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(54) Color control system for a printing press

(57) A system and method for controlling the ink feed in a printing press in order to achieve and maintain target values of color. More particularly, the invention relates to a system for controlling the ink control devices using an adaptive controller.

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Description

FIELD OF THE INVENTION

[0001] The present invention relates generally to a system and method for controlling the ink feed in a web-offset printing press in order to achieve and maintain target values of color. More particularly, the invention relates to a system for controlling the ink feed using a fuzzy logic adaptive closed loop controller.

BACKGROUND OF THE INVENTION

[0002] A web-offset printing press operates to print a multi-color image by combining several single color images through superimposed printing on a moving substrate or web. A typical four color printing process includes black, cyan, magenta and yellow ink. The color quality of the printed image is determined by the degree to which the colors of the printed image match a desired or exemplary reference image, which is often provided or endorsed by the print customer. One way to evaluate color involves visual examination of the printed image by a trained pressman. Another way to evaluate color is to measure the optical density of a solid color bar printed on the substrate. In general terms, the actual color quality is compared to the desired quality, and the amount of ink fed to the substrate is adjusted as necessary.

[0003] In particular, the printing press includes an inking assembly for each color of ink used in the printing process. Each inking assembly includes an ink reservoir as well as a blade disposed along the outer surface of an ink fountain roller. The amount of ink supplied to the roller train of the press and ultimately to a substrate such as paper is adjusted by changing the spacing between the edge of the blade and the outer surface of the ink fountain roller. The blade is divided into a plurality of blade segments, and the position of each blade segment relative to the ink fountain roller is independently adjustable by movement of an adjusting screw, or ink key, to thereby control the amount of ink fed to a corresponding strip or zone of the substrate extending in the longitudinal direction. A typical printing press includes 24-30 ink keys which operate to control ink to an ink key zone having a dimension of approximately 1.2-2.5 inches.

[0004] Ink is also spread laterally from one ink key zone to adjacent zones on the substrate due to the movement of vibrator rollers, which oscillate in a lateral direction relative to the longitudinal direction of travel of the substrate.

[0005] In order to preset the initial positions of the ink keys, it is common for a printing press operator to visually examine printed copies or proofs of the image to be printed and to note the amount of color necessary in respective zones of the image to be printed. Based on this visual examination as well as experience with the press, ink, and type of substrate (typically paper), the operator may preset the ink keys to approximate the settings that will be required once the press is running. As an example, low-tack yellow ink has a low pigment strength and requires a greater amount of ink to produce an image with a given optical density. As another example, uncoated paper requires more ink than does coated paper to achieve an image having a given optical density.

[0006] Once the printing press is started, the rate of ink flow from the ink fountain to the web must be controlled by adjusting the ink keys for each of the ink colors. The time spent for the ink key adjustment until the desired solid ink density for each zone is achieved on press is termed makeready. Again, ink key adjustment is typically achieved based on visual examination and manual adjustment by an experienced press operator. After makeready, it is common for a press operator to continually monitor the printed output and to make appropriate ink key adjustments in order to achieve appropriate quality control of the color of the printed image. For example, if the color in a zone is too weak, the operator adjusts the corresponding ink key to allow more ink flow to that zone; if the color is too strong, the corresponding ink key is adjusted to decrease the ink flow. Also during runtime, further color adjustments may be necessary to compensate for changing press conditions, or to account for the personal preferences of the customer.

[0007] The above-described visual inspection and manual adjustment techniques used in connection with ink key presetting, makeready, and runtime are relatively inaccurate, expensive, and time-consuming. Additionally, such techniques require a high level of operator expertise.

[0008] In general, methods for determining a more accurate initial estimate for required ink amounts for presetting the ink keys are known. These methods typically rely on a measurement of plate coverage to achieve more accurate results than those obtained by visual estimation. Plate coverage is the ratio of the inked area to the total plate area, and provides a measure of the amount of ink required to print the desired image. By dividing the printing plate into zones corresponding to the ink keys, and determining the plate coverage of each zone, an initial ink key setting for each zone can be determined.

[0009] Additionally, methods other than visual inspection of the printed image are known for monitoring color quality once the press is running. These methods typically include measuring the optical density of a printed image. Optical density of various points of a printed image can be measured by using a densitometer or scanning densitometer either off-line or on-line of the web printing process. Optical density measurements are performed by illuminating a test image with a light source and measuring the intensity of the light reflected from the image. Optical density (D) is defined as:

$$D = -\log_{10}(R)$$

where R is the reflectance, or ratio of reflected light intensity to incident light intensity.

[0010] Since substrate material is wasted until acceptable color is achieved, an accurate and quick method of determining ink key settings will minimize the required time and material costs. Especially for print jobs of short duration, start-up waste can be a major percentage of total time and materials required.

[0011] Typically, a conventional proportional-integral-derivative (PID) controller is the most widely used controller in industry. The conventional PID controller was developed in the 1940's based on the classical linear time-invariant system. Theoretically, such a controller would work well in a printing application to control ink feed rate provided that the entire printing process was linear and time invariant. In other words, the color density would need to be proportional to the ink key settings and the factors affecting the entire printing process would need to remain unchanged.

SUMMARY OF THE INVENTION

[0012] A conventional PID controller does not work well if the controlled system is highly nonlinear or includes uncertain factors in the working environment. Because web offset printing is a very complicated process, there are many known and unknown factors which affect the measured solid ink density (SID) values such that the overall system is nonlinear. Known factors affecting the SID values include the make and model of the printing press, ink and color variations, fountain solution pH value, operating temperature variations, differences in paper stock, age and speed of the press, etc. Consequently, it is not desirable to control color using a PID controller alone having a fixed set of PID parameters because such a controller is unable to account for all the different operating conditions.

[0013] It is an object of the present invention to provide a system and method for accurate control of color on a web-offset press utilizing adaptive control which overcomes the disadvantages of a conventional PID controller. With adaptive control, the gain parameters Kp, Ki, and Kd of a PID controller are tuned to "optimal" values in real time.

[0014] It is an additional object of the present invention to accomplish adaptive control with the use of fuzzy logic.

[0015] Other features and advantages of the invention will become apparent to those of ordinary skill in the art upon review of the following detailed description and drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

[0016]

Fig. 1 is a block diagram of a web-offset printing system in accordance with the present invention;
Fig. 2 is an illustration of an inking assembly including an ink fountain roller, ink reservoir, and ink keys;
Fig. 3(a) is a side view of the inking assembly of Fig. 2, taken along line 3-3, when the ink key is partially open;
Fig. 3(b) is a side view of the inking assembly of Fig. 2, when the ink key is closed.
Fig. 4 is a schematic of a roller train of a lower printing unit of a Harris M1000B printing press;
Fig. 5 is a schematic illustration of an ink key control system in accordance with the present invention;
Fig. 6 is a schematic of the relationship between a PID controller and a fuzzy logic parameter tuner;
Fig. 7 is a block diagram of a general fuzzy inference system;
Fig. 8 is an illustration of a Mamdani fuzzy inference system;
Fig. 9 is an illustration of five input membership functions;
Fig. 10 is an illustration of five output membership functions;
Fig. 11 is an example of an ink key spread matrix; and
Fig. 12 is an example of a pseudo-inverse spread matrix.

[0017] Before one embodiment of the invention is explained in detail, it is to be understood that the invention is not limited in its application to the details of construction and the arrangement of components set forth in the following description or illustrated in the drawings. The invention is capable of other embodiments and of being practiced or being carried out in various ways. Also, it is to be understood that the phraseology and terminology used herein is for the purpose of description and should not be regarded as limiting.

DESCRIPTION OF THE PREFERRED EMBODIMENT

[0018] The following description of the preferred embodiment specifically refers to a Harris M1000B printing press using 24 ink keys as an example. It should be noted that the invention is applicable to other models of printing presses, to printing presses of other manufacturers, and to printing presses having a different number of ink keys, such as 22 or 36.

[0019] Fig. 1 illustrates a web-offset printing system 10 for printing a multi-color image upon a moving web 12. In the preferred embodiment, four printing units 14, 16, 18, and 20 each print one color of the image upon the web 12. The location of printing units 14, 16, 18, and 20 relative to each other is determined by the printer, and may vary. Each printing unit 14, 16, 18, 20 includes a printing plate cylinder and a blanket cylinder. This type of printing is commonly referred to as web-offset printing. In particular, each printing unit includes an upper blanket cylinder 22, an upper printing plate cylinder 24, a lower blanket cylinder 26, and a lower printing plate cylinder 28 to permit printing on both sides of web 12. In printing system 10, colors 31, 32, 33, and 34 on printing units 14, 16, 18, and 20, respectively, are typically black (K), cyan (C), magenta (M), and yellow (Y). Cyan, magenta, and yellow are three subtractive primary color inks which are used to reproduce the color image. The black ink is used to sharpen features and to replace the overprints of the three primary ink colors.

