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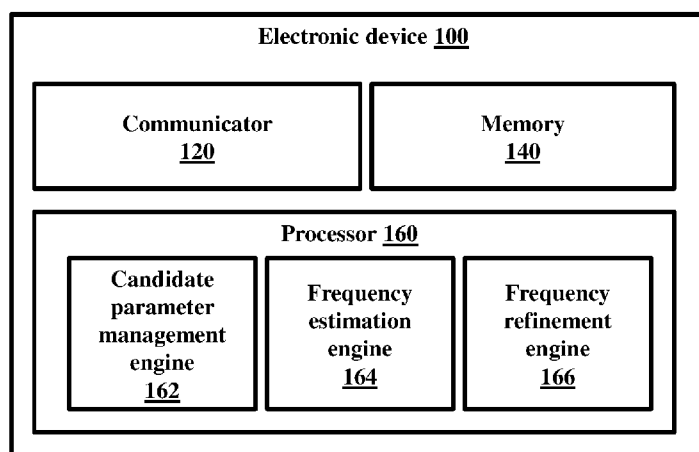


FIG. 1

(57) Abstract: Embodiments herein provide a method for estimating frequencies of multiple sinusoids by an electronic device (100). The method includes receiving, by the electronic device (100), a received signal comprising a plurality of sinusoids, where each sinusoid in the received signal comprises at least one unknown parameter and determining, by the electronic device (100), whether a candidate parameter is less than zero, where the candidate parameter is a function of an estimated SNR and an estimated threshold SNR. Further, the method includes performing, by the electronic device (100), one of: estimation of frequencies of the plurality of sinusoids in the received signal using a first frequency estimation technique, in response to determining that the candidate parameter is less than zero, and estimation of the frequency of the sinusoids using a second frequency estimation technique, in response to determining that the candidate parameter is greater than zero.

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**METHOD AND ELECTRONIC DEVICE FOR ESTIMATING
FREQUENCIES OF MULTIPLE SINUSOIDS USING LINE
SPECTRAL PAIRS-FILED**

FIELD OF INVENTION

[0001] The embodiments herein relates to frequency estimation. More particularly relates to a method and electronic device for estimating frequencies of multiple sinusoids. The present application is based on, and claims priority from an Indian Application Number **202041012231** filed on **20th March, 2020**
5 the disclosure of which is hereby incorporated by reference herein.

BACKGROUND OF THE INVENTION

[0002] In general, many applications such as Radio Detection and Ranging (RADAR), sound navigation ranging (SONAR), etc., requires estimation of frequencies of multiple sinusoids in presence of noise. Also, the problem of estimating a direction of arrival (DoA) of multiple plane waves is
10 also the frequency estimation in disguise. The DoA problem plays an important role in cellular and wireless communication, especially in the context of 5G networks where antenna arrays are deployed for accurately estimating an angle of arrival. All practical frequency estimators display a threshold effect, *i.e.*, the variance of the estimates increases sharply whenever a signal to noise ratio
15 (SNR) falls below a certain value (which is estimator dependent). Maximum-likelihood estimator (MLE) has the lowest threshold among existing practical estimators but is computationally the most burdensome. Subspace-based methods are computationally less burdensome but have a higher threshold than the MLE. Any estimator that has the same threshold as that of MLE but with a
20 less computational burden has a very significant practical impact.

[0003] The above information is presented as background information only to help the reader to understand the present invention. Applicants have

made no determination and make no assertion as to whether any of the above might be applicable as prior art with regard to the present application.

OBJECT OF INVENTION

[0004] The principal object of the embodiments herein is to provide a method and electronic device for estimating frequencies of multiple sinusoids.

5 [0005] Another object of the embodiments herein is to process a received signal comprising a plurality of sinusoids.

[0006] Another object of the embodiments herein is to determine an estimated SNR and an estimated threshold SNR of the received signal based on eigenvalues of a correlation matrix.

10 [0007] Another object of the embodiments herein is to determine whether a candidate parameter is less than zero based on a difference between an estimated SNR and an estimated threshold SNR.

[0008] Another object of the embodiments herein is to perform estimation of frequencies of the plurality of sinusoids in the received signal
15 using a first frequency estimation technique when the candidate parameter is less than zero.

[0009] Another object of the embodiments herein is to perform estimation of the frequency of the sinusoids using a second frequency estimation technique when the candidate parameter is greater than zero.

20

SUMMARY

[0010] Accordingly, the embodiments herein provide a method for estimating frequencies of multiple sinusoids by an electronic device (100). The method includes processing, by the electronic device (100), a received signal comprising a plurality of sinusoids, where each sinusoid in the received signal comprises at least one unknown parameter and determining, by the electronic device (100), whether a candidate parameter is less than zero, where the candidate parameter is a function of an estimated SNR and an estimated threshold SNR. Further, the method includes performing, by the electronic device (100), one of: estimation of the frequencies of the plurality of the sinusoids in the received signal using a first frequency estimation technique, in response to determining that the candidate parameter is less than zero, and estimation of the frequencies of the sinusoids using a second frequency estimation technique, in response to determining that the candidate parameter is greater than zero.

[0011] In an embodiment, the method for determining, by the electronic device (100), whether the candidate parameter is less than zero includes estimating, by the electronic device (100), a correlation matrix of the received signal and determining, by the electronic device (100), eigenvalues of the correlation matrix. Further, the method includes determining, by the electronic device (100), an estimated SNR of the received signal based on the eigenvalues of the correlation matrix and determining, by the electronic device (100), an estimated threshold SNR, where the estimated threshold SNR is dependent on the eigenvalues of the correlation matrix; and determining, by the electronic device (100), whether the candidate parameter is less than zero based on a difference between the estimated SNR and the estimated threshold SNR.

[0012] In an embodiment, the frequencies of the plurality of sinusoids in the received signal are estimated using the first frequency estimation technique. The method includes determining, by the electronic device (100), a

linear prediction (LP) polynomial using one of: Root-MUSIC technique, Min-Norm technique and Kumaresan-Tufts technique. Further, the method also includes determining, by the electronic device (100), initial frequency estimates from roots of the LP polynomial and generating, by the electronic device (100),
5 line spectral frequencies (LSF) of the LP polynomial, wherein the LSF of the LP polynomial represents roots of two polynomials derived from the LP polynomial. Furthermore, the method includes determining, by the electronic device (100), a set of LSF that are second closest to the initial frequency estimates and determining, by the electronic device (100), a set of LSF that are
10 third closest to the initial frequency estimates. Furthermore, the method also includes combining, by the electronic device (100), the set of LSF that are second closest to the initial frequency estimates and the set of LSF that are third closest to the initial frequency estimates to form a combined set of LSF, where a size of the combined set of LSF is at most double a number of sinusoids. The
15 method also includes determining, by the electronic device (100), a plurality of possible sets of candidate frequencies from the combined set of LSF and discarding, by the electronic device (100), outlier frequency estimates from the plurality of possible set of candidate frequencies to obtain a first pared down set and discarding, by the electronic device (100), redundant frequency
20 estimates from the first pared down set to obtain a final pared down sets of candidate frequencies. The method then includes performing by the electronic device (100), a refinement of the final pared down sets of candidate frequencies using a gradient descent technique; and estimating, by the electronic device (100), the frequencies of the plurality of sinusoids by choosing an optimal
25 frequency estimate based on likelihood values from the refined final pared down sets of the gradient descent step.

[0013] In an embodiment, discarding, by the electronic device (100), outlier frequency estimates from the plurality of possible sets of candidate frequencies to obtain the first pared down set includes determining, by the

electronic device (100), the outlier frequency estimates based on likelihood values associated with each set of the plurality of possible sets of candidate frequencies; and discarding, by the electronic device (100), the outlier frequency estimates from the plurality of possible sets of candidate frequencies
5 to obtain the first pared down set of candidate frequency estimates.

[0014] In an embodiment, discarding, by the electronic device (100), the redundant frequency estimates from the first pared down set to obtain the final pared down sets of candidate frequencies includes determining by the electronic device (100), the redundant frequency estimates based on nearness
10 of frequency criterion; and discarding, by the electronic device (100), the redundant frequency estimates from the first pared down set to obtain the final pared down sets of candidate frequencies.

[0015] In an embodiment, the second frequency estimation technique is one of: an Estimation of Signal Parameters via Rotational Invariance Technique
15 (ESPRIT), a Multiple Signal Classification (MUSIC), a Minimum-Norm (Min-Norm).

[0016] Accordingly, the embodiments herein provide an electronic device (100) for estimating frequencies of multiple sinusoids. The electronic device (100) includes a communicator (120), memory (140) and a processor
20 (160) coupled to the memory (140). The processor (160) is configured to process a received signal comprising a plurality of sinusoids, where each sinusoid in the received signal comprises at least one unknown parameter and determine whether a candidate parameter is less than zero, where the candidate parameter is a function of an estimated SNR and an estimated threshold SNR.
25 Further, the processor (160) is configured to perform one of: estimation of frequencies of the plurality of sinusoids in the received signal using a first frequency estimation technique, in response to determining that the candidate parameter is less than zero, and estimation of the frequency of the sinusoids

using a second frequency estimation technique, in response to determining that the candidate parameter is greater than zero.

