted for the amplitude spectrum of the base signal pairs had been a Doppler induced effect. We shall now want to give cognizance to variations in the amplitude spectrum caused by frequency dependent alternative mechanisms of the propagation medium and also the frequency dependent reflective properties of the target.

A preferential loss of higher frequency components as a function of the length of propagation path is a common characteristic of most propagation media. Particular media can exhibit "window" effects where for certain frequencies or polarizations (if applicable) anomalous attenuation or lack of attenuation will occur. For all of the permissible base signals having either three or four fundamental properties, these effects should not alter in most circumstances the basic smooth and unimodal character of the amplitude spectrum and so the mathematical approximations presented would retain their validity. No effects would be induced on phase as it is measured here, since the base signals share a common amplitude spectrum.

Empirical studies have been cited in a number of previous discussions as a means for establishing standards or functional relationships necessary of the determination of certain of the target identification parameters. Clearly, by empirical studies it is possible to determine the absolute amplitudes and changes in form of the amplitude spectrum of member signals caused by a propagation medium so that these effects might be removed from consideration. Other differences of magnitude and form must then be diagnostic in some sense of the target, describing its quality.

Peculiarities of the target figure or shape, as well as perhaps a transitional character in reflective properties can cause modifications to the amplitude spectrum which might be unique to certain targets hence facilitating positive identifications, or else simply assisting in their categorization. We must also note in the context of target quality determination that phase plays some role, since it is a 180° phase shift or reversal of arithmetic sign of the echo which allows distinction between "hard" and "soft" targets where the magnitudes of the reflective contrasts between the target and the propagation medium are equal. The terminology "hard" is being applied to targets of materials in which the signal propagation velocity exceeds its velocity in the propagation medium.

Target quality information is contained in the member signals of the signal train as well as in the entire train itself. Hence as in the case of most of the other parameters describing the target, a measure of redundancy is again present.

Figure 1 illustrates the most elementary embodiment described, but can be used to help explain the most encompassing embodiment yet to be described. All variations discussed so far have included a single transmitter 1, a single receiver 4, and single reflecting target 3. In fact, any number of any of these elements can be used, yet allowing all other essentials of the embodiment so that the plurality of reflecting targets can each be identified individually according to their parameters with whatever degree of redundancy the particular embodiment allows. Figure 10 exemplifies such an embodiment based on the variation shown in Figure 1.

In Figure 10, three distinct reflecting targets 3, two distinct receivers 4A, 4B and a single transmitter 1 are shown. The propagation signal 1A is taken as in Figure 1 and the propagation medium 2 is also as in Figure 1. Each reflecting target returns an echo to each of the receivers 4A, 4B. The processing sequences 5 are essentially as depicted in greater detail in Figure 2 except that each of the reflecting targets 3 now corresponds to a sequence of peaks in the step 5E of Figure 2 which may have any arbitrary relation in time, one sequence to another, hence necessitating also the inclusion of some logic to separate the member peaks in each sequence so that target parameter estimates 6 can be made for each reflecting target. Since there are two receivers, the target parameter estimator 6 may now include angular information, even though the embodiment of Figure 1 upon which we based this illustrative case made no provision for the inclusion of such information.

Figure 11 suggests a familiar analytic basis by which the use of information from the two receivers 4A, 4B may be handled to yield angular information. The transmitter T and receivers R1, R2 of Figure 11 occupy known relative positions which are either fixed or changing in a known manner in time. The processing sequence 5 as described by Figure 2 is capable of estimating for each reflecting target only a range and relative velocity. For any estimated range, the permissible target locus is an ellipse with the transmitter and receiver which detected the particular echo at its focii. The intersection of the elliptical locii will define the target position and thus provide the angular information about the target expressed in the coordinate network of the transmitter and two receivers. Ambiguity of position can be eliminated as indicated again in Figure 11 by designing the configuration so that certain positions are disallowed, as for example those to the left of the dashed line AA′.

It is important to mention that the logic by which the sequences corresponding to the differing targets are isolated can include clues about the consistency of relative velocity of the targets and even amplitude spectral information which in fact was not mentioned in the embodiment of Figure 1, but was described in a later, more sophisticated technique.

To further illustrate the technique in which a plurality of elements is permissible, we address now the embodiment of Figure 3, where angular information in one angle is one of the target identification parameters, and appeal also to a computer simulation. For illustrative purposes we adopt the configuration shown in Figure 3 but declare only a plurality of reflecting targets, specifically three. The signal 1A from transmitter 1 is taken to be made of a Klauder base signal pair as described by equation 4 and depicted in Figure 13.

Amplitudes of the three reflecting targets are taken to be in the ratio 4, 3, 2 while their initial ranges, angular information and relative velocities tabulate as:

	Range (Arbitrary Units)	Angular Information	Relative Velocity (Ratio)	Amplitude (Ratio)
Target 1	150	0°	1	4
Target 2	300	–45°	1.05*	3
Target 3	450	90°	.95#	2

*approaching
#fleeing

Note that we are using the same convention for encoding the angular information as indicated in Figure 15. We are assuming constant phase distortions imparted by the phase lens 7 of Figure 3. Also, the target relative velocities are expressed as 1+their ratios with the speed at which echo location signal 1A travels.

Since target 2 is approaching the detection system centroid while target 3 is fleeing, the expected target ranges for the assigned target relative velocities are 280 and 475 units respectively. Also, if all other parameters can be correctly identified, then a reconstruction of the received return signal 4 may be accomplished for comparison with the observed one. Such a sequence is shown by the computer simulations of Figure 19 where the noise-free and noisy case are extended.

In Figure 19, curve A is the observed received return signal 4. The reconstruction from detected parameters is denoted curve D. Curve B shows the sum of the squares of the correlation components, curves C1 and C2. Referring to Figure 4 which describes the particular processing sequence 5, we recognize curve C1 as 5D1, curve C2 as 5D2 and curve B as 5E. The family of curves with primes represent results from the circumstance where background noise is present. For this case note that there is little difficulty in distinguishing among the reflecting targets.

Hence these illustrations make clear certain of the advantages of the embodiments which function in environments with a plurality of reflecting targets and utilize where beneficial, pluralities of transmitters and receivers.

A more general variation encompassing all other variations described and which can be developed starting with any of these techniques, employs a base signal pair in the processing sequence which can differ from the base signal pair of the outgoing signal design. The two distinct base signal pairs shall be designated as the design signal pair and the processing signal pair respectively. For any given design signal pair, the admissible processing signal pairs must represent only rotations of the design signal pair phase spectra by a constant angle, and all differences in amplitude spectrum must be constrained such that the product of the design signal pair common amplitude spectrum and the processing signal pair common amplitude spectrum in itself has a form appropriate to a base signal pair.

All embodiments described heretofore employed a common base signal pair for the design of the outgoing signal train and the processing sequence. In fact, this restriction need not exist and we may with appropriate planning use an outgoing signal train developed with Klauder signals, yet select a processing base signal pair constructed with Gabor signals to extract desired target parameters (refer to equations 2B and 2C which specifically define Klauder and Gabor signals).

Should the particular application not make use of phase encoding of information as the variation described in Figures 1 and 2, then both the design signal pair and processing signal pair would each have to satisfy only the first three fundamental properties which were outlined in that discussion. Where angular resolution via phase encoding is called upon, the fourth fundamental property introduced in the discussion of the application depicted in Figures 3 and 4 is needed. A modified form of this property may now be taken for both the design signal pair and the processing signal pair. Calling such a signal pair $f_k(t)$, $f_j(t)$ the revised statement of Property IV reads:

IV. $f_k(t)$ and $f_j(t)$ must be transformable to respective odd and even form about the central coordinate value in their interval of definition of duration α, by a constant shift of phase at all frequencies where the coordinate origin of definition of such phase is again taken at the same central coordinate value.

In reviewing the mathematical discussions of the simpler techniques it becomes clear that the processing sequences as described can progress up to determinations of phase using differing design and processing signal pairs having three or four fundamental properties as the application requires, and having permissible departures in amplitude spectra. Specifically, in the embodiment shown in Figure 1, any appropriate processing signal pair allows completion of the entire processing sequence shown in Figure 2 in its entirety with no modification. Alternatively, in the embodiment shown in Figure 3 where phase encoding is employed to achieve angular resolution, the processing sequence of Figure 4 would be unmodified through element 5F. Thereafter, an adjustment in phase related to the constant shift of the processing base pair relative to the design base pair described by the revised Property IV would be required.

