SUPPORTING STRUCTURE FOR FREEFORM SURFACES IN BUILDINGS

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ABSTRACT

Supporting structure (8) for curved envelope geometries in buildings consisting of support elements (4) which are each combined to form N-sided polygons (N=3,4,...) which span the envelope geometry and which each enclose a planar surface element (5), and the support elements (4) of adjacent N-sided polygons each form a common node region (3) in which the support elements (4) abut, and the plane of a surface element (5) and the respective planes of the surface elements (5) of two N-sided polygons which adjoin in non-parallel spatial directions lie in different planes in at least one portion of the envelope geometry. According to the invention, the support elements (4) in this portion each form 4- or 6-sided polygons, and the support elements (4) each have a longitudinal axis (L) which extends in a straight line between in each case two node regions (3) and runs parallel to the imaginary intersection line of the surface element planes associated with said portion, wherein the cross section of the support element (4) normal to their longitudinal axis in each case has a relative twist angle of 0° along the entire longitudinal axis of the support element (4).
SUPPORTING STRUCTURE FOR FREEFORM SURFACES IN BUILDINGS

[0001] The invention relates to a supporting structure for curved envelope geometries in buildings, consisting of support elements which are each combined to form N-gons (N=3, 4, ... ) which span the envelope geometry and which each enclose a planar surface element, and the support elements of adjacent N-gons each form a common node region in which the support elements abut, and the plane of a surface element and the respective planes of the surface elements of two N-gons which adjoin in non-parallel spatial direction lie in different planes in at least one section of the envelope geometry, according to the preamble of claim 1.

[0002] The invention further relates to a method for determining a supporting structure for curved envelope geometries in buildings, consisting of support elements in which a predetermined curved envelope geometry is approximated by a continuous network of N-gons (N=3,4, ... ) which define a planar mesh, with respectively adjoining N-gons having a common node, and the plane of a surface element and the respective planes of the surface elements of two N-gons adjoining in non-parallel spatial directions lie in different planes in at least one section of the envelope geometry, according to the preamble of claim 6.

[0003] Curved envelope geometries of this kind are used in building construction for realizing freeform surfaces in which the curvature is different in two different spatial directions, such as in the case of domed buildings or also more complex surface shapes. Freeform surfaces of this kind are also known as non-developable surfaces, and are designed at first in the course of architectural planning in the computer model as continuous surfaces. In constructional implementation, the continuous freeform surfaces are approximated by a plurality of individual surface elements which are held in a supporting structure. It is thus possible for example to also realize complex freeform surfaces with multilayered planar glass elements which are fastened above, between and beneath a supporting structure made of steel for example. Said supporting structure is formed from the individual support elements which are each composed into N-gons, i.e. triangles, quadrilaterals, hexagons, etc. The N-gons span the supporting structure, with the support elements abutting in the node region where they are fastened to each other.

[0004] One possibility of approximation of freeform surfaces by individual surface elements is to approximate the freeform surface by curved surface elements which are each held by planar support elements for cost reasons. One example for this is the bubble-like construction of the “Kunsthaus Graz”, where a plastically deformable material (namely plexiglass) was used. The curvature required for the individual surface elements was provided by thermal deformation. The supporting structure is realized in a constructional respect by support elements made of forming tubes, with the respectively required spatial curves of the support elements being generated by twisting in the node region of mutually adjacent forming tubes.

[0005] Such a procedure comes with a number of disadvantages. The choice of material for the surface elements is subject to limitations due to the required deformability. Specifically, the necessity arose for example in the mentioned example of the “Kunsthaus Graz” to create an additional, fire-resistant layer due to the thermal deformation of the oriented plexiglass in order to neutralize the adverse fire properties of the plexiglass. Moreover, it was noticed after realization that the thermal behavior as a result of the very high coefficient of thermal expansion (approx. six times that of steel) of the oriented plexiglass leads under insulation to the “sagging” of the plexiglass plates, and therefore separate supports had to be provided in order to ensure that the shape was retained. Permanent changes in the form of bumps or dents can be recognized through irregular reflections.

[0006] A further principal disadvantage in realizing freeform surfaces by means of individual curved surface elements is also given in cases where a multilayered structure of the building shell in order to house necessary building infrastructure such as piping and the like. The user will only notice the optically visible layers, i.e. the inner and outer shell of the building. For the building’s functionality however, a large number of layers relating to the physics of the building must be provided between the optically visible layers by maintaining the shape of the outer shell of the building. Planar intermediate layers are necessary for this infrastructure which needs to be introduced with much effort between the optically visible, curved freeform surfaces. This concept is also pursued for example in the buildings of Frank O’Gehry, as a result of which his complex building shapes were able to be built in an economically viable way.

[0007] A further principal disadvantage also arises in practice from the considerable quantities of data which need to be processed in the course of planning and which need to be exchanged between architect and specialist planners. In order to describe a freeform surface, it is necessary to reproduce the spatial position and the shaping of support and surface elements in a spatial system of coordinates, with there hardly being any possibilities for reducing data due to the often different shape of every single support and surface element. In practice, “point clouds”, i.e. individual data points in a large number, need to be processed, thus causing problems especially in the use of different CAD or FEM software packages, e.g. in the data transfer between individual specialist planners such as from the architect to the planner for the supporting framework. If a freeform surface is realized instead with the help of planar surface elements, local systems of coordinates can be defined for the individual surface elements in which the boundary points of the surface element are distinctly definable already by two coordinates such as an x and y coordinate, and a z coordinate can easily be determined by the surface normal on the surface element. These local systems of coordinates are distinctly defined relative to the global system of coordinates of the overall structure. This enables a simple exchange of data with the help of an output file for example which shows the position of the normal of the supporting structure in the node region and the associated local coordinates of the planar surface elements.