[0017] These and other aspects of the embodiments herein will be better appreciated and understood when considered in conjunction with the following description and the accompanying drawings. It should be understood, however, 5 that the following descriptions, while indicating preferred embodiments and numerous specific details thereof, are given by way of illustration and not of limitation. Many changes and modifications may be made within the scope of the embodiments herein without departing from the spirit thereof, and the 10 embodiments herein include all such modifications.

BRIEF DESCRIPTION OF FIGURES

[0018] This invention is illustrated in the accompanying drawings, throughout which like reference letters indicate corresponding parts in the various figures. The embodiments herein will be better understood from the following description with reference to the drawings, in which:

[0019] FIG. 1 is a block diagram of an electronic device (100) for estimating frequencies of multiple sinusoids, according to the embodiments as disclosed herein;

[0020] FIG. 2 is a flow chart for a method for estimating frequencies of multiple sinusoids by the electronic device (100), according to the embodiments as disclosed herein;

[0021] FIG. 3 is a graph plot illustrating Line Spectral Pair (LSP) and roots of an M -th order linear prediction polynomial $A(z)$ for three sinusoids, according to the embodiments as disclosed herein;

[0022] FIG. 4 is a graph plot illustrating Mean Squared Error (MSE) of proposed LSF-based method in comparison to existing methods, according to the embodiments as disclosed herein;

[0023] FIG. 5 is a graph plot illustrating an estimated and actual threshold values for a two sinusoids in an example scenario, according to the embodiments as disclosed herein;

[0024] FIG. 6A is a graph plot illustrating a histogram of estimated threshold SNRs from noisy data, according to the embodiments as disclosed herein;

[0025] FIG. 6B is a graph plot illustrating a histogram of estimated SNR, according to the embodiments as disclosed herein;

[0026] FIG. 7A is a graph plot illustrating probability of a candidate parameter being greater than zero for a two-sinusoid example, according to the embodiments as disclosed herein;

[0027] FIG. 7B is a graph plot illustrating conditional probability of the candidate parameter being greater than zero given that an estimated frequency is an outlier for a two-sinusoid example, according to the embodiments as disclosed herein; and

- 5 [0028] FIG. 8 illustrates a Mean Squared Error (MSE) plot for three sinusoids with random amplitudes and frequencies averaged over 50k trials, according to the embodiments as disclosed herein.

DETAILED DESCRIPTION OF INVENTION

[0030] Various embodiments of the present disclosure will now be described in detail with reference to the accompanying drawings. In the following description, specific details such as detailed configuration and components are merely provided to assist the overall understanding of these
5 embodiments of the present disclosure. Therefore, it should be apparent to those skilled in the art that various changes and modifications of the embodiments described herein can be made without departing from the scope and spirit of the present disclosure. In addition, descriptions of well-known functions and
10 constructions are omitted for clarity and conciseness.

[0031] Also, the various embodiments described herein are not necessarily mutually exclusive, as some embodiments can be combined with one or more other embodiments to form new embodiments.

[0032] Herein, the term “or” as used herein, refers to a non-exclusive
15 or, unless otherwise indicated. The examples used herein are intended merely to facilitate an understanding of ways in which the embodiments herein can be practiced and to further enable those skilled in the art to practice the embodiments herein. Accordingly, the examples should not be construed as limiting the scope of the embodiments herein.

[0033] As is traditional in the field, embodiments may be described and illustrated in terms of blocks which carry out a described function or functions. These blocks, which may be referred to herein as managers, units,
20 modules, hardware components or the like, are physically implemented by analog and/or digital circuits such as logic gates, integrated circuits, microprocessors, microcontrollers, memory circuits, passive electronic
25 components, active electronic components, optical components, hardwired circuits and the like, and may optionally be driven by firmware and software. The circuits may, for example, be embodied in one or more semiconductor chips, or on substrate supports such as printed circuit boards and the like. The

circuits constituting a block may be implemented by dedicated hardware, or by a processor (e.g., one or more programmed microprocessors and associated circuitry), or by a combination of dedicated hardware to perform some functions of the block and a processor to perform other functions of the block. Each block of the embodiments may be physically separated into two or more interacting and discrete blocks without departing from the scope of the disclosure. Likewise, the blocks of the embodiments may be physically combined into more complex blocks without departing from the scope of the disclosure.

10 **[0034]** Accordingly, the embodiments herein provide a method for estimating frequencies of multiple sinusoids by an electronic device (100). The method includes processing, by the electronic device (100), a received signal comprising a plurality of sinusoids, where each sinusoid in the received signal comprises at least one unknown parameter and determining, by the electronic device (100), whether a candidate parameter is less than zero, where the candidate parameter is a function of an estimated SNR and an estimated threshold SNR. Further, the method includes performing, by the electronic device (100), one of: estimation of the frequencies of the plurality of sinusoids in the received signal using a first frequency estimation technique, in response to determining that the candidate parameter is less than zero, and estimation of the frequency of the sinusoids using a second frequency estimation technique, in response to determining that the candidate parameter is greater than zero.

25 **[0035]** Unlike the conventional methods and systems, the proposed method reduces the number of candidate frequencies to at most $2p$ points (where p is the number of sinusoids) and has the same performance as the maximum likelihood (ML) estimator.

[0036] Unlike the conventional methods and systems, the proposed method does not follow one technique of estimating the frequency throughout, rather determines the threshold SNR and switches to the best method depending

on the threshold SNR. When the SNR is below the threshold SNR, the proposed method estimates the frequency as described in this specification. Further, when the SNR is above the threshold SNR, the proposed method switches to methods like ESPRIT.

5 **[0037]** Referring now to the drawings, and more particularly to FIGS. 1 through 8, where similar reference characters denote corresponding features consistently throughout the figures, there are shown preferred embodiments.

[0038] FIG. 1 is a block diagram of an electronic device (100) for estimating frequencies of multiple sinusoids, according to the embodiments as
10 disclosed herein.

[0039] Referring to the FIG. 1, the electronic device (100) may for example, but not limited to: a mobile phone, a smart phone, Personal Digital Assistant (PDA), a tablet, or the like. In an embodiment, the electronic device (100) includes a communicator (120), a memory (140) and a processor (160).

15 **[0040]** In an embodiment, the communicator (120) is configured to receive a received signal comprising a plurality of sinusoids. Each sinusoid in the received signal includes at least one unknown parameter. The unknown parameter may be, for example, an amplitude and frequency associated with multiple sinusoids needs to be estimated in diverse fields such as radar, sonar,
20 etc.

[0041] In an embodiment, the memory (140) can include non-volatile storage elements. Examples of such non-volatile storage elements may include magnetic hard discs, optical discs, floppy discs, flash memories, or forms of electrically programmable memories (EPROM) or electrically erasable and programmable (EEPROM) memories. In addition, the memory (140) may, in
25 some examples, be considered a non-transitory storage medium. The term “non-transitory” may indicate that the storage medium is not embodied in a carrier wave or a propagated signal. However, the term “non-transitory” should not be interpreted that the memory (140) is non-movable. In some examples,

the memory (140) is configured to store larger amounts of information than the memory. In certain examples, a non-transitory storage medium may store data that can, over time, change (*e.g.*, in Random Access Memory (RAM) or cache).

[0042] In an embodiment, the processor (160) includes a candidate
5 parameter management engine (162), a frequency estimation engine (164) and a frequency refinement engine (166).

[0043] In an embodiment, the candidate parameter management engine
(162) is configured to process the received signal comprising the plurality of
sinusoids. The candidate parameter management engine (162) is also
10 configured to estimate a correlation matrix of the received signal and determine
eigenvalues of the correlation matrix. Further, the candidate parameter
management engine (162) is configured to determine an estimated SNR of the
received signal based on the eigenvalues of the correlation matrix and
determine an estimated threshold SNR. The estimated threshold SNR is
15 dependent on the eigenvalues of the correlation matrix. Furthermore, the
candidate parameter management engine (162) is configured to determine
whether the candidate parameter is less than zero based on a difference between
the estimated SNR and the estimated threshold SNR.

[0044] In an embodiment, the frequency estimation engine (164) is
20 configured to estimate the frequencies of the plurality of sinusoids in the
received signal using a first frequency estimation technique, in response to
determining that the candidate parameter is less than zero. In this case, the
frequency estimation engine (164) produces a set of initial estimates obtained
using a technique that is one of: Root MUSIC, Min-Norm, or Kumaresan-Tufts
25 (KT) method. These initial estimates are then passed on to the frequency
refinement engine (166) for further processing.