For purposes of having a more concrete illustration, consider an outgoing signal train developed using as the design signal pair $f_k(t)$, $f_j(t)$ defined by equation 5 with the constraint of equation 6 and subject to equation 24. In the Fourier frequency domain following equation 25, we have the counterparts

$$f_k(w)=F(w)e^{\mp i(\theta+\pi/2)} \qquad 45$$

$$F_j(w)=F(w)e^{\mp i\theta}$$

If we envisage a phase encoding mechanism as in the technique described by Figures 3 and 4, the received return signal 5A having undergone a constant phase shift now termed β for the particular angular resolution, would have a form similar to equation 27 but specifically

$$f_k(s(t)-\alpha/2)-T,\theta+\beta)+f_j(s(t)-\alpha/2-\tau)-T,\theta+\beta) \qquad 46$$

In the Fourier frequency domain the received return signal 5A (also equation 46) is now

$$F(w/s)e^{\mp i(\theta+\beta)}e^{\mp i\ w/s\ T/s}\{e^{\mp i(w/s\ \alpha/2+\pi/2)}+e^{\mp i(w/s[\alpha/2+\tau])}\}. \qquad 47$$

(Compare with equation 28).

Let us select a processing signal pair $f_k(t)$, $g_j(t)$ defined analogously to $f_k(t)$, $f_j(t)$ having as Fourier frequency domain counterparts

$$G_k(w)=G(w)e^{\mp i(\gamma+\pi/2)}$$
$$G_j(w)=G(w)e^{\mp i\gamma} \qquad 48$$

A(t), B(t) or 5D1, 5D2 of the processing sequence of Figure 4 would then have as frequency domain equivalents

$$A(w)=G(w)F(w/s)e^{\mp i(\theta+\beta-\gamma)}e^{\mp i\ w/s\ T/s}\{e^{\mp(w/s\ \alpha/2)}+e^{\mp i(w/s[\alpha/2+\tau]-\pi/2)}\}$$
$$B(w)=G(w)F(w/s)e^{\mp(\theta+\beta-\gamma)}e^{\mp i\ w/s\ T/s}\{e^{\mp i(w/s\ \alpha/2+\pi/2)}+e^{\mp i(w/s[\alpha/2+\tau])}\} \qquad 49$$

(Compare with equation 29).

From equation 49 onward this illustrative analysis may proceed in parallel with the development based on Figures 3 and 4 with two provisions. First, the product G(w)F(w/s) must define an amplitude spectrum which is essentially smooth and unimodal so that a signal pair analogous to $f_k'(t,\theta)$, $f_j'(t,\theta)$ of equation 30 may be defined. Second, the phase encoded angular information relating to β can be determined only after compensating for the phase rotation of the design signal pair by θ and the processing signal pair rotation by γ. Note that the relative rotation between these two pairs is again constant and equal to $\theta-\gamma$. In particular, alternates 5G1 and 5G2 of Figure 4 would both yield phase determinations in this case of $-(\theta+\beta+\gamma)$ which would give a value for β when corrected as necessary for the known phase rotations θ and γ. (See for example equation 34).

The nature of the amplitude spectral differences permitted between the design signal pair and the processing signal pair is now greatly clarified. G(w) and F(w/s) must for all realistic Doppler variations governed by s overlap sufficiently in frequency w so that the product G(w)F(w/s) has a band width sufficiently broad to correspond to a signal of finite duration and impulsive character when transformed to the time domain with a constant zero phase spectrum. (This requirement is analogous in some measure to fundamental Property II). Also, the product G(w) F(w/s) must have a character essentially as demanded in fundamental Property I.

It should be apparent to anyone with some background in signal processing that the permissible differences in the amplitude spectra of the design signal pair and the processing pair can often be used to great advantage. Techniques utilizing some "conditioning" of amplitude spectra in association with a correlation or convolution process are widely used and even standard in the treatment of signals for detection and other applications (see for example Phillip E. Panter, *Modulation, Noise and Spectral Analysis,* McGraw Hill, 759 P, 1965). By analogy, similar enhancement techniques can be designed to function in the present context.

As a most elementary example of such a method, one may consider a practical environment which is attenuative in nature and preferentially removes the high frequency content of F(w/s) as the propagation distance to the target increases. For some approximation to the expected target range, G(w) might conversely give appropriate emphasis to the high frequencies so that the product F(w/s)G(w) is again almost flat. Such a method would improve both the resolution obtainable in the relative velocity determination and range calculation.

Hence this embodiment endows great flexibility in all the alternative variations making possible a number of advantages which can arise from a judicious manipulation of the amplitude and phase spectral character of the design signal pair and the processing signal pair. Introduction of a time variation in the definition of the processing signal pair might enhance detectability through amplitude spectral "whitening" while also compensating for the changing constant phase characteristic introduced by a propagation medium. The scope and significance of such possibilities can be appreciable.

Attention is drawn to my copending British Patent Application No. 8018281. (Serial No. 1604183).

WHAT I CLAIM IS:—

1. A method of determining the velocity of a signal-reflecting object positioned in a medium of known signal-propagation characteristics, relative to a signal transmitter and a signal receiver, comprising the steps of:

a) transmitting from the transmitter, at respective instants of time having a predetermined time interval therebetween two signals, each signal comprising a pair of base signals having a common continuous amplitude spectrum below low and high frequency limits, such base signals having a zero value before and after a finite time interval and such base signals of each pair being in phase quadrature relative to each other, so that the object can reflect the two signals;

b) receiving at the receiver the two reflected signals;

c) measuring the time interval between the two received signals; and

d) determining the relative velocity of the object in dependence upon (1) the measured time interval between the received signals, and (2) the predetermined time interval between the transmitted signals.

2. A method according to Claim 1, wherein at least one of the signals may be transformed to a symmetric signal of constant phase relative to a time reference by adding a constant phase angle to the phase at each frequency.

3. A method according to Claim 1, wherein each of the signals may be transformed to a symmetric signal of constant phase relative to a time reference by adding a constant phase angle to the phase at each frequency.

4. A method according to Claim 1, wherein said two instances of time are predetermined, and further comprising the steps of:

a) measuring the arrival time of at least one reflected signal;

b) determining the signal transit time from said measured arrival time and said predetermined time instant of transmission;

c) correcting said transit time as a function of said determined velocity; and

d) determining the distance of said object relative to said transmitter and said receiver as a function of (1) said corrected transit time and (2) said propagation characteristics of said medium.

5. A method according to Claim 1, wherein at least one of the signals has a known initiation time, the range of the object being ascertained by the further steps of:

a) measuring the arrival time of the signal of known initiation time;

b) correcting the signal transit time of the signal of known initiation time for the determined relative velocity of the object; and

c) ascertaining the range of the object from the corrected signal transit time.

6. A method substantially as hereinbefore described with reference to any one of the examples which is illustrated in the accompanying drawings and which is in accordance with Claim 1.

7. Apparatus substantially as hereinbefore described with reference to any one of the examples which is illustrated in the accompanying drawings and which is in accordance with Claim 1.