[0008] A further advantage of the use of planar surface elements is the unrestricted choice of material for the surface and rod-like elements because no special elastic properties or plastic deformability are required. Moreover, the cutting to size of planar surface elements can be made more easily than in the case of curved surface elements. This reduces the overall construction costs considerably for structural shapes with freeform surfaces.

[0009] The implementation of freeform surfaces demanded by architects in a structural shape that can be executed in construction with the help of planar surface elements which approximate the freeform surface in the best possible way is
linked to difficulties. It is the object to find a distribution of support elements which is realistic technically and economically and which after the insertion of the planar surface elements reproduces the predetermined freeform surface in a continuous manner and with the appearance of an aesthetically continuous progression.

[0010] According to the state of the art, one mostly chooses a supporting structure for this purpose in which the support elements are arranged in form of the triangle. Because the modeling of freeform surfaces by means of a continuous network of triangles can be solved mathematically very well and can be realized from a constructional standpoint in a comparatively simple way.

[0011] The use of triangles as an elementary basic structure of the supporting structure also comes with disadvantages however. Especially, it will not be possible to find a distribution of the support elements with the help of triangles in which the support elements need not be subjected to any torsion in the geometrical sense, i.e. a twisting of the longitudinal axis in the node region for example, during the mounting between two node regions. Only support elements with a circular cross section can be positioned successively in a "torsion-free" manner, in the geometrical sense. When using non-circular cross sections, torsion (in the geometrical sense) will occur in the supporting structure in the node region. This leads to node regions which are unsatisfactory in regard to aesthetics and statics. Moreover, this also leads to the problem that multi-layered structure cannot be realized or only be realized with considerable additional effort. It is therefore necessary to provide a separate support system for each layer, thus increasing the material costs and mounting work several times over.

[0012] It is therefore the object of the invention to find a constructional implementation of freeform surfaces which reduces the technical and economic requirements and satisfies aesthetic demands. In particular, mounting work and costs shall be kept as low as possible. It is a further object of the invention that the supporting structure also offers the possibility of an uncomplicated multi-layered configuration for approximation of freeform surfaces, i.e. the mounting in parallel offset of several planar surface elements. These objects are achieved by the measures of claim 1.

[0013] Claim 1 relates at first to a supporting structure for curved envelope geometries of a building, consisting of support elements which are each combined to form N-gons (N=3, 4, ... ) which span the envelope geometry and which each enclose a planar surface element, and the support elements of adjacent N-gons each form a common node region in which the support elements abut, and the plane of a surface element and the respective planes of the surface elements of two N-gons which adjoin in non-parallel spatial direction lie in different planes in at least one section of the envelope geometry. The property that the plane of a surface element and the respective planes of the surface elements of two N-gons which adjoin in non-parallel spatial directions lie in different planes reflects the fact that the supporting structure is provided for realizing freeform surfaces in which the curvature of the approximating freeform surface is different in two different spatial directions. It is provided in accordance with the invention that the support elements form in this section 4-gons or 6-gons each, and the support elements each have a longitudinal axis which extends in a straight line between two node regions each and runs parallel to the imaginary line of intersection of the surface element planes associated with the same, with the cross section of the support elements normal to their longitudinal axis in each case having a relative twist angle of 0° along the entire longitudinal axis of the support element. The "associated surface element planes" of a support element are the planes of the surface elements which are supported by the respective support element.

[0014] The choice of quadrilaterals or hexagons as an elementary basic shape of supporting structure in contrast to the triangular network structure as known in the state of the art is of decisive importance because the applicants have recognized a quadrilateral or hexagonal network structure for the approximation of freeform surfaces has remarkable mathematical properties which are highly advantageous for a constructional implementation of freeform surfaces. In particular, a form of geometrical approximation of freeform surfaces can be found for quadrilateral or hexagonal network structures which ensure parallel displacement ability of the surface elements, with the respectively parallel displaced surface elements having boundary lines which are parallel to the respective original boundary lines, and thus lead to a continuous overall surface, as will be explained below. The practical consequences of parallel displacement ability ("offset") and their relevance for building engineering have not yet been recognized in the state of the art.

[0015] Moreover, construction costs can be reduced with the help of a quadrilateral or hexagonal network structure in comparison with triangular network structures because the cutting to size of triangular surface elements causes more work than the one for quadrilateral surface elements for example. Moreover, a supporting structure consisting of quadrilateral or hexagonal basic shapes requires less material because the applicants were able to show that in the case of equivalent approximations of freeform surfaces with the help of triangular and quadrilateral network structures the realization by means of quadrilateral network structures requires a lower number of support elements than the one with the triangular basic shapes.

[0016] As a result of parallel displacement ability enabled by the choice of quadrilaterals or hexagons as the basic form of the supporting structure, there is the possibility to provide a further feature in accordance with the invention concerning the support elements, which are support elements which extend between the node areas in a straight line and without torsion (in the geometric sense). This is expressed by the feature that the support elements each comprise a longitudinal axis which extends in a straight line between two node regions each and runs parallel to the imaginary line of intersection of the surface element planes associated with the same, and the cross section of the support elements normal to their longitudinal axis in each case have a relative twist angle of 0° along the entire longitudinal axis of the support element. In the case of a support element which is subject to torsion in the geometric sense, the cross section along the longitudinal axis has a twisting in contrast to this, which thus has a twist angle of unequal 0°. As a result of this feature in accordance with the invention, the support elements transversally to the plane of the surface element can be provided higher, so that a multi-layered configuration in one and the same supporting structure is enabled. The support elements can be arranged in a sufficiently high way in order to create space for the building infrastructure between the boundary outer layers. As a result of the higher arranged support elements and the use of quadrilateral or hexagonal surface elements, the installation of the electrical and technical systems and a structural-
multilayered configuration are facilitated. Moreover, the use of straight support elements without torsion (in the geometric sense) and bending facilitates mounting, thus reducing the assembly costs.