[0045] Further, the frequency estimation engine (164) is configured to
estimate the frequency of the sinusoids using a second frequency estimation
technique, in response to determining that the candidate parameter is greater

than zero. The second frequency estimation technique is one of: an Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT), a Multiple Signal Classification (MUSIC), a Minimum-Norm (Min-Norm).

[0046] When the candidate parameter is less than zero, the frequency
5 refinement engine (166) is configured to generate a set of $2p$ candidate frequencies by carrying out the following sequence of steps: (a) converting the linear prediction (LP) polynomial obtained using Root-MUSIC to a corresponding line spectral frequencies (LSFs) representation, and (b) choosing the second closest LSF and the third closest LSF to each of the p initial
10 frequency estimates given by Root-MUSIC. This results in the set of $2p$ candidate frequencies. Furthermore, the frequency refinement engine (166) is configured to a) generate plurality of possible sets of p frequencies from the $2p$ candidate set, b) discard some of the sets based on outlier rejection and redundancy removal, c) improve the remaining sets by carrying out gradient
15 descent and d) choose the final estimates using likelihood values.

[0047] Although the FIG. 1 shows the hardware elements of the electronic device (100) but it is to be understood that other embodiments are not limited thereon. In other embodiments, the electronic device (100) may include less or more number of elements. Further, the labels or names of the
20 elements are used only for illustrative purpose and does not limit the scope of the invention. One or more components can be combined together to perform same or substantially similar function.

[0048] FIG. 2 is a flow chart 200 for a method for estimating frequencies of multiple sinusoids by the electronic device (100), according to
25 the embodiments as disclosed herein.

[0049] Referring to the FIG. 2, at step 202, the electronic device (100) processes the received signal comprising the plurality of sinusoids. For example, in the electronic device (100) illustrated in the FIG. 1, the

communicator (120) is configured to process the received signal comprising the plurality of sinusoids.

[0050] At step 204, the electronic device (100) estimates the correlation matrix of the received signal. For example, in the electronic device (100) illustrated in the FIG. 1, the processor (160) is configured to estimate the correlation matrix of the received signal.

[0051] At step 206, the electronic device (100) determines the eigenvalues of the correlation matrix. For example, in the electronic device (100) illustrated in the FIG. 1, the processor (160) is configured to determine the eigenvalues of the correlation matrix.

[0052] At step 208, the electronic device (100) determines the estimated SNR of the received signal based on the eigenvalues of the correlation matrix. For example, in the electronic device (100) illustrated in the FIG. 1, the processor (160) is configured to determine the estimated SNR of the received signal based on the eigenvalues of the correlation matrix.

[0053] At step 210, the electronic device (100) determines the estimated threshold SNR. For example, in the electronic device (100) illustrated in the FIG. 1, the processor (160) is configured to determine the estimated threshold SNR.

[0054] At step 212, the electronic device (100) determines whether the candidate parameter is less than zero based on the difference between the estimated SNR and the estimated threshold SNR. For example, in the electronic device (100) illustrated in the FIG. 1, the processor (160) is configured to determine whether the candidate parameter is less than zero based on the difference between the estimated SNR and the estimated threshold SNR.

[0055] At step 214, the electronic device (100) estimates the frequencies of the plurality of sinusoids in the received signal using the first frequency estimation technique, in response to determining that the candidate parameter is less than zero based on the difference between the estimated SNR

and the estimated threshold SNR. For example, in the electronic device (100) illustrated in the FIG. 1, the processor (160) is configured to estimate the frequencies of the plurality of sinusoids in the received signal using the first frequency estimation technique, in response to determining that the candidate
5 parameter is less than zero based on the difference between the estimated SNR and the estimated threshold SNR.

[0056] At step 216, the electronic device (100) estimates the frequency of the sinusoids using the second frequency estimation technique, in response to determining that the candidate parameter is greater than zero based on the
10 difference between the estimated SNR and the estimated threshold SNR. For example, in the electronic device (100) illustrated in the FIG. 1, the processor (160) is configured to estimates the frequency of the sinusoids using the second frequency estimation technique, in response to determining that the candidate parameter is greater than zero based on the difference between the estimated
15 SNR and the estimated threshold SNR.

[0057] The various actions, acts, blocks, steps, or the like in the method may be performed in the order presented, in a different order or simultaneously. Further, in some embodiments, some of the actions, acts, blocks, steps, or the like may be omitted, added, modified, skipped, or the like without departing
20 from the scope of the invention.

[0058] FIG. 3 is a graph plot illustrating Line Spectral Pair (LSP) and roots of an M -th order linear prediction polynomial $A(z)$ for three sinusoids, according to the embodiments as disclosed herein.

[0059] Referring to the FIG. 3, the root MUSIC estimates the
25 frequencies incorrectly when the SNR = 24 dB. The proposed method estimates the frequencies correctly for this example.

[0060] In the proposed method, the observed data consist of p complex sinusoids but is corrupted by additive white Gaussian noise (AWGN). Therefore, observed signal model is,

$$x[n] = \sum_{l=1}^p v_l e^{j2\pi f_l n} + w[n] \quad n=0, 1, \dots, N-1 \quad (1)$$

where $v_l (= |v_l| e^{j\theta_l})$ and f_l are unknowns; p is assumed to be known.

Further, the problem of finding Direction of Arrival (DoA) of signals in array processing is the same mathematical problem as above frequency estimation.

5 **[0061]** Conventionally, an efficacy associated with a frequency estimator is determined by plotting a mean square error (MSE) of the frequency estimates versus SNR. A characteristic feature of the MSE vs SNR plot is the threshold effect, wherein a sudden increase in MSE is observed when the SNR falls below a certain value called as threshold SNR. The lower the threshold,
10 the better is the frequency estimator. Further, the method of maximum-likelihood (ML) has the lowest threshold compared to other conventional frequency estimators but is computationally the most burdensome (mainly due to the coarse p -dimensional search needed for the initial guess).

15 **[0062]** In the proposed method, when the candidate parameter is less than zero, an M -th order linear prediction polynomial $A(z)$ is estimated using the Root-MUSIC method, and further, the $A(z)$ is used to determine the standard LSF polynomials $P(z)$ and $Q(z)$.

Let $\theta_k \in [0, 1)$ for $k = 1, 2, \dots, 2M + 2$ represent the angles of the unit circle roots of $P(z)$ and $Q(z)$ in ascending order (successive root angles in the
20 counter-clockwise direction). The pair (θ_k, θ_{k+1}) is called as a line spectral pair (LSP) and θ_k is called as a line spectral frequency (LSF).

25 **[0063]** According to the conventional methods and systems, “close LSPs” correspond to regions near which the true signal roots are highly likely to be present. Therefore, in the proposed method the overall search space is populated with points that are derived from the close LSPs using a simple closeness criterion.

In the next step, the electronic device (100) generates at most $2p$ frequency candidates as:

1. Find Root-MUSIC estimates $\tilde{f}_k, k = 1, 2, \dots, p$

2. Initialization: $i=1, \Omega_1=\Omega_2=\emptyset$
3. Find LSP (θ_l, θ_{l+1}) near to $\check{f}_l$ such that $\theta_l \leq \check{f}_l \leq \theta_{l+1}$
4. Choose second closest LSF to $\check{f}_l$ i.e.,

$$\Omega_1 = \Omega_1 \cup \arg \max_{\theta \in \{\theta_l, \theta_{l+1}\}} |\theta - \check{f}_l|$$

5. Choose third closest LSF to $\check{f}_l$ i.e.,

$$\Omega_2 = \Omega_2 \cup \arg \min_{\theta \in \{\theta_{l-1}, \theta_{l+2}\}} |\theta - \check{f}_l|$$

6. $i \leftarrow i + 1$ and repeat steps 3–5 for $i \leq p$
7. $\Omega = \Omega_1 \cup \Omega_2$

[0064] Further, the electronic device (100) ignores the closest LSF to $\check{f}_l$ to reduce the number of frequency candidates without compromising the performance. Referring to the FIG. 3, consider a signal containing three sinusoids with frequencies $f_1 = 0.852$, $f_2 = 0.879$ and $f_3 = 0.908$ to demonstrate the selection of p points. For this example root-MUSIC method is unable to resolve the two closely spaced signal frequencies f_1 and f_2 ($\check{f}_1 = 0.75$ and $\check{f}_2 \approx 0.5(f_1 + f_2)$). However, the second and the third closest LSP frequencies of $\check{f}_2$ (θ_5 and θ_7) are better estimates of the f_1 and the f_2 respectively than the other LSPs. In the case of $f_3 = 0.907$, the root MUSIC method estimates accurately. Further, the second closest LSP frequency θ_9 is very close to the f_3 . Finally the root MUSIC method outlines the chooses $\Omega_1 = \{\theta_2, \theta_5, \theta_9\}$ and $\Omega_2 = \{\theta_3, \theta_7\}$ where (θ_7 appears twice but duplicates are discarded). Let Ω be the set containing $K > p$ frequency candidates (e.g., $K = 2p$) which provides ${}^K C_p$ possible estimates,