[0020] Each printing unit 14, 16, 18, and 20 includes an associated inking assembly 36 which is shown in Fig. 2. Inking assembly 36 operates to supply ink to a roller train which includes a plate cylinder and a blanket cylinder and then to the web 12. In particular, inking assembly 36 includes an ink reservoir 38 disposed adjacent an ink fountain roller 40 (also known as the ink ball) which extends laterally across the web. A blade 42 extends along the ink fountain roller 40 and is segmented so that the spacing of each segment relative to the ink fountain roller 40 can be independently adjusted to control the ink fed to a respective ink key zone on the web 12. As shown in Figs. 3(a) and 3(b), each blade segment 44 has an edge 46 which is moved toward and away from the outer surface 48 of the ink fountain roller 40 by adjustment of an associated ink flow adjustment device 50.

[0021] More specifically, a portion of the ink fountain roller 40 forms one main wall of the ink reservoir 38. The other principal wall of the reservoir 38 is provided by the blade segments 44. Ink passes from the ink reservoir 38 through the space between the surface of the ink fountain roller 40 and the lower edge 46 of the blade segment 44, and the spacing of the blade edge 46 to the ink fountain roller 40 acts to control the thickness of the ink film provided to the outer surface 48 of ink fountain roller 40.

[0022] A plurality of the ink flow adjustment devices 50 are disposed at equally-spaced lateral locations along the inking assembly 36 to press against the blade segments 44 at those locations to establish and adjust the size of the space between the roller 40 and the blade segment 44. Each ink flow adjustment device 50 includes an ink key 54 having screw threads engaging threads in a fixed portion of the frame of the inking assembly 36. The ink key 54 has a tip portion 56 which pushes against the associated blade segment 44 to deflect it and to thereby provide locally adjustable control of the spacing and the ink feed.

[0023] The ink key 54 is driven by a bi-directional actuator motor 58 which operates to move the ink key 54 toward and away from the ink fountain roller 40. A potentiometer 60 has a movable arm mechanically connected with the ink key 54. The potentiometer 60 has a pair of outside electrical terminals and an inside electrical terminal located between the outside electrical terminals. The inside terminal of the potentiometer is mechanically connected to the movable arm of the potentiometer 60. The position of the movable arm of the potentiometer 60 depends upon the position of the ink key 54. The potentiometer 60 is energized at its outside electrical terminals so that an electrical signal indicative of the position of the ink key is produced at the inside electrical terminal of the potentiometer. The motor 58 is responsive to a signal on line 66 to position the ink key 54 as desired.

[0024] Fig. 4 is an illustration of a side view of a roller train 96 of a lower printing unit of a Harris M1000B printing press. Ink is supplied from the inking assembly 36 via the ink fountain roller 40 to a ductor roller 98 which continuously moves back and forth from contact with the ink fountain roller 40 and roller 100. The amount of ink on the ink fountain roller itself is also adjustable by changing the angle that the ink fountain roller 40 rotates each stroke. This occurs by adjusting a conventional ratchet assembly (not shown) as is known in the art. The rotation angle, along with the positions of the blade segments 44, determine the amount of ink transferred to the ductor roller 98. The relationship between the rotation angle and the amount of ink transferred to ductor roller 98 is assumed to be linear. Ink is supplied from roller 100 to the various other rollers 102-124 as shown in Fig. 4. The arrows of Fig. 4 indicate the direction of rotation of rollers 98-124. Rollers 100, 104, 114, and 118 are vibrator rollers which oscillate back and forth in a lateral direction with respect to the longitudinal direction of travel of the web 12, thereby operating to spread ink from one ink key zone to adjacent ink key zones.

[0025] With reference to Fig. 5, the general operation of an ink key control system 200 of the present invention is described. In general, the control system 200 operates to adjust the position of the ink keys 54 to control the position of the blade segments 44 in order to control the amount of ink fed to corresponding ink key zones on the web 12 of the printing press. The control system 200 includes controller 204 and a color density measuring system 208. Although various ways to measure SID values can be utilized, preferably, the color density measuring system 208 generates measured solid ink density (SID) values for color bar patches in a color bar oriented transversely across the web 12. In terms of feedback control, the controller 204 operates to maintain the SID values for each color patch within a desired range. The measured SID values are called the controlled variables, and are the ultimate control target of the controller 204.

[0026] In particular, a measured SID value is compared to a desired or setpoint SID value and an SID error signal (SID_err) is generated. The controller 204 includes a fuzzy logic parameter tuner 212, a conventional Proportional-Integral-Differential (PID) controller 216, and optionally, a decoupling computation unit 220. The fuzzy logic parameter tuner 212 adaptively adjusts gain parameters in the PID controller 216. The PID controller 216 provides signals to the decoupling computation unit 220. The decoupling computation unit 220 takes into account the effects of the ink key coupling due to the lateral movement of the vibrator rollers 100, 104, 114 and 118 and provides signals to drive the motors 58 to independently control the position of each ink key 54. In operation without the decoupling computation unit 220, the signals from the PID controller 216 are directly provided to the ink keys 54.

[0027] Optionally, the controller 204 can also interface with a ratchet assembly 224 to control the angle of rotation of the ink fountain roller 40.

[0028] More specifically, the color density measuring system 208 includes a color CCD video camera mounted on a transport bar that spans across the web. The system 208 reports values of solid ink density of solid color patches within a color bar that is oriented transversely across the web 12. A strobe light is flashed at an appropriate time so that the color CCD camera obtains an image of a portion of the color bar on the web 12. The image of the color bar is processed through an algorithm to calculate an accurate SID value for each individual color patch. These SID values are fed to the controller 204. The camera is moved laterally across the web 12 in a series of steps to acquire sequential images in all the ink key zones across the web 12. A color density measuring system 208 which accurately measures the optical density of a printed image while the press is running is the color measuring system (CMS) described in U.S. Pat. App. Serial No. 08/434,928, invented by John C. Seymour, Jeffrey P. Rappette, Frank N. Vroman, Chia-Lin Chu, Bradly S. Moersfelder, Michael A. Gill and Karl R. Voss, entitled "SYSTEM AND METHOD FOR MONITORING COLOR IN A PRINTING PRESS", which is hereby incorporated by reference.

[0029] The controller 204 performs several functions. First, the controller 204 receives the measured SID value from the color density measuring system 208 and calculates:

$$SID\_err(j,k,t) = SID\_set\_point(j,k,t) - Measured\_SID(j,k,t)$$

where:

- j: color index (j = C, M, Y, or K)
- k: ink key index across the web (k = 1, ..., 24)
- t: sampling time index (t = 1, 2, ...)

[0030] The controller 204 also calculates the trend of the SID_err increment, i.e., the difference between the current SID_err at time t and the previously sampled SID error at time (t-1):

$$SID\_derr(j,k,t) = SID\_err(j,k,t) - SID\_err(j,k,t-1)$$

where j, k, and t are defined above.

[0031] Fig. 6 illustrates the relationship between the PID controller 216 and the fuzzy logic parameter tuner 212. As shown, the two calculated signals, SID_err(j,k,t) (or the SID error signal for color j and ink key k at time t) and SID_derr(j,k,t) (or the change in the SID error signal for color j and ink key k at time t) are fed both to the fuzzy logic parameter tuner 212 and the PID controller 216. The PID controller 216 computes the ink key settings to achieve the desired set point SID values for each ink key zone and for each ink color, without accounting for the coupling of the ink keys. The function of the fuzzy logic parameter tuner 212 is to adjust the three gain parameters in the PID controller 216 adaptively to compensate for the variations in press performance. There are two ways to adjust the values of the PID parameters: 1) a direct output of current PID gain parameter values by the fuzzy logic parameter tuner, or 2) an indirect or incremental adjustment of the PID parameters. The second method is preferred because it is more stable and reduces drastic swings in parameter values over time.

[0032] Hereafter, the following notations are used:

Kp(j,k,t), Ki(j,k,t), and Kd(j,k,t) are the proportional, integral, and differential gain parameters, respectively, used by the PID controller 216 for color j and ink key k at time t. These gain parameters are tuned in real time by the fuzzy logic parameter tuner 212.

d_Kp(j,k,t), d_Ki(j,k,t), and d_Kd(j,k,t) are the incremental adjustments of the Kp, Ki, and Kd parameters, respectively, for color j and ink key k at time t.

ink_key(j,k,t) is the command ink key setting for color j and ink key k at time t, without taking into account ink key coupling.

[0033] The overall output of the PID controller 216 is the unadjusted command ink key setting. Since the color density measuring system 208 reports the SID values sequentially, the PID controller 216 can be implemented sequentially. The overall output of the PID controller 216 is the linear combination of proportional, integral, and differential terms, as follows:

$$\text{ink\_key}(j,k,t) = Kp(j,k,t) * \text{SID\_err}(j,k,t) + \left[\sum_{l=1}^{t} Ki(j,k,l) * \text{SID\_err}(j,k,l)\right] * \Delta T + K_{INT} + Kd(j,k,t) * \frac{\text{SID\_derr}(j,k,t)}{\Delta T}$$

where: K_{INT} = ink_key(j,k,0) .

