July 15, 1930. H. NYQUIST 1,770,422

PHASE COMPENSATING NETWORK

Filed Feb. 25, 1926 6 Sheets-Sheet 1

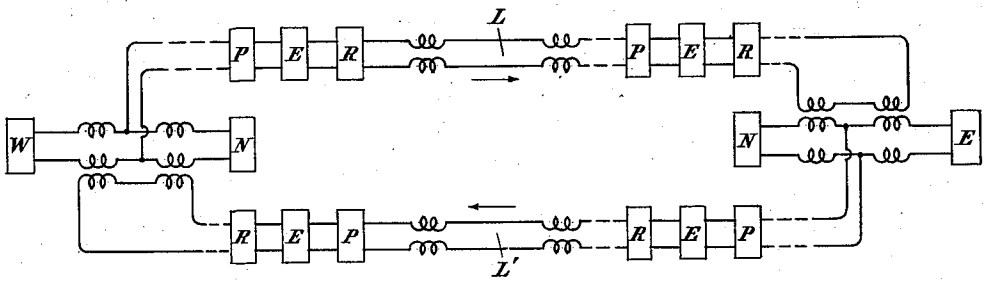



Fig. 1

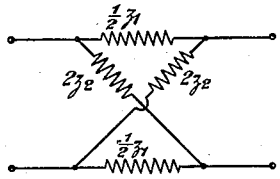



Fig. 2

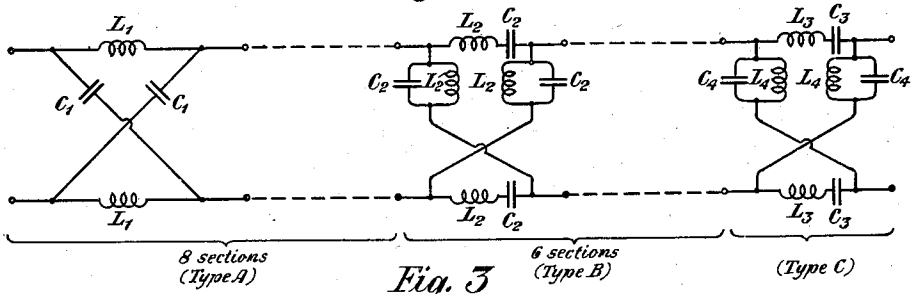



Fig. 3

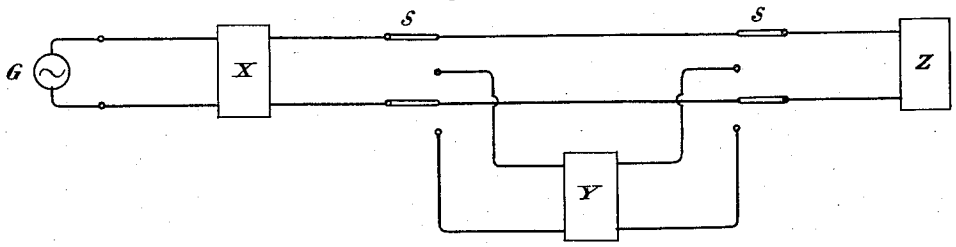



Fig. 5

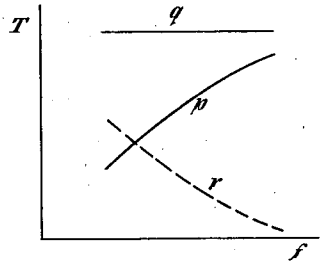



Fig. 6

INVENTOR
H. Nyquist
BY
ATTORNEY

July 15, 1930. H. NYQUIST 1,770,422

PHASE COMPENSATING NETWORK

Filed Feb. 25, 1926 6 Sheets-Sheet 2

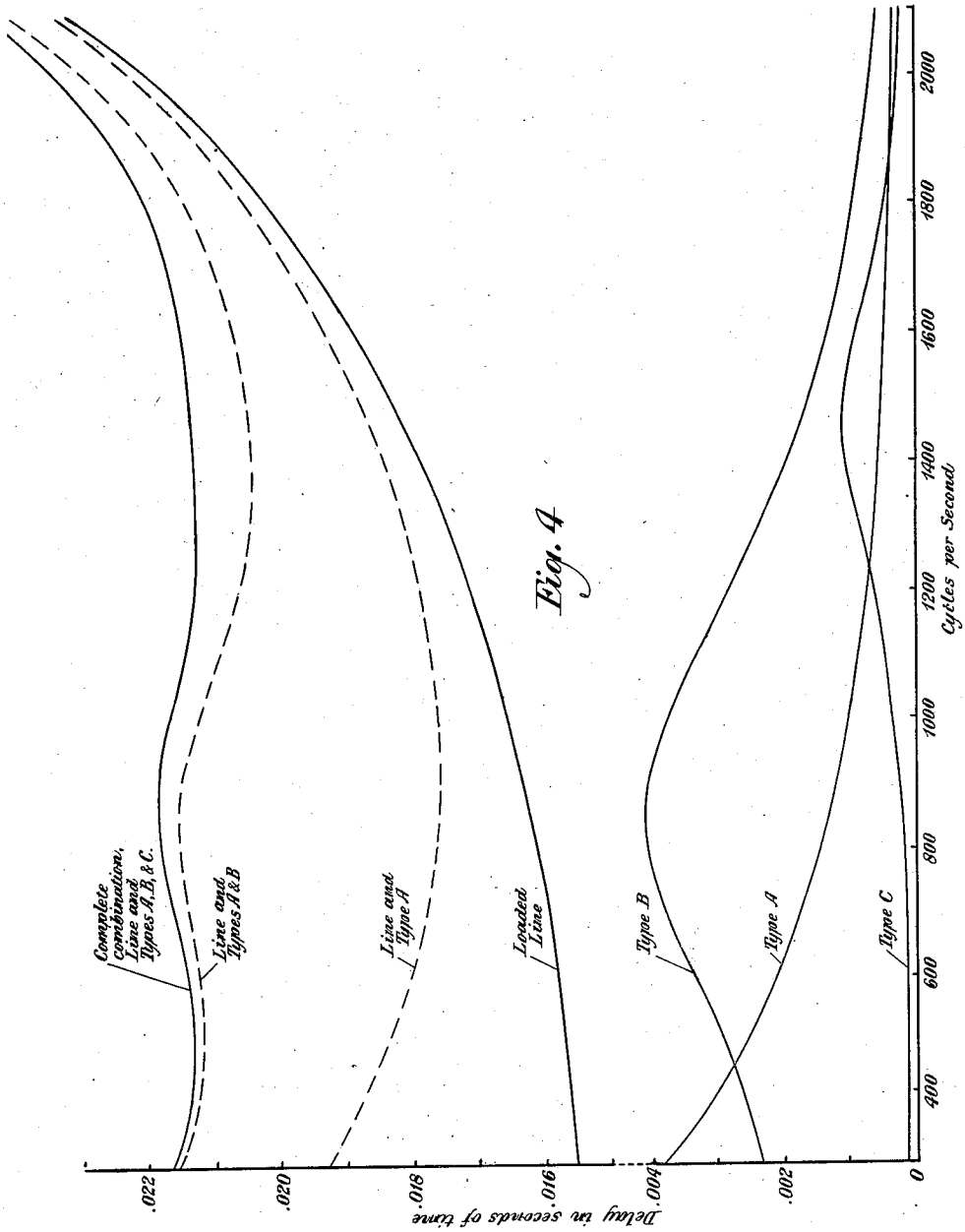



Fig. 4

INVENTOR
H. Nyquist

BY

ATTORNEY

July 15, 1930. H. NYQUIST 1,770,422

PHASE COMPENSATING NETWORK

Filed Feb. 25, 1926 6 Sheets-Sheet 3

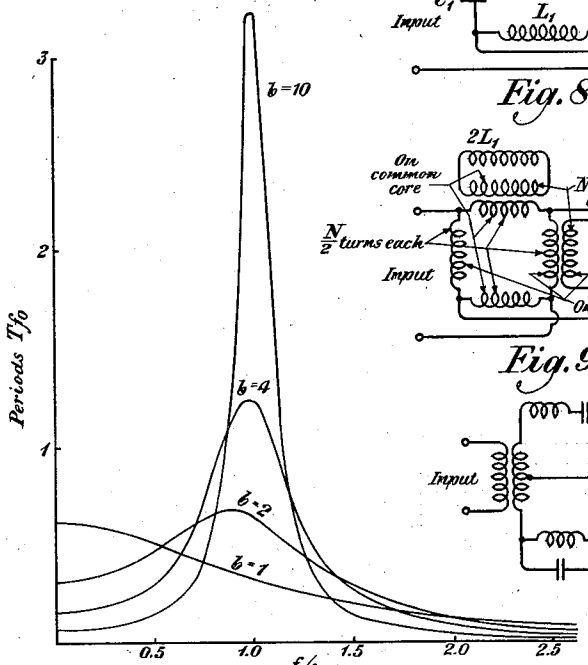



Fig. 7

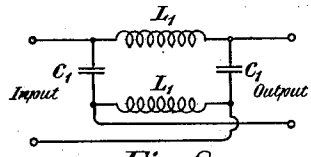



Fig. 8




Fig. 9

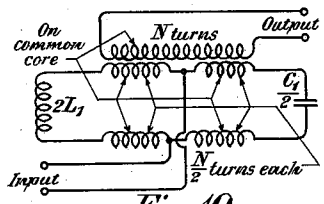



Fig. 10

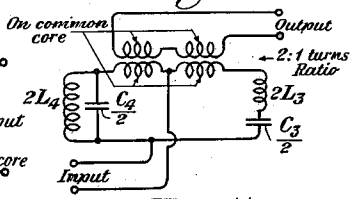



Fig. 11

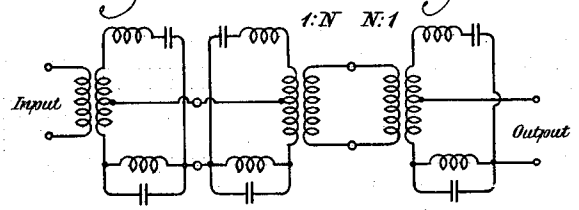



Fig. 12

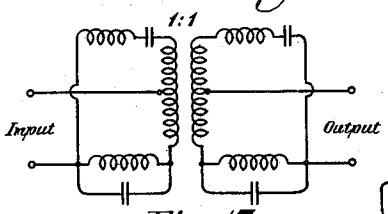



Fig. 13

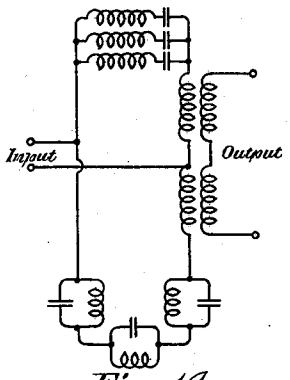



Fig. 14

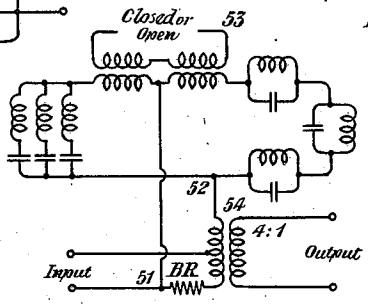



Fig. 15

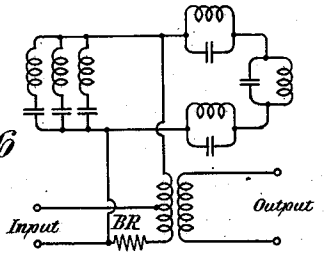



Fig. 16

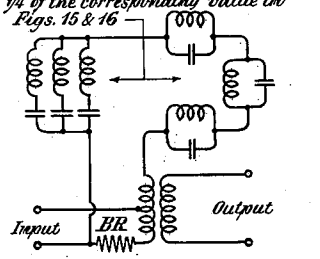



Fig. 17

INVENTOR
H. Nyquist
BY
ATTORNEY

July 15, 1930. H. NYQUIST 1,770,422

PHASE COMPENSATING NETWORK

Filed Feb. 25, 1926 6 Sheets-Sheet 4

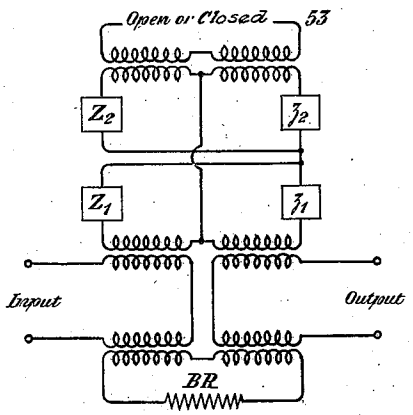



Fig. 18

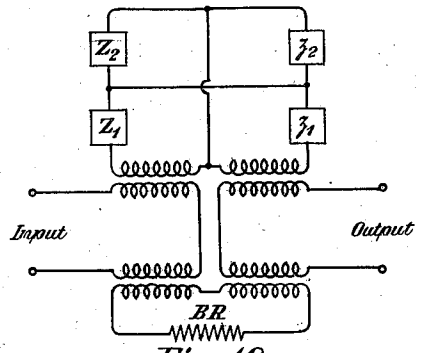



Fig. 19

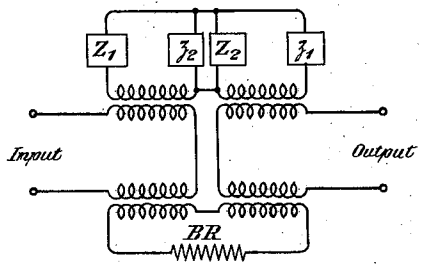



Fig. 20

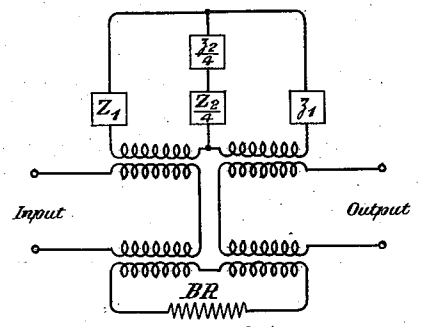



Fig. 21

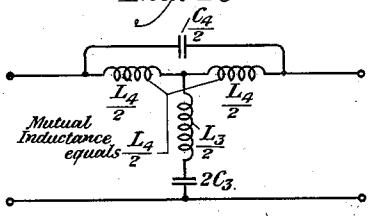



Fig. 22

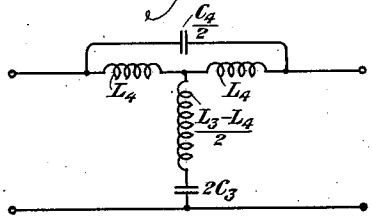



Fig. 24

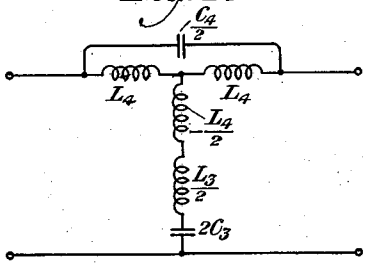



Fig. 23

INVENTOR
H. Nyquist
BY
ATTORNEY

July 15, 1930. H. NYQUIST 1,770,422