[0017] One possibility for ensuring parallel displaceability of planar surface elements in a supporting structure approximating a freeform surface is according to claim 2 that the angular sum of respectively opposing angles is equal in the point of intersection of the imaginary lines of intersection of four surface element planes adjoining in a node region. Notice must be taken that the result of a geometric approximation of a freeform surface predetermined by the architect is at first a network of lines, with the quadrilaterals being represented by a quadrilateral polyline which will also be referred to as “mesh”, and two adjoining polygons each have a common boundary line, and four adjoining polylines each have a common node in which the respectively common boundary lines intersect, as will be explained below in closer detail. In constructional implementation however, the support elements shall be applied along the common boundary lines, so that the surface elements held in the support elements usually no longer abut physically in order to divide a common boundary line, but are spaced from one another. An imaginary line of intersection can still be formed which is defined by the imaginary extension of the respective surface element planes. This imaginary line of intersection corresponds to the aforementioned common boundary line of the polylines of the geometrical approximation. The point of intersection of the imaginary lines of intersection of four surface element planes adjoining in a node region corresponds to the aforementioned node of four polylines. In the nodes, four angles are obtained between the four adjoining polylines, with the supporting structure having to be arranged in accordance with claim 2 in such a way that the sum total of two mutually opposite angles each is equal. This is a sufficient condition for parallel displaceability of the surface elements held in the supporting structure and the support elements can be arranged without torsion (in the geometric sense), which will be explained below in closer detail. In this case, the polygon network underlying such a supporting structure is also known as a “conical network”; as will also be explained below in closer detail.

[0018] A further possibility for ensuring parallel displaceability of planar surface elements in a supporting structure approximating a freeform surface is according to claim 3 that the angular sum of respectively opposite angles between the surface normals of two adjoining surface element planes of four surface element planes adjoining in a node region is equal. In this case, the polygon network underlying such a supporting structure is also known as a “dual-isothermal net-work”; as will also be explained below in closer detail.

[0019] It is provided according to claim 4 that the support elements have a rectangular cross-sectional shape or can be inscribed into a rectangular cross-sectional shape. The longitudinal axis of these support elements extends between two node regions each, and the transversal axis stands along the entire longitudinal extension of the support element both normally to the longitudinal axis as well as normally to the imaginary line of intersection of the surface element planes associated with the same. The longitudinal axis need not necessarily be an axis of symmetry of the support element. It is relevant that it extends along the support element in a straight line between two node regions and parallel to the imaginary line of intersection of the surface element planes associated with the same. Rectangular cross sections also have an aesthetic advantage because they appear to be more slender than circular cross sections, and can also be arranged in a more slender way because only the height of the supports normally to the line of intersection of the surface element planes associated with the same is relevant for the bending load as a result of the load by the surface elements acting along the transversal axis. Moreover, other forms of support elements can be advantageous depending on the technical requirements, such that the support elements can have an L-like cross-sectional shape.

[0020] It is provided according to claim 5 that at least two surface elements are held on the support elements. The advantage of the features in accordance with the invention is utilized in that a multilayered arrangement by mutually parallel offset surface elements on one and the same supporting structure is easily possible. Between the at least two surface element planes as provided for in accordance with claim 5, space for the installations of additional building infrastructure is created such as pipelines or structural-physical layers. Moreover, the intermediate layer area can fulfill tasks for climatic properties of the structural shape such as the circulation of air masses for rear ventilation and thermal insulation.

[0021] Claim 6 relates to a method for determining a supporting structure for curve envelope geometries in buildings, consisting of support elements in which a predetermined curve envelope geometry is approximated by a continuous network of N-gons (N=3, 4, ... ) which each define a planar mesh, with adjoining N-gons having a common node, and in at least one section of the envelope geometry the mesh plane of an N-gon and the respective mesh planes of two N-gons adjoining in non-parallel spatial directions being situated in different planes. It is provided in accordance with the invention that the approximation of the predetermined curved structural shape occurs with the help of a first continuous network of 4-gons or 6-gons which can be transferred to a further continuous network of 4-gons or 6-gons by parallel displacement in a direction normally to the mesh plane of the respective 4-gon or 6-gon, with two respective adjoining N-gons having a common boundary line which determines the progression of the longitudinal axis of one support element associated with these N-gons, and the dimensions of a support element perpendicular to said boundary line being determined by the distance of the respective boundary line of the first network to that of the further, parallel displaced network. Parallel displacement of a network of 4-gons for example means that each mesh of a 4-gon of the first network is displaced parallel in a direction normal to the mesh plane of the respective 4-gon. In accordance with the invention, the network of 4-gons thus displaced in parallel in this manner must result again in a continuous network of 4-gons with respectively planar mesh planes. This leads to the advantage that the distance of a boundary line of two adjoining 4-gons of the first network to the boundary line of the further, parallel displaced network obtained by parallel displacement of the same can be used for determining the dimensions of a support element perpendicular to said boundary line. Since the progression of the longitudinal axis of this support element is also determined by said boundary line in accordance with the invention, i.e. it extends parallel to the same, straight support elements without torsion (in the geometric sense) are thus obtained by the determining of the support frame in accordance with the invention.
Claim 7 relates to the determination of a supporting structure with the help of an underlying conical network, such that the angular sum of respectively opposite angles is equal in their common node between the boundary lines of four adjoining 4-gons.