Let $\Psi_0 = \{\mathbf{f} | f_k \in \Omega, k = 1, 2, \dots, p\}$. In the conventional methods such as the EPUMA, the final estimate $\hat{\mathbf{f}}$ would be that $\mathbf{f}$ that minimizes the standard cost function, i.e.,

$$\hat{\mathbf{f}} = \arg \min_{\mathbf{f} \in \Psi_0} L(\mathbf{f}) \quad (2)$$

Here $L(\mathbf{f}) = \mathbf{x}^H (\mathbf{I} - \mathbf{S}(\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H) \mathbf{x}$, $\mathbf{S} = [\mathbf{e}_1 \ \mathbf{e}_2 \ \dots \ \mathbf{e}_p]$

$$\mathbf{e}_i = [1 \ e^{j2\pi f_i} \dots e^{j2\pi(N-1)f_i}]^T \text{ and } \mathbf{x} = (x[0], x[1], \dots, x[N-1])^T$$

[0065] The value of $\hat{\mathbf{f}}$ obtained from Eq. (2) is a poorer estimate than the one obtained by using the ML method. However, if the gradient descent step is performed with each $\mathbf{f} \in \Psi_0$ i.e., $\mathbf{f}_k \rightarrow \mathbf{f}_k^{\text{gd}} (k = 1, 2, \dots, |\Psi_0|)$ then the
 5 estimate obtained so with the least value of $L(\cdot)$ will be virtually the same as the estimate obtained from the ML method.

[0066] Since $|\Psi_0| = K_{\text{Cp}}$, the number of gradient-descent steps may be reduced without losing out on the performance. The number of $\mathbf{f}_k$ required to be passed on to the gradient descent step is achieved in two parts: first, the
 10 electronic device (100) limits to $\Psi_1 \subseteq \Psi_0$, where

$$\Psi_1 = \{\mathbf{f}_k | k = 1, 2, \dots, |\Psi_0|, L(\mathbf{f}_k) < (1 + \alpha)L(\hat{\mathbf{f}})\} \quad (3)$$

[0067] The electronic device (100) in the equation (3) eliminates the $\mathbf{f}_k \in \Psi_0$ with large cost that are very unlikely to lead to the final estimate. The proposed method is established using $\alpha = 0.2p$ in the experiments. Further,
 15 redundant frequency estimates i.e., elements in Ψ_1 that are very close to each other are eliminated by the electronic device (100) due to the fact that two different initial guesses that are close to each other are likely to yield the same final solution in the gradient descent step. That is, if $\mathbf{f}^a, \mathbf{f}^b \in \Psi_1$ are such that $\|\hat{\mathbf{f}} - \mathbf{f}\|_{\infty} < \Delta_f$, then only $\mathbf{f}^a$ (where $L(\mathbf{f}^a) < L(\mathbf{f}^b)$) is retained. The nearness
 20 is defined in terms of parameter Δ_f where the $\Delta_f = \frac{1}{N}$ in the various experiments described in the specification. The pared down set is $\Psi_2 \subseteq \Psi_1$. Further, for each $\mathbf{f}_k \in \Psi_2$, $\mathbf{f}_k \rightarrow \mathbf{f}_k^{\text{gd}}$ is obtained, which leads to the final estimate:

$$\hat{\mathbf{f}}^{\text{gd}} = \arg \min_{\mathbf{f}_k^{\text{gd}}} \{L(\mathbf{f}_k^{\text{gd}}), k = 1, 2, \dots, |\Psi_2|\} \quad (4)$$

[0068] FIG. 4 is a graph plot illustrating Mean Squared Error (MSE) of
 25 the proposed LSF-based method in comparison to existing methods, according to the embodiments as disclosed herein.

[0069] Referring to the FIG. 4, the threshold SNR generated using the proposed LSF-based method and the threshold SNR generated using existing methods such as Enhanced Principal singular- vector Utilization Modal Analysis (EPUMA) match the threshold SNR generated using the optimal ML method for the two-sinusoid example, and effectively reduces the threshold SNR compared to methods such as ESPRIT. The proposed reduction-in-threshold technique can be applied to existing methods such as for example EPUMA.

Consider the two sinusoids example: $N = 25$, $f_1 = 0.5$, $f_2 = 0.$, $|v_1| = |v_2| = 1$, and $\phi_1 - \phi_2 = \frac{3\pi}{2}$.

[0070] From the FIG. 4, the threshold SNRs of both the proposed LSF-based method and the EPUMA are lowered and equal the threshold SNR of the ML method. Further, the lowering of the threshold SNR is achieved with significantly fewer number of gradient descent steps. For example, consider the SNR to be 16 dB which is below the threshold SNR generated using the ESPRIT method the observed values were $|\Psi_0| = 4.22$, $|\Psi_1| = 1.29$, and $|\Psi_2| = 1.02$ (averaged over 50k experiments).

[0071] In the proposed method, the computational burden on the electronic device (100) is further reduced by switching to any conventional method for estimating frequency such as for example but not limited to ESPRIT when the SNR is above the threshold SNR. The reason for switching to the conventional method for estimating frequency is that for the conventional method for estimating frequency has SNRs which are computationally less burdensome and yet has similar performance. For example, in the FIG. 4 when the SNR is above 20 dB, it is advantageous to switch to the conventional method for estimating frequency such as the ESPRIT.

[0072] Generally, the switch can be done only when both the SNR and the threshold SNR are known to the electronic device (100). However, conventionally, the SNR and the threshold SNR are unknown and hence the

proposed method estimates the SNR and the threshold SNR then initiate a switch when appropriate. The SNR of the data is estimated from the eigenvalues of a sample correlation matrix. Further, the threshold SNR is data dependent, *i.e.*, the threshold SNR is a function of frequency, phase and
 5 amplitude of the sinusoids.

[0073] The SNR is defined as $-10 \log_{10}(\sigma^2)$ where σ^2 is the noise variance. The theoretical autocorrelation matrix $\mathbf{R}_{xx}$ corresponding to sinusoidal signals corrupted by white noise has eigenvalues $\lambda_k = \lambda_k^s + \sigma^2$ for $1 \leq k \leq p$. The eigenvalues are arranged as follows: $\lambda_1 \geq \dots \geq \lambda_p > \sigma^2 =$
 10 $\sigma^2 = \dots = \sigma^2$. where λ_k^s are the eigenvalues of a signal correlation matrix.

[0074] In practice the SNR is estimated using:

$$\hat{\sigma}^2 = \frac{1}{M-p} \sum_{i=p+1}^M \hat{\lambda}_i \quad (5)$$

where $\hat{\lambda}_i$'s are the eigenvalues of the $M \times M$ estimated $\hat{\mathbf{R}}_{xx}$.

[0075] The total signal power P_s is obtained from the eigenvalues using:

$$P_s = \frac{1}{M} \sum_{i=1}^p \lambda_i^s = \frac{1}{M} \sum_{i=1}^p (\lambda_i - \sigma^2) \quad (6)$$

[0076] The probability of a frequency estimate being an outlier depends on weakest signal's power when compared to the noise power. Since $\lambda_i = \lambda_i^s + \sigma^2$, the weakest signal's power can be estimated using $\tilde{P} = \frac{1}{M} (\hat{\lambda}_p - \hat{\sigma}^2)$ Eq. (6) indicates that $\tilde{P}$ is the power contribution of p -th eigenvalue of signal correlation matrix. Intuitively, if $\tilde{P} > \hat{\sigma}^2$ *i.e.*, when the weakest signal's power is greater than the noise power, then the estimated SNR is expected to be above the threshold. In this regime, there will be few outliers. In general, let $\tilde{P}/\hat{\sigma}^2 \geq \beta$ guarantee no outlier, where β is a user defined parameter which depends on the frequency estimation method selected. The estimated threshold SNR is the
 20 SNR at which $\hat{\sigma}^2$ equals $\tilde{P}/\beta$. That is,

$$\text{Estimated threshold SNR} = -10 \log_{10}(\tilde{P}/\beta) \quad (7)$$

[0077] FIG. 5 is a graph plot illustrating the estimated and actual threshold values for a two sinusoids in an example scenario, according to the embodiments as disclosed herein.

[0078] Referring to the FIG. 5, the estimated threshold SNR determined using $\hat{\mathbf{R}}_{xx}$ of the noiseless signal for $\beta = 1$ is plotted against different values of $\phi_1 - \phi_2$ for the two sinusoid scenario. The FIG. 5 also shows the actual threshold for the ESPRIT. The match between the actual threshold for the ESPRIT and the estimated threshold is very close except when $\phi_1 - \phi_2 = \frac{\pi}{2}$.