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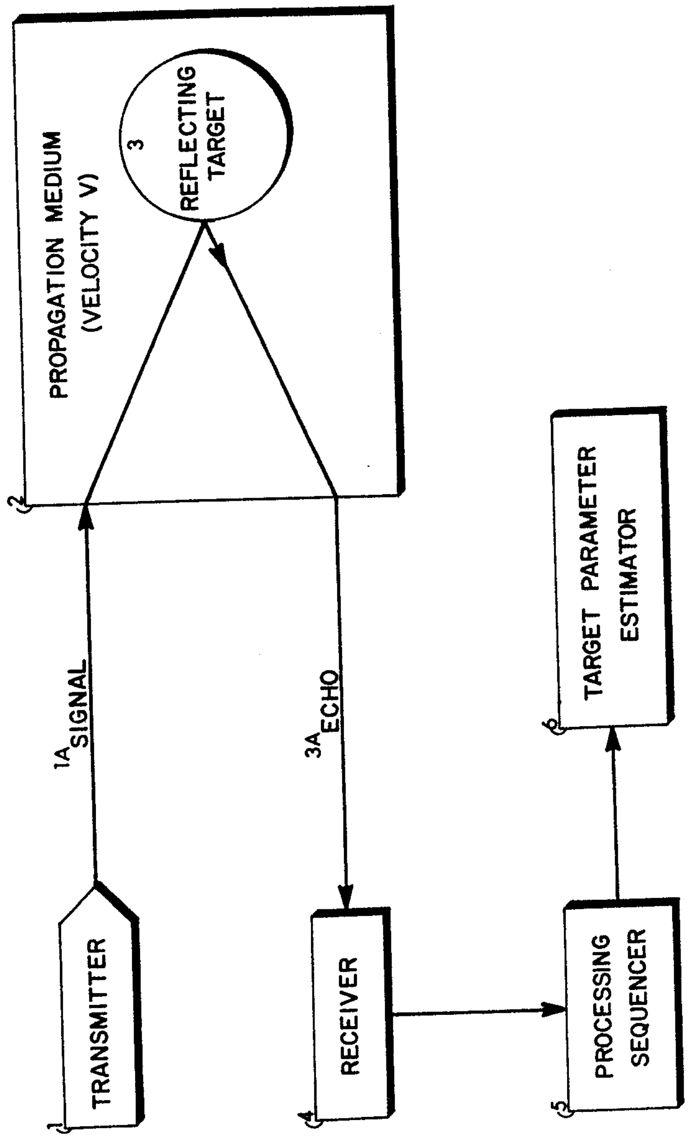

PROPAGATION MEDIUM
(VELOCITY V)
3
REFLECTING
TARGET
2
1A SIGNAL
3A ECHO
TRANSMITTER
1
RECEIVER
4
PROCESSING
SEQUENCER
5
TARGET PARAMETER
ESTIMATOR
6

FIG. 1

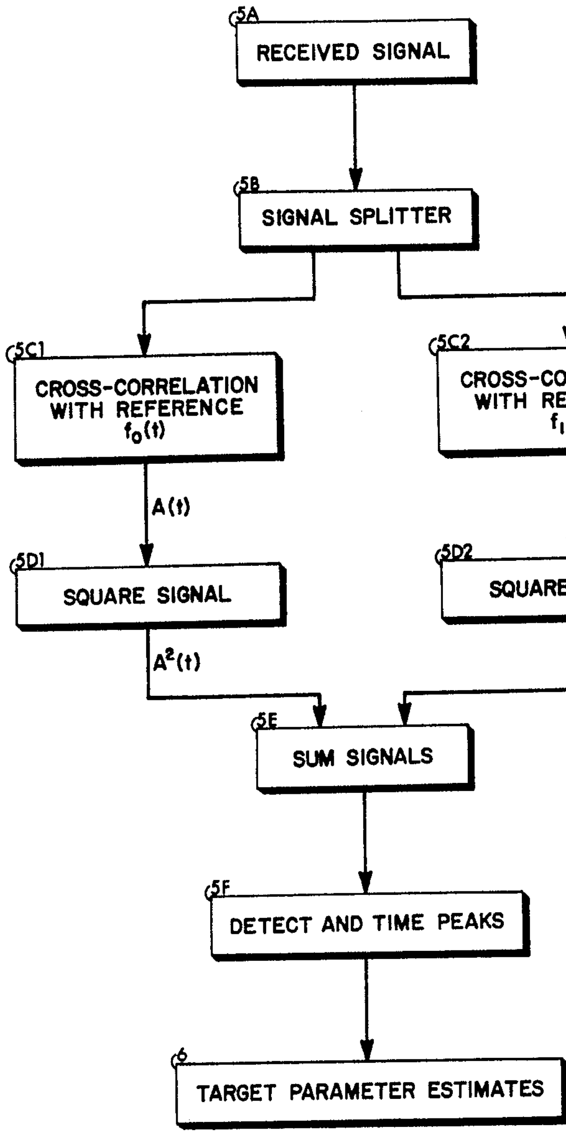

5A
RECEIVED SIGNAL
5B
SIGNAL SPLITTER
5C1
CROSS-CORRELATION WITH REFERENCE $f_0(t)$
5C2
CROSS-CORRELATION WITH REFERENCE $f_1(t)$
A(t)
B(t)
5D1
SQUARE SIGNAL
5D2
SQUARE SIGNAL
$A^2(t)$
$B^2(t)$
5E
SUM SIGNALS
5F
DETECT AND TIME PEAKS
6
TARGET PARAMETER ESTIMATES

FIG. 2

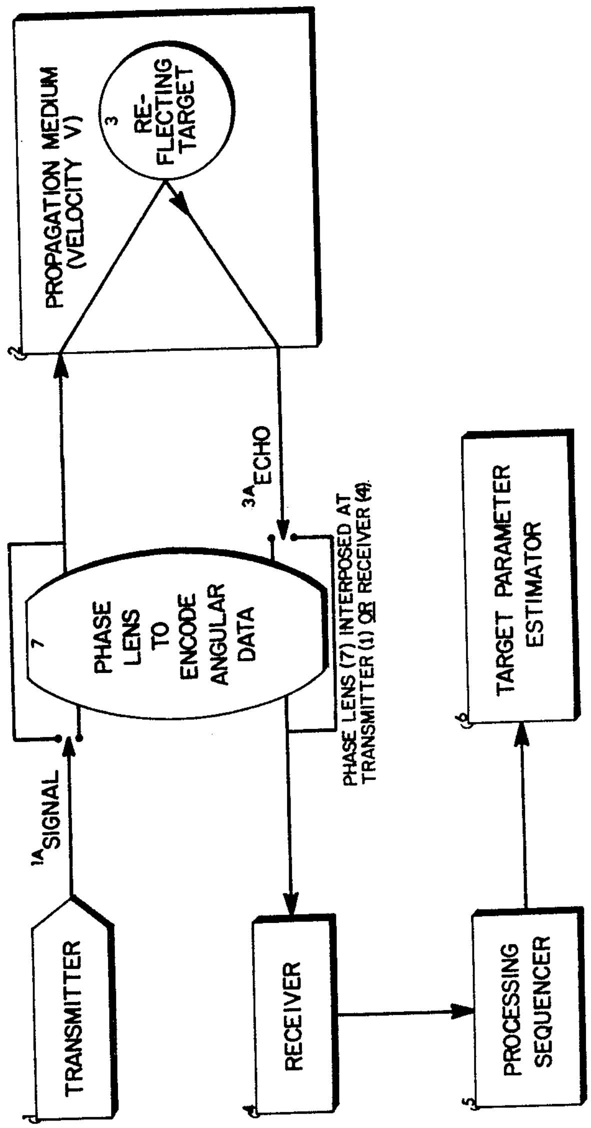

TRANSMITTER
1A SIGNAL
7
PHASE LENS TO ENCODE ANGULAR DATA
PROPAGATION MEDIUM (VELOCITY V)
3
RE-FLECTING TARGET
3A ECHO
PHASE LENS (7) INTERPOSED AT TRANSMITTER (1) OR RECEIVER (4)
RECEIVER
PROCESSING SEQUENCER
TARGET PARAMETER ESTIMATOR