[0034] In adaptive control, the parameters Kp, Ki, and Kd of the PID controller 216 are tuned to some "optimal" value in real time. One way to accomplish adaptive control is utilizing fuzzy logic.

[0035] Fuzzy logic is based on fuzzy set theory and operates to map an input space to an output space. When used in conjunction with adaptive control, fuzzy logic incorporates the operation knowledge of human experts into a control loop. Fuzzy logic is also useful for modelling nonlinear functions of arbitrary complexity. Fuzzy logic can be blended with conventional control techniques, such as conventional PID control. The embodiment described herein is an example of an indirect fuzzy logic control system. An indirect fuzzy logic control system is used in conjunction with, for example, a conventional PID controller and has the advantage that the control design is separated from the adaptive mechanism. In contrast, a direct fuzzy logic controller generally uses a static incremental process model to relate the error in the calculated control action to the deviation in the desired behavior.

[0036] Fuzzy logic includes the concept of fuzzy sets. A fuzzy set is one that does not have clear and crisp boundaries but instead describes a somewhat vague concept. Examples of fuzzy sets are:

The set of old people;
The set of tall people;
The set of high temperatures;
The set of excellent drivers;
The set of poor restaurant service; and
The set of hot weather.

[0037] The degree that an item belongs to the fuzzy set is measured by its membership function. A membership function is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership), which is a value between 0 and 1. As an example, a man of age 69 may belong to the fuzzy set of "old people" with a membership value of .8 (the degree of belonging to the set). A membership function can be represented by curves of various shapes including, for example, triangular, gaussian, bell shaped, sigmoidal, and polynomial-based curves, as well as others.

[0038] Another feature of fuzzy sets is that they do not obey the rule of "mutually exclusive." An item can belong to two or more different fuzzy sets simultaneously. Using the same example above, a man of age 69 could belong to the fuzzy set "young people" with a membership function value of only .2 at the same time he belongs to the fuzzy set "old people" with a membership function value of .8.

[0039] A fuzzy inference system, such as that depicted in Fig. 7, is capable of implementing a nonlinear mapping from its input space to an output space. The mapping is accomplished by a number of fuzzy if-then rules, each of which describes the local behavior of the mapping and which reflects certain knowledge of human experts' decision making process. For example, the following rules are an example of a method for determining the size of a tip at a restaurant:

Rule 1: If the food quality is excellent and the service quality is average, then the tip is moderately generous.
Rule 2: If the food quality is poor and the service quality is below average, than the tip is minimal.

[0040] The rules establish a simple input-output inference system, where "food quality" and "service quality" are the input fuzzy variables, and the single output fuzzy variable is "the amount of the tip". The antecedent of a rule defines a fuzzy region in the input space, while the consequent specifies a fuzzy region in the output space.

[0041] A fuzzy inference system basically includes the functions of fuzzification, inferencing, aggregation, and defuzzification. One way to accomplish the above steps is known as the Mamdani fuzzy inference system, which is known in the art. Some of the processing steps involved in the Mamdani fuzzy inference system are illustrated in Fig. 8. The Mamdani inference system includes output membership functions (shown as C1 and C2) which are also fuzzy sets.

[0042] Because the inputs to the fuzzy inference system are common crisp values, they must undergo a fuzzification process in order to apply fuzzy if-then rules. Similarly, the results of the multiple fuzzy if-then rules must be aggregated and then defuzzified to generate a crisp output.

[0043] Fuzzification is accomplished with the use of a plurality of input membership functions, wherein the membership values of each membership function are determined for a given input variable. The next step is determining which of the if-then rules are activated for the given input variables. An if-then rule is activated if the membership values of the fuzzy variables included in its antecedent are nonzero. Interpreting an if-then rule includes evaluating the antecedent (which involves fuzzifying the input and applying any necessary fuzzy operators) and applying that result to the consequent. If there are two or more fuzzy variables in the antecedent of a rule, the fuzzy operators must be applied. For example, the output of the statement A AND B, where A and B are within the range (0,1) is determined by min (A, B) (i.e., the minimum of the two values). This is illustrated in the left portion of Fig. 8. Similarly, the output of the statement A OR B, where A and B are within the range (0,1) can be determined by max (A, B) (i.e, the maximum of the two values).

[0044] The outputs of the activated rules are aggregated. The output fuzzy sets are aggregated by combining them into a single output fuzzy set, typically using the max operator, as shown in the right portion of Fig. 8. The resulting set is defuzzified, or resolved to a single number.

[0045] Various defuzzification methods are known. Defuzzification is the conversion of a fuzzy quantity to a precise quantity. Four known defuzzification methods are described in "Fuzzy Logic with Engineering Applications" by Timothy J. Ross, copyright 1995 by McGraw-Hill, Inc. Preferably, the centroid method, also known as the center of area or center of gravity method, is utilized to perform the defuzzification.

[0046] The design and implementation of the fuzzy logic parameter tuner 212 for the ink key control is accomplished as follows. As previously stated, the basic principle is to build the fuzzy inference system for parameter tuning of the PID parameters. The two fuzzy input variables are SID_err(j,k,t) and SID_derr(j,k,t). Each input variable is fuzzified into a plurality of membership functions. For example, each input variable can be fuzzified into five membership functions, as illustrated in Fig. 9. It should be noted that a different number of membership functions can be employed such as 4, 6 or 7.

[0047] In the control system 200 described herein, the membership functions are selected to be triangular, and are such that an input has a nonzero value for at most two membership functions simultaneously. The membership functions are as follows:

NL	(negatively large)
NM	(negatively medium)
ZE	(zero)
PM	(positively medium)
PL	(positively large)

[0048] There are two fuzzy output variables, FOp and FOi. The output sets in the preferred embodiment also include five membership functions, as illustrated in Fig. 10.

[0049] The following are examples of the if-then rules for the five membership function inference system:

1. If (sid_err is NL) and (sid_derr is NL) then (FOp is NL)(FOi is PL)
2. If (sid_err is NL) and (sid_derr is NM) then (FOp is NL)(FOi is PL)
3. If (sid_err is NL) and (sid_derr is ZE) then (FOp is PM)(FOi is PL)
4. If (sid_err is NL) and (sid_derr is PM) then (FOp is PM)(FOi is PM)
5. If (sid_err is NL) and (sid_derr is PL) then (FOp is ZE)(FOi is ZE)
6. If (sid_err is NM) and (sid_derr is NL) then (FOp is NL)(FOi is PL)
7. If (sid_err is NM) and (sid_derr is NM) then (FOp is NM)(FOi is PM)
8. If (sid_err is NM) and (sid_derr is ZE) then (FOp is PM)(FOi is PM)
9. If (sid_err is NM) and (sid_derr is PM) then (FOp is ZE)(FOi is ZE)
10. If (sid_err is NM) and (sid_derr is PL) then (FOp is ZE)(FOi is ZE)
11. If (sid_err is ZE) and (sid_derr is NL) then (FOp is NM)(FOi is NL)
12. If (sid_err is ZE) and (sid_derr is NM) then (FOp is NM)(FOi is NM)

13. If (sid_err is ZE) and (sid_derr is ZE) then (FOp is ZE)(FOi is ZE)
14. If (sid_err is ZE) and (sid_derr is PM) then (FOp is NM)(FOi is NM)
15. If (sid_err is ZE) and (sid_derr is PL) then (FOp is NM)(FOi is NL)
16. If (sid_err is PM) and (sid_derr is NL) then (FOp is ZE)(FOi is NM)
17. If (sid_err is PM) and (sid_derr is NM) then (FOp is ZE)(FOi is ZE)
18. If (sid_err is PM) and (sid_derr is ZE) then (FOp is PM)(FOi is PM)
19. If (sid_err is PM) and (sid_derr is PM) then (FOp is NM)(FOi is PM)
20. If (sid_err is PM) and (sid_derr is PL) then (FOp is NL)(FOi is PL)
21. If (sid_err is PL) and (sid_derr is NL) then (FOp is ZE)(FOi is ZE)
22. If (sid_err is PL) and (sid_derr is NM) then (FOp is PM)(FOi is PM)
23. If (sid_err is PL) and (sid_derr is ZE) then (FOp is PM)(FOi is PL)
24. If (sid_err is PL and (sid_derr is PM) then (FOp is NL)(FOi is PL)
25. If (sid_err is PL) and (sid_derr is PL) then (FOp is NL)(FOi is PL)

[0050] The fuzzy output variables are then used in the following equations:

$$FAp(j,k,t) = FAp(j,k,t\text{-}1) + alphaP(j) * FOp(j,k,t)$$

where: FAp is the fuzzy accumulator output for the proportional term, and FOp is the fuzzy tuner output for the proportional term.

$$FAi(j,k,t) = FAi(j,k,t\text{-}1) + alphaI(j) * FOi(j,k,t)$$

where: FAi is the fuzzy accumulator output for the integral term and FOi is the fuzzy tuner output for the integral term.