[0079] FIG. 6A is a graph plot illustrating a histogram of estimated threshold SNRs from noisy data, according to the embodiments as disclosed herein.

[0080] FIG. 6B is a graph plot illustrating a histogram of estimated SNR, according to the embodiments as disclosed herein.

[0081] Generally, the noisy data is used for estimating the threshold SNRs and hence the estimated SNR will be a random quantity. Referring to the FIG. 6A, the histograms of the estimated threshold SNR for two SNRs are shown (SNR 1 = 20 dB and SNR 2 = 25 dB, $\phi_1 - \phi_2 = \frac{3\pi}{2}$). In both the scenarios, the mean of the estimated threshold SNR is close to the actual threshold SNR value.

[0082] Referring to the FIG. 6B, the estimated SNR which is obtained from the noisy data will also have a spread. Nevertheless, the actual SNR and the mean value of the SNR match closely.

[0083] FIG. 7A is a graph plot illustrating probability of a candidate parameter being greater than zero for a two-sinusoid example, according to the embodiments as disclosed herein.

[0084] FIG. 7B is a graph plot illustrating conditional probability of the candidate parameter being greater than zero given that an estimated frequency is an outlier for a two-sinusoid example, according to the embodiments as disclosed herein.

[0085] The candidate parameter Γ_β compares the estimated threshold and estimated SNR of the plurality of sinusoids. The FIG. 7A illustrates a scenario of $P(\Gamma_\beta > 0)$ and the FIG. 7B illustrates a scenario of $(P(\Gamma_\beta > 0)|\hat{f}$ is an outlier) for the two-sinusoid example with $\phi_1 - \phi_2 = \frac{3\pi}{2}$.

5 Consider the candidate parameter,

$$\Gamma_\beta = \text{Estimated SNR} - \text{Estimated Threshold.}$$

Based on the equation (5) and the equation (7), the candidate parameter is:

$$\Gamma_\beta = 10 \log_{10} \left(\frac{\frac{1}{M} \left(\lambda p - \frac{1}{M-p} \sum_{i=p+1}^M \lambda_i \right)}{\frac{\beta}{M-p} \sum_{i=p+1}^M \lambda_i} \right) \quad (8)$$

[0086] In the proposed method, the electronic device (100) determines the value of the candidate parameter and based on the whether the value of the candidate parameter is greater than '0' or not, the appropriate method for estimation of the frequencies is selected. Therefore, in the proposed method the electronic device (100) selects the ESPRIT method to determine the estimated frequencies when the $\Gamma_\beta > 0$ to save computation.

15 [0087] Consider that $\hat{\mathbf{f}}$ is the estimated frequency using the ESPRIT method.

When the value of the $\Gamma_\beta > 0$, then the estimated SNR is above the estimated threshold SNR and hence the $\hat{\mathbf{f}}$ is probably not an outlier. For the illustrative example with $\phi_1 - \phi_2 = \frac{3\pi}{2}$, the plot of $P(\Gamma_\beta > 0)$ against the estimated SNR is as shown in the FIG. 7A with β as a parameter. The desirable value of $P(\Gamma_\beta > 0)$ is large *i.e.*, ideally 1 when the estimated SNR is above the estimated threshold SNR which allows the electronic device (100) to bypass the usage of computationally expensive methods.

[0088] A high value of the $P(\Gamma_\beta > 0)$ is desirable for the estimated SNRs which are above the estimated threshold SNR. However, given that $\hat{\mathbf{f}}$ is

an outlier, the probability of the $\Gamma_\beta > 0$ must very low (ideally 0), *i.e.*, $P(\Gamma_\beta > 0)| \text{outlier}) \approx 0$.

[0089] The FIG. 7B shows the plot of the $P(\Gamma_\beta > 0)| \text{outlier})$ for $\beta = 0.5, 1$. An estimate is considered as an outlier if $\|\hat{\mathbf{f}} - \mathbf{f}\|_\infty > \frac{1}{N}$. While the FIG. 7A shows that the $P(\Gamma_{0.5} > 0) > P(\Gamma_1 > 0)$, the FIG. 7B shows that the $(P(\Gamma_1 > 0)| \hat{\mathbf{f}} \text{ is an outlier}) < (P(\Gamma_{0.5} > 0)| \hat{\mathbf{f}} \text{ is an outlier})$ in the estimated SNR range of 18–20 dB. Therefore, the proposed method uses $\beta = 1$ for the experiments, which also has the interpretation of yielding the actual threshold for the two-sinusoid case (as described in the FIG. 5).

[0090] FIG. 8 illustrates a Mean Squared Error (MSE) plots for three sinusoids with random amplitudes and frequencies averaged over 50k trials, according to the embodiments as disclosed herein.

[0091] Referring to the FIG. 8, the MSE plots for the three sinusoids where the each of the sinusoids has random amplitudes and frequencies and averaged over 50k trials is provided. The proposed method provides the same threshold as the maximum likelihood (ML) performance.

[0092] The effectiveness of the Γ_β -based switching to the ESPRIT is demonstrated in the FIG. 8 using the three-sinusoid example for $N = 25$. In the proposed demonstration, v_i and f_i are chosen randomly for 1000 realizations: for every $1 \leq i \leq p$, $|v_i| \in u[0.5, 1]$, $\phi_i \in u[0, 1)$ and $f_i \in u[0, 1)$'s with a minimum difference between the two frequencies as $\frac{1}{2N}$. For each random parameter setting, $\mathbf{f}$ is estimated using 50 noisy trials, leading to a total of 50,000 realizations. The user-defined parameters are $\Delta_f = \frac{1}{N}$ and $\beta = 1$. The FIG. 8 is the MSE of the estimates.

[0093] The FIG. 8 ascertains that the proposed method provides the same threshold as that of the ML technique. Further, when the proposed reduction-in-threshold method is applied to the EPUMA, EPUMA's threshold gets lowered to that of ML. Moreover, Γ_β -based switching also results in

computational savings because of switching to the conventional methods such as the ESPRIT based on the threshold SNR.

[0094] From the experiments, the observation is that whenever the ESPRIT is chosen, the estimated frequency is not an outlier. However, the ESPRIT may not be always chosen when the SNR is above the threshold SNR. Further, the probability of the ESPRIT not being chosen ($1 - P(\Gamma_1 > 0)$) in the cases rapidly decreases with the increasing SNR. For the two-sinusoid example, $1 - P(\Gamma_1 > 0)$ at SNR levels 21, 23, and 25 dB were found to be 0.38, 0.03, and 0.

[0095] The foregoing description of the specific embodiments will so fully reveal the general nature of the embodiments herein that others can, by applying current knowledge, readily modify or adapt for various applications such specific embodiments without departing from the generic concept, and, therefore, such adaptations and modifications should and are intended to be comprehended within the meaning and range of equivalents of the disclosed embodiments. It is to be understood that the phraseology or terminology employed herein is for the purpose of description and not of limitation. Therefore, while the embodiments herein have been described in terms of preferred embodiments, those skilled in the art will recognize that the embodiments herein can be practiced with modification within the spirit and scope of the embodiments as described herein.

STATEMENT OF CLAIMS

We claim:

1. A method for estimating frequencies of multiple sinusoids by an electronic device (100), comprising:

processing, by the electronic device (100), a received signal comprising a plurality of sinusoids, wherein each sinusoid in the received signal comprises at least one unknown parameter;

determining, by the electronic device (100), whether a candidate parameter is less than zero, wherein the candidate parameter is a function of an estimated SNR and an estimated threshold SNR; and

performing, by the electronic device (100), one of:
estimation of frequencies of the plurality of sinusoids in the received signal using a first frequency estimation technique, in response to determining that the candidate parameter is less than zero, and

estimation of the frequency of the sinusoids using a second frequency estimation technique, in response to determining that the candidate parameter is greater than zero.

2. The method as claimed in claim 1, wherein determining, by the electronic device (100), whether the candidate parameter is less than zero comprises:

estimating, by the electronic device (100), a correlation matrix of the received signal;

determining, by the electronic device (100), eigenvalues of the correlation matrix;

determining, by the electronic device (100), an estimated SNR of the received signal based on the eigenvalues of the correlation matrix;

determining, by the electronic device (100), an estimated threshold SNR, wherein the estimated threshold SNR is dependent on the eigenvalues of the correlation matrix; and

5 determining, by the electronic device (100), whether the candidate parameter is less than zero based on a difference between the estimated SNR and the estimated threshold SNR.