Claim 8 relates to the further possibility of the determination of a support frame with the help of an underlying, dual-isothermal network, such that the angular sum of respectively opposite angles between the surface normals of two adjoining mesh planes of four surface element planes adjoining in a node is equal.

Claim 9 aims at a multilayered configuration of the curved structural shape, such that in a section of the supporting structure at least one second continuous network of 4-gons is determined which each define a planar mesh plane, with the 4-gons of the second network being formed by parallel displacement of the 4-gons of the first network in a direction normal to the mesh plane of the respective 4-gon.

Preferred embodiments of the invention will be explained below in closer detail by reference to the enclosed drawings, wherein:

FIG. 1 shows an illustration of two parallel 4-gon networks with respectively planar mesh planes;

FIG. 2 shows an illustration of a section of a supporting structure in accordance with the invention in an architectural application;

FIG. 3 shows the construction of a parallel displaced network N_4 from a base network N and the parallel network p(N) in a two-dimensional view;

FIG. 4 shows the construction of a parallel displaced network N_4 from a base network N and the parallel network p(N) in a three-dimensional view;

FIG. 5a shows an illustration of a conical node;

FIG. 5b shows an illustration of two adjacent conical nodes;

FIG. 6a shows an illustration of a Schramm circle packing on the sphere and the isothermal network p(N);

FIG. 6b shows an illustration of a dual-isothermal node;

FIG. 6c shows an illustration on the required angular relationship in dual-isothermal networks;

FIG. 7 shows a 4-gon network (in the background) and a refined 4-gon network with respective planar mesh planes (in the foreground);

FIG. 8 shows an example of an architectural application of the 4-gon network according to FIG. 7;

FIGS. 9 to 12 show an embodiment for determining support elements on the basis of a 4-gon network generated according to the method in accordance with the invention;

FIG. 13 shows a schematic, two-dimensional illustration of the rectangular cross-sectional shape of the construction space in the node region;

FIG. 14 shows a schematic, three-dimensional illustration of the rectangular cross-sectional shape of the construction space in the node region;

FIGS. 15a to 15b show possible architectural applications of a supporting structure;

FIG. 16 shows a schematic illustration for explaining the construction of the support elements;

FIG. 17 shows support elements which converge in a node region of a dual-isothermal network;

FIG. 18 shows an illustration of a multilayered arrangement on the basis of an example of a dual-isothermal network;

FIGS. 19a to 19b show an illustration of the geometric supporting structure of a hexagonal network with support trapezoids of constant height, and

FIGS. 20a to 20c in FIG. 20b show an offset pair N, N, with constant gon distance and constant surface distance which was obtained from the supporting structure of FIG. 20a, and of which the associated planar surface support system is shown in FIG. 20c in the form of a diagram.

The following will illustrate how a supporting structure as in accordance with the invention can be implemented from the start of planning of a freeform surface up to the realized structural shape with the help of the method in accordance with the invention. The starting point is a computer-generated freeform surface, with the architect having to take into account aesthetic and well-proportioned shaping. Architectural planning of the freeform surface will also include its structure from individual surface elements and the design of the supporting structure. For the architect, the question will arise as to the configuration of the individual surface elements with respect to type, size and shape primarily with respect to the visual impression of the overall structure. However, technical and economic viability will have to be taken into account. The freeform surface is made up of the individual surface elements which compile the freeform surface in an uninterrupted manner.

The object must be achieved at first in the computer model to subdivide a predetermined continuous freeform surface into N-gons in such a way that the freeform surface is represented in a continuous way. The boundary lines of the N-gon are progressions which delimit a surface content which is designated as a mesh. The mesh plane should be planar and represents the plane of the future surface element such as a glass plate. The term “mesh 1” shall be used below in connection with the geometric approximation of a freeform surface by a network made of N-gons, and the term “surface element” shall be used in connection with the physical cover element in constructional implementation which is inserted into the support elements and extends in the respective mesh plane of the geometric model.

As is generally known, only the plane or a simply curved surface can be subdivided into a plurality of similar triangles, quadrilaterals or polygons. Once the surface comprises a second curvature for forming a space, i.e. it becomes a freeform surface, a mesh network made of the same triangles for example is only possible in a number of few special cases. The manner of subdivision of a sphere for example, i.e. a shape that can be described in a comparatively simple way, belongs to the oldest tasks of an engineer. One possible solution for the sphere are the geodesic cupolas of Buckminster Fuller for example, which are an example for a surface division of a sphere with similar hexagons. Even more difficult is the finding of suitable solutions for approximating more complex freeform surfaces, as are demanded in connection with randomly shaped, multiply curved surfaces of contemporary architecture.

The object of the invention is approximations of freeform surfaces by quadrilateral or hexagonal networks with planar meshes. Quadrilateral network structures are discussed first below. Such a quadrilateral network is continuously made up of planar quadrilaterals in such a way that precisely two quadrilaterals abut along each inside edge of the network. In an inside vertex of the network, generally precisely four quadrilaterals abut. Such a vertex is generally designated below as node X and is called regular, otherwise
the node X is known as singular. Only a quadrilateral mesh I is possible by edges or boundary lines 2 of boundary polygons. Only one or two meshes I abut at regular edges of boundary polygons. When reference is made below to a “network” without any other specifications, a quadrilateral network with planar meshes I is always meant.

[0050] It shall be explained below how a quadrilateral network N can be found within the framework of the invention as an approximation of a surface F, which is mostly a freeform surface, which in constructional realization has the advantages in accordance with the invention.

[0051] In this respect, a quadrilateral network N is always imagined as an approximation of a surface F, which is mostly a freeform surface. The network can be refined in such a way that the lateral surfaces will become increasingly smaller and move continually close to F. As a boundary position, a curve network K to F is thus obtained. If the planarity of the meshes I is obtained in the refining, a so-called conjugated curve network K to F is obtained in the boundary.