[0051] The alphaP and alphaI terms each take a proportion of its associated fuzzy tuner output and add that to the fuzzy accumulator. This step is intended to make the tuning process more stable.

[0052] The equations used to update the PID parameters are as follows:

$$Kp(j,k,t) = FAp(j,k,t) * MaxPGain(j) * COVERAGE(j,k)$$

$$Ki(j,k,t) = FAi(j,k,t) * MaxIGain(j) * COVERAGE(j,k)$$

$$Kd(j,k,t) = FAi(j,k,t) * MaxDGain(j) * COVERAGE(j,k)$$

where:

Kp(j,k,t), Ki(j,k,t), Kd(j,k,t) are the gain parameters for the PID controller;

MaxPGain(j), MaxIGain(j), and MaxDGain(j) are empirically determined constants for each ink color; and

COVERAGE(j,k) is the plate coverage value for color j for each ink key k.

[0053] In the preferred embodiment, COVERAGE(j,k) is set to .20 for all keys for all colors. However, the actual values of plate coverage for each ink key zone, if available, can be used to achieve faster convergence. Also, note that the FAi term is used in the calculation of Kd. However, a separate FAd term can be determined, using a FOd term as an output of the inference rules.

[0054] From the above, it follows that the incremental adjustment of the gain parameters are:

$$d\_Kp(j,k,t) = alphaP(j) * FOp(j,k,t) * MaxPGain(j) * COVERAGE(j,k)$$

$$d\_Ki(j,k,t) = alphaI(j) * FOi(j,k,t) * MaxIGain(j) * COVERAGE(j,k)$$

$$d\_Kd(j,k,t) = alphaI(j) * FOi(j,k,t) * MaxDGain(j) * COVERAGE(j,k)$$

[0055] A list of the exemplary values pertaining to the Harris M1000B printing press used in the preceding equations are as follows:

BLACK (j=K)

MaxPGain = 15
MaxIGain = 45
MaxDGain = 20
AlphaP = .05
AlphaI = .10

CYAN (j=C)

MaxPGain = 20
MaxIGain = 30
MaxDGain = 25
AlphaP = .08
AlphaI = .10

MAGENTA (j=M)

MaxPGain = 20
MaxIGain = 35
MaxDGain = 25
AlphaP = .08
AlphaI = .15

YELLOW (j=Y)

MaxPGain = 20
MaxIGain = 60
MaxDGain = 25
AlphaP = .15
AlphaI = .30

[0056] The initial values of Kp, Ki, Kd can be determined by the known Ziegler-Nichols method.

[0057] As a further example, an input set including six membership functions instead of five can be defined. In this case, the six input membership functions could be the same as the five membership functions previously defined, with the exception that ZE is divided into two functions, termed PZE (positive zero) and NZE (negative zero). With five output sets, the fuzzy logic adaptive controller 216 could use the following inference rules:

1. If (sid_err is NL) and (sid_derr is NL) then (pgain is NL)(igain is PL)(dgain is PL)
2. If (sid_err is NL) and (sid_derr is NM) then (pgain is NL)(igain is PL)(dgain is PM)
3. If (sid_err is NL) and (sid_derr is NZE) then (pgain is PM)(igain is PM)(dgain is ZE)
4. If (sid_err is NL) and (sid_derr is PZE) then (pgain is PM)(igain is PM)(dgain is ZE)
5. If (sid_err is NL) and (sid_derr is PM) then (pgain is PM)(igain is PM)(dgain is NM)
6. If (sid_err is NL) and (sid_derr is PL) then (pgain is ZE)(igain is ZE)(dgain is NL)
7. If (sid_err is NM) and (sid_derr is NL) then (pgain is NL)(igain is PL)(dgain is PL)
8. If (sid_err is NM) and (sid_derr is NM) then (pgain is NM)(igain is PM)(dgain is PM)
9. If (sid_err is NM) and (sid_derr is NZE) then (pgain is PM)(igain is PM)(dgain is ZE)
10. If (sid_err is NM) and (sid_derr is PZE) then (pgain is PM)(igain is ZE)(dgain is ZE)
11. If (sid_err is NM) and (sid_derr is PM) then (pgain is ZE)(igain is ZE)(dgain is ZE)
12. If (sid_err is NM) and (sid_derr is PL) then (pgain is ZE)(igain is NM)(dgain is NM)
13. If (sid_err is NZE) and (sid_derr is NL) then (pgain is NM)(igain is PL)(dgain is PL)
14. If (sid_err is NZE) and (sid_derr is NM) then (pgain is NM)(igain is PM)(dgain is PM)
15. If (sid_err is NZE) and (sid_derr is NZE) then (pgain is ZE)(igain is ZE)(dgain is ZE)
16. If (sid_err is NZE) and (sid_derr is PZE) then (pgain is ZE)(igain is ZE)(dgain is ZE)
17. If (sid_err is NZE) and (sid_derr is PM) then (pgain is NM)(igain is NL)(dgain is PM)
18. If (sid_err is NZE) and (sid_derr is PL) then (pgain is NM)(igain is NL)(dgain is PL)
19. If (sid_err is PZE) and (sid_derr is NL) then (pgain is NM)(igain is NL)(dgain is PL)
20. If (sid_err is PZE) and (sid_derr is NM) then (pgain is NM)(igain is NM)(dgain is PM)

21. If (sid_err is PZE) and (sid_derr is NZE) then (pgain is ZE)(igain is ZE)(dgain is ZE)
22. If (sid_err is PZE) and (sid_derr is PZE) then (pgain is ZE)(igain is ZE)(dgain is ZE)
23. If (sid_err is PZE) and (sid_derr is PM) then (pgain is NM)(igain is PM)(dgain is PM)
24. If (sid_err is PZE) and (sid_derr is PL) then (pgain is NM)(igain is PL)(dgain is PL)
25. If (sid_err is PM) and (sid_derr is NL) then (pgain is ZE)(igain is NM)(dgain is NM)
26. If (sid_err is PM) and (sid_derr is NM) then (pgain is ZE)(igain is ZE)(dgain is ZE)
27. If (sid_err is PM) and (sid_derr is NZE) then (pgain is PM)(igain is ZE)(dgain is ZE)
28. If (sid_err is PM) and (sid_derr is PZE) then (pgain is PM)(igain is PM)(dgain is ZE)
29. If (sid_err is PM) and (sid_derr is PM) then (pgain is NM)(igain is PM)(dgain is PM)
30. If (sid_err is PM) and (sid_derr is PL) then (pgain is NM)(igain is PL)(dgain is PL)
31. If (sid_err is PL) and (sid_derr is NL) then (pgain is ZE)(igain is ZE)(dgain is NL)
32. If (sid_err is PL) and (sid_derr is NM) then (pgain is PM)(igain is PM)(dgain is NM)
33. If (sid_err is PL) and (sid_derr is NZE) then (pgain is PM)(igain is PM)(dgain is ZE)
34. If (sid_err is PL) and (sid_derr is PZE) then (pgain is PM)(igain is PM)(dgain is ZE)
35. If (sid_err is PL) and (sid_derr is PM) then (pgain is NL)(igain is PL)(dgain is PM)
36. If (sid_err is PL) and (sid_derr is PL) then (pgain is NL)(igain is PL)(dgain is PL)

[0058] As previously stated, the effective ink key settings from the PID controller 212 can be used to directly control the ink keys, or can be further processed by the decoupling computation unit 220 to generate adjusted or actual ink key settings.

[0059] The problem of ink key coupling is due to the spread of ink by the movement of the vibrator rollers. If the controller 204 determines that the ink flow to a particular ink key zone should be increased, because the increased ink amount spreads to adjacent ink key zones, increasing the ink flow to one zone will also increase the ink flow to neighboring zones. In order to compensate for this, the ink flow to neighboring keys must be decreased. This will have an effect on the neighboring ink keys as well.

[0060] Before describing one method to compensate for ink spread, it is necessary to describe different ways the color control system can operate to control the ink keys with sequential SID readings of the color patches. One side of a web has 24 ink key zones, which correspond to 24 SID measurements. One method to implement the system is to wait until all 24 SID measurements are obtained, and then change all 24 ink key readings at once. However, this method is slow. Another way to implement the system is to change an ink key immediately after the corresponding SID measurement is obtained, without accounting for the effects of neighboring ink keys. In this case, the method will eventually stabilize, but it does not take into account the effects of neighboring ink keys.