3. The method as claimed in claim 1, wherein estimating, by the electronic device (100), the frequencies of the plurality of sinusoids in the received signal using the first frequency estimation technique comprises:

10 determining, by the electronic device (100), a linear prediction (LP) polynomial using one of: Root-MUSIC technique, Min-Norm technique and Kumaresan-Tufts technique;

determining, by the electronic device (100), initial frequency estimates from roots of the LP polynomial;

15 generating, by the electronic device (100), line spectral frequencies (LSF) of the LP polynomial, wherein the LSF of the LP polynomial represents roots of two polynomials derived from the LP polynomial;

20 determining, by the electronic device (100), a set of LSF that is second closest to the initial frequency estimates;

determining, by the electronic device (100), a set of LSF that is third closest to the initial frequency estimates;

25 combining, by the electronic device (100), the set of LSF that are second closest to the initial frequency estimates and the set of LSF that are third closest to the initial frequency estimates to form a combined set of LSF, wherein a size of the combined set of LSF is at most double a number of sinusoids;

determining, by the electronic device (100), a plurality of possible sets of candidate frequencies from the combined set of LSF ;

discarding, by the electronic device (100), outlier frequency estimates from the plurality of possible set of candidate frequencies to obtain a first pared down set;

5 discarding, by the electronic device (100), redundant frequency estimates from the first pared down set to obtain a final pared down sets of candidate frequencies;

performing, by the electronic device (100), a refinement of the final pared down sets of candidate frequencies using a gradient descent technique; and

10 estimating, by the electronic device (100), the frequencies of the plurality of sinusoids by choosing an optimal frequency estimate based on likelihood values from the refined final pared down sets of the gradient descent step.

4. The method as claimed in claim 3, wherein discarding, by the electronic
15 device (100), outlier frequency estimates from the plurality of possible sets of candidate frequencies to obtain the first pared down set comprises:

determining, by the electronic device (100), the outlier
20 frequency estimates based on likelihood values associated with each set of the plurality of possible sets of candidate frequencies; and

discarding, by the electronic device (100), the outlier frequency estimates from the plurality of possible sets of candidate frequencies to obtain the first pared down set of candidate frequency estimates.

5. The method as claimed in claim 3, wherein discarding, by the electronic
25 device (100), the redundant frequency estimates from the first pared down set to obtain the final pared down sets of candidate frequencies comprises:

determining by the electronic device (100), the redundant frequency estimates based on nearness of frequency criterion; and

discarding, by the electronic device (100), the redundant frequency estimates from the first pared down set to obtain the final pared down sets of candidate frequencies.

- 5 6. The method as claimed in claim 1, wherein the second frequency estimation technique is one of: an Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT), a Multiple Signal Classification (MUSIC), a Minimum-Norm (Min-Norm).
- 10 7. An electronic device (100) for estimating frequencies of multiple sinusoids, the electronic device (100) comprising:
- a memory (140);
- a processor (160) coupled to the memory (140);
- a communicator (120) coupled to the memory (140) and the processor (160) and configured to:
- 15 process a received signal comprising a plurality of sinusoids, wherein each sinusoid in the received signal comprises at least one unknown parameter;
- the processor (160) configured to:
- determine whether a candidate parameter is less than zero, wherein the candidate parameter is a function of an
- 20 estimated SNR and an estimated threshold SNR; and
- perform one of:
- estimation of the frequencies of the plurality of sinusoids in the received signal using a first frequency estimation technique, in response to determining that the
- 25 candidate parameter is less than zero, and
- estimation of the frequency of the sinusoids using a second frequency estimation technique, in response to determining that the candidate parameter is greater than zero.

8. The electronic device (100) as claimed in claim 7, wherein the processor (160) is configured to determine whether the candidate parameter is less than zero comprises:
- estimate a correlation matrix of the received signal;
 - 5 determine eigenvalues of the correlation matrix;
 - determine an estimated SNR of the received signal based on the eigenvalues of the correlation matrix;
 - determine an estimated threshold SNR, wherein the estimated threshold SNR is dependent on the eigenvalues of the correlation
 - 10 matrix; and
 - determine whether the candidate parameter is less than zero based on a difference between the estimated SNR and the estimated threshold SNR.
9. The electronic device (100) as claimed in claim 7, wherein the processor (160) is configured to estimate frequencies of the plurality of sinusoids in the received signal using the first frequency estimation technique comprises:
- 15 determine a linear prediction (LP) polynomial using one of: Root-MUSIC technique, Min-Norm technique and Kumaresan-Tufts
 - 20 technique;
 - determine initial frequency estimates from roots of the LP polynomial;
 - generate line spectral frequencies (LSF) of the LP polynomial, wherein the LSF of the LP polynomial represents roots of two
 - 25 polynomials derived from the LP polynomial;
 - determine a set of LSF that are second closest to the initial frequency estimates;
 - determine a set of LSF that are third closest to the initial frequency estimates;

combine the set of LSF that are second closest to the initial frequency estimates and the set of LSF that are third closest to the initial frequency estimates to form a combined set of LSF, wherein a size of the combined set of LSF is at most double a number of sinusoids;

5 determine a plurality of possible sets of candidate frequencies from the combined set of LSF;

 discard outlier frequency estimates from the plurality of possible set of candidate frequencies to obtain a first pared down set;

 discard redundant frequency estimates from the first pared down
10 set to obtain a final pared down sets of candidate frequencies;

 perform a refinement of the final pared down sets of candidate frequencies using a gradient descent technique; and

 estimate the frequencies of the plurality of sinusoids by
choosing an optimal frequency estimate based on likelihood values
15 from the refined final pared down sets of the gradient descent step.

10. The electronic device (100) as claimed in claim 9, wherein the processor (160) is configured to discard outlier frequency estimates from the plurality of possible sets of candidate frequencies to obtain the first pared down set comprises:

20 determine the outlier frequency estimates based on likelihood values associated with each set of the plurality of possible sets of candidate frequencies; and

 discard the outlier frequency estimates from the plurality of possible sets of candidate frequencies to obtain the first pared down set
25 of candidate frequency estimates.

11. The electronic device (100) as claimed in claim 9, wherein the processor (160) is configured to discard the redundant frequency estimates from the first pared down set to obtain the final pared down sets of candidate frequencies comprises:

determine the redundant frequencies estimates based on nearness of frequency criterion; and

5 discard the redundant frequency estimates from the first pared down set to obtain the final pared down sets of candidate frequencies to obtain the second pared down set of candidate frequency estimates.

12. The electronic device (100) as claimed in claim 7, wherein the second frequency estimation technique is one of: an Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT), a Multiple Signal Classification (MUSIC), a Minimum-Norm (Min-Norm).