PHASE COMPENSATING NETWORK

Filed Feb. 25, 1926 6 Sheets-Sheet 5

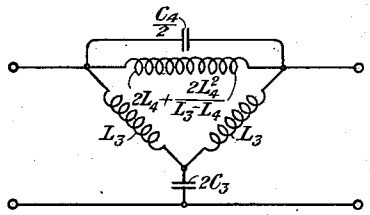



Fig. 25

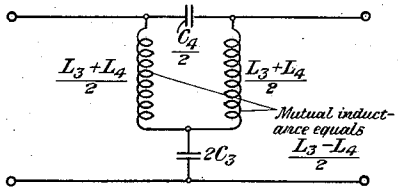



Fig. 26

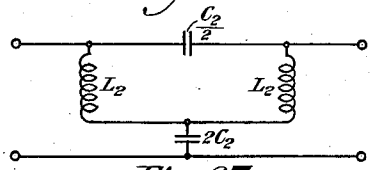



Fig. 27

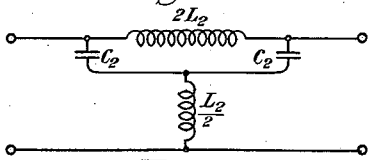



Fig. 27a

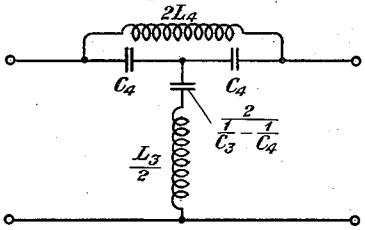



Fig. 27b

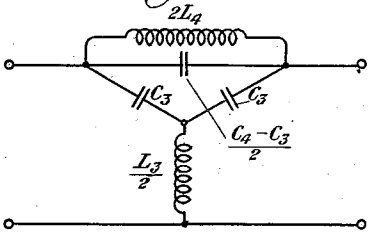



Fig. 27c

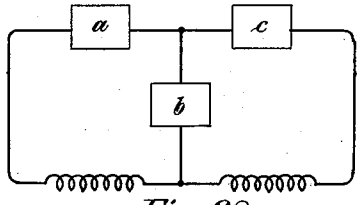



Fig. 28

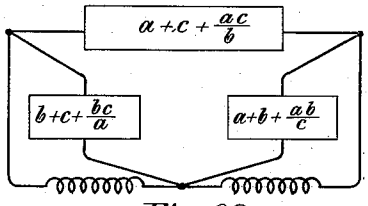



Fig. 29

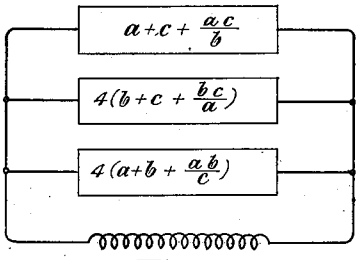



Fig. 30

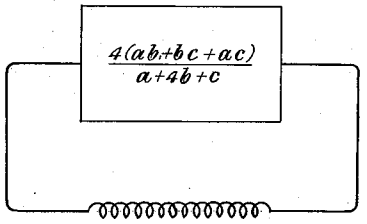



Fig. 31

INVENTOR
H. Nyquist

BY

ATTORNEY

July 15, 1930. H. NYQUIST 1,770,422

PHASE COMPENSATING NETWORK

Filed Feb. 25, 1926 6 Sheets-Sheet 6

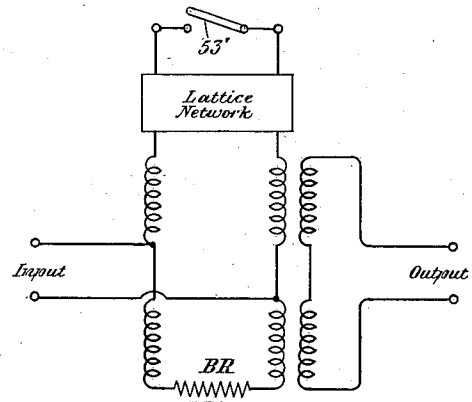



Fig. 32

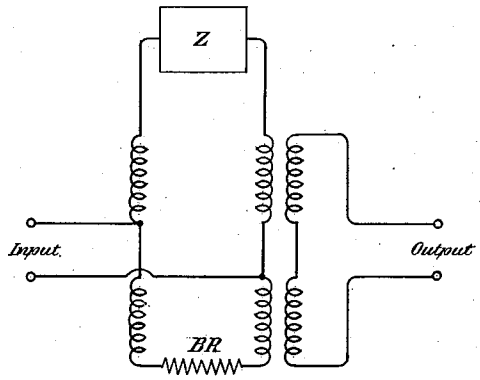



Fig. 33

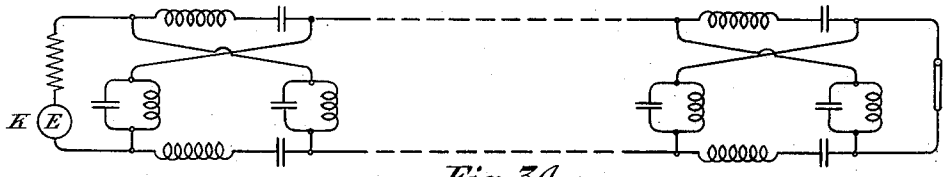



Fig. 34

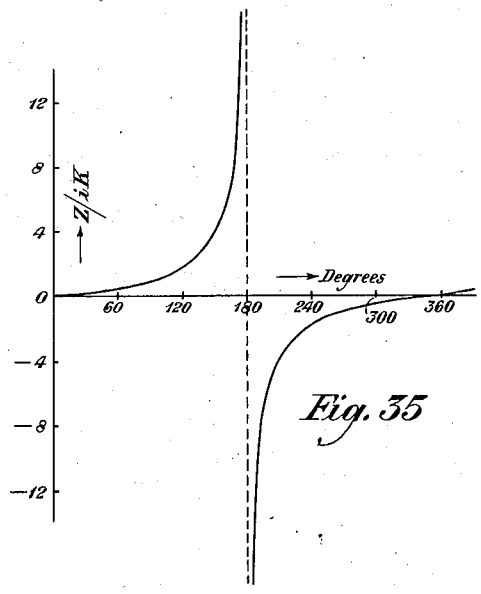



Fig. 35

INVENTOR
H. Nyquist

BY

ATTORNEY

Patented July 15, 1930 1,770,422

# UNITED STATES PATENT OFFICE

HARRY NYQUIST, OF MILLBURN, NEW JERSEY, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK

## PHASE-COMPENSATING NETWORK

Application filed February 25, 1926. Serial No. 90,656.

An object of my invention is to provide a new and improved electrical transmission system in which the frequency components of the current within a certain desired frequency range will be in the same phase relation with one another at the receiving end as at the transmitting end. Another object of my invention is to make the received composite wave form in such a system of the same shape as the transmitted wave form. Another object is to provide for a desired displacement in time of the respective frequency components of a composite alternating current. Another object is