[0052] Important methods for generating quadrilateral networks with planar meshes I and especially such which have advantageous properties for architecture are based on the concept of parallel (parallel-relating) networks. Parallel networks M, N are such in which the meshes I of the network M can be mapped on the meshes of the other network N by maintaining all neighborhood relationships in such a way that the planes of respective meshes are parallel. Since in this transformation or parallel displacement, meshes I with a common edge 2 are mapped again on meshes I with a common edge 2, respective edges 2 are parallel in the networks M and N (see FIG. 1) due to the parallelism of respective mesh planes. It may occur that all meshes I of the one network, e.g. M, are convex, but there are meshes I with self-intersections in the parallel network N.

[0053] One can assume a network M and construct new networks N by parallel displacement of the side surfaces by maintaining the vertex conditions. The degrees of freedom occurring thereby can be used for the design. FIG. 1 illustrates a construction method which produces the parallel relationship of two polygons (bold). The remainder of N (right illustration in FIG. 1) is inevitably obtained by parallel drawing to the respective edges of M (left illustration in FIG. 1).

[0054] Such networks N are relevant for the construction of special networks for which there is a parallel network p(N) which approximates a convex surface S (e.g. a sphere). This regularizes the network N in the sense that too many vertex angles are avoided in the meshes I and thus too narrow quadrilaterals. The deeper reason is that the following occurs under refining in the sense stated above and by maintaining the planar meshes I of N and p(N): N has a curve network K on a surface F as a boundary position; p(N) has a curve network p(K) on the surface S as a boundary position. The networks K and p(K) relate parallel with respect to each other and therefore K is the network of the relative curvature lines of F with respect to the “relative sphere” S. If S is a sphere, then K is the network of ordinary curvature lines and therefore rectangular. The network of the curvature lines describes the directions of the strongest and weakest normal curvature of a surface. It is suitable to provide the spectator with a good imagination of the shape, thus increasing relevance for architecture. Below, a network N with a parallel network p(N) which approximates a convex surface S is also known as a general curve network.

[0055] In accordance with the invention, the concept of parallel displaceability of a network N, M for generating a so-called “offset” for determining a supporting structure and for realizing multilayered superstructures of shells in architecture is used. An offset N to a quadrilateral network N shall be a parallel quadrilateral network, with requirements being placed on the distances of respective meshes I, edges 2 or nodes X depending on the application.

[0056] FIG. 2 shows a sectional view of a supporting structure in accordance with the invention. Two offsets occur in this case which are explained by reference to a network mesh I which is shown in FIG. 2 as a surface element 5. On the one hand, there is a quadrilateral of the second layer (e.g. glass construction) which extends in a parallel plane to the planar quadrilateral of the basic mesh I and in which also the bracings 7 extend. With the help of the spacer elements 6, a third plane is defined at a slightly larger distance which is indicated by the connecting lines between the end points of the spacer elements 6. FIG. 2 also shows a typical construction of the support elements 6 in which the glass panes are placed. The following situation can be found in these support elements 4 from a geometrical standpoint, with the support elements 4 appearing in the geometrical model at first as two-dimensional support quadrilaterals, which hereinafter shall also be referred to as support trapezoids. Respective (parallel) edges of basic network N and offset network N are connected by planar quadrilaterals (support quadrilaterals). The distance which connects respective nodes X and X, from base to offset is a common edge n(X) of the four support quadrilaterals converging in this node. The common edge n(X) can be imagined as a counterpart to the surface normal of a smooth surface. The quantity of all support quadrilaterals shall be referred to below as a supporting structure. It is advantageous in the construction of offsets and supporting structures to use oriented networks in which one differentiates between two sides (an outside skin and an inside skin). In this way, the normals n(X) are also oriented in a uniform manner (to the outside or inside). The node regions 3 of the constructional implementation are located at the positions of node X of the geometrical model.

[0057] A quadrilateral network N with planar meshes I is related parallel to such a network p(N), with p(N) approximating a convex surface S. Within S, a point Z can be chosen which plays the role of a central point and governs the distribution of the distances of the offsets. The convexity of S is not absolutely necessary, but certainly so that a point Z exists from which all surfaces of (p(N)) are visible. Offsets N of N with respect to p(N) are built in the following way: Each mesh plane Q of network N is displaced in parallel to a new position Q, within the terms of the given orientation. The distance of Q to Q, must be equal to the distance of Z to p(Q). In this respect, p(Q) designates the plane of the mesh I of the parallel network p(N) which belongs to Q (see FIG. 3). Based on the Minkowski sum of convex polyhedrons, one can thus regard the obtained offset network N, as the sum total of N and p(N), in short N, = N + p(N). In forming the sum total, all distances can be multiplied with a uniform factor λ (or previously carry out a scaling of p(N)). When writing the scaling of p(N) with factor λ as λ p(N), then the offsets to be built with Z and p(N) are the networks N, = N + λ p(N).

[0058] Notice must be taken that the distances of respective nodes of X and X, of base and offset are given by the distances of Z to the nodes of the parallel network p(N), and the same applies to the respective edges.
It is obvious that in this offset construction there are certain freedoms in the distribution of the distances. They lie in the choice of the center Z. When all mesh planes of p(N) have the same distance from Z, offsets in constant surface distance are thus obtained. The initial network N must not be random however, but must already relate in parallel to a matching network p(N).

If respective quadrilateral surfaces of base and offset should have a constant distance, the parallel network p(N) must have quadrilateral surfaces at a constant distance from Z. That is why p(N) of a sphere S with center Z must be circumscribed in a contacting manner, i.e. all mesh planes of p(N) must make contact with the sphere S. A value of approximately 1 can be assumed for the radius of sphere S.