FIG. 3

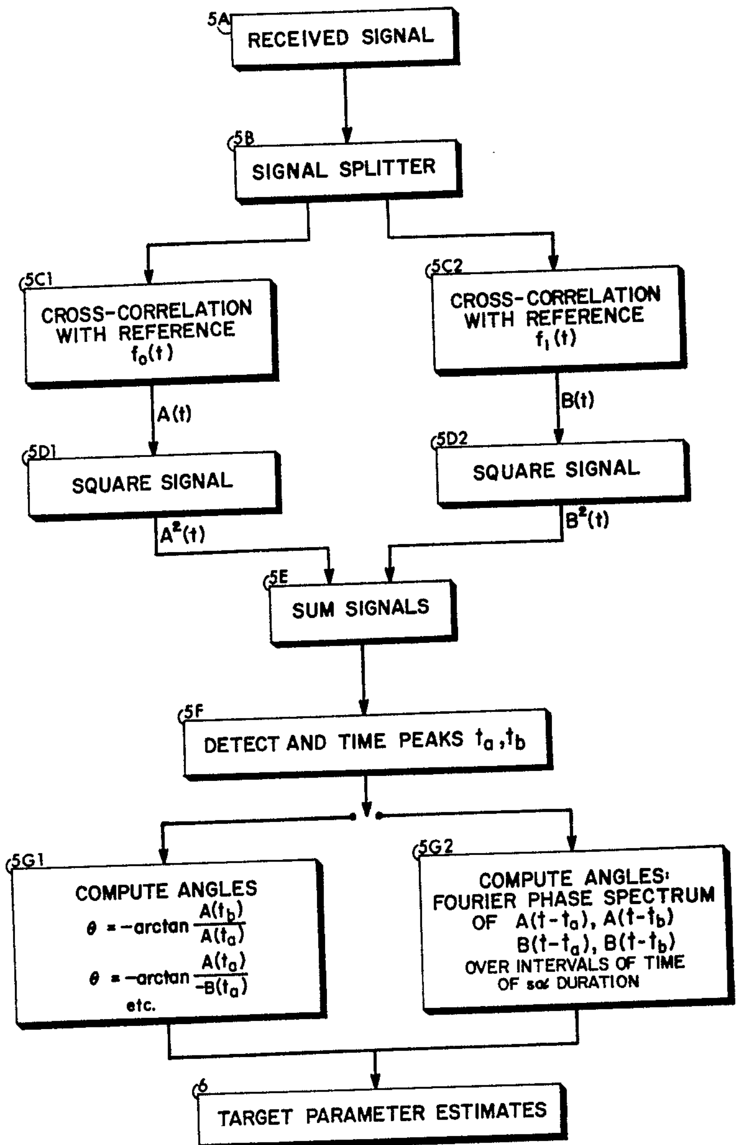

5A
RECEIVED SIGNAL
5B
SIGNAL SPLITTER
5C1
CROSS-CORRELATION WITH REFERENCE $f_0(t)$
5C2
CROSS-CORRELATION WITH REFERENCE $f_1(t)$
A(t)
B(t)
5D1
SQUARE SIGNAL
5D2
SQUARE SIGNAL
$A^2(t)$
$B^2(t)$
5E
SUM SIGNALS
5F
DETECT AND TIME PEAKS t_a, t_b
5G1
COMPUTE ANGLES
$\theta = -\arctan \frac{A(t_b)}{A(t_a)}$
$\theta = -\arctan \frac{A(t_a)}{-B(t_a)}$
etc.
5G2
COMPUTE ANGLES: FOURIER PHASE SPECTRUM OF $A(t-t_a)$, $A(t-t_b)$ $B(t-t_a)$, $B(t-t_b)$ OVER INTERVALS OF TIME OF $s\alpha$ DURATION
6
TARGET PARAMETER ESTIMATES

FIG. 4

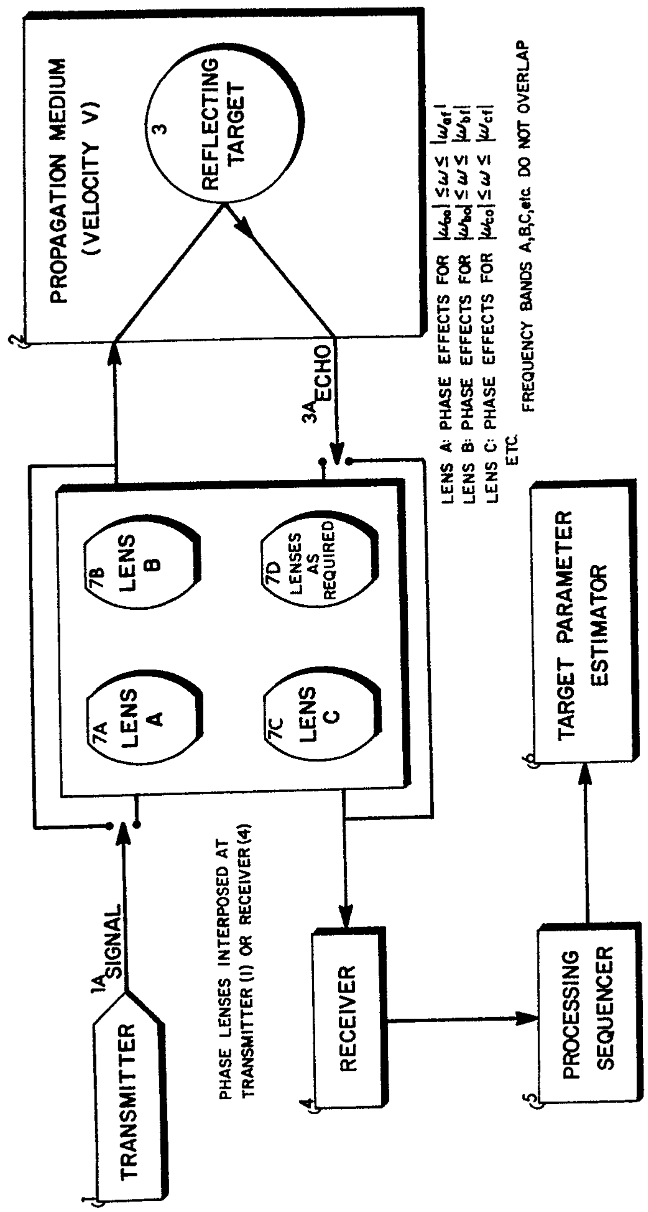

PROPAGATION MEDIUM (VELOCITY V)
2
3
REFLECTING TARGET
3A ECHO
1A SIGNAL
TRANSMITTER
1
7A LENS A
7B LENS B
7C LENS C
7D LENSES AS REQUIRED
PHASE LENSES INTERPOSED AT TRANSMITTER (1) OR RECEIVER (4)
RECEIVER
4
PROCESSING SEQUENCER
5
TARGET PARAMETER ESTIMATOR
6
LENS A: PHASE EFFECTS FOR $|\omega_{a0}| \le \omega \le |\omega_{a f}|$
LENS B: PHASE EFFECTS FOR $|\omega_{b0}| \le \omega \le |\omega_{b f}|$
LENS C: PHASE EFFECTS FOR $|\omega_{c0}| \le \omega \le |\omega_{c f}|$
ETC.
FREQUENCY BANDS A,B,C,etc. DO NOT OVERLAP

FIG. 5

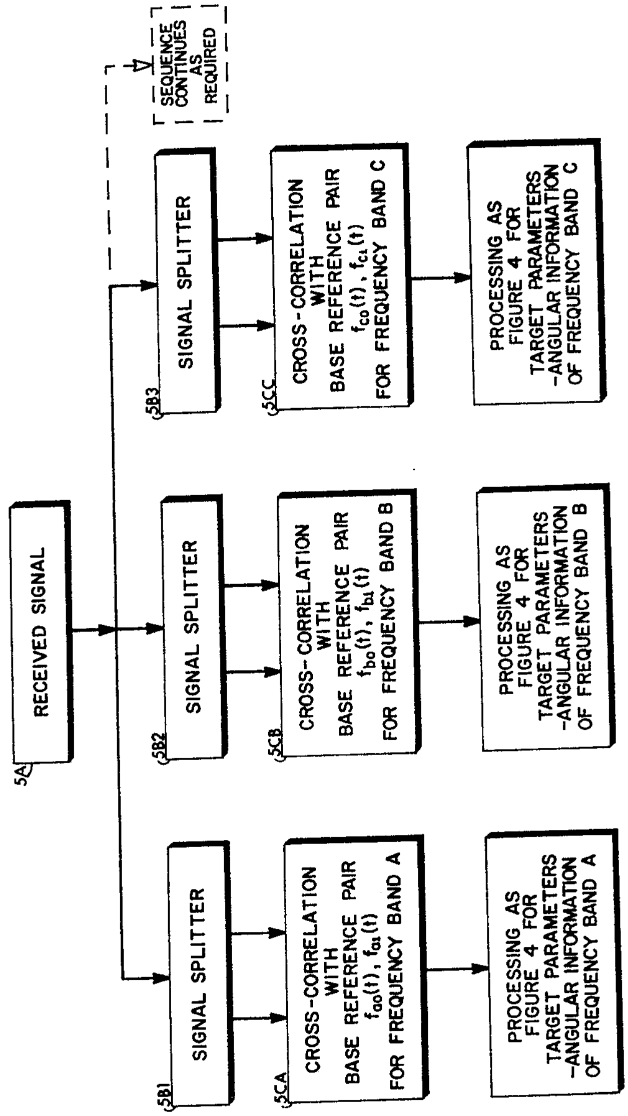

RECEIVED SIGNAL
5A
SIGNAL SPLITTER
5B1
5B2
5B3
SEQUENCE CONTINUES AS REQUIRED
CROSS-CORRELATION WITH BASE REFERENCE PAIR $f_{ao}(t)$, $f_{a1}(t)$ FOR FREQUENCY BAND A
CROSS-CORRELATION WITH BASE REFERENCE PAIR $f_{bo}(t)$, $f_{b1}(t)$ FOR FREQUENCY BAND B
CROSS-CORRELATION WITH BASE REFERENCE PAIR $f_{co}(t)$, $f_{c1}(t)$ FOR FREQUENCY BAND C
5CA
5CB
5CC
PROCESSING AS FIGURE 4 FOR TARGET PARAMETERS -ANGULAR INFORMATION OF FREQUENCY BAND A
PROCESSING AS FIGURE 4 FOR TARGET PARAMETERS -ANGULAR INFORMATION OF FREQUENCY BAND B
PROCESSING AS FIGURE 4 FOR TARGET PARAMETERS -ANGULAR INFORMATION OF FREQUENCY BAND C