[0061] An ink key distribution function or ink key spread function can be determined which represents the spread of ink from a source of ink which is the width of an ink key zone. The ink key spread function can be represented by a vector whose elements are representative of ink amounts in a corresponding zone. One way to determine an ink key spread vector is to open one ink key and see how ink is spread into adjacent ink key zones. For example, one such test resulted in the following vector V:

$$V = [\ .007\ .009\ .016\ .043\ .196\ .460\ .196\ .043\ .016\ .009\ .007\ ]$$

[0062] Vector V is obtained by averaging the experimentally obtained data over the width corresponding to each ink key zone, and then scaling so that the addition of all vector elements adds up to 1. The elements in vector V can then be interpreted as the fraction of ink which is distributed to a specific ink key zone. Each ink key results in its own distribution of ink, which is proportional to the ink key opening. For the Harris M1000B press, 46% of the ink provided by a given ink key is passed directly into its corresponding ink key zone, 20% is passed to the immediate neighboring zones, and 4% is passed to the next set of neighbors, and so on.

[0063] The effects of the vibrator rollers are taken into account by the decoupling computation unit 220 of Fig. 1. Mathematically, this is a deconvolution in which one seeks to find the ink key settings given an ink key distribution function and the effective ink key settings. In the preferred embodiment of the color control system 200, the SID measurements for respective ink key zones reach the PID loop serially in time rather than all at once.

[0064] A matrix equation can be written which relates actual and effective ink key openings:

$$E = S\ A$$

where E is a vector representing the effective ink key openings, and A is a vector representing the actual ink key openings, and S is an ink key spread matrix, determined from vector V. E and A are both a 24 by 1 element vectors. S is a 24 by 24 element matrix. (The size is determined by the fact that there are 24 ink keys on the Harris M1000B press). If the ink spread is invariant across the ink keys, then matrix S is a Toeplitz matrix, that is, a matrix in which each row is a shifted version of the row above. Each row contains the elements of the vector V. Matrix S is illustrated in Fig. 11.

[0065] The above equation can be rewritten to solve for A:

$$A = S^{-1} E$$

[0066] Provided matrix S is invertible, a solution for actual ink key openings is obtainable. However, inverted matrix S^{-1} may be slightly ill-conditioned. This means that the matrix may amplify noise. Additionally, the inverted matrix includes entries in each of the 24 columns. Thus to multiply E by a row of S^{-1} requires the use of all 24 entries.

[0067] One approach is to approximate S^{-1} with a matrix M^{-1} which approximates what S^{-1} does. That is, M^{-1} approximates an inverse spread function. One approximation of M^{-1} is illustrated in Fig. 12. Matrix M^{-1} is a symmetric matrix, and the numbers used to derive this matrix are .518, .196 and .045. In other words, for any ink key zone, it is assumed that 51.8% of the ink remains in that zone, 19.6% goes to immediate neighbors, and 4.5% goes to the neighbors two zones away. Using M^{-1} instead of S^{-1}, because there are at most 5 entries in a row of M^{-1}, it is necessary to obtain at most 5 SID measurements at a given time before an ink key change can be implemented. The numbers .518, .196 and .045 are a particular set of spread coefficients that will produce convergence of the control loop.

[0068] Use of the matrix M^{-1} may introduce edge effects in the calculated ink key settings for the ink keys on each end. The edge effects are due to the fact that at an end, an increased ink amount for an ink key will affect the amount of ink fed to the adjacent keys on one side only. One approach to more accurately computing the ink key settings for the ink keys on the ends may be accomplished by modifying the element values in the matrix M^{-1}. For example, the ink that theoretically would be fed to a side of the web is accounted for by including that amount in the amount of ink fed to the end ink key zone. In other words, the element in the first row, first column of M^{-1} would be increased by adding [(.196 + .045)/.518]. Similarly, the element in the second row, first column of M^{-1} would be increased by adding (.045/.518). The ink key settings for the affected ink key settings on the other side of the web would be taken into account by modifying the elements in the last column of the last and second to last rows. The element in the last row, last column would be increased by adding [(.196 + .045)/.518]. Also, the element in the second to last row, last column would be increased by adding (.045/.518). Various other refinements are possible to account for edge effects.

[0069] In the preferred embodiment, the control loop operates with the following constraints: if the measured SID value is within .1 of the desired SID value, then the PID controller 216 operates without using the fuzzy logic parameter tuner 212 to tune the PID gain parameters, because of concern that the rule set is not optimized at that range. Additionally, there is a dead band zone. For example, if the SID value is within .07 of the desired SID value, the PID controller 216 does not operate to make further adjustments to the ink key settings.

[0070] Because both the ink key settings and the ratchet assembly rotation angle control the amount of ink fed to the respective ink key zones, it is possible to change the ink key settings and/or the ratchet setting R in the ratchet assembly. In theory, any ratchet setting is acceptable. In practice, however, there are constraints on the ratchet setting. Ratchet settings which are too low may require ink key openings which are beyond the physical limits of the ink key. On the other hand, setting the ratchet too high leads to very low ink key openings, and a greater sensitivity of ink film thickness to changes in ink key opening. This reduces the precision in the ink key opening.

[0071] The optimal condition is met when the ratchet setting is as low as possible without forcing the ink key openings beyond a certain fraction of the physical limit. This fraction is necessary to allow room for subsequent adjustment.

[0072] One complication which may occur is that the control algorithm may call for an ink key setting which is beyond the physical limits of an ink key. For example, the requested ink key setting may be for an opening greater than 100%, or for a setting which is negative. In the simplest implementation, requested ink key openings which are out of range are merely clipped, so that they do not go beyond the extreme values.

[0073] In the preferred embodiment, there are separate actions for an ink key being requested to move above 100%, and for an ink key being requested to move to less than zero. In the former case, it may still be possible to attain the proscribed density by increasing the ratchet setting. To accomplish this, the ratchet setting is increased by such an amount as to bring the requested ink key setting within the physical limits.

[0074] Since the ratchet setting and the ink key opening are multiplicative, the correction is straightforward. If, for example, the requested ink key opening is 120%, the current ratchet setting must be increased to at least 1.2 times its current value. In this case, the new ink key opening would be set to 100%. Alternatively, it may be preferred to increase the ratchet setting 10% higher in order to allow for some further range of adjustment.

[0075] When the ratchet setting is changed, all the ink key openings must be compensated accordingly. If the ratchet setting is increased by multiplying by Q, the ink key openings must all be decreased by dividing by Q.

[0076] It is understood that the invention is not confined to the particular construction and arrangement of parts herein illustrated and described, but embraces all such modified forms thereof as may come within the scope of the following claims. It will be apparent that many modifications and variations are possible in light of the above teachings. It therefore is to be understood that within the scope of the appending claims, the invention may be practiced other than is specifically described. Alternative embodiments and variations of the method taught in the present specification may suggest themselves to those skilled in the art upon reading of the above description.

[0077] For example, the above color control system was designed to be implemented in conjunction with a Telecolor II type ink fountain. However, such a system will also be useful in connection with other ink fountains.

Claims

1. An adaptive control system for use in conjunction with a printing process for controlling the amount of ink fed to a substrate, the system comprising:

   a controller for calculating an ink feed amount based upon a measured ink color value and a target ink color value, the controller using at least one gain parameter; and
   an adaptive tuner which communicates with the controller and adjusts the at least one gain parameter in response to the operation of the printing process.

2. The adaptive control system of claim 1 wherein the adaptive tuner is a fuzzy logic adaptive tuner.

3. The adaptive control system of claim 1 wherein the controller implements fuzzy logic.

4. The adaptive control system of claim 1 wherein the controller includes a PID controller.

5. The adaptive control system of claim 4 wherein the adaptive tuner is a fuzzy logic adaptive tuner.

6. The adaptive control system of claim 4 wherein the at least one gain parameter includes at least one of the P, I and D gain parameters of the PID controller.

7. The adaptive control system of claim 1 wherein the ink color value is a density value.

8. A method for controlling the amount of ink fed to a substrate of a printing press by an ink metering device, said method comprising the steps:

   selecting at least one controller gain parameter;
   adaptively adjusting the at least one controller gain parameter to compensate for variations in printing press performance;
   calculating a setting of the ink metering device using the adjusted at least one controller gain parameter; and
   communicating the setting to the ink metering device.

9. The method of claim 8 wherein the at least one controller gain parameter includes the P, I and D gain parameters of a PID controller.

10. The method of claim 8 wherein the at least one controller gain parameter is adjusted using fuzzy logic.

11. The method of claim 8 wherein the setting is calculated based upon a measured ink color value and a target ink color value.

12. The method of claim 11 wherein the ink color value is a density value.

13. An adaptive tuner for use with a controller to control the amount of ink fed by an ink metering device to the substrate of a printing press, the tuner including an adjustment mechanism for determining an adjustment to at least one gain parameter of the controller in response to changes in printing press performance.

