Oct. 13, 1925.

O. J. ZOBEL

TERMINATING NETWORK FOR FILTERS

Filed April 30, 1920

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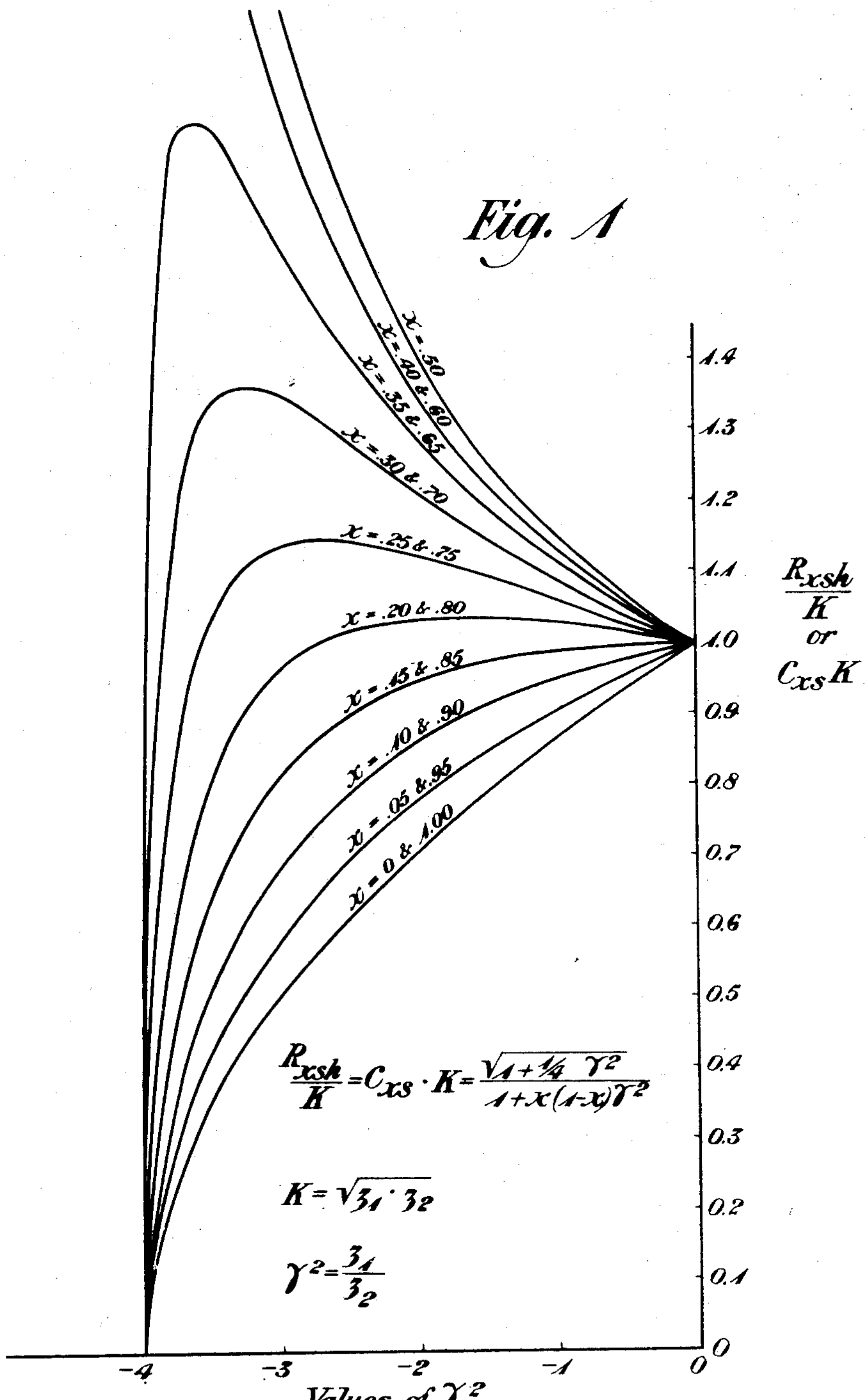
Fig. 1



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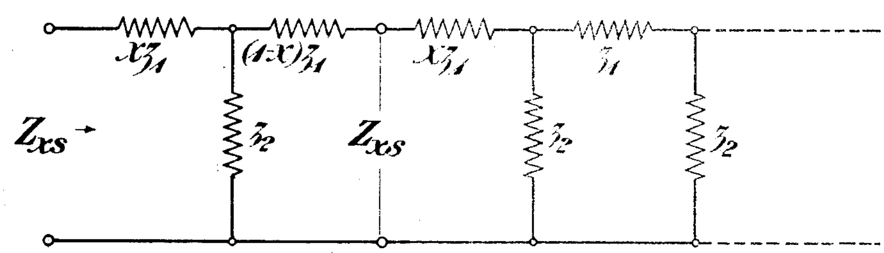