to provide for a suitable phase shift of currents of different frequencies in a circuit so as to bring them into a desired phase relation. Another object is to provide for a relative phase shift of the frequency components in a transmission line to compensate for a normal phase shift in the line and to restore the components at the receiving end of the line to the same phase relation as though no line phase shift were present. Another object is to provide a transducer to operate in tandem with a transmission line that shall compensate the distortion due to differential phase shift on the line. Still another object is to provide a transducer to compensate for distortion due to a greater phase shift at high and low frequencies than at an intermediate frequency. In the following specification with the accompanying drawings I disclose specific examples of practice according to my invention. It will be understood that the specification relates largely to these particular cases and that the invention is defined in the appended claims.

By the word "transducer" as employed in this specification, I mean any apparatus having a pair of input terminals for applied electromotive force and a pair of output terminals by which electromotive force may be applied to another element, the output being a function of the input.

Referring to the drawings, Figure 1 is a symbolic diagram of a four-wire transmission system embodying my invention; Fig. 2 is a diagram of a general network of which special forms may be employed in embodying my invention; Fig. 3 is a diagram of a particular network used in an embodiment of my invention here disclosed by way of illustration; Fig. 4 is a diagram of delay-frequency characteristics to which reference will be made in explaining the principle of my invention, and the procedure for embodying it in a particular case; Fig. 5 is a diagram showing the relation of my improved delay network to other elements of the system; Fig. 6 is a diagram showing delay as a function of frequency for certain transducers; Fig. 7 gives a family of delay-frequency characteristics that may afford guidance in the design of a suitable network in a particular case; Figs. 8, 9 and 10 show network sections equivalent to type A of Fig. 3; Fig. 11 shows a network section equivalent to type C of Fig. 3; Fig. 12 shows a plurality of sections, each like Fig. 11, connected in tandem; Fig. 13 shows an equivalent for the combination of the two right-hand network sections of Fig. 12; Fig. 14 shows a network section like Fig. 11 but with impedance elements made structurally complex to secure flexibility of design; Fig. 15 shows a network somewhat like Fig. 14 but arranged as a delay excursion circuit; Figs. 16 and 17 show simplified special case equivalents for Fig. 15; Fig. 18 shows a delay excursion circuit arrangement by which the currents are made twice to traverse two sections of network like the two sections at the left in Fig. 12; Figs. 19, 20 and 21 show simplified special case equivalents for Fig. 18; Figs. 22 to 26, inclusive, show bridged-T network sections equivalent to type C of Fig. 3; Figs. 27 and $27^a$ show bridged-T sections equivalent to type B of Fig. 3; Figs. $27^b$ and $27^c$ show bridged-T sections equivalent to type C of Fig. 3; Figs. 28 to 31, inclusive, show successive stages in the derivation of a network equivalent to Fig. 20 or Fig. 21; Figs. 32 to 34, inclusive, are diagrams illustrating an alternative procedure for obtaining an equivalent delay excursion circuit; and Fig. 35 is a curve diagram to be considered in connection with Fig. 34.