14. The adaptive tuner of claim 13 wherein the adjustment mechanism utilizes a measured ink density value and a target ink density value.

15. The adaptive tuner of claim 13 wherein the at least one gain parameter includes at least one of the P, I and D gain parameters of a PID controller.

16. The adaptive tuner of claim 13 wherein the adjustment mechanism implements fuzzy logic.

17. A method for adaptively controlling the settings for at least one ink control device in a printing press wherein the ink control device controls the amount of ink supplied to a zone of a first roller, wherein ink is transferred from the first roller to a roller train and then to a substrate to print an image, the method comprising:

(a) measuring the ink color value of a zone;
(b) communicating the measured ink color value to a controller having at least one gain parameter and to an adaptive tuner;
(c) calculating a new setting of the ink control device corresponding to the zone using the controller and the adaptive tuner such that the adaptive tuner calculates an adjustment to the at least one gain parameter based upon the performance of the printing press;
(d) changing the setting of the ink control device to the new setting; and
(f) repeating steps (b) through (e) until the measured ink color value is within a predetermined range.

18. The method of claim 17 wherein the ink control device is an ink key.

19. The method of claim 17 wherein the ink color value is a density value.

20. The method of claim 17 wherein the ink control device includes a ratchet assembly.

21. The method of claim 17 wherein the adaptive tuner implements fuzzy logic.

22. The method of claim 17 wherein the controller includes a PID controller.

23. The method of claim 17 wherein the at least one gain parameter includes at least one of the P, I and D gain parameters of a PID controller.

24. An adaptive control system for controlling ink fed to a substrate in a printing press, said control system comprising:

an ink color measurement device operationally adjacent the substrate;
an ink feed meter which controls the amount of ink fed to the substrate; and
an adaptive controller in communication with the measurement device to receive ink color values therefrom and in communication with the meter to communicate thereto changes to be made in the amount of ink fed to the substrate so that the measured ink density value converges toward a target ink density value, the adaptive controller calculating the changes to be made in the amount of ink fed to the substrate based upon operation of ink feed meter.

25. The adaptive control system of claim 24 wherein the ink value measurement device is an ink density measurement device.

26. The adaptive control system of claim 24 wherein the adaptive controller is a fuzzy logic adaptive controller.

27. The adaptive control system of claim 24 wherein the adaptive controller includes a controller and a parameter tuner.

28. The adaptive control system of claim 27 wherein the parameter tuner is a fuzzy logic parameter tuner.

29. The adaptive control system of claim 27 wherein the parameter tuner adjusts at least one gain parameter of the controller and the at least one gain parameter includes at least one of the P, I and D gain parameters of a PID controller.

30. The adaptive control system of claim 24 wherein the ink feed meter includes an ink key.

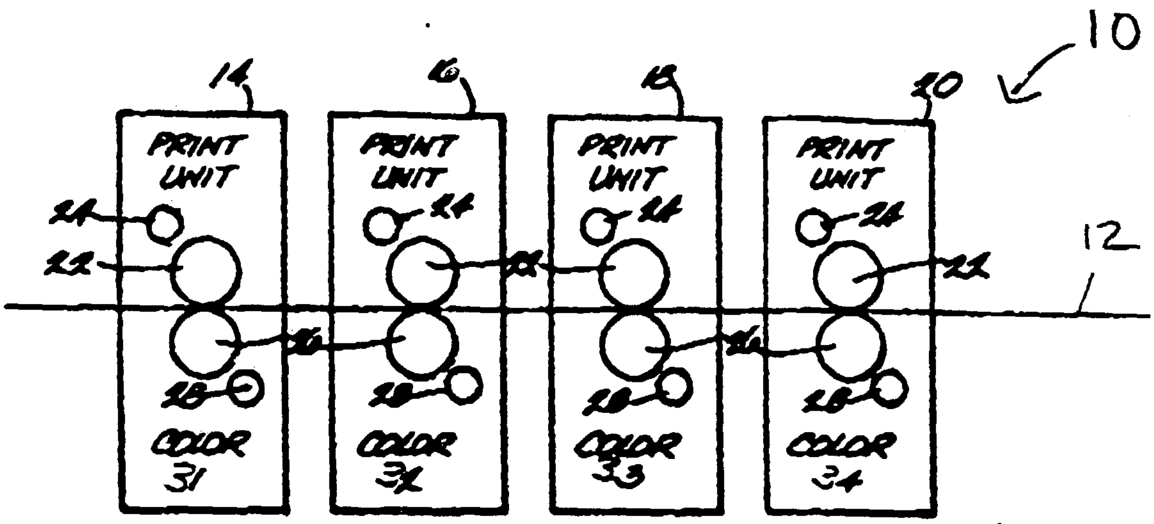
10
12
PRINT UNIT
COLOR 31
PRINT UNIT
COLOR 32
PRINT UNIT
COLOR 33
PRINT UNIT
COLOR 34

Fig. 1

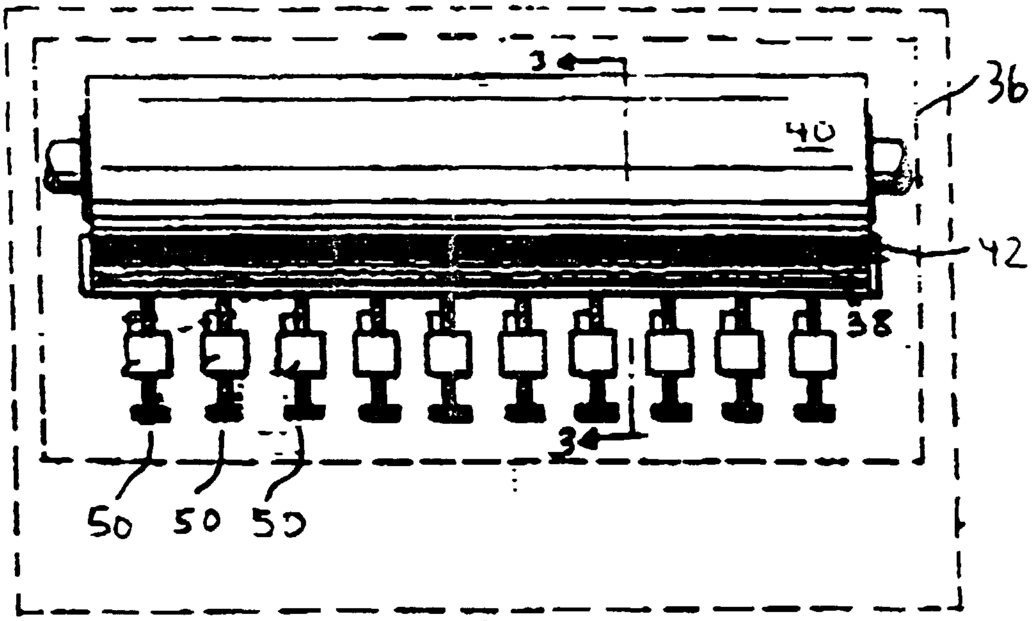
3
3
40
36
42
38
50 50 50

FIG. 2

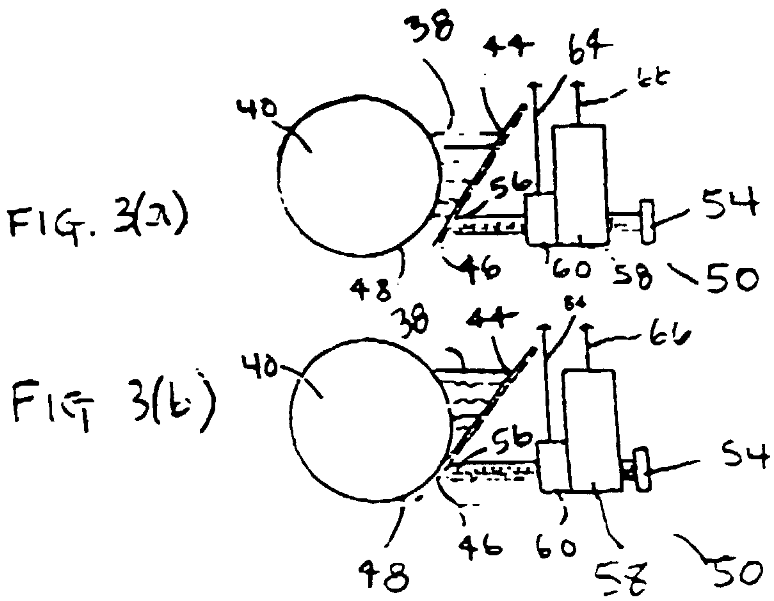
FIG. 3(a)
38
44
64
40
56
54
48
46
60
58
50
FIG 3(b)
38
44
64
66
40
56
54
48
46
60
58
50