FIG. 6

1604182 COMPLETE SPECIFICATION

18 SHEETS *This drawing is a reproduction of the Original on a reduced scale*

Sheet 7

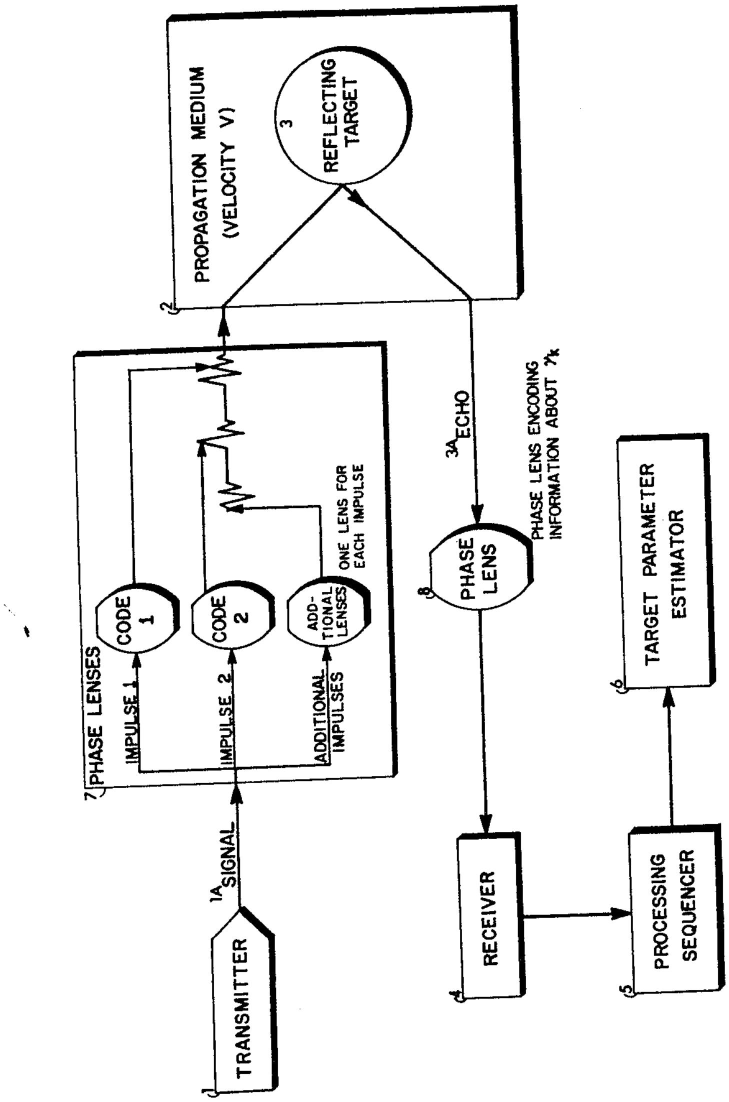


FIG. 7

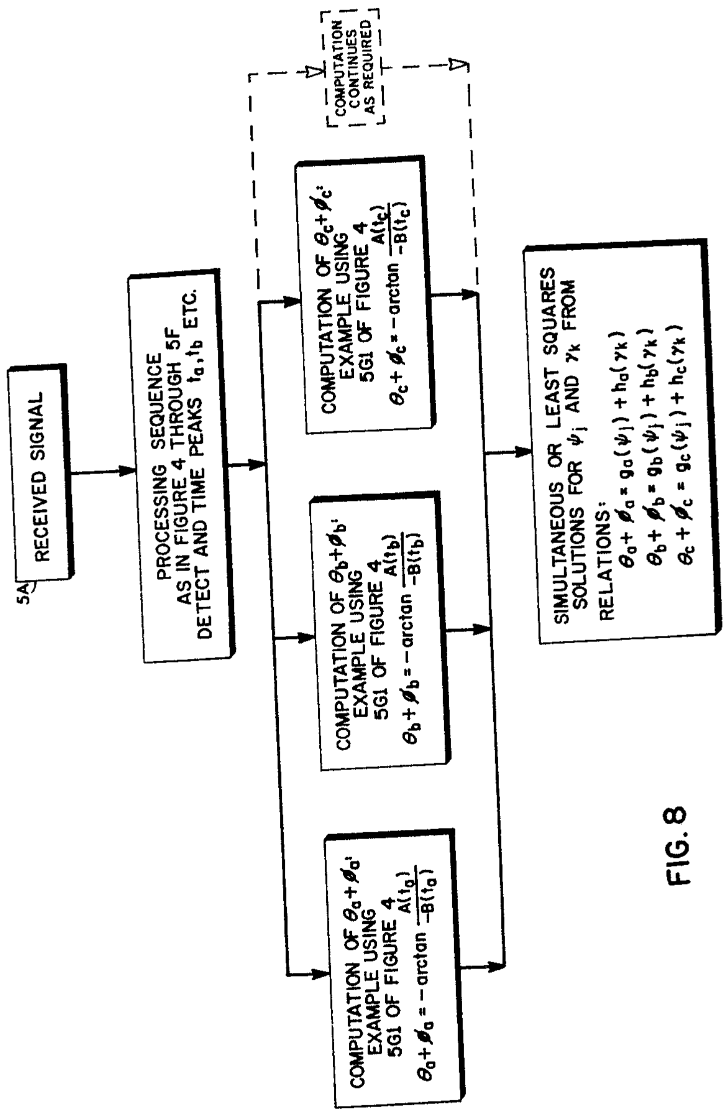

5A
RECEIVED SIGNAL
PROCESSING SEQUENCE AS IN FIGURE 4 THROUGH 5F DETECT AND TIME PEAKS t_a, t_b ETC.
COMPUTATION OF $\theta_a+\phi_a$: EXAMPLE USING 5G1 OF FIGURE 4
$\theta_a+\phi_a = -\arctan \frac{A(t_a)}{-B(t_a)}$
COMPUTATION OF $\theta_b+\phi_b$: EXAMPLE USING 5G1 OF FIGURE 4
$\theta_b+\phi_b = -\arctan \frac{A(t_b)}{-B(t_b)}$
COMPUTATION OF $\theta_c+\phi_c$: EXAMPLE USING 5G1 OF FIGURE 4
$\theta_c+\phi_c = -\arctan \frac{A(t_c)}{-B(t_c)}$
COMPUTATION CONTINUES AS REQUIRED
SIMULTANEOUS OR LEAST SQUARES SOLUTIONS FOR ψ_j AND γ_k FROM RELATIONS:
$\theta_a+\phi_a = g_a(\psi_j) + h_a(\gamma_k)$
$\theta_b+\phi_b = g_b(\psi_j) + h_b(\gamma_k)$
$\theta_c+\phi_c = g_c(\psi_j) + h_c(\gamma_k)$