The stations W and E of Fig. 1 are connected in a four-wire system by the two loaded lines L and L′ in each of which occasional repeaters R are interposed. On the

input side of each repeater R there are interposed a network P and an attenuation equalizer E intended to correct for distortion due to differential phase shift and attenuation on the line. Voice frequency currents put on the line L at station W go to the first repeater station, and then they pass through the phase distortion equalizer P which compensates for the differential retardation of the components of the various frequencies by further retarding them unequally so that they are brought to the same time relation to each other as at the sending end. Then the currents pass through the attenuation equalizer E and then through the amplifier R, and so on at each repeater station.

I will first assign specific values for the constants of a certain line and will give the specific design for the network of my invention in this instance, and thereafter I will discuss the principles on which this design is based and point out certain other examples of practice of the invention.

The line chosen for this first example is taken as a one-way No. 19 gauge medium heavy loaded cable circuit, 154 miles long, with a repeater at the middle and at the receiving end, and with a phase compensating network at the receiving end. The constants of the loaded line are:

Resistance $R_0=95.5$ ohms per mile at 1000 cycles per second;

Inductance $L_0=0.154$ henry per mile;

Capacity $C_0=0.065$ microfarad per mile;

Interval between loads=1.136 miles.

Assume that a wave train of a certain pure frequency $f(=w/2\pi)$ is applied suddenly at one end of this line. As is well known, the received current at the other end will build up somewhat gradually. Let the time from the instant the sending current is applied until the received current reaches one-half its full steady-state value be represented by the letter T. This time T as a function of frequency $f$ is shown by the curve marked "Loaded line" in Fig. 4. It will be seen that the delay is greater for the higher frequencies of the essential voice range.

The appropriate phase equalizing network of Fig. 3 consists of sections of three different types, eight sections of type A, six sections of type B and one section of type C. All these sections are special cases of the general crossed or lattice network of Fig. 2. The respective inductances and capacities in the network of Fig. 3 have the values given in the following table:

$L_1=0.5$ henry;
$C_1=0.2$ microfarad;
$L_2=0.25$ henry;
$C_2=0.1$ microfarad;
$L_3=0.42$ henry;
$L_4=0.059$ henry;
$C_3=0.024$ microfarad;
$C_4=0.17$ microfarad.

The eight sections of type A by themselves have a delay-frequency characteristic as shown by the curve marked "Type A" in Fig. 4. When they are in tandem with the line, the resultant characteristic is shown by the dotted curve marked "Line and type A."

It will be seen that the characteristic for the loaded line alone slopes up to the right and is concave up, and the characteristic for the type A network slopes down to the right and is concave up, and the resultant characteristic shown in dotted lines has an intermediate minimum. The additional sections of type B and type C are designed so that, by themselves, they give intermediate maxima, as shown in Fig. 4, and the entire combination gives an approximately horizontal characteristic curve as designated "Complete combination" in Fig. 4. This means that for all the frequency components within the essential voice range the delay in transmission is approximately the same, that is, between about 0.021 and 0.022 second of time.

Referring to Fig. 5, let X be any transducer between the source G and the receiver Z. The delay-frequency characteristic of the transducer X over a certain frequency range will have a certain form, for example, as shown by the curve $p$ in Fig. 6. Suppose it is desired that the currents shall go to receiver Z with their components of different frequency in the same time relation as at the generator G, in other words, so that all the components will be delayed equally and at the receiver the delay-frequency curve will be a horizontal line such as $q$. This effect will be obtained by interposing a compensator whose characteristic $r$ is complementary to $p$ as shown in Fig. 6.

If the characteristic of the transducer X slopes up to the right as at $p$ in Fig. 6, then the compensator Y should have a characteristic sloping down to the right as at $r$. Sections of crossed type network, such as those of type A in Fig. 3, answer to this requirement, as will be seen by comparing curve $r$ in Fig. 6 with the curve for type A in Fig. 4.

For a series of network sections like the one shown generally in Fig. 2, and with the impedance values indicated thereon, the propagation constant $\Gamma$ and the characteristic impedance K are given by the following formulas:

$$\cosh \Gamma=1+2z_1/(4z_2-z_1) \quad (1)$$

$$K=\sqrt{z_1 z_2} \quad (2)$$

It is desirable that the characteristic impedance shall be a real constant K, and approximately the same as the impedance (resistance) of the elements with which the network is connected on the input and output sides. Assuming that the impedances $z_1$ and $z_2$ are dissipationless, that is made up only of

reactance elements, this result is secured by making

$$z_1 = Kz \text{ and } z_2 = K/z \quad (3)$$

where K is a real constant and where $z$ is a pure reactance. A network of the type of Fig. 2 with K constant is called a "constant K" network. Substituting in 1, it follows that

$$\cosh \Gamma = 1 + \frac{2z}{\frac{4}{z} - z} \quad (4)$$

A familiar formula of hyperbolic trigonometry is

$$\tanh \Gamma/2 = \sqrt{\frac{\cosh \Gamma - 1}{\cosh \Gamma + 1}}. \quad (5)$$

Substituting from (4), this reduces to

$$\tanh \Gamma/2 = z/2. \quad (6)$$

In general, the propagation constant $\Gamma$ may be put equal to $\alpha + i\beta$, where $\alpha$ is the attenuation constant and $\beta$ is the phase shift constant. The structure for $z$ is of reactance elements only and on this basis it follows from Equation (6) that $\alpha = 0$, and that

$$i \tan \beta/2 = z/2. \quad (7)$$

For the sections of type A of Fig. 3, this gives

$$\beta = 2 \tan^{-1} w\sqrt{L_1 C_1}, \quad (8)$$

where $w = 2\pi f$, $f$ being the frequency.