Every network N that is parallel to such a network p(N), shall be referred to below as a conical network. This designation is the result of the following geometric characterization: The four mesh planes abutting in a regular inner node X of a sphere S, of radius 1 which is obtained from the inscribed sphere S of p(N) by suitable parallel displacement. That is why these four planes contact through X the circular cone with tip X which is circumscribed in the sphere S, in a contacting manner (see Figs. 5a and 5b). In this case, a conical node X is also referred to here. The rotational axis u(X) of the cone contains the nodes X, of offsets N, = P(N) at distance d which correspond to the node X.

Conical networks thus have the practical property to have offsets at a constant surface distance. Since p(N) approximates a sphere S, conical networks can also be regarded as approximations of networks of lines of curvature. For calculation, the following optimization method is especially suitable in addition to the construction from p(N).

In order to realize the invention, an algorithm is proposed which yields the following:

Input is a quadrilateral network N whose meshes do not deviate too strongly from planar meshes. Such networks can be generated automatically or interactively from conjugated curve networks of the underlying surface F.

Output is a quadrilateral network with planar meshes which lies as close as possible to F.

The algorithm works with a numeric optimization which runs iteratively. It occurs by step-by-step displacement of nodes X by maintaining connectivity. At the beginning of the iteration, the following target function \( F = f_{\text{planar}} + w_1 f_{\text{planar}} + w_2 f_{\text{smooth}} \) is minimized by a penalty process. The meaning of these functions is the following: Since a quadrilateral is planar and convex precise in the case when \( \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 2\pi \), the sum total of the interior angles, \( f_{\text{planar}} \), is arranged as the sum total of terms \( \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - 2\pi \), with the sum total extending over all quadrilaterals of the network. The function \( f_{\text{smooth}} \) shall assume small values for smooth and aesthetic networks. Any known smoothing function can principally be used. The function \( f_{\text{smooth}} \) holds the network close to the given reference surface F or also in the input network N during the optimization. The sum total of the distances of respective nodes X of the current and the improved network is used in the simplest of cases. The known tangential distance method has proven to be better however. The weights \( w_1 \) and \( w_2 \) need to be reduced in the course of optimization in order to increase the influence of the planarity term \( f_{\text{planar}} \). In order to obtain a numerically high-quality planarity of the meshes I, a Lagrange iteration is used after the penalty process which minimizes the target function \( w_1 f_{\text{planar}} + w_2 f_{\text{smooth}} \) under a quantity of constraints. The constraints are the equations \( \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 2\pi \) for the individual quadrilateral meshes I.

The algorithm can only be used in a useful way when the input network reflects the geometry of a conjugated curve network, which means in particular that it was obtained from such a one. To ensure that it is easier to have an input which fulfills these requirements, the proposed optimization method can be combined with a subdivision algorithm which works on quadrilateral networks. One predetermines a coarse and simple quadrilateral network with (nearly) planar meshes I (see example in the background of Fig. 7) and then switches between subdivision and optimization. The network in the foreground of Fig. 7 was generated with such a method, which is also the base of a specific design of a stop for a means of public transport according to Fig. 8.

Furthermore, the following characterization of a conical node X must be built into the optimization: The interior angles \( \omega_1, \omega_2, \omega_3, \omega_4 \) which occur about a conical node X successively fulfill the equation

\[
\omega_1 + \omega_2 + \omega_3 + \omega_4 = \theta_1 + \theta_2 + \theta_3 + \theta_4.
\]

In the penalty process, a term \( (\omega_1 + \omega_2 + \omega_3 + \omega_4) \theta \) is added to the function \( f_{\text{planar}} \) per node X. In the optimization with constraints, the constraint (K) is added to each node X. The network in the foreground of Fig. 7 was calculated by alternating between Catmull-Clark subdivision and optimization by including the condition (K) per node X.

If one wishes to use support elements 4 with constant profiles for a support frame 8 and to perform this with optimized nodes X (four planes through an axis), offsets are necessary at a constant distance from the edges (height of support elements 4). This is a stronger limitation than the demand for constant surface distance or constant vertex distance, and therefore it is not possible to approximate any random shapes with such networks. There is still a large variety of such networks however, which are obtained in the following manner: Due to the constant distance of edges 2 from base network N and offset network N, there must be a parallel network p(N) whose edges 2 continually have the same distance from Z. Accordingly, all edges of p(N) touch a sphere S with center Z. Every quadrilateral mesh of p(N) lies in a plane E whose cutting circle with S touches all sides of mesh 1. Accordingly, each mesh 1 of p(N) has an incircle lying on sphere S. Adjoining meshes 1 generate contacting incircles. In summary, the quantity of the incircles therefore forms a circle packing on the sphere (see Fig. 6a). It is a so-called Schramm circle packing because four common tangents of contingence each go through a common point (node X of p(N)). The network p(N) is known as an isothermal network.

Networks N which relate in parallel to such a quadrilateral network p(N), are called dual-isothermal networks because they represent the Laguerre-geometric (and thus to a certain extent dual) counterparts to the isothermal networks. It also follows from this however that only such surfaces can be approximated with such networks in which the Gaussian image of the lines of curvature is an isothermal network. It also follows from this however that only such surfaces can be approximated with such networks in which the Gaussian image of the lines of curvature is an isothermal network. This means a certain limitation in the design. The class of surfaces especially contains the minimal surfaces. Networks with a constant edge distance which approximate minimal surfaces are known in the state of the art without the present property.
of the offsets and their meaning for architecture having been recognized. Important for the practical implementation is the known construction of the networks p(N) which are also known as Koebe's polyeder.