FIG. 8

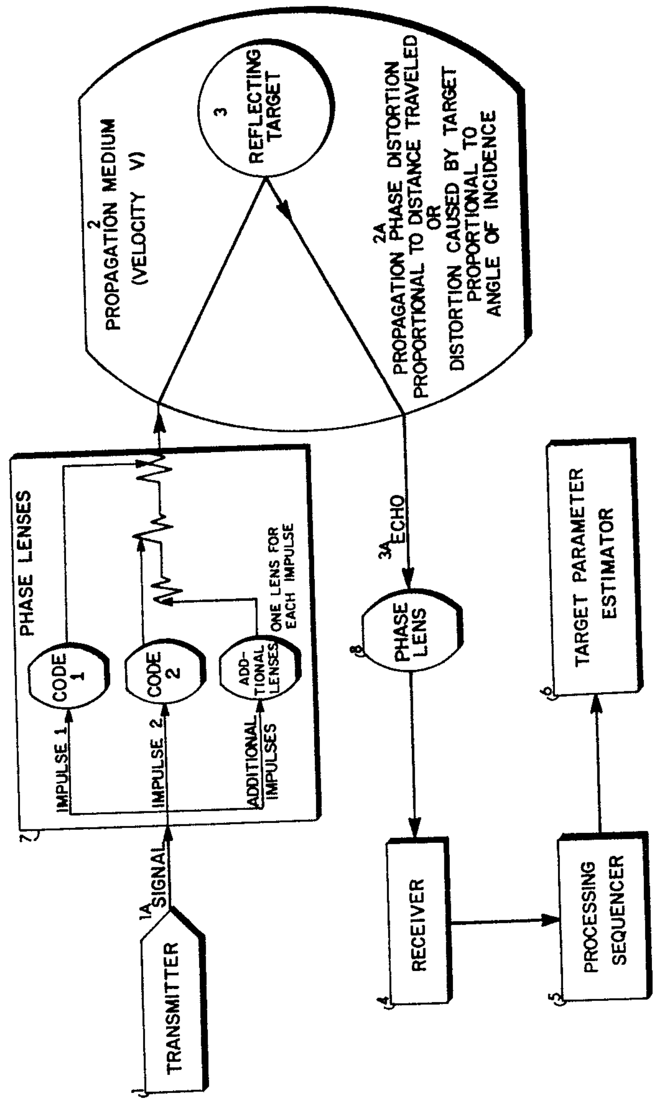

TRANSMITTER
1
1A SIGNAL
7
PHASE LENSES
IMPULSE 1
IMPULSE 2
ADDITIONAL IMPULSES
CODE 1
CODE 2
ADDITIONAL LENSES
ONE LENS FOR EACH IMPULSE
2 PROPAGATION MEDIUM (VELOCITY V)
3 REFLECTING TARGET
2A PROPAGATION PHASE DISTORTION PROPORTIONAL TO DISTANCE TRAVELED OR DISTORTION CAUSED BY TARGET PROPORTIONAL TO ANGLE OF INCIDENCE
3A ECHO
8 PHASE LENS
4 RECEIVER
5 PROCESSING SEQUENCER
6 TARGET PARAMETER ESTIMATOR

FIG. 9

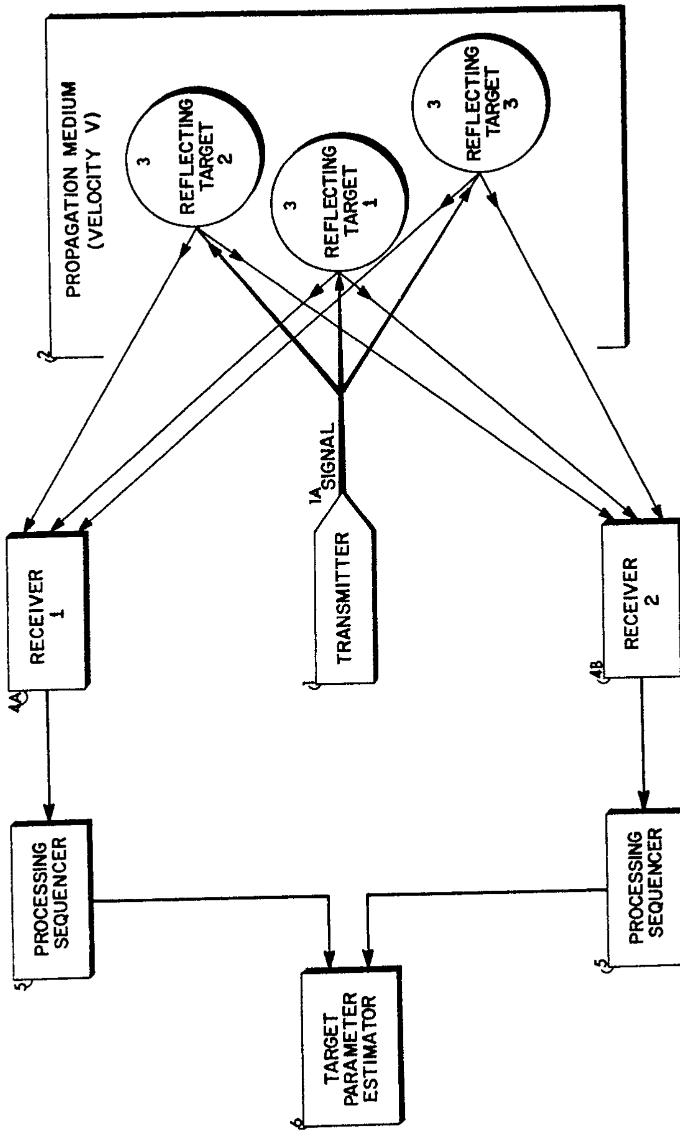
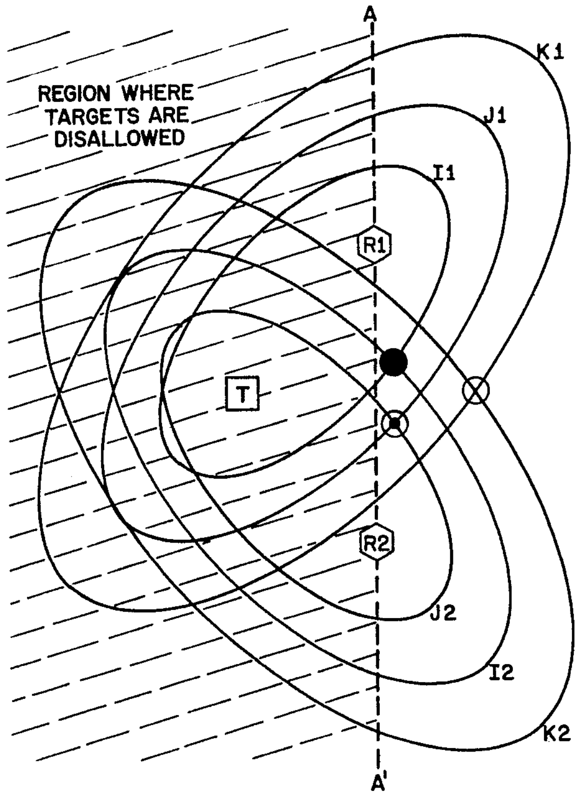

PROPAGATION MEDIUM (VELOCITY V)
2
3
REFLECTING TARGET 2
3
REFLECTING TARGET 1
3
REFLECTING TARGET 3
1A
SIGNAL
RECEIVER 1
4A
TRANSMITTER
1
RECEIVER 2
4B
PROCESSING SEQUENCER
5
PROCESSING SEQUENCER
5
TARGET PARAMETER ESTIMATOR
6

FIG. 10


A
K1
J1
I1
REGION WHERE TARGETS ARE DISALLOWED
R1
T
R2
J2
I2
K2
A'