10

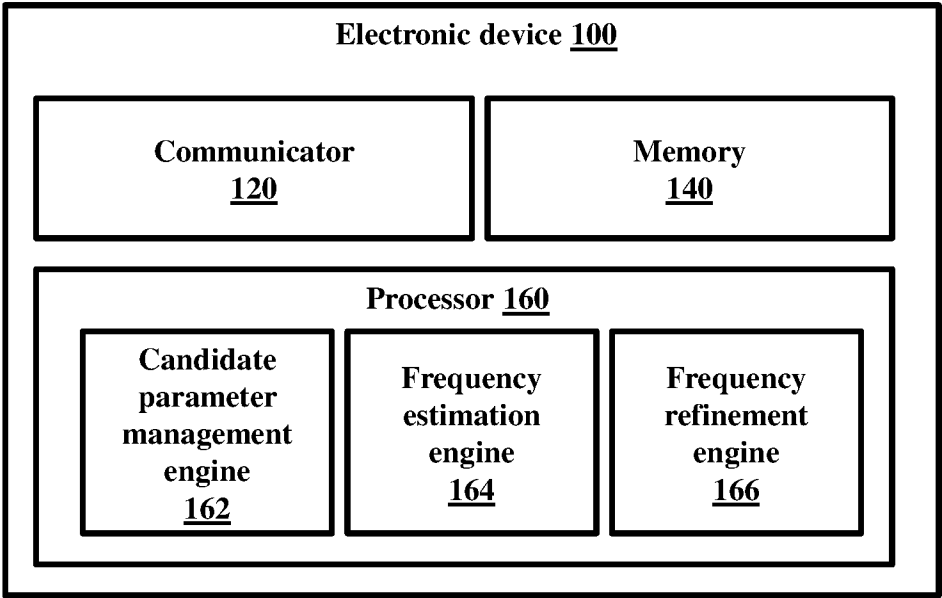


FIG. 1

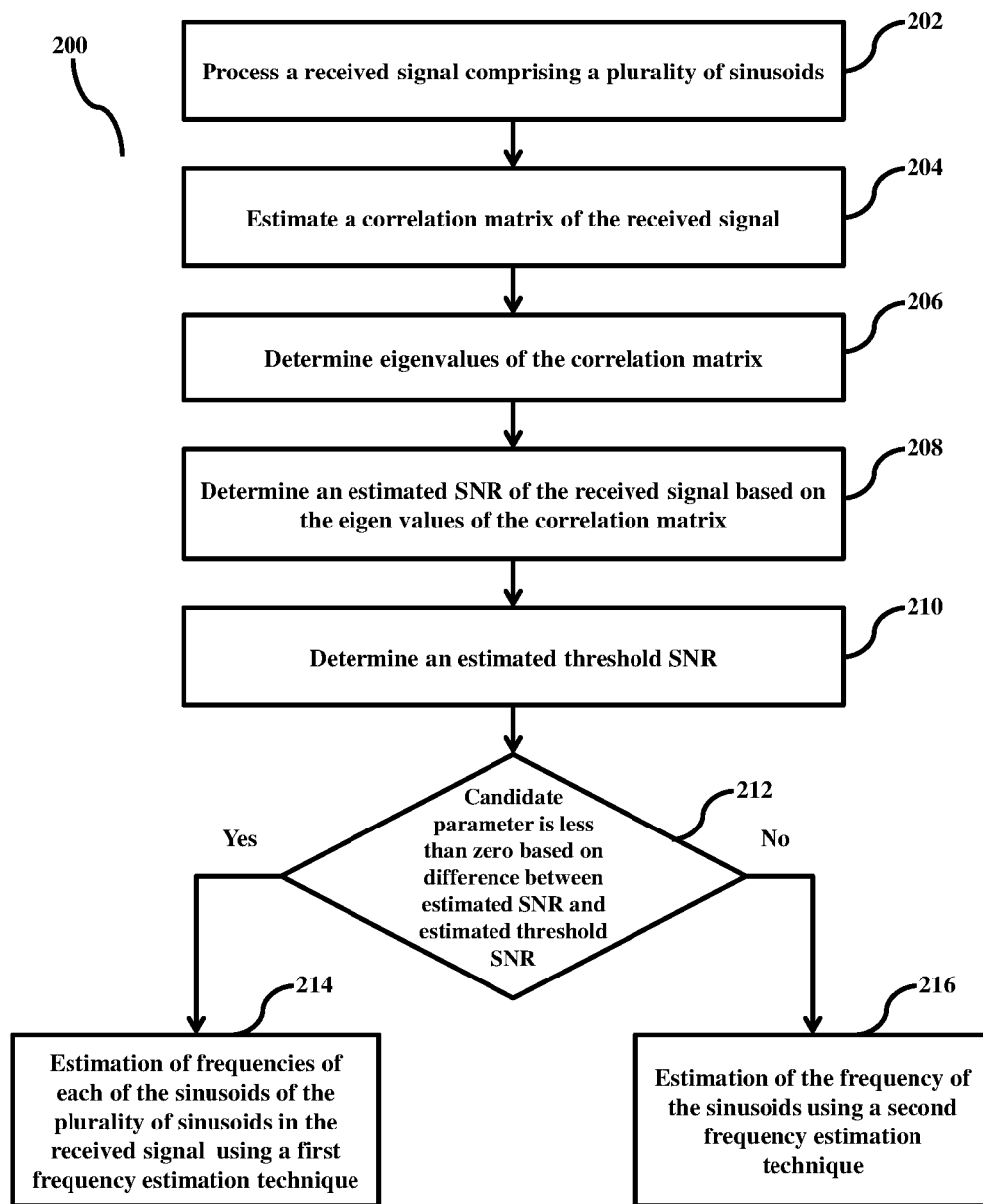


FIG. 2

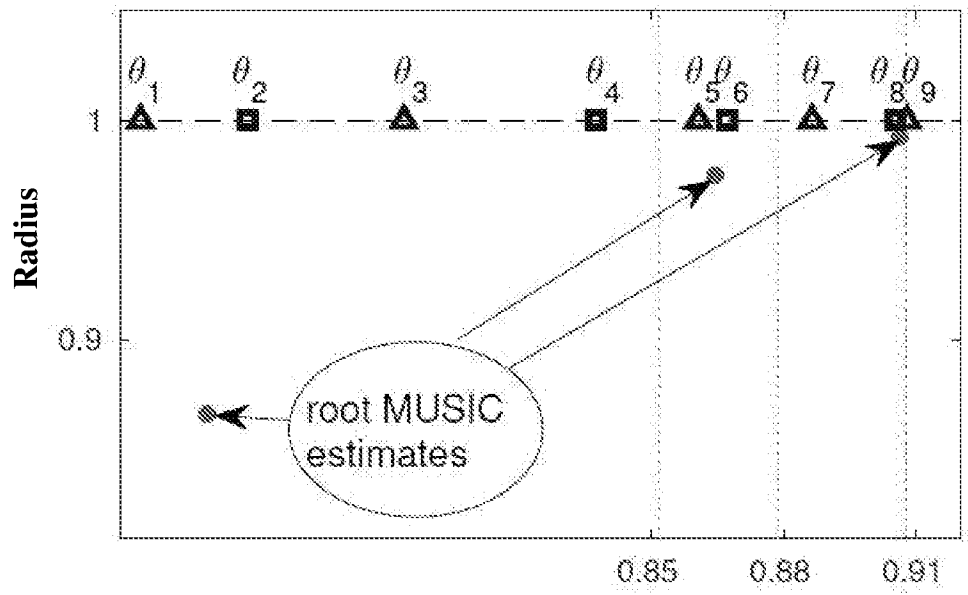


FIG. 3

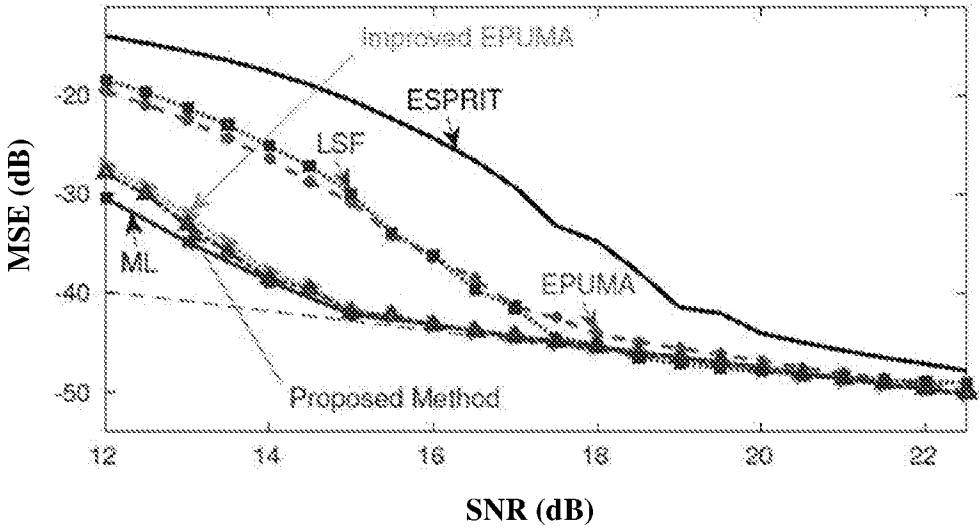


FIG. 4

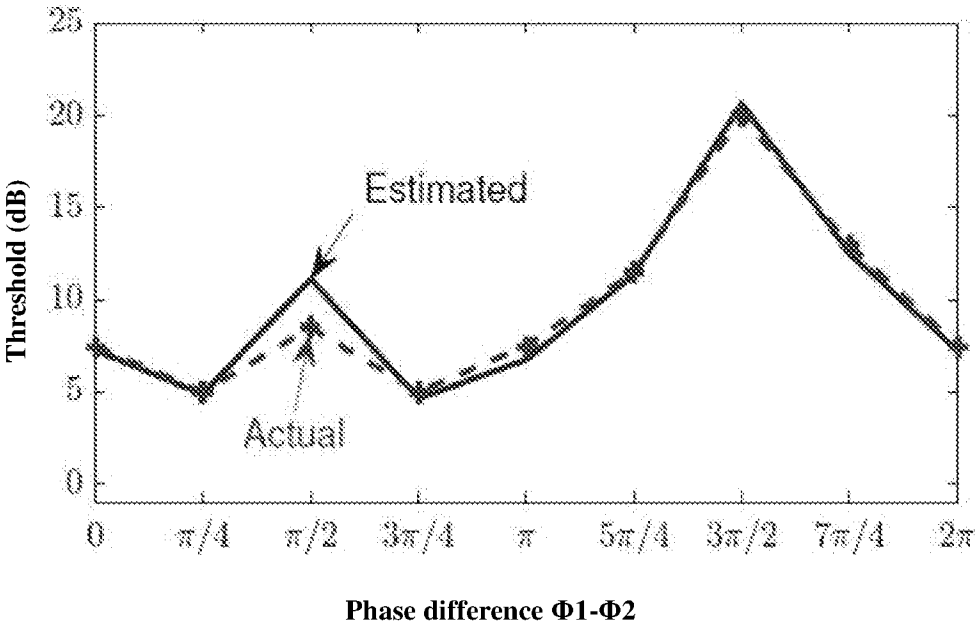


FIG. 5

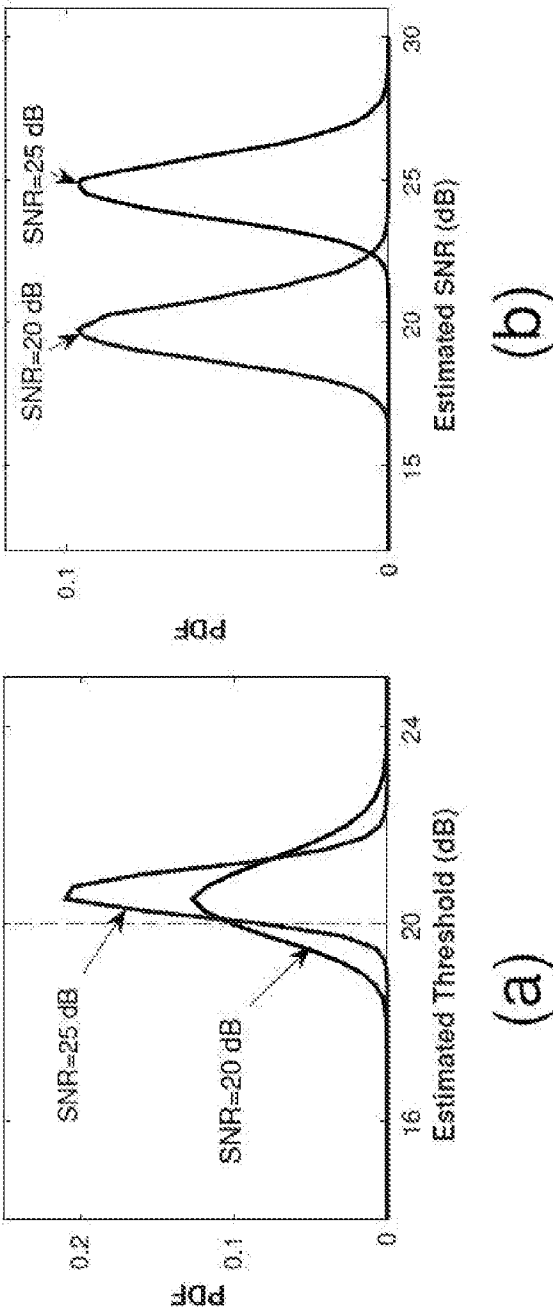


FIG. 6

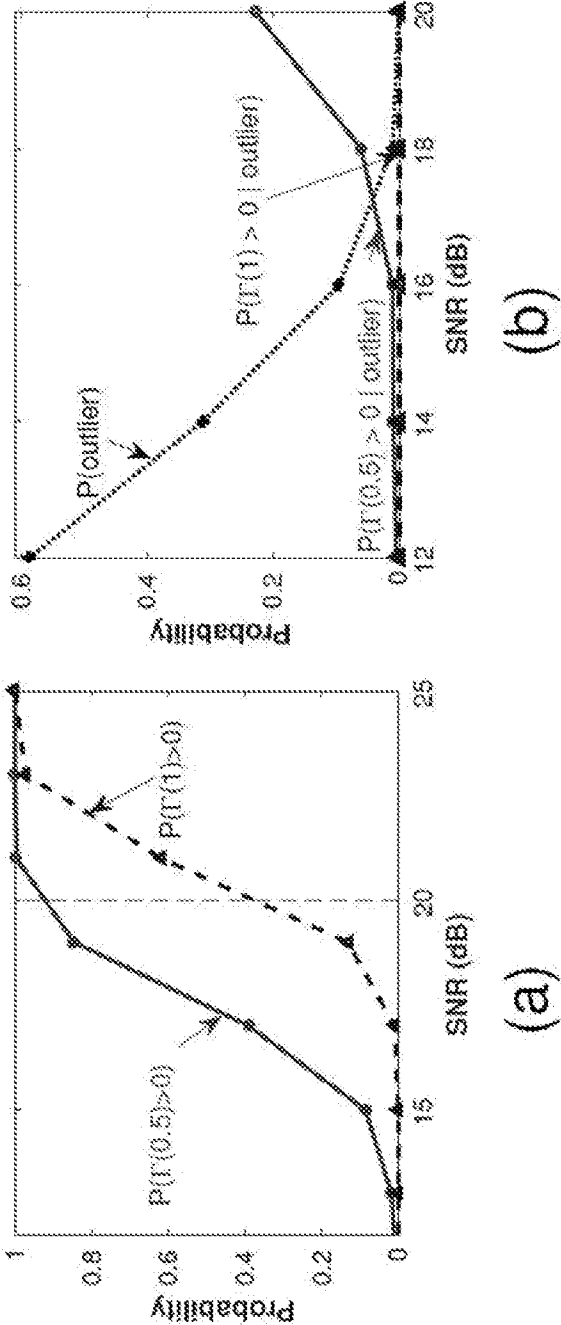


FIG. 7

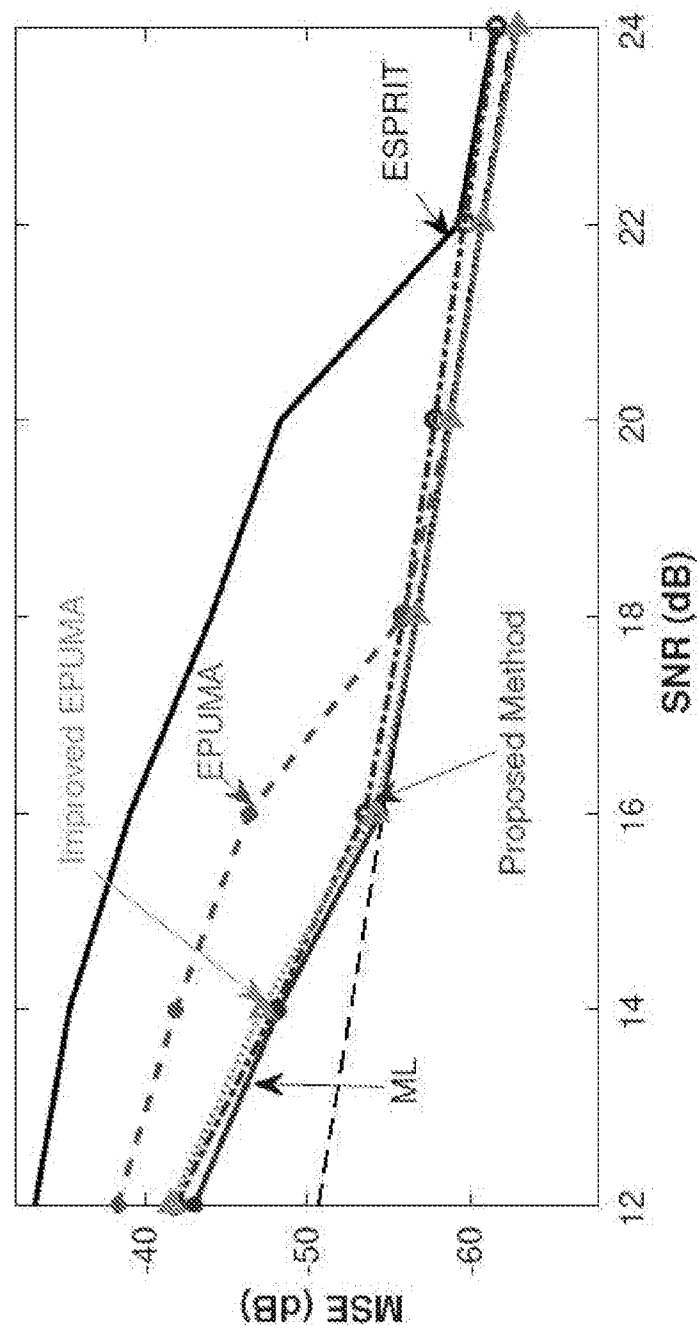


FIG. 8

INTERNATIONAL SEARCH REPORT

International application No.
PCT/IN2021/050280

A. CLASSIFICATION OF SUBJECT MATTER

G01R23/02, G01R23/16, G06F17/00 Version=2021.01

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

G01R, G06F

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

DATABASES: IPO INTERNAL DATABASE, TOTALPATENT ONE

KEYWORDS: MULTIPLE, SINUSOIDS, FREQUENCY, ESTIMATION, SNR, THRESHOLD

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	US10270547 B2 (University of Notre Dame US Department of Navy) 23 APRIL 2019 (23-04-2019) WHOLE DOCUMENT	1-12
Y	VISHNU PALAKKAL ET AL., "Improving the Estimation of Sinusoidal Frequencies and Direction-of-Arrival Using Line Spectral Frequencies", IEEE Signal Processing Letters (Volume:25, Issue: 12, Dec. 2018) 16 OCTOBER 2018 (16-10-2018) WHOLE DOCUMENT	1-12



Further documents are listed in the continuation of Box C.



See patent family annex.

* Special categories of cited documents:

"A" document defining the general state of the art which is not considered to be of particular relevance

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"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search

06-07-2021

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06-07-2021

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