[0072] A characterization of dual-isothermal networks can also occur by angular conditions. In each vertex of the network, the edges 2 originating from there lie on a circular cone. The angle occurring along an edge 2 between the surfaces (angle between the normal lines, measured in the interval (0,π)) is designated below as dihedral angle along said edge 2, with φ₁, φ₂, φ₃, φ₄ being the dihedral angles occurring along the edges 2 about a node X, with the following applying (see FIG. 6b in this connection)

\[ \tan(\alpha/2) \tan(\beta/2) = \tan(\alpha/2) \tan(\beta/2). \]  

(D1)

[0073] This condition must be fulfilled in each node of a dual-isothermal network. There are further angular conditions which are associated with the edges 2 of the network. When A and B are the end points of an edge 2 and the angles occurring there to the adjacent edges 2 opening into A and B on both sides of said edge 2 are designated with α₁, α₂, β₁, β₂ (see FIG. 6c), then the following relationship exists between these angles:

\[ \tan(\alpha/2) \tan(\beta/2) = \tan(\alpha/2) \tan(\beta/2). \]  

(D2)

[0074] The validity of (D1) in each node X and the validity of (D2) in each edge 2 of a quadrilateral network with planar meshes 1 are necessary and sufficient for the presence of a dual-isothermal network.

[0075] FIG. 14 shows a node region 3 of a dual-isothermal network, formed by support elements 4 with a rectangular cross section. The longitudinal axes of the support elements 4 form the same angle with the axis of the quadrilateral mesh 1. When the circular cone mentioned above. In the case of consistently elliptical (local convex) nodes X in the geometric model (all edges 2 on the same semicircle), the glass planes can be applied directly to the edges of the support elements 4 because these edges themselves delimit planar quadrilateral meshes 1.

[0076] If one wishes to achieve a fixed distance between the respective nodes X and X₀ from base and offset, the nodes of the parallel network p(N) must lie on a sphere S. Since every quadrilateral mesh 1 of p(N) lies in a plane E, the vertices of mesh 1 lie on the intersecting circle of E and S. Every quadrilateral mesh 1 of p(N) is therefore a quadrilateral inscribed in a circle, i.e. it comprises a circumscribed circle. By parallel displacement of the sides of a quadrilateral inscribed in a circle, the sides of a quadrilateral inscribed in a circle are obtained again. Since the sides of each mesh 1 of N are parallel to the respective sides of the respective mesh 1 of p(N), every quadrilateral in N must also be a quadrilateral inscribed in a circle, i.e. it comprises a circumscribed circle. That is why N is also designated as a circular network. Since p(N) approximates a sphere, the circular networks must also be regarded as approximations of networks of lines of curvature. Their offsets Nₓ=N[1-p(N)] at distance d have nodes X which lie at a distance d from the respective nodes of N.

[0077] The optimization method as described above is suitable for calculating circular networks. One only needs to replace the planarity condition \( \alpha₁+\alpha₂+\alpha₃+\alpha₄=2\pi \) per mesh 1 by the two conditions \( \alpha₁+\alpha₃=\pi \) and \( \alpha₂+\alpha₄=\pi \) which characterize a convex quadrilateral inscribed in a circle.

[0078] There are also networks N whose offsets have both constant surface distance as well as constant vertex distance. They relate in parallel to the networks p(N) whose surfaces touch a sphere S₁ with center Z and whose vertices lie on a concentric sphere S₂. This is precisely the case when the quadrilateral meshes 1 of p(N) have circumscribed circles with a fixed radius. Such a circular pattern is constructed from a spherical network of rhombuses. FIG. 20b shows such an offset pair N, Nₓ, with a constant vertex distance and constant surface distance which was obtained from the support structure of FIG. 20a. FIG. 20c shows the associated planar surface support system in the form of a diagram.

[0079] The statements made on parallel-related networks, their offsets and supporting structures apply to random networks with planar meshes 1, even when the meshes are N-gons. However, they become trivial for triangular networks: Two triangles with parallel sides are similar, and therefore parallel-related triangular networks are always similar. On the other hand, the case of hexagonal networks is of interest to the invention. Similar to a regular paving with hexagons (honeycomb), three meshes 1 and three edges 2 are in a regular vertex of such a network (FIG. 19a). Since only convex hexagons are usually interesting for practical application, only surfaces with positive Gaussian curvature can be approximated with hexagonal networks with planar meshes 1.

[0080] Such networks are of special interest in which the offsets have special properties. A node X is always conical (because three planes always touch a circular cone) and therefore there are always offsets at constant surface distance. If one wishes to achieve constant edge distance, there must be a parallel-related hexagonal network p(N) whose edges 2 touch a sphere. Such a network can be constructed by means of known algorithms for Koebe's polyeder. The associated supporting structure of N has the property again that the occurring trapezoids have a constant height. In a supporting structure 8 in accordance with the invention, the support elements 4 with a constant cross section can be used. The longitudinal axes of the support elements 4 open under the same angles into the nodes axes A and the support elements 5 can be mounted directly on the inner edges of the supporting structure 8.

[0081] The result is a quadrilateral network with planar meshes 1 which approximates a freeform surface F. FIG. 9 shows such a network schematically, which approximately might concern a conical network. In the following FIGS. 10 to 12, it is now shown in an exemplary way how the progression of the support elements 4 can be examined on the basis of such a network of a geometrical model.

[0082] The starting point for the construction of the support elements 4 is a quadrilateral network N with planar meshes 1 (FIG. 9). In each node point X of the network there is a common straight line (node axis A) on which the respective nodes X of the offset networks are situated (FIG. 10). In a conical network N, the node axis A is the axis of the circular cone which is touched by the adjacent mesh planes. In the case of a dual-isothermal network N, the node axis A is the axis of the circular cone which contains the adjacent edges.