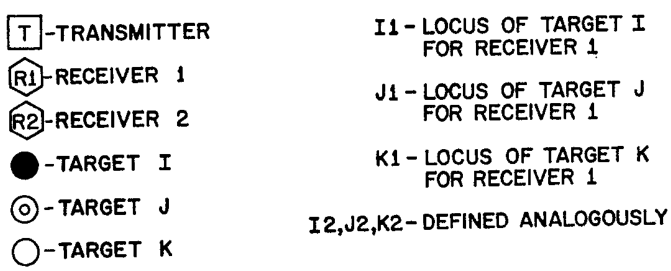
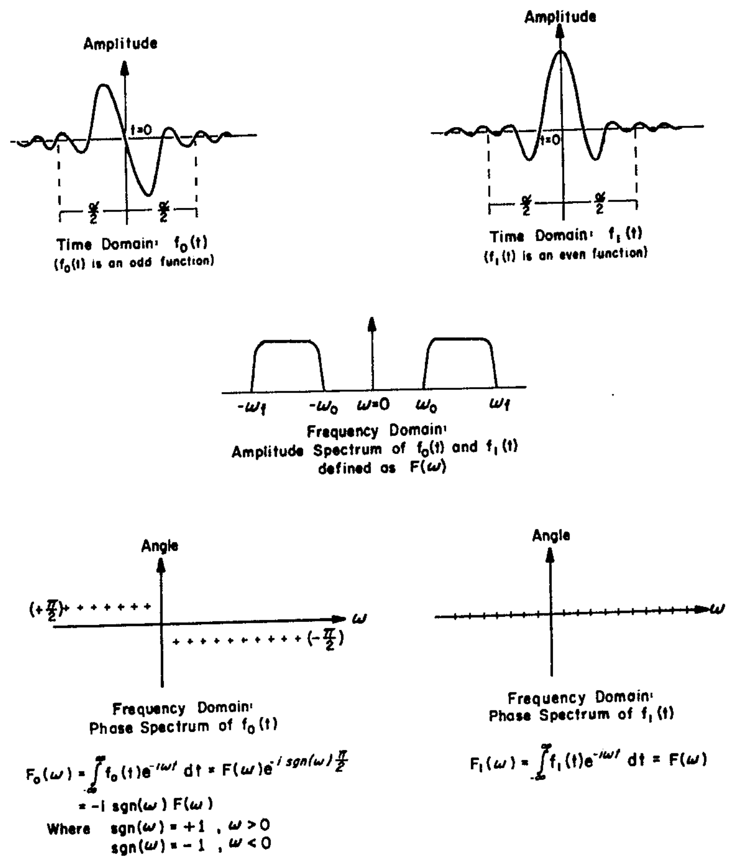

T -TRANSMITTER
R1 -RECEIVER 1
R2 -RECEIVER 2
-TARGET I
-TARGET J
-TARGET K
I1 - LOCUS OF TARGET I FOR RECEIVER 1
J1 - LOCUS OF TARGET J FOR RECEIVER 1
K1 - LOCUS OF TARGET K FOR RECEIVER 1
I2,J2,K2 - DEFINED ANALOGOUSLY

FIG. 11



TIME DOMAIN AND COMPLEX FREQUENCY DOMAIN (IN POLAR FORM) REPRESENTATIONS OF THE FUNCTION PAIR $f_0(t)$, $f_1(t)$

FIG. 12

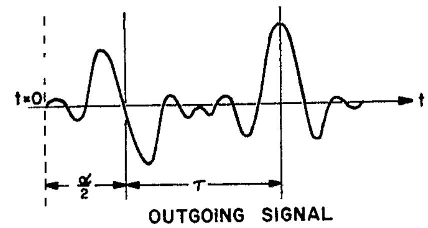
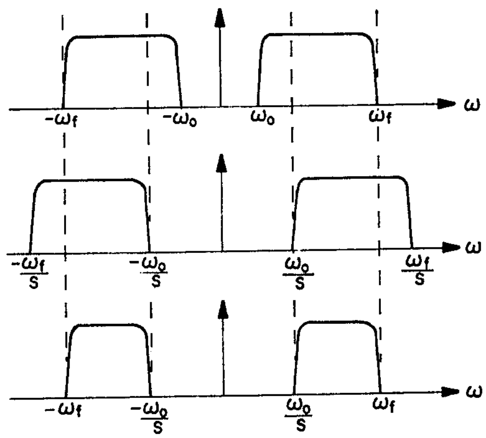


FIG. 13



Target in Figure is approaching transceiver. The return signal is shrunken with all frequencies shifting linearly to higher ones.

GRAPHICAL EVOLUTION OF THE PRODUCT $|F(\omega)||F(\frac{\omega}{S})|$
THE FREQUENCY DOMAIN EQUIVALENT OF $k_0(st) * k_0(t)$ OR $k_1(st) * k_1(t)$

FIG. 14

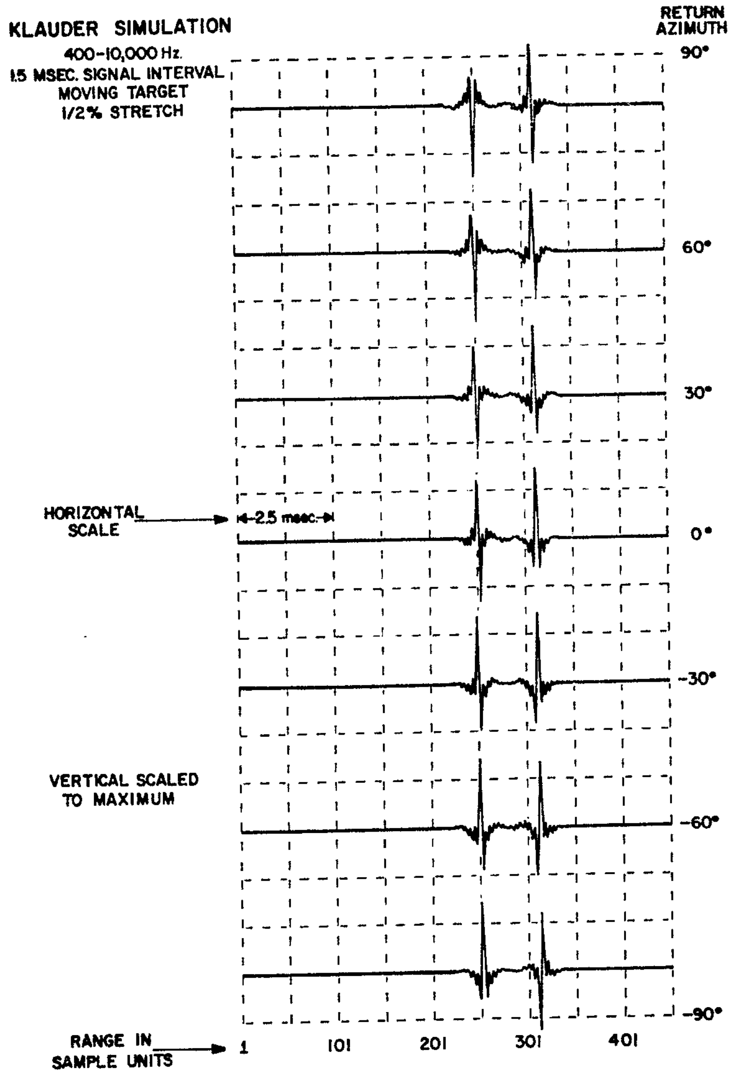

KLAUDER SIMULATION
400-10,000 Hz.
1.5 MSEC. SIGNAL INTERVAL
MOVING TARGET
1/2% STRETCH
RETURN AZIMUTH
90°
60°
30°
0°
-30°
-60°
-90°
HORIZONTAL SCALE
2.5 msec.
VERTICAL SCALED TO MAXIMUM
RANGE IN SAMPLE UNITS
1
101
201
301
401