It is approximately true that

$$T = d\beta/dw. \quad (9)$$

Hence by differentiating (8), the result is obtained that

$$T = \frac{2\sqrt{L_1 C_1}}{1 + w^2 L_1 C_1} \quad (10)$$

By the aid of this equation, delay frequency characteristics can be drawn for respective values of the product $L_1C_1$ and from them it can readily be determined what is the best value of $L_1C_1$ and how many network sections are necessary to get such compensation as should be effective for the type A sections. In this way, the number of sections for type A in Fig. 3 has been fixed at eight, and the value of $L_1C_1$ at $10^{-7}$. The value of $L_1$ and the dependent value of $C_1$ have been determined so that

$$\sqrt{L_1/C_1}$$

equals the desired value of K, 1580 ohms.

For the sections of type B or C, let the impedance $z$ be made up of a series resonant circuit so that

$$\frac{z}{2} = iw\frac{b}{2w_0} + \frac{bw_0}{i2w}, \quad (11)$$

where $b$ and $w_0$ are parameters to which we may assign proper values, $w_0/2\pi = f_0$ being the resonance frequency. For the type B network,

$$b = 2 \text{ and } \sqrt{L_2/C_2} = K,$$

and the truth of Equation (11) will become apparent on noticing that

$$w_0 = 1/\sqrt{L_2 C_2},$$

and substituting for $L_2$ and $C_2$ in terms of $b$ and $w_0$ in the equation

$$\frac{z}{2} = \frac{z_1}{2K} = \frac{iwL_2 + 1/iwC_2}{K} = iw\sqrt{L_2C_2} + \frac{1}{iw\sqrt{L_2C_2}}$$

For the type C network,

$$w_0 = \frac{1}{\sqrt{L_3C_3}} = \frac{1}{\sqrt{L_4C_4}};$$

$$K = \frac{2}{b}\sqrt{\frac{L_3}{C_3}} = \frac{b}{2}\sqrt{\frac{L_4}{C_4}}; \text{ and } b = 2\sqrt[4]{\frac{L_3C_4}{C_3L_4}}.$$

truth of Equation (11) will become apparent in this case by substituting for $L_3$ and $C_3$ in terms of $b$ and $w_0$ in the equation

$$\frac{z}{2} = \frac{z_1}{2K} = \frac{iwL_3 + 1/iwC_3}{K} = \frac{iwb\sqrt{L_3C_3}}{2} + \frac{b}{2iw\sqrt{L_3C_3}}.$$

As before, with $\Gamma = \alpha + i\beta$, from Equations (6) and (11) it follows that

$$\tanh \Gamma/2 = \tanh i\beta/2 = (ib/2)\,(w/w_0 - w_0/w) \quad (12)$$

whence

$$\beta = 2 \tan^{-1} (b/2)\,(w/w_0 - w_0/w) \quad (13)$$

Differentiating (13) and substituting in (9),

$$T = \frac{(b/w_0)\,(1 + w_0^2/w^2)}{1 + (b^2/4)\,(w/w_0 - w_0/w)^2} \quad (14)$$

As $w$ increases from 0 to infinity, the numerator decreases always, but the denominator passes through a minimum at $w = w_0$. If $b$ is made sufficiently large, the fraction in (14) has a maximum near $w_0$. This is shown in Fig. 7, which also shows that by giving to $b$ increasing values, T can be made to increase at its maximum with accompanying decrease of values away from its maximum, the area of the curve remaining constant. By increasing the number of sections of the network, T can be increased over the whole range for $w$. Furthermore, $w_0$ may be chosen to put the maximum point at the right or left as may be desired.

Referring to Fig. 4, it is seen that after the compensation effected by type A, this gives a minimum at or near 1000 cycles, or $w_0 = 2\pi 1000$. In Fig. 7 a series of curves is constructed with coordinates $f/f_0$ and $Tf_0$ instead of coordinates $f$ and T as in Fig. 4. This is somewhat more convenient, and the curves of Fig. 7 can be utilized for any value of $w_0$.

Plotted either way, the area of each curve for a single section is unity. Hence we can estimate from the diagrams what value of $b$ will be best and how many sections will be needful. In this way, the value $b=2$ is chosen and six sections of type B are found to be appropriate. Having chosen respective values for K, $w_0$ and $b$, it follows from (3) and (11) that

$$z_1/2=zK/2=iwbK/2w_0+bw_0K/i2w;$$

and using the lettering of Fig. 3,

In type C, series $L_3=bK/2w_0$
series $C_3=2/bw_0K$.
In type B, since $b=2$,
series $L_2=K/w_0$
series $C_2=1/w_0K$.

Also,

$$2z_2=\frac{2K}{z}=\frac{1}{iw\frac{b}{2Kw_0}+\frac{bw_0}{i2wK}}$$

and in type C, shunt $C_4=b/2Kw_0$
shunt $L_4=2K/bw_0$.
In type B, since $b=2$,
shunt $C_2=1/Kw_0$
shunt $L_2=K/w_0$.

It will be noticed that the value $b=2$ enables us to use inductances and capacities of the same values in the series and shunt combinations; this is the case for type B.

The dotted curve in Fig. 4 marked "Line and types A and B" gives the delay-frequency characteristic for the corresponding combination. This shows a dip at about 1400 cycles, and by the principles and methods already explained, the design is determined for another section designated type C and this gives the resultant characteristic as shown in Fig. 4.

It has already been mentioned that each section of type B or type C contributes about a unit area to the curves of Fig. 4 or 7. Each type A section contributes only about half a unit, but this is not necessarily to be looked on as a disadvantage, for the type A sections have only half as many reactance elements as the others.

The general procedure is first to plot the delay-frequency characteristic of the transducer to be compensated as in the case of the curve marked "Loaded line" in Fig. 4. Then add sections like type A in proper number and with properly chosen reactance values so as to bring the ends of the characteristic up somewhere nearly to the same level. Then add sections like type B or C to raise the minimum dips in the curve up to nearly the level of the ends, and make the curve have nearly the same altitude all across the essential frequency range as for the uppermost curve of Fig. 4.

Referring again to the fact that each section of type B or type C contributes about a unit of area to the resultant curve of Fig. 4 or Fig. 7, (and half a unit for each section of type A) it will readily be seen that this affords a guide as to the number of sections that may need to be employed. Having given the "Loaded line" curve of Fig. 4, an ideal characteristic can be drawn higher up and the area between the two characteristics will give the number of sections of network that must be employed. The shorter the frequency range is made, the less the number of sections that will be required, but shortening the frequency range may impair the quality in one way while the addition of phase correcting network sections improves it in another way. Assuming that economy of apparatus, particularly network sections, is a desideratum, then it may be said that without unduly shortening the frequency range a resultant characteristic can be obtained which may be allowed to depart a little from the horizontal, and will require no more network sections than would be required to get a fully horizontal characteristic for a less frequency range. In other words, starting with a characteristic like that marked "Loaded line" the optimum characteristic with proper economy of network sections will be a characteristic which extends over the whole desired frequency range but which slopes up a little at the right and thus requires a less number of network sections than would be necessary to attain a completely horizontal characteristic over the same frequency range.