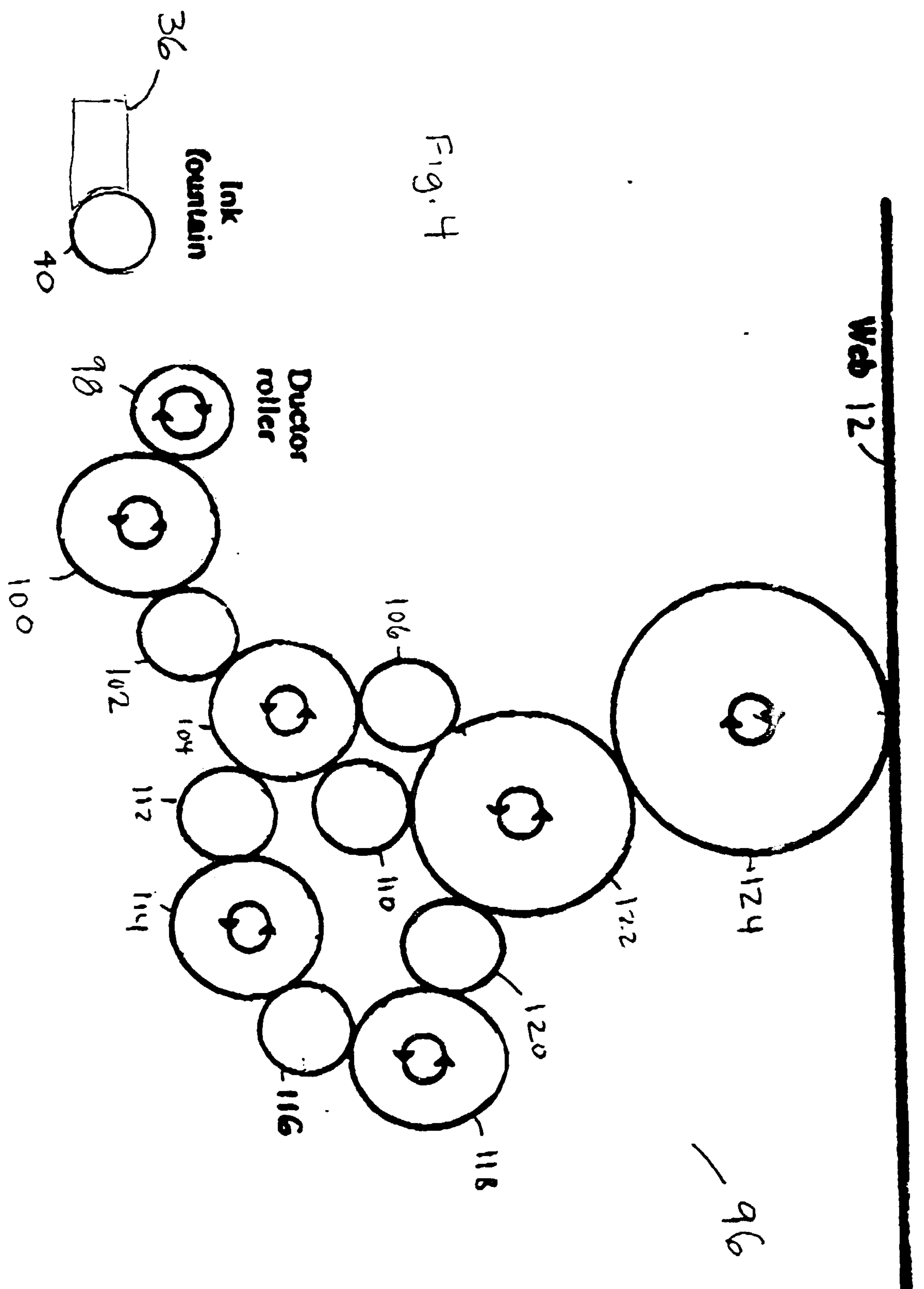
Ink fountain
Ductor roller
Web 12
36
40
98
100
102
104
106
110
112
114
116
118
120
122
124
96

Fig. 4

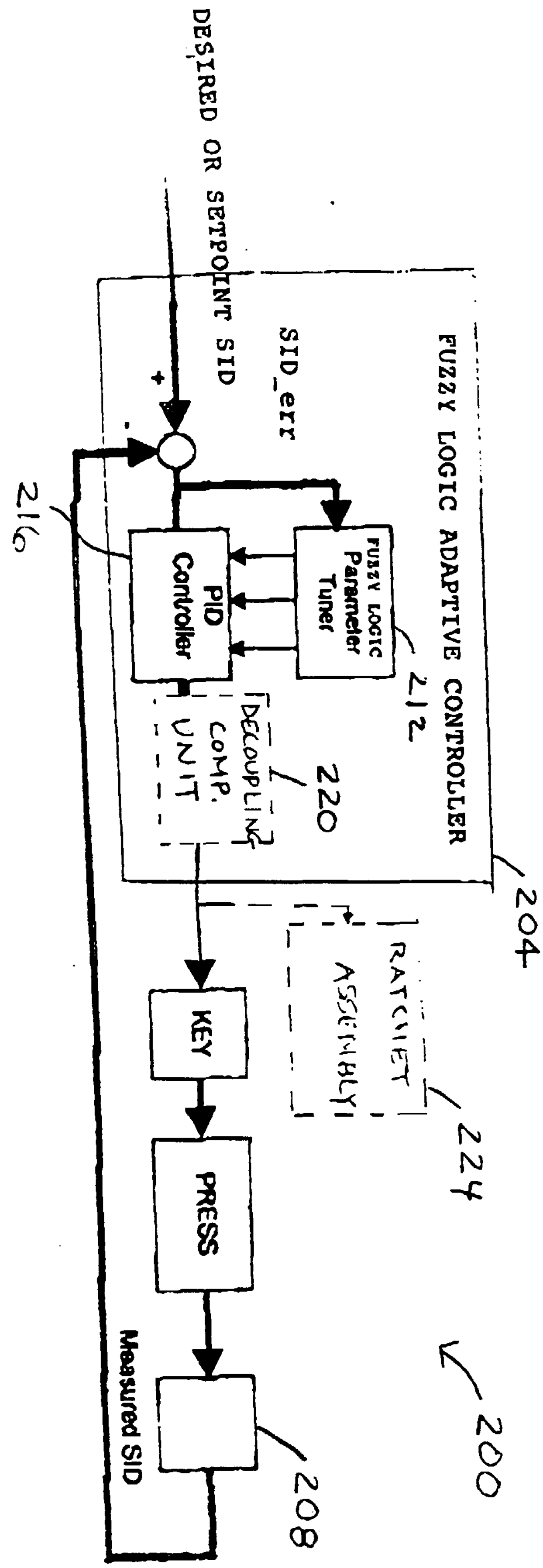
DESIRED OR SETPOINT SID
SID_err
+
-
FUZZY LOGIC ADAPTIVE CONTROLLER
FUZZY LOGIC Parameter Tuner
PID Controller
DECOUPLING COMP. UNIT
212
216
220
204
KEY
PRESS
RATCHET ASSEMBLY
224
Measured SID
208
200