FIG. 15

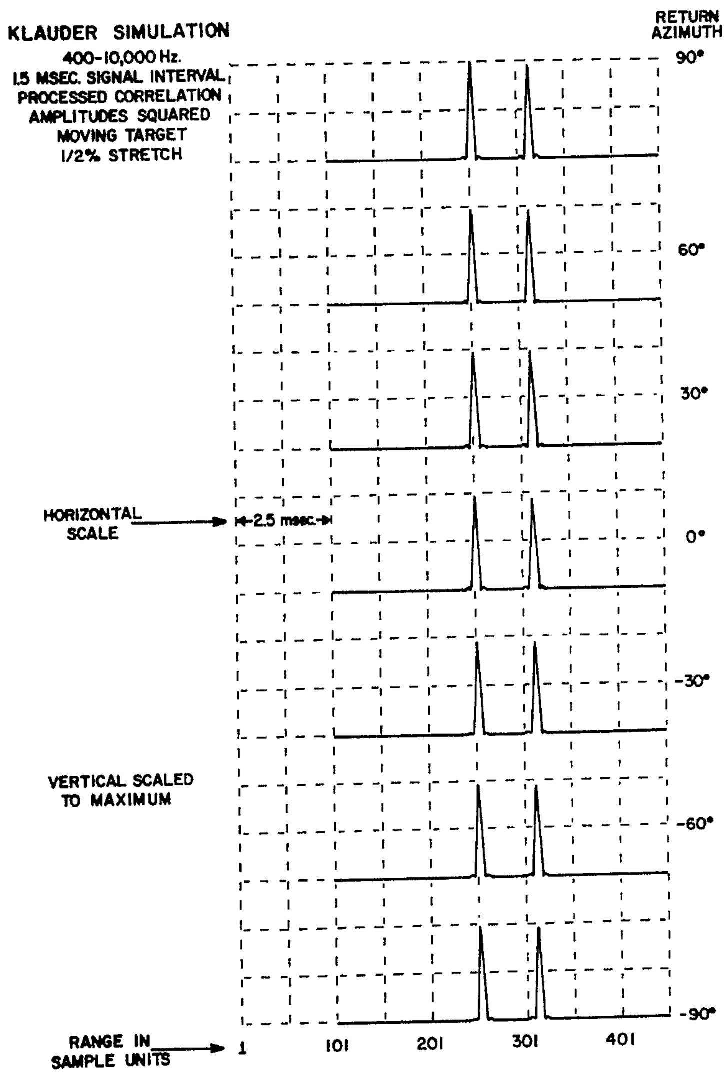
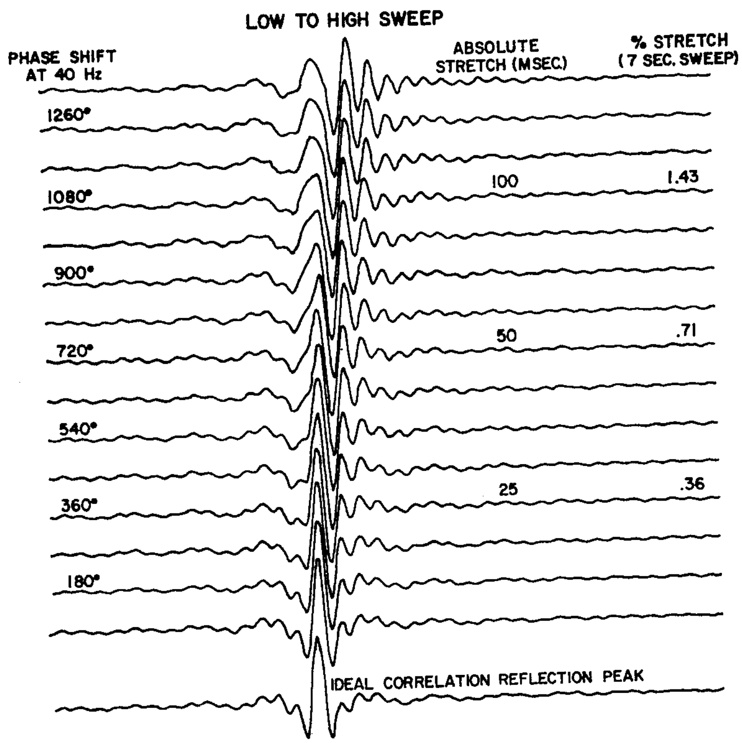

KLAUDER SIMULATION
400-10,000 Hz.
1.5 MSEC. SIGNAL INTERVAL
PROCESSED CORRELATION
AMPLITUDES SQUARED
MOVING TARGET
1/2% STRETCH
RETURN AZIMUTH
90°
60°
30°
0°
-30°
-60°
-90°
HORIZONTAL SCALE
2.5 msec.
VERTICAL SCALED TO MAXIMUM
RANGE IN SAMPLE UNITS
1
101
201
301
401

FIG. 16



SINE CHIRP CORRELATIONS

FIG.17

1604182 COMPLETE SPECIFICATION

18 SHEETS *This drawing is a reproduction of the Original on a reduced scale*

Sheet 17

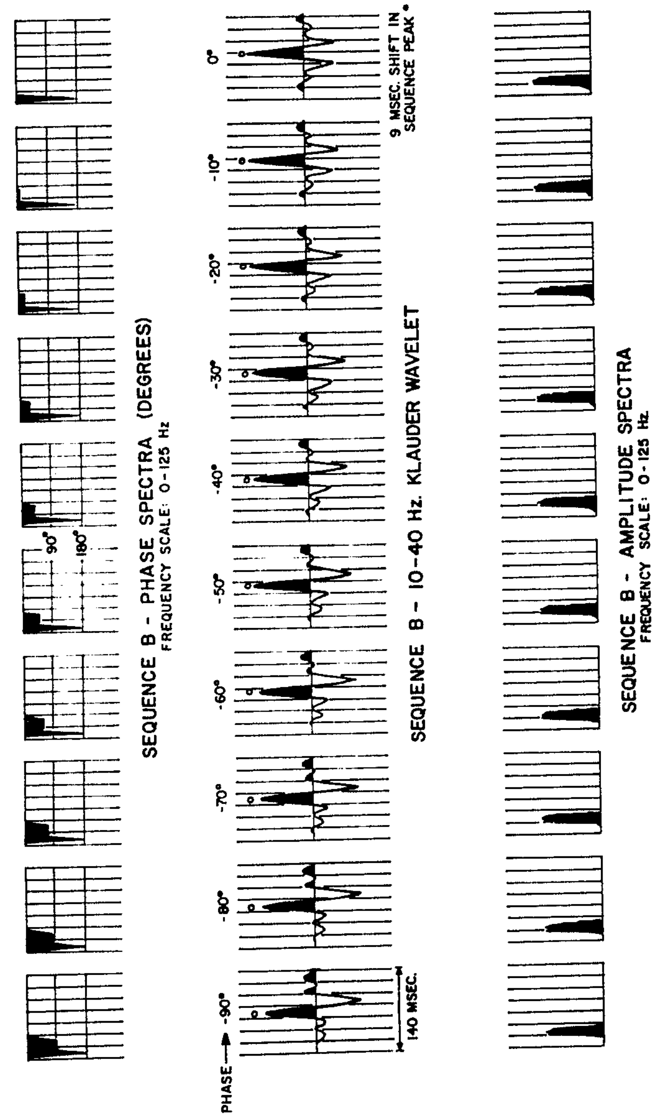
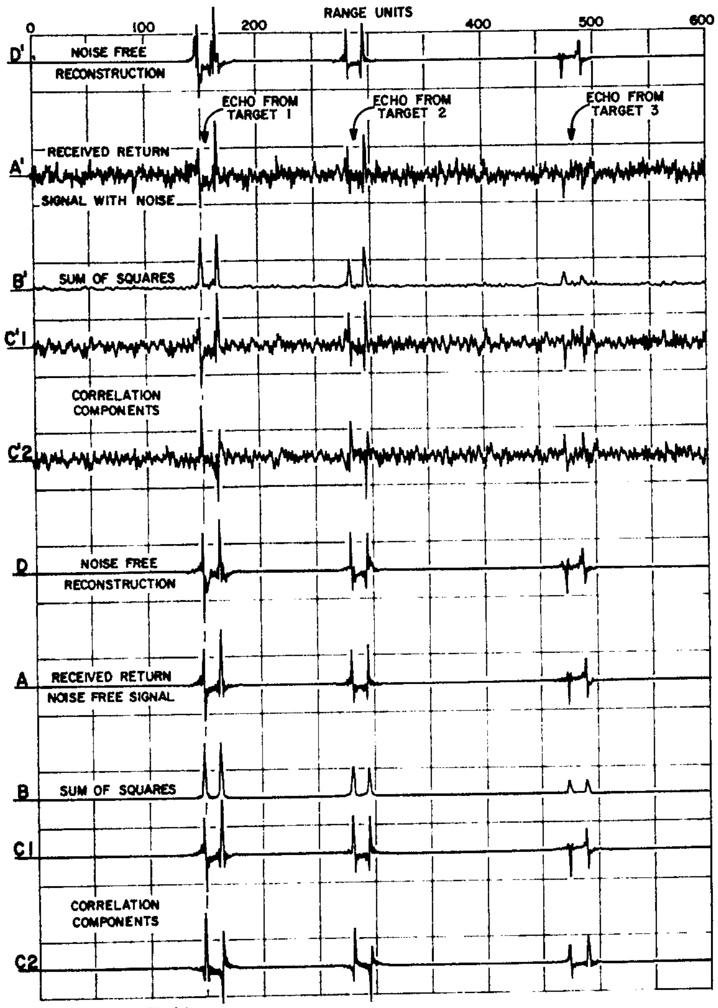


FIG. 18



Echolocation Computer Simulation For Three Moving Targets (With And Without Noise) Using Klauder Base Signal Pair And Constant Phase Encoding

FIG. 19