In the derivation of equivalents for network sections like those shown in Fig. 3, it will be convenient to represent type A of Fig. 3 as in Fig. 8. Fig. 9 is derived from Fig. 8 by the use of "perfect" transformers. The addition of these transformers has the advantage of permitting single reactance elements to be used where pairs of reactance elements were required as in Fig. 8.

Another transformer arrangement that accomplishes the same result in reducing the number of separate reactance elements is shown in Fig. 10, where a so-called hybrid coil or three-winding transformer is employed. Here, as in Fig. 9, the equivalence with Fig. 8 will be exact only when perfect transformers are employed.

Whereas Fig. 10 corresponds to type A of Fig. 3, Fig. 11 corresponds to type C of Fig. 3. Another distinction between Fig. 11 and Fig. 10 is that the number of distinguishable component coils in the hybrid coil has been reduced from five to three.

In Fig. 12 three network sections like Fig. 11 have been connected in tandem in a manner apparent from the drawing. Between the middle section and the right-hand section, it will be seen that there are two transformers connected together without any intermediate apparatus. A first step in the simplification

of these two sections is to make them like Fig. 13 where the intermediate transformer windings are omitted and the two transformers are consolidated into one transformer.

The reactance elements of a single network section can be made composite in such a way as to make the single section equivalent to a plurality of simpler sections in tandem. This principle can be employed to reduce the number of transformers in such an arrangement as that of Fig. 12, where the plurality of transformers may be objectionable in that they add somewhat to the cost and introduce some electrical effects due to departure from the condition for perfect transformers. Thus in Fig. 14 a single section involving only one transformer is shown with its reactance elements made composite, so that with suitable values the single section of this figure becomes substantially equivalent to the three sections of Fig. 12.

In Fig. 15 I have shown how the network of Fig. 14 may be employed in a delay excursion circuit. The balance between the network above the points 51 and 52 and the balancing resistance BR is not perfect, and there is a reflection whether the transformer secondary is closed or open at 53, but the value of BR is chosen so that there is no reflection there. While the direct wave produces a null effect in the secondary of the transformer 54, the unbalanced reflected wave is effective and gives a delayed output in the circuit of said secondary. By a suitable design of the network, the delay in the output can be made to have different desired values for the different frequencies so as to compensate for delay in a connected transducer.

Reflection will occur at 53 whether the circuit there is closed or open, the only difference being a difference of phase. If the connection at 53 is closed, it becomes equivalent to Fig. 16, but if the connection at 53 is open, it becomes equivalent to Fig. 17.

Another arrangement of an excursion circuit with a hybrid coil is shown in Fig. 18 where the balancing resistance BR is on one side and on the other side is a pair of network sections like the middle and left section of Fig. 12 except that the component impedance elements are shown symbolically in Fig. 18. Assuming that the circuit is closed at 53, Fig. 18 simplifies to Fig. 19, then to Fig. 20.

On the other hand, if the circuit of Fig. 18 is to be left open at 53, then Fig. 18 reduces to Fig. 21. It will be apparent that in all these cases represented by Figs. 15 to 21 it is possible to retain the simple resistance BR as a balancing network; and in the resultant Figs. 20 and 21 the networks shown between the input terminals at the left and the output terminals at the right are all equivalent to four sections of lattice type network like any of the types of Fig. 3.

The bridged-T network section of Fig. 22 is equivalent to the lattice type C network section of Fig. 3. The equivalence is dependent upon giving the reactance elements the values indicated by the legends in Fig. 22 as compared with type C in Fig. 3.

In Fig. 23 a substitution is made for the two coils of Fig. 22 having mutual inductance, and a system of three equivalent coils is introduced with no mutual inductance between the members. One of these coils has a negative inductance which, of course, has no physical counterpart. But provided that $L_3$ is greater than $L_4$, the network of Fig. 23 can readily be realized physically in the network of Fig. 24.

Fig. 25 is derived from Fig. 24 by the well-known substitution of a delta for the star connection.

Another equivalent for Fig. 24 is obtained in Fig. 26 by substituting for the star a transformer of proper inductance values to be the equivalent of the star.

If $L_3$ is made equal to $L_4$, then each of the four Figs. 23, 24, 25 and 26 reduces to Fig. 27, which is an equivalent bridged-T network for the lattice type B network of Fig. 3.

The bridged-T network of Fig. 22 has the advantage over type C of Fig. 3 in that the total number of coils has been reduced from 4 to 2 and the total number of condensers has been reduced from 4 to 2. Another advantage of Fig. 22 over some equivalent networks that have been discussed is that no hybrid coil is involved. In Fig. 26, only a single coil is used, subject to the condition that it has an inductance coupling less than unity, and thus it embodies two magnetic circuits and is theoretically the equivalent of two coils. Fig. 27 has the practical advantage that the two coils are equal and separate, which simplifies construction and design.

Fig. 27[a] is an alternative equivalent for Fig. 27 and Figs. 27[b] and 27[c] are further equivalents for type C of Fig. 3.

Referring to Figs. 20 and 21, it may be advantageous to consolidate the impedance elements represented by the symbols in the four boxes in each figure. Thus the upper part of Fig. 20 first becomes like Fig. 28 where $Z_1=a$, $z_1=c$ and $1/Z_2+1/z_2=1/b$. The changes indicated by these three equations are easily realizable physically.

Fig. 28 can also be taken to represent Fig. 21 where $Z_1=a$, $z_1=c$ and $Z_2/4+z_2/4=b$, and these equations are physically realizable. Now, for the star connection of $a$, $b$ and $c$ in Fig. 28, the equivalent delta is substituted in Fig. 29. The network of Fig. 29 consists of three parts, one bridged across the entire transformer winding, and two bridged each across one-half of the winding. It is well known that the latter two parts are equivalent to impedances of four times the

values of said parts bridged across the whole of the winding. This consideration leads to Fig. 30 as a further equivalent for Fig. 28, and in Fig. 31 the three impedances shown in multiple in Fig. 30 are consolidated to a single impedance. If the resulting network of Fig. 31 can be realized physically, it is substituted back in Fig. 20 or 21, as the case may be.