Fig. 2

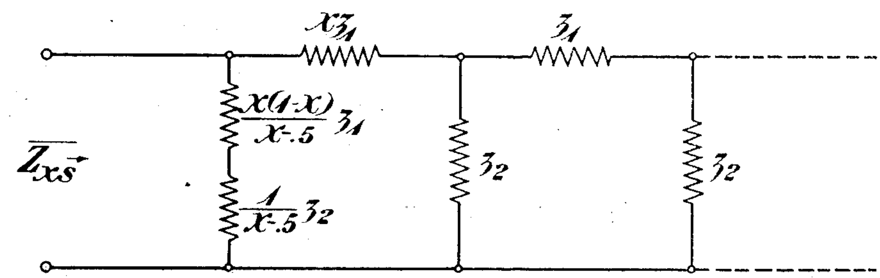

Fig. 3

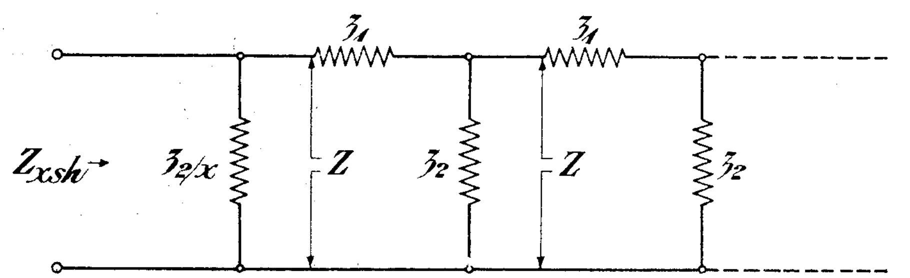

Fig. 4

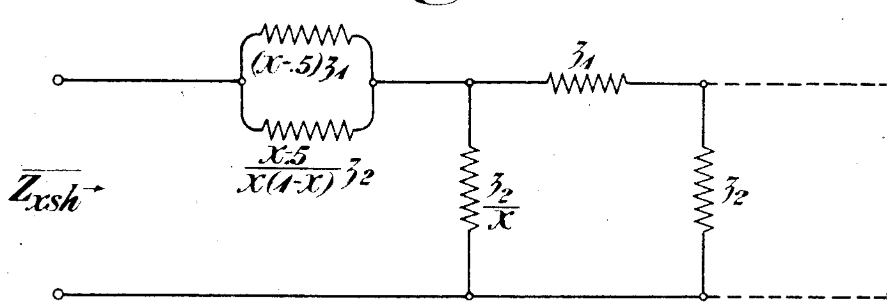

Fig. 5

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Mid-Series "Equivalent" Types

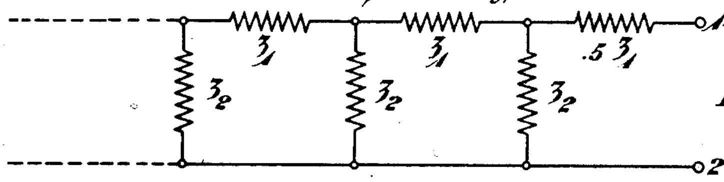

Fig. 6

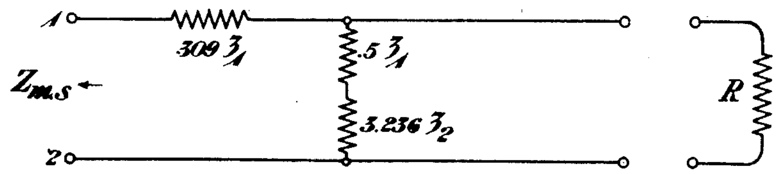

Fig. 7

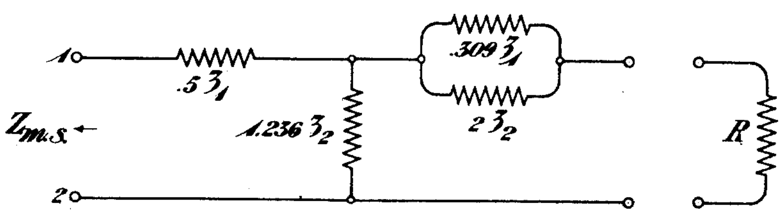

Fig. 8

Mid-Shunt "Equivalent" Types

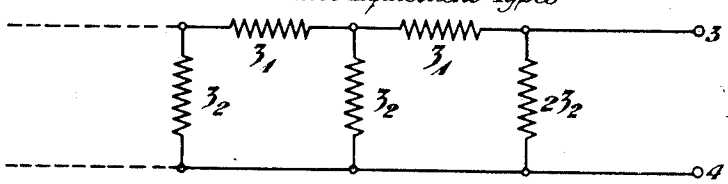

Fig. 9

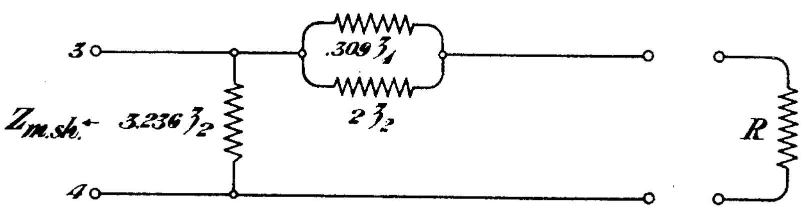

Fig. 10

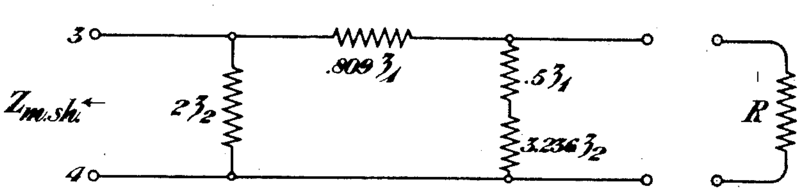

Fig. 11

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Mid-Series "Equivalent" Types

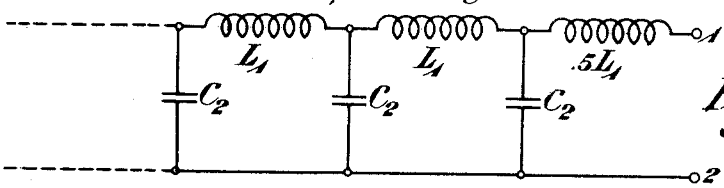

Fig. 12

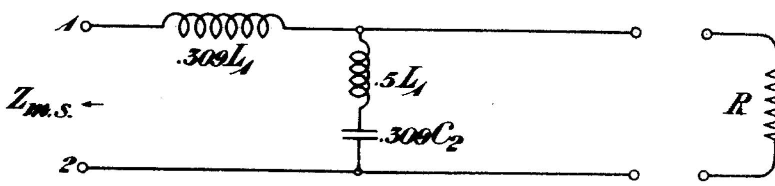

Fig. 13

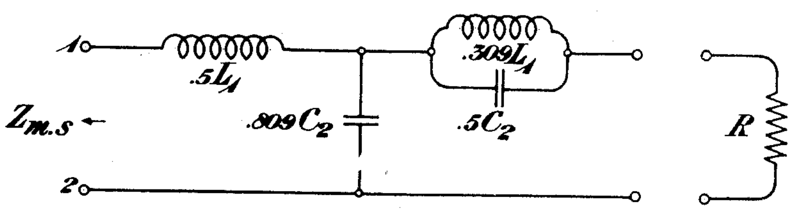

Fig. 14

Mid-Shunt "Equivalent" Types

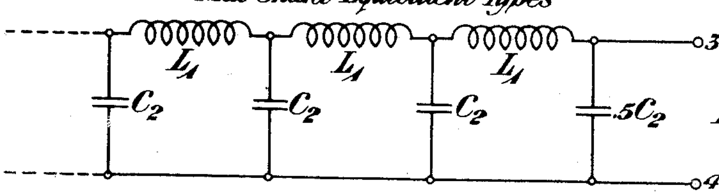

Fig. 15

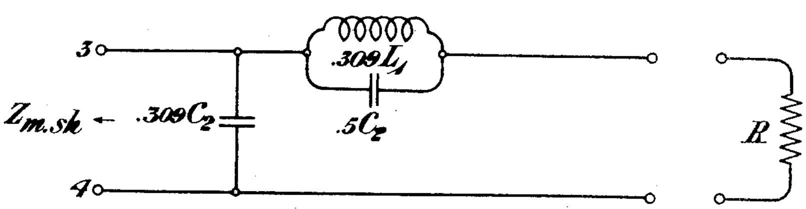

Fig. 16

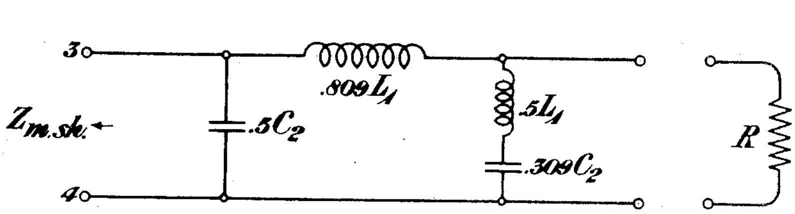

Fig. 17

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Mid-Series "Equivalent" Types

