

(No Model.)

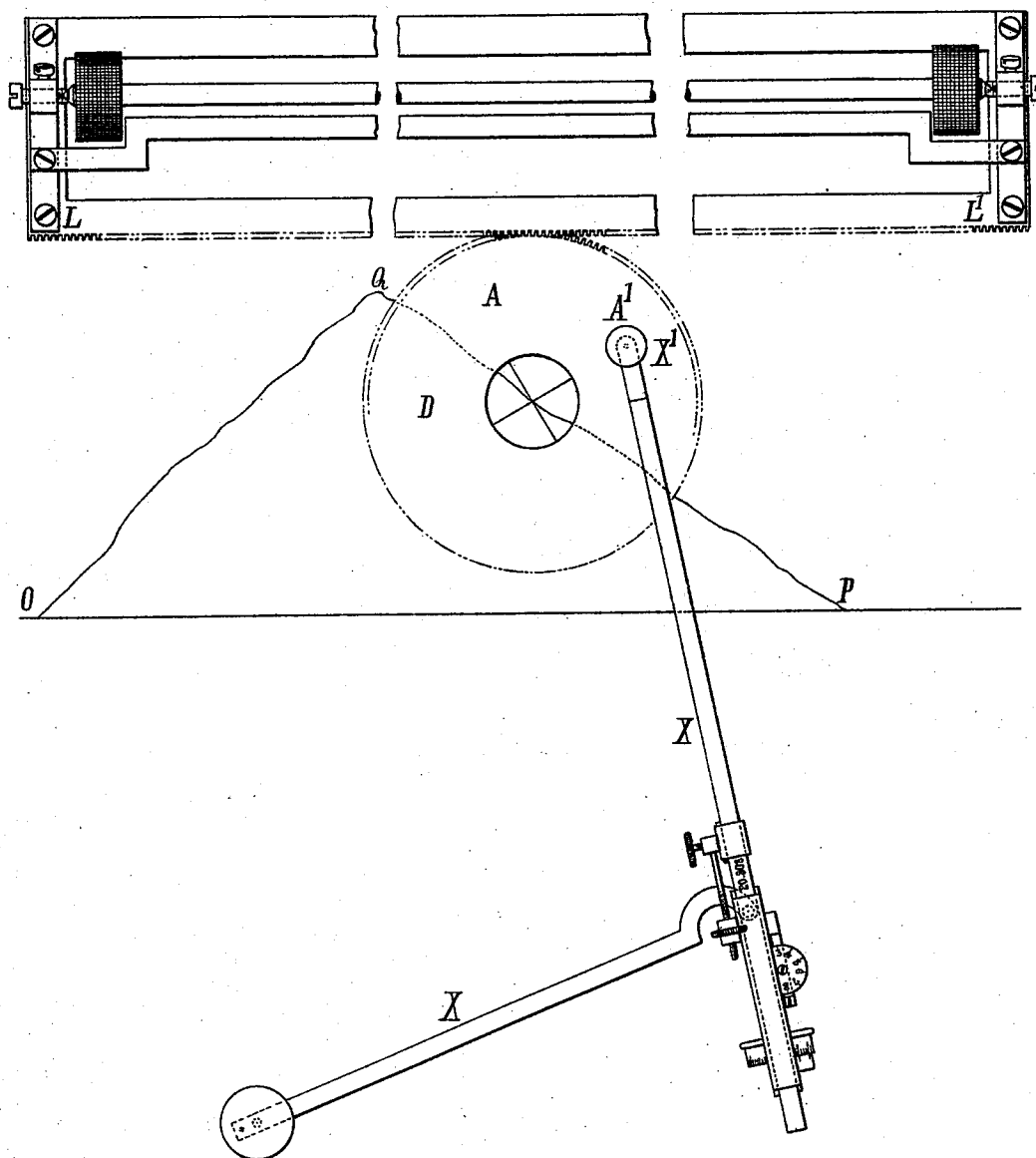
2 Sheets—Sheet 1.

G. U. YULE.
APPARATUS FOR HARMONIC ANALYSIS.

No. 532,625.

Patented Jan. 15, 1895.

Fig. 1.



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(No Model.)

2 Sheets—Sheet 2.

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