Alternative to the procedure pursued in connection with Figs. 28 to 31 we may obtain an equivalent delay excursion circuit by proceeding as will now be explained in connection with Figs. 32 to 35. In Fig. 32 a delay excursion network is shown consisting of a series of lattice type network sections connected to the hybrid coil. It is desired to compute and design an impedance Z as in Fig. 33 that will be equivalent to this sectional network.

Fig. 34 shows a number of lattice type sections connected in tandem. These sections may be dissimilar, the only requirement being that they have a common impedance K. A resistance equal to K is connected to one end and the other end is either short circuited or left open. At this end there is complete reflection, whereas at the end where the resistance is located, there is no reflection at all, the whole of the returning wave being absorbed in the resistance.

Now let a sinusoidal voltage E be applied at the input of Fig. 34. The value of the current into the network is E/2K. Let the phase change suffered by this current in traversing the network once be $\beta/2$. Then the total phase change for the reflected wave having traversed the network in both directions is $\beta$ when the far end is short-circuited, and differs from $\beta$ by $\pi$ radians when the far end is open-circuited. Hence in the two respective cases the current is:

$$(E/2K)\ (\cos\beta - i\sin\beta)$$

or

$$-(E/2K)\ (\cos\beta - i\sin\beta).$$

Accordingly the total current flowing into the resistance from the network is equal to

$$(E/2K)\ (1+\cos\beta - i\sin\beta)$$

or

$$(E/2K)\ (1-\cos\beta + i\sin\beta)$$

according as the switch at the right of Fig. 34 is closed or open. It follows that the total steady-state impedance is respectively

$$\frac{2K}{1+\cos\beta - i\sin\beta}$$

or

$$\frac{2K}{1-\cos\beta + i\sin\beta}.$$

Hence the impedance of the network is obtained by subtracting K from either of the foregoing expressions giving a result which simplifies respectively to

$$Z=\frac{i\sin\beta}{1+\cos\beta}K=iK\tan\beta/2.$$

or

$$Z=\frac{-i\sin\beta}{1-\cos\beta}K=-iK\cot\beta/2.$$

Fig. 35 gives a plot of $Z/iK=\tan\beta/2$ for the first of these alternative expressions, that is, for the case of the switch closed at the right of Fig. 34.

Now, $\beta$ can be readily determined from the known delay which the network should introduce. Then, if $\beta$ is plotted as a function of the frequency, it is, of course, an easy matter to convert from the curve of Fig. 35 to Z as a function of frequency. It will be apparent that the function Z will be imaginary for all values of frequency, and that it passes continuously from very large negative values of reactance through zero to very large positive values. There is a discontinuity at the point where Z reaches infinity, and for greater values of frequency, it reappears as a large negative value. With this definite guide to what is required, the network Z of Fig. 33 can be determined approximately and made to replace the series of lattice sections of Fig. 32.

What is claimed is:

1. In combination, a transducer giving different delays for different frequencies over a certain frequency range and a delay compensator giving compensatory delays for the respective frequencies, said delay compensator being connected in tandem with the said transducer and comprising a plurality of sections with one or more transformers incorporated in them, at least one such transformer having at least three circuits connected therewith and a reactance element in one such circuit whereby it is made effective in duplicate in each of the other circuits.

2. In combination, a transducer giving different delays for different frequencies over a certain frequency range and a delay compensator giving compensatory delays for the respective frequencies, said delay compensator being connected in tandem with the said transducer and comprising a plurality of sections with certain consecutive sections consolidated by the incorporation of transformers between them.

3. In combination, a transducer giving different delays for different frequencies over a certain frequency range and a delay compensator giving compensatory delays for the respective frequencies, said delay compensator being connected in tandem with the said transducer and comprising a plurality of sections with one or more three-winding transformers incorporated between them.

4. In combination, a transducer giving different delays for different frequencies, a

three-winding transformer connected with said transducer, and balanced members connected with said transformer, one of them consisting of a differential delay compensating network.

5. In combination, a transducer giving different delays for different frequencies, a three-winding transformer connected with said transducer, and balanced members connected with said transformer, one member comprising means completely to absorb waves sent into it, and the other consisting of a differential delay compensating network and comprising means to produce reflection of waves sent into it.

6. In combination, a transducer giving different delays for different frequencies over a certain frequency range and a delay compensator connected with said transducer and consisting of a single section network comprising a plurality of reactance elements, these elements having adjusted values to make the network equivalent to a series of lattice type sections of network.

7. In combination, a transducer giving different delays for different frequencies over a certain frequency range and a delay compensator connected with said transducer and consisting of a single network section having complex impedance elements with their components of respective values to make the section as a whole equivalent to a tandem series of lattice type network sections each with comparatively simple impedance elements.

8. The method of effecting delay compensation of electric wave currents which are delayed differently for different frequencies which consists in transmitting the said currents and subjecting them to forces to effect approximately half the desired change and reflecting them back to effect the remaining desired change.

9. In combination, a hybrid coil with input and output terminals and with an absorbing network and a reflecting network balanced against each other, the reflecting network being designed to give certain desired different delays at respective frequencies.

10. The method of producing a desired degree of phase compensation in electric wave currents which consists in sending them repeatedly over the same course subject to compensating forces and thereby cumulating the change due to going once over the said course.

11. In combination, a pair of input terminals, a pair of output terminals, a phase compensating network between said pairs of terminals, a transducer having a non-uniform delay characteristic connected with one said pair, and means to direct input currents through the network and reflect them back through it to the output terminals with compensation of delay so as to give an overall uniform delay characteristic.

12. In combination, a transducer giving different delays for different frequencies over a certain frequency range and a delay compensator in combination therewith, said delay compensator comprising at least one section and composed of a certain number of reactance elements, the said reactance elements being adjusted in value to make the delay-frequency characteristic of the said compensator the same as of a series of lattice type network sections composed of a greater number of reactance elements and designed to compensate the delay in the said transducer.

In testimony whereof, I have signed my name to this specification this 23rd day of February, 1926.

HARRY NYQUIST.