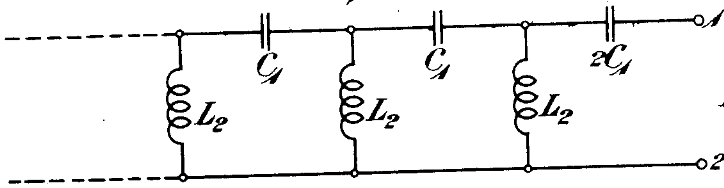

Fig. 18

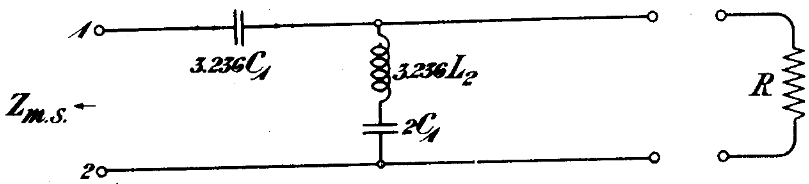

Fig. 19

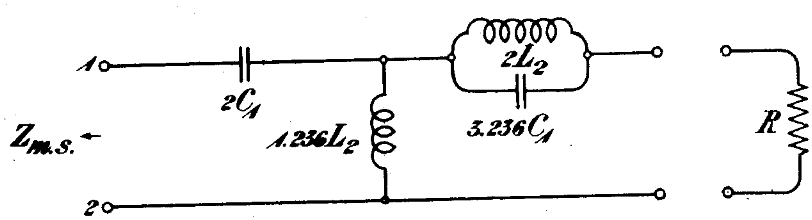

Fig. 20

Mid-Shunt "Equivalent" Types

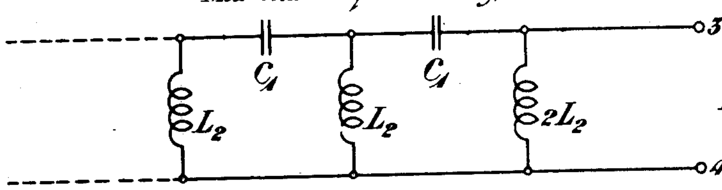

Fig. 21

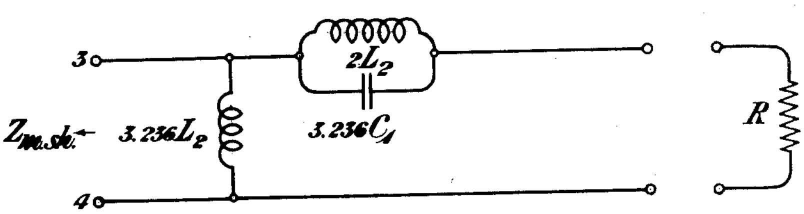

Fig. 22

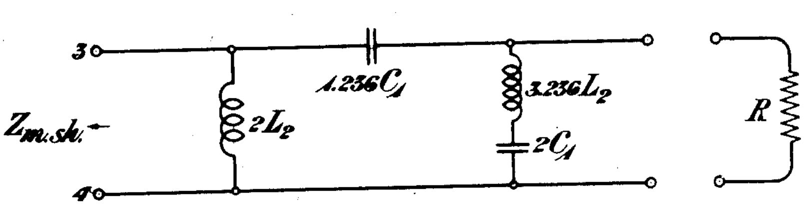

Fig. 23

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Mid-Series Equivalent Types

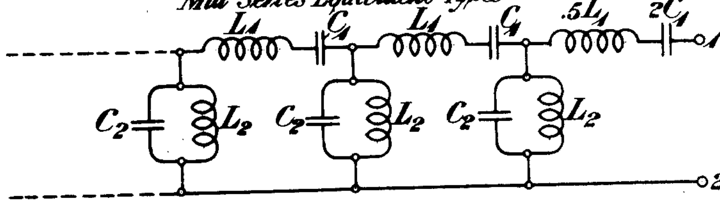

Fig. 24

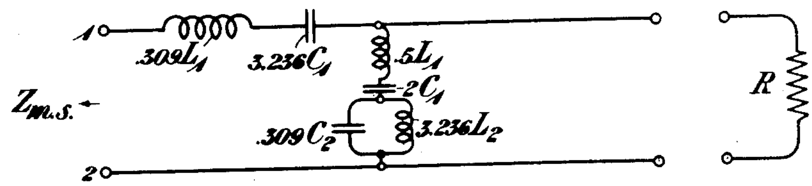

Fig. 25

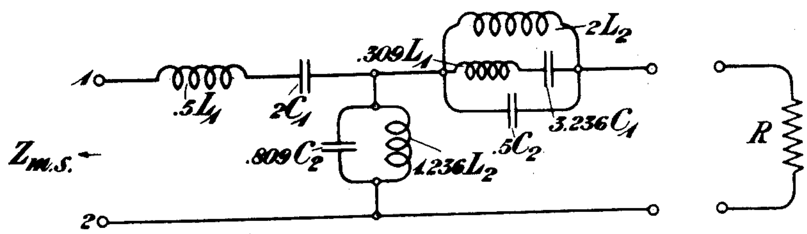

Fig. 26

Mid-Shunt "Equivalent" Types

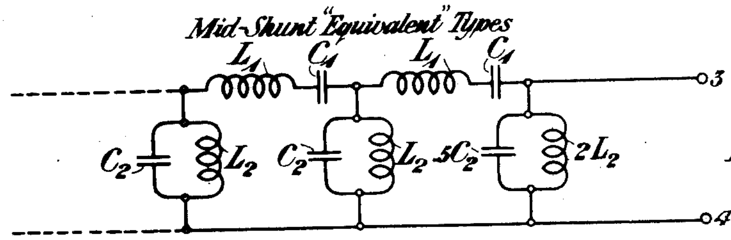

Fig. 27

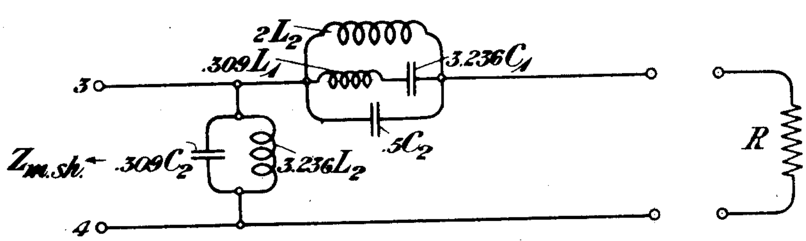

Fig. 28

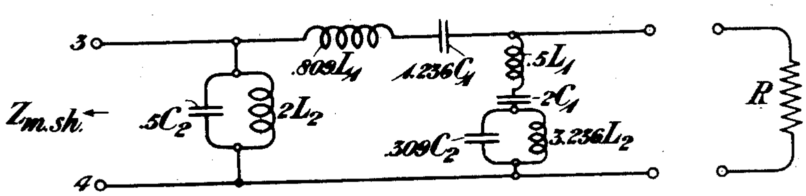

Fig. 29

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Patented Oct. 13, 1925. **1,557,229**

UNITED STATES PATENT OFFICE.

OTTO J. ZOBEL, OF MAPLEWOOD, NEW JERSEY, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

TERMINATING NETWORK FOR FILTERS.

Application filed April 30, 1920. Serial No. 377,964.

To all whom it may concern:

Be it known that I, OTTO J. ZOBEL, residing at Maplewood, in the county of Essex and State of New Jersey, have invented certain Improvements in Terminating Networks for Filters, of which the following is a specification.

This invention relates to selective circuits of the type known as wave filters, and more particularly to terminating arrangements for such filters.

Wave filters, as heretofore known in the art, consist of networks having a plurality of similar periodic sections, each section including a series and a shunt impedance element. Wave filters of this general type and their properties have been described in U. S. patents to George A. Campbell Nos. 1,227,113 and 1,227,114 issued May 22, 1917. In general when a wave filter of the type above referred to is connected to a line whose impedance is practically a constant resistance, large impedance irregularities are introduced between the wave filter and the line in the range of frequencies to be transmitted, since the characteristic impedance of the filter for any termination varies greatly with frequency. These impedance irregularities at the frequencies to be transmitted are objectionable, not only from the standpoint of maximum energy transferred from the line to the wave filter, but also from the standpoint of repeater balance.

The wave filters forming the subject matter of this invention have a singly periodic structure consisting of a plurality of sections, as indicated in Fig. 2, each section comprising a series impedance element z_1 and a shunt impedance element z_2. These impedance elements are reactances, that is, they are made up of inductances and capacities in a manner more fully hereafter described. While the discussion of these wave filters is primarily on the basis that the elements are non-dissipative in character, the inevitable introduction of dissipation will not materially alter the designs obtained. In order to minimize the transmission losses in the wave filter, there should be provided as large time constants for the inductances and capacities as is practicable in each specific case.

The computation for a filter on the assumed basis that its reactances are non-dissipative is justified both by theoretical investigations and by practical tests. It is well known that the resistance of a coil or a condenser can be made very small compared to its inductive reactance or capacity reactance, and therefore the performance of such a coil or condenser may be computed approximately with entire neglect of such slight resistance.

When the magnitudes of the inductances and capacities have been obtained in this basis and the types of coils and condensers giving these magnitudes have been decided upon, the corresponding amounts of resistance necessarily introduced are then accurately taken account of in practice when computing the current losses thru the wave-filter. The effect of this dissipation is principally to cause small allowable current losses within the bands of free transmission.

I have discovered that certain types of these filters have a property which was not pointed out by Campbell in his patents, and one which, in practice, is of considerable importance, namely that the product of the series and shunt impedances of any section is equal to the square of a constant which constant may, by proper design of the filter, be made equal to the resistance of the line with which the filter is associated. Filters having this property are known as "constant k filters."

One of the objects of the present invention is to provide such termination for a "constant k" wave filter, that the impedance of the filter, over practically the entire range of transmitted frequencies, may be made practically a constant resistance equal to k, k being preferably equal to the resistance of the line with which the filter is to be associated. Another object of the invention is to provide such a terminating arrangement for a filter that the filter will be given practically infinite attenuation just outside the transmitting range, thereby increasing the sharpness of the cutoff. Other and further objects of the invention will more fully appear hereinafter.

There are in general two types of terminations. In one type, known as the x-series termination, where x may have any value from 0 to unity, the network terminates in a series element whose impedance is x times the impedance of a full series element, that is, if z_1 be taken to designate the impedance of the series element, the terminating element will have an impedance $x.z_1$. In the other type of termination, known as the x-shunt termination, the network terminates in a shunt element whose admittance is x times the admittance of a full shunt element z_2; that is, it has an impedance $\frac{z_2}{x}$.

I have discovered that a part of the x series characteristic admittance of any singly periodic recurrent network, whether constant k or not, may be annulled by the addition of a shunt annulling element, provided x *is* greater than .5. If the network under consideration be a wave filter where z_1 and z_2 are pure reactances, this annulled part is, in the transmitting range, the susceptance. The part remaining is, in the transmitting range of the wave filter, a pure conductance, C_{xs} varying with frequency. I have also found that, for a termination of the network, such that x has a value somewhere in the neighborhood of .8, the conductance C_{xs} will be substantially $\frac{1}{k}$ over most of the transmitting range. Hence, for types of filters in which k is constant, the total impedance in the transmitting range will, under these conditions, with the annulling element provided, be practically equal to k=constant.

I have also found that a part of the x-shunt characteristic impedance may be annulled by an addition of a series annulling element when x is greater than .5. For a wave filter this annulled part is, in the transmitting range, the reactance. There remains in this range a resistance, R_{xsh}, variable with frequency. This resistance is substantially equal to k over most of the transmitting range where x has a value in the neighborhood of .8. Therefore, if k is constant, the total impedance in the transmitting range will, if the annulling element be provided, again be practically equal to k=constant.

The invention will now be more fully understood from the following description, when read in connection with the accompanying drawings, in which

Fig. 1 shows a number of curves illustrating the characteristics of certain filter terminations.

Fig. 2 is a simplified diagram of a wave filter having an x series termination.

Fig. 3 is a diagram of a shunt annulling element for use in connection with the filter in Fig. 2.

Fig. 4 is a simplified diagram of a filter having an x shunt termination.

Fig. 5 is a diagram showing a series annulling element for use in connection with the filter in Fig. 4.

Fig. 6 is a simplified diagram of a filter terminating in mid-series.

Figs. 7 and 8 are diagrams of terminating arrangements or annulling elements for the filter in Fig. 6.

Fig. 9 is a simplified diagram of a filter terminating in mid-shunt.

Figs. 10 and 11 are diagrams of annulling elements for use in connection with the filter of Fig. 9.

Fig. 12 is a diagram of a low pass filter terminating in mid-series.

Figs. 13 and 14 illustrate the corresponding annulling elements.

Fig. 15 is a diagram of a low pass filter terminating in mid-shunt.

Figs. 16 and 17 illustrate the corresponding annulling elements.

Fig. 18 is a diagram of a high pass filter terminating in mid-series.

Figs. 19 and 20 illustrate the corresponding annulling elements.

Fig. 21 is a diagram of a high pass filter terminating in mid-shunt.

Figs. 22 and 23 illustrate the corresponding annulling elements.

Fig. 24 is a diagram of a band filter terminating in mid-series.

Figs. 25 and 26 illustrate the corresponding annulling elements.

Fig. 27 illustrates a band filter having a mid-shunt termination, the proper annulling elements being illustrated in Figs. 28 and 29.

A description will now be given of two methods of terminating a "constant k" wave filter, so that the resulting corrected impedance is substantially a constant resistance k throughout the transmitting range of the wave filter. For the best correction one method terminates the wave filter in .809-series (that is, in a series element whose impedance is .809 times that of a full series element) and adds in shunt, a shunt annulling element consisting of $.5z_1$ in series with $3.236z_2$. The other method terminates the network in .809-shunt (that is, in a shunt element whose admittance is .809 times that of a full shunt element) and adds in series, a series annulling element consisting of $.309z_1$ in shunt with $2z_2$. (See Figs. 6 to 11 inclusive). The theory underlying these impedance corrective designs is as follows:

It is well known that a smooth line having uniform series impedance distributions z_1

and uniform shunt impedance distributions z_2 per unit length, has the characteristic impedance

$$k=\sqrt{z_1 \cdot z_2} \quad (1)$$

and a propagation constant

$$\gamma=\sqrt{\frac{z_1}{z_2}} \quad (2)$$

A wave filter such as illustrated in Fig. 6, having series element z_1 and shunt element z_2 has a characteristic impedance at any termination which is a function of both the product and ratio of z_1 and z_2 and has a propagation constant which is a function of their ratio. Hence it has been found convenient to express both the characteristic impedance and propagation constant of the wave filter in terms of k and γ, the parameters of the corresponding smooth line. As pointed out in the Campbell patent above referred to, free transmission occurs in such wave filters, (if infinitely long) for a range of frequencies corresponding to the range

$$\gamma^2=0 \text{ to } \gamma^2=-4. \quad (3)$$

Let us now determine the x-series characteristic admittance, A_{xs}, of a wave filter, having a series element z_1 and a shunt element z_2 per section. Referring to the diagram of Fig. 2 the x-series impedance Z_{xs} of such a filter with an infinity of recurrent sections may be expressed as follows:

$$Z_{xs}=xz_1+\frac{z_2(Z_{xs}+(1-x)z_1)}{Z_{xs}+z_2+(1-x)z_1} \quad (4)$$

This expression, by simple algebraic transformation may be expressed

$$Z_{xs}=(x-.5)z_1\pm\sqrt{z_1z_2+\frac{1}{4}z_1^2} \quad (5)$$

Now since the admittance is a reciprocal of the impedance, we have

$$A_{xs}=\frac{1}{Z_{xs}}=\frac{1}{\pm\sqrt{z_1z_2+\frac{1}{4}z_1^2}+(x-.5)z_1} \quad (6)$$

Multiplying both numerator and denominator by

$$\pm\sqrt{z_1z_2+\frac{1}{4}z_1^2}-(x-.5)z_1$$

we get

$$A_{xs}=\frac{\pm\sqrt{z_1z_2+\frac{1}{4}z_1^2}+(.5-x)z_1}{z_1z_2+x(1-x)z_1^2} \quad (7)$$

Replacing $\frac{z_1}{z_2}$ by γ^2 and $z_1 \cdot z_2$ by k^2 we have

$$A_{xs}=\frac{\pm\sqrt{1+\frac{1}{4}\gamma^2}}{1+x(1-x)\gamma^2}\cdot\frac{1}{k}+\frac{(.5-x)\gamma}{1+x(1-x)\gamma^2}\cdot\frac{1}{k} \quad (8)$$

In the transmitting range of the wave filter, where, as explained in the Campbell patent, γ^2 lies between 0 and -4, the admittance may be considered as being made up as two components, the conductance, C_{xs}, and the susceptance, S_{xs}. This may be expressed as follows:

$$A_{xs}=C_{xs}+iS_{xs}. \quad (9)$$

Since, as explained in the Campbell patent, at transmitted frequencies γ lies between 0 and $\pm i2$ and is imaginary it follows that, from the formula of equation 8, the first half of the right hand portion of equation 8 represents the conductance (the conductance being positive), and the last half represents the susceptance (the susceptance being always contained in the imaginary factor). Hence we have

$$C_{xs}=+\frac{\sqrt{1+\frac{1}{4}\gamma^2}}{1+x(1-x)\gamma^2}\cdot\frac{1}{k} \quad (10)$$

and

$$iS_{xs}=\frac{(.5-x)\gamma}{1+x(1-x)\gamma^2}\cdot\frac{1}{k} \quad (11)$$

Now it is apparent that the susceptance may be annulled in equation 8 by a shunt annulling element of equal value and opposite in sign to the value of the susceptance given in equation 11. The admittance A_{sh} of the shunt element must then be

$$A_{sh}=\frac{(x-.5)\gamma}{1+x(1-x)\gamma^2}\cdot\frac{1}{k} \quad (12)$$

Since the impedance Z_{sh} of the annulling element is the reciprocal of the admittance, or $\frac{1}{A_{sh}}$, equation 12 may be rewritten by substituting the values of γ and k as follows:

$$Z_{sh}=\frac{z_2}{x-.5}+\frac{x(1-x)}{x-.5}z_1 \quad (13)$$

From the form of this equation it is apparent that the shunt annulling element consists of two parts in series—one part having an impedance

$$\frac{z_2}{x-.5}$$

and the other part having an impedance

$$\frac{x(1-x)}{x-.5}z_1.$$

This only holds true when x is greater than .5 because if x is less than .5 these quantities become negative and cannot be realized physically. The circuit arrangement of a filter terminating in x-series, and having a shunt annulling element conforming to equation 13 is illustrated schematically in Fig. 3. In the arrangement of Fig. 3, if x is greater than .5 and the shunt annulling element is provided, there is left only the first part of the admittance of the filter, which, in the transmitting range, is the conductance C_{xs}. The conductance coefficient may be written as follows:

$$C_{xs}\cdot k = \frac{\sqrt{1+\frac{1}{4}\gamma^2}}{1+x(1-x)\gamma^2} \quad (14)$$

(See equation 10). This coefficient, is plotted in Fig. 1, for different values of x, and as will be apparent this coefficient is substantially constant over the greater portion of the transmission range, when x has a value of about .8. In the "constant k" type of filter the conductance will then be nearly equal to $\frac{1}{k}$ and the impedance to k = constant in the transmitting range.

We will now determine the proper impedance corrective design for a filter having an x-shunt termination. Such a filter is indicated schematically in Fig. 4, and from this figure it is apparent that the x-shunt impedance Z_{xsh} may be written as follows:

$$Z_{xsh} = \frac{\frac{z_2}{x}\cdot Z}{\frac{z_2}{x}+Z} = \frac{z_2 Z}{z_2+xZ} \quad (15)$$

The impedance Z of a filter having a full termination may, from Fig. 4, be expressed as follows:

$$Z = z_1 + \frac{z_2 Z}{z_2+Z} \quad (16)$$

Substituting this value of Z in formula 15, we have, by simple algebraic transformation

$$Z_{xsh} = \frac{z_2\sqrt{z_1z_2+\left(\frac{z_1}{2}\right)^2}+\frac{z_1z_2}{2}}{z_2+x\sqrt{z_1z_2+\left(\frac{z_1}{2}\right)^2}+\frac{xz_1}{2}} \quad (17)$$

Multiplying both numerator and denominator by

$$z_2+\frac{xz_1}{2}-x\sqrt{z_1z_2+\left(\frac{z_1}{2}\right)^2}$$

and simplifying, we have

$$Z_{xsh} = \frac{z_2\sqrt{z_1z_2+\frac{1}{4}\cdot z_1^2}+(.5-x)z_1z_2}{z_2+x(1-x)z_1} \quad (18)$$

Substituting the values of γ and k, this equation becomes

$$Z_{xsh} = \frac{\sqrt{1+\frac{\gamma^2}{4}}}{1+x(1-x)\gamma^2}\cdot k + \frac{(.5-x)\gamma}{1+x(1-x)\gamma^2}\cdot k \quad (19)$$

From the form of this equation it is apparent that the second component is a reactance and the first component a resistance. The reactance component may obviously be annulled by a series element whose impedance is equal to the second component and opposite in sign. Thus

$$z_s = \frac{(x-.5)\gamma}{1+x(1-x)\gamma^2}\cdot k \quad (20)$$

Substituting the values of γ and k in equation 20, we have, by simple algebraic transformation

$$z_s = \frac{(x-.5)z_1\frac{(x-.5)}{x(1-x)}\cdot z_2}{(x-.5)z_1+\frac{x-.5}{x(1-x)}\cdot z_2} \quad (21)$$

From equation 21 it is apparent that the annulling element may consist of two parallel elements, whose impedances are

$$(x-.5)z_1$$

and

$$\frac{x-.5}{x(1-x)}\cdot z_2$$

respectively where x is greater than .5.

This arrangement is illustrated in Fig. 5, and when provided it is obvious that only the first part of the impedance of the filter remains, this part in the transmitting range being the resistance R_{xsh}. The resistance coefficient $\frac{R_{xsh}}{k}$ is the same coefficient as given in equation 14, and from Fig. 1 is nearly unity over the greater part of the transmitting range, when x has a value of about .8. Hence with a termination of .8 shunt the resistance R_{xsh} will, in the "constant k" types of filters, be substantially equal to k in the transmitting range.

The network here disclosed in connection with Figures 4 and 5 serves equally well at the input end or the drop end of the filter, as may be shown by an explanation similar to that for Figs. 2 and 3.

Since, in practice, it is customary to terminate filters and other forms of networks in mid-series or mid-shunt terminations instead of .8 terminations, it is desirable that annulling elements be provided for mid-shunt and mid-series terminations. Fig. 6 illustrates a filter terminating in mid-series, and Figs. 7 and 8 illustrate the annulling elements to correspond thereto. These annulling elements are so designed as to be made up of a few simple standard elements, which may be used in the construction of any type of corrective network to be used with wave filters having either mid-series or mid-shunt characteristic impedances equivalent to those of the "constant k" type. In the design illustrated x has been given a value of .809 which not only has the advantage of being near .8, which, as shown in Fig. 1 is the most desirable value, but also permits of a similarity in the elements of both the shunt and series annulling networks. This value of x is chosen because then

$$.5z_1 = \frac{x(1-x)}{x-.5} z_1. \quad (22)$$

In Fig. 7 the shunt annulling element is made up of two elements having values $.5z_1$ and $3.236z_2$, these values corresponding to those given by equation 13. In addition a series element having a value of $.309z_1$ is also provided. This series element is simply employed so that it, when taken with the mid-series termination of $.5z_1$ in Fig. 6, gives in effect a termination of $.809z_1$.

In Fig. 8 the mid-series terminating filter of Fig. 6 is built out by adding a series element of $.5z_1$, thus making a full series element for the section, and the shunt element having the value $1.236z_2$ is provided, since this corresponds to a .809 shunt termination. This permits of using the elements of a series annulling element, such as is ordinarily used with an x-shunt terminating filter. These elements, as indicated, have the values $.309z_1$ in parallel with $2z_2$. A comparison of Figs. 7 and 8 shows that the two annulling devices are constructed from identical units.

Fig. 9 illustrates a filter terminating in mid-shunt while Figs. 10 and 11 illustrate the corresponding annulling elements to be used therewith. In Fig. 10 the shunt element, having the value $3.236z_2$, when placed in parallel with the shunt termination of $2z^2$ in Fig. 9, gives an effective shunt termination of $\frac{z_2}{.809}$. The series annulling elements themselves are the same as in Fig. 8, and have the values given by the formula 21. In Fig. 11 the mid-shunt termination of Fig. 9 is built out to a full shunt section by means of the shunt element $2z_2$, and the filter is given in effect a .809 series termination by means of the series element $.809z_1$. The shunt annulling elements then used are the same as those shown in Fig. 7.

The principles employed in Figs. 6 to 11 inclusive, are shown applied to a low pass type of filter in Figs. 12 to 17 inclusive. The design of the networks of these figures will be apparent without further description. Similarly Figures 18 to 23 inclusive give the corresponding designs for high pass filters terminating either in mid-series or mid-shunt, and Figs. 24 to 29 inclusive, give the designs for single band filters terminating either in mid-series or mid-shunt.

In all of these cases it will be seen that two alternative annulling networks are provided for each mid-series and each mid-shunt termination, and that the elements of the second network of each pair will always give the elements of the first network of each pair.

The annulling elements have an additional useful property besides impedance correction in that they cause infinite attenuation just outside the transmitting region. This occurs in both forms of annulling networks, if $x=.809$ when

$$\gamma^2 = \frac{z_1}{z_2} = -6.47$$

for, under these circumstances, the shunt annulling element Z_{sh} becomes resonant, that is,

$$.5z_1 + 3.236z_2 = 0.$$

Also the series annulling Z_s becomes anti-resonant, that is

$$\frac{.309z_1 \cdot 2z_2}{.309z_1 + 2z_2} = \infty$$

or expressed in another way $.309z_1 + 2z_2 = 0$. This result also follows from the perfectly general property of these annulling elements wherein, for the same value of x, their product is

$$z_s \cdot z_{sh} = z_1 \cdot z_2 = k^2 \quad (23)$$

It will be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated without departing from the spirit of the invention, as defined in the following claims.

What I claim is:

1. In a transmission circuit a wave filter comprising a plurality of like sections, each section comprising series and shunt impedance elements, at least one of said elements including both inductance and capacity, and a terminating network unlike said sections associated with said filter, said network being so proportioned as to neutralize the reactance component of the impedance of the filter over practically the entire range of free transmission and to equalize the corresponding resistance component to a constant value.

2. In a transmission circuit a wave filter comprising a plurality of like sections, each section comprising series and shunt impedance elements, at least one of said elements including both inductance and capacity, and a terminating network unlike said sections associated with said filter, said network being so proportioned as to equalize the reactance component and the resistance component of the impedance of the filter each to a constant value over practically the entire range of free transmission.

3. In a transmission circuit a wave filter comprising a plurality of like sections, each section comprising series and shunt impedance elements, at least one of said elements including both inductance and capacity, and a terminating network unlike said sections associated with said filter, said network being so proportioned as to neutralize the re-

actance component of the impedance of the filter over practically the entire range of free transmission.

4. In a transmission circuit a wave filter comprising a plurality of like sections, each section comprising series and shunt impedance elements, at least one of said elements including both inductance and capacity, and a terminating network unlike said sections associated with said filter, said network being so proportioned as to neutralize the susceptance component of the admittance of the filter, within its free transmitting range.

5. In a transmission circuit a wave filter comprising a plurality of like sections, the end section of the series being only a fractional part of the other sections, and a terminating network associated with said end section, said network being so proportioned with respect to the filter as to neutralize the reactance component of the impedance of the filter over practically the entire range of free transmission and to equalize the corresponding resistance component to a constant value.

6. In a transmission circuit a wave filter comprising a plurality of like sections, the end section of the series being only a fractional part of the other sections, and a transmitting network associated with said end section, said network being so proportioned that the reactance and resistance components of the impedance of the filter over substantially the entire range of free transmission will each be equalized respectively to a constant value.

7. In a transmission circuit a wave filter comprising a plurality of like sections, the end section of the series being only a fractional part of the other sections, and a terminating network associated with said end section, said network being so proportioned that the reactance component of the impedance will be neutralized over a substantial range of frequencies.

8. In a transmission circuit a wave filter comprising a plurality of like sections, the end section of the series being only a fractional part of the other sections, and a terminating network associated with said end section, said network being so proportioned that the susceptance component of the admittance will be substantially neutralized over the range of free transmission.

9. In a transmission circuit a wave filter comprising a plurality of like sections, means to build out the terminating section so that the filter may be given either a fractional series or a fractional shunt termination, and an annulling network to be connected either in series or in shunt with the termination of the filter for neutralizing the reactance component of the impedance of the filter, the elements of the building out means and the annulling network being so proportioned that the same elements may be combined whether the termination is fractional series or fractional shunt.

10. In a transmission system a wave filter comprising a plurality of like sections, each section comprising series and shunt impedance elements, at least one of said elements including both inductance and capacity, and a terminating network applied to the last section of the wave filter, said network comprising means to present substantially an infinite impedance at a frequency just outside of the free range of transmission, thereby giving the filter a very sharp cut-off.

11. In a transmission system a wave filter comprising a plurality of like sections, each section comprising series and shunt impedance elements, at least one of said elements including both inductance and capacity, and a terminating network applied to the last section of the wave filter, said network being so proportioned as to render the impedance of the wave filter substantially constant over the range of free transmission and comprising means to present practically an infinite impedance to a frequency just outside the range of free transmission, thereby giving the filter a sharp cutoff.

12. A wave filter having a fractional end section and a net involving the fractional value in its design, said net being so proportioned as to neutralize the reactance component of the impedance of the filter over the free transmitting range, the said fractional end section being taken at such a fractional value that the combination gives the nearest approximation to a constant resistance for the resistance component of the impedance thereof.

13. A wave filter having alternately disposed series impedances and shunt impedances, the product of the series impedance value by the shunt impedance value being constant, said filter having a fractional end section and a net involving the fractional value in its design, said net being so proportioned as to neutralize the reactance component of the impedance of the filter over the free transmitting range, the said fractional end section being taken at such a fractional value that the combination gives the nearest approximation to a constant resistance for the resistance component of the impedance thereof.

14. In combination, a wave filter and means to neutralize its reactance component and equalize its resistance component to a substantially constant value over the free transmitting range of the filter.

15. In combination, a wave filter and a terminal network to neutralize the reactance component of the impedance within the free transmitting range and to equalize the resistance component of the impedance within the free transmitting range.

16. In combination, a wave filter, a piece of apparatus of substantially constant resistance connected thereto, and an interposed network to neutralize the reactance component of the impedance of the filter and approximately equalize its resistance component to the resistance of said piece of apparatus.

17. In combination, a wave filter having recurrent alternately disposed series impedances z_1 and shunt admittances $1/z_2$ and at one end the series element of value X times one of the foregoing elements where X is a real number less than unity, and at the same end a network whose impedance value is a function of X and which is so proportioned as to neutralize the reactance component of the impedance of the filter.

18. In combination, a wave filter having alternately disposed series impedances z_1 and shunt admittances $1/z_2$ and at one end a series element which is 0.809 times one of the foregoing elements and in combination an impedance so proportioned as to neutralize the reactance component of the impedance of said filter.

19. A wave filter having a fractional end section, the value of the fraction being chosen most nearly to make the resistance component constant and also having a terminal network to neutralize the reactance component corresponding to the value chosen for the said fraction.

20. A wave filter having alternately disposed series impedances and shunt admittances with a fractional element being a fraction of one of the foregoing at one end and in combination therewith a network to neutralize the reactance of the said filter.

In testimony whereof, I have signed my name to this specification this 28th day of April 1920.

OTTO J. ZOBEL.