[0083] The quadrilateral network N and an associated offset network Nₓ determine a geometric supporting structure which is made up of planar quadrilaterals (FIG. 11). Each of these quadrilaterals is a trapezoid, bordered by an edge XY of N, the respective parallel edge X₀Y₀ of Nₓ, and nodes X of N and X₀ corresponding to the connecting lines X₀X and Y₀Y. In order to build the supporting structure, the support elements 4 must be arranged along the geometric supporting structure. In order to produce a building space in which the support elements 4 lie, the support trapezoids can be extruded.
to both sides by a desired width normal to their planes, thus resulting in cuboid building spaces for the support elements 4 with an oblique cut in the node region 3 (FIG. 12). Depending on the static arrangement of the support system, a support layer can be continuous and the second support layer will abut against the first support layer (also see FIG. 14).

[0084] FIG. 13 shows a schematic, two-dimensional illustration of the rectangular cross-sectional shape of the building space. FIG. 14 shows a respective three-dimensional illustration of the building space in the area of the node region 3, and FIGS. 15a to 15d show possible architectural applications of a supporting structure 8 in steel, wood and concrete (not true to scale). The building space need not have a rectangular cross-sectional shape, but must not exceed the maximum possible building space.

[0085] In preparing the maximum dimensions of the cross sections of support elements 4, the extremal positions of the angles between the surfaces of the base network and the planes of the supporting structure need to be noted (see FIG. 16).

[0086] In the case of a dual-isothermal network, the heights of the support trapezoids are constant and thus also the distances of the line bearings to the support element 4. Further advantages of dual-isothermal networks are the following: One can also mount the panels of the covering, i.e. the surface element 5, or further layers on the insides of the support elements 4 because they also form planar meshes 1 (also see FIG. 18). This only applies in the case of convex nodes X. FIG. 17 shows a convex node region 3 of a dual-isothermal network, formed by support elements 4 with a rectangular cross section. The longitudinal axes L of the supporting elements 4 form the same angle with the node axis A (axis of the circular cone as mentioned above) (also see FIG. 14).

[0087] It is thus possible with the help of the invention to find a constructional implementation of freeform surfaces with the help of a supporting structure 8 which reduces the technical and economic demands. In particular, the amount of work and costs for mounting can be kept as low as possible. It is further possible that the supporting structure 8 also offers the possibility of a multilayer arrangement for approximating freeform surfaces, i.e. the parallel offset mounting of several surface elements 5 to the support elements 4 of a single supporting structure 8.

1. A supporting structure (8) for curved envelope geometries in buildings, consisting of support elements (4) which are each combined to form N-gons which span the envelope geometry and which each enclose a planar surface element (5), and the support elements (4) of adjacent N-gons each form a common node region (3) in which the support elements (4) abut, and the plane of a surface element (5) and the respective planes of the surface elements (5) of two N-gons which adjoin in non-parallel spatial direction lie in different planes in at least one section of the envelope geometry, wherein the support elements (4) form in this section 4-gons or 6-gons each, and the support elements (4) each have a longitudinal axis (L) which extends in a straight line between two node regions (3) each and runs parallel to the imaginary line of intersection of the surface element planes associated with the same, with the cross section of the support elements (4) normal to their longitudinal axis in each case having a relative twist angle of 0° along the entire longitudinal axis (L) of the support element (4).

2. A supporting structure according to claim 1, wherein the angular sum of respectively opposing angles is equal in the point of intersection of the imaginary lines of intersection of four surface element planes adjoining in a node region (3).

3. A supporting structure according to claim 1, wherein the angular sum of respectively opposite angles between the surface normals of two adjoining surface element planes of four surface element planes adjoining in a node region (3) is equal.

4. A supporting structure according to claim 1, wherein the support elements (4) have a rectangular cross-sectional shape or can be inscribed into a rectangular cross-sectional shape.

5. A supporting structure according to claim 1, wherein at least two surface elements (5) are held on the support elements (4).

6. A method for determining a supporting structure (8) for curved envelope geometries in buildings, consisting of support elements (4) in which a predetermined curved envelope geometry is approximated by a continuous network of N-gons (N=3, 4, . . . ) which each define a planar mesh (1), with respectively adjoining N-gons having a common node (X), and the mesh plane of an N-gon and the respective mesh planes of two N-gons adjoining in non-parallel spatial directions lie in different planes in at least one section of the structural shape, wherein the approximation of the predetermined curved envelope geometry occurs with the help of a first continuous network of 4-gons or 6-gons which can be transferred to a further continuous network of 4-gons or 6-gons by parallel displacement in a direction normal to the mesh plane of the respective 4-gon or 6-gon, with two respective adjoining N-gons having a common boundary line (2) which determines the progression of the longitudinal axis of one support element (4) associated with said N-gons, and the dimensions of a support element (4) perpendicular to said boundary line (2) being determined by the distance (d) of the respective boundary line (2) of the first network to that of the further, parallel displaced network.

7. A method for determining a supporting structure (8) according to claim 6, wherein the angular sum of respectively opposite angles between the boundary lines (2) of four adjoining 4-gons is equal in their common node (X).

8. A method for determining a supporting structure (8) according to claim 6, wherein the angular sum of respectively opposite angles between the surface normals of two adjoining mesh planes of four surface element planes adjoining in a node (X) is equal.

9. A method for determining a supporting structure (8) according to claim 6, wherein in a section of the supporting structure (8) at least one second continuous network of 4-gons or 6-gons is determined which each define a planar mesh plane, with the 4-gons or 6-gons of the second network being formed by parallel displacement of the 4-gons or 6-gons of the first network in a direction normal to the mesh plane of the respective 4-gon or 6-gon.

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