Fig. 5

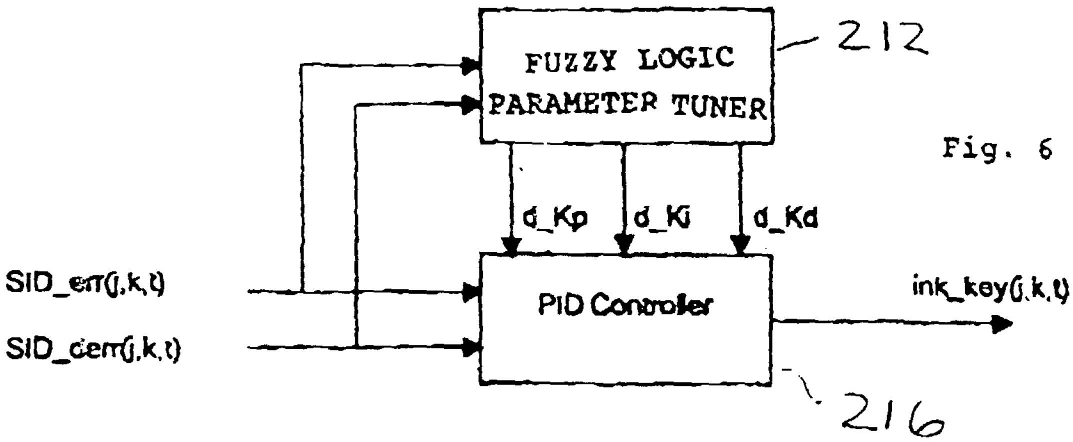

Fig. 6

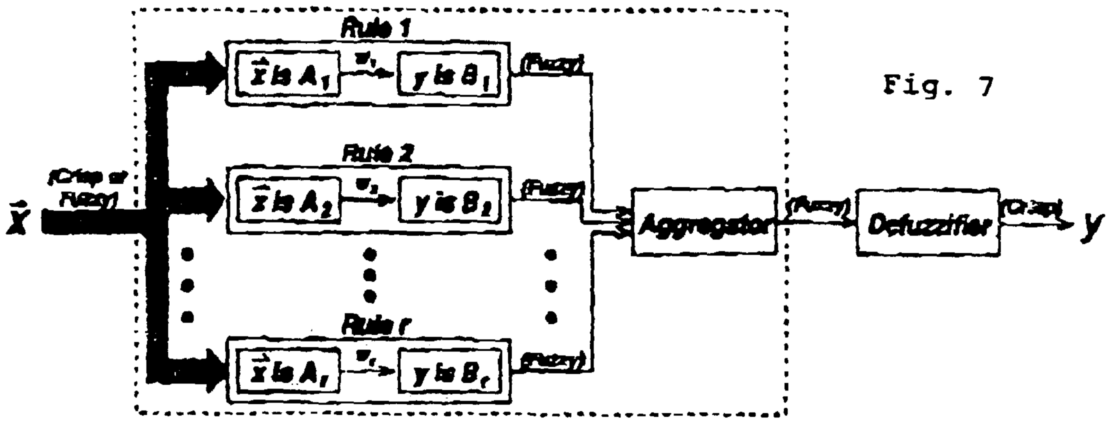

Fig. 7

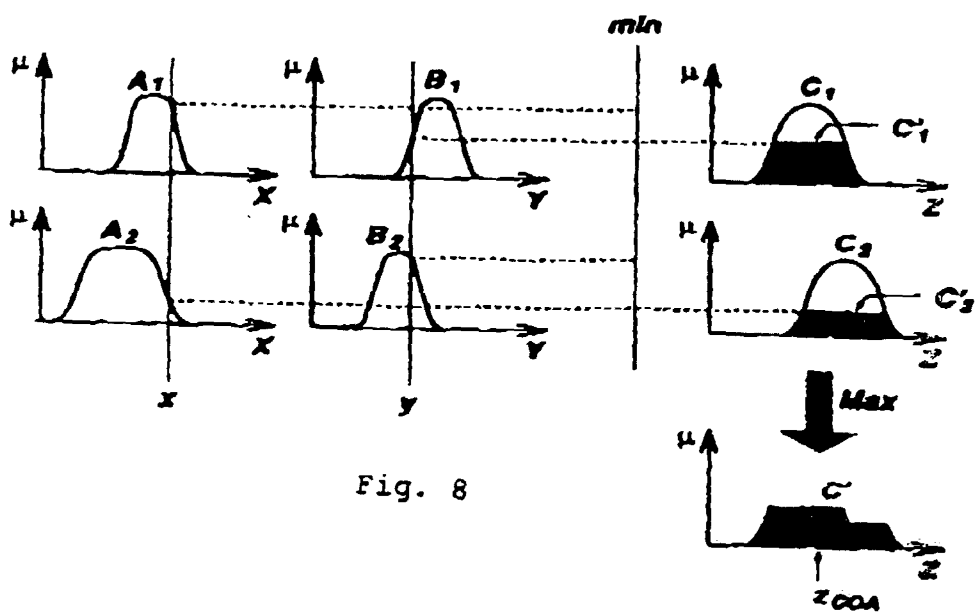

Fig. 8

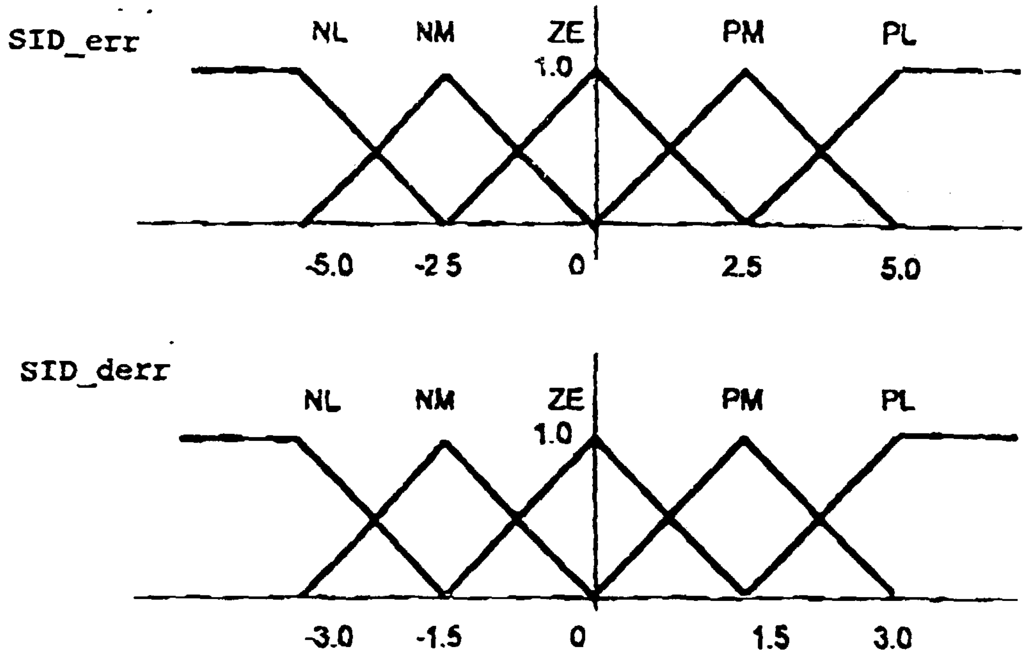
SID_err
NL
NM
ZE
PM
PL
1.0
-5.0
-2 5
0
2.5
5.0
SID_derr
NL
NM
ZE
PM
PL
1.0
-3.0
-1.5
0
1.5
3.0

Fig. 9

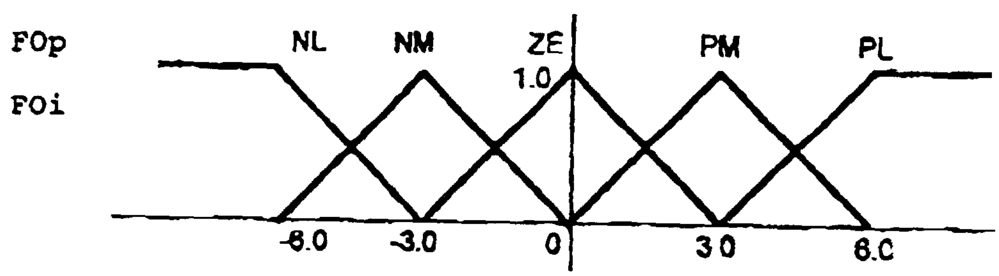
FOp
FOi
NL
NM
ZE
PM
PL
1.0
-6.0
-3.0
0
3 0
6.0

Fig. 10

.460	.196	.043	.016	.009	.007	.000	.000	.000	. . .		. . .		
.196	.460	.196	.043	.016	.009	.007	.000	.000	. . .		. . .		
.043	.196	.460	.196	.043	.016	.009	.007	.000	. . .		. . .		
.016	.043	.196	.460	.196	.043	.016	.009	.007	. . .		. . .		
.009	.016	.043	.196	.460	.196	.043	.016	.009	.007		. . .		
.007	.009	.016	.043	.196	.460	.196	.043	.016	.009	.007			
.000	.007	.009	.016	.043	.196	.460	.196	.043	.016	.009	.007		
.000	.000	.007	.009	.016	.043	.196	.460	.196	.043	.016	.009	.007	
⋮	⋮												
⋮	⋮												
⋮						.007	.009	.016	.043	.196	.460	.196	.043
⋮						.000	.007	.009	.016	.043	.196	.460	.196
.000	...	...				...	.000	.007	.009	.016	.043	.196	.460

Fig. 11

$$
M^{-1} = \begin{bmatrix}
\frac{1}{.518} & \frac{-.196}{.518} & \frac{-.045}{.518} & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
\frac{-.196}{.518} & \frac{1}{.518} & \frac{-.196}{.518} & \frac{-.045}{.518} & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
\frac{-.045}{.518} & \frac{-.196}{.518} & \frac{1}{.518} & \frac{-.196}{.518} & \frac{-.045}{.518} & 0 & \cdot & \cdot & \cdot & 0 \\
0 & \frac{-.045}{.518} & \frac{-.196}{.518} & \frac{1}{.518} & \frac{-.196}{.518} & \frac{-.045}{.518} & 0 & \cdot & \cdot & 0 \\
\cdot & & & & \cdot & \cdot & & & & \cdot \\
\cdot & & & & \cdot & \cdot & & & & \cdot \\
0 & \cdot & \cdot & \cdot & 0 & 0 & 0 & \frac{-.045}{.518} & \frac{-.196}{.518} & \frac{1}{.518}
\end{bmatrix}
$$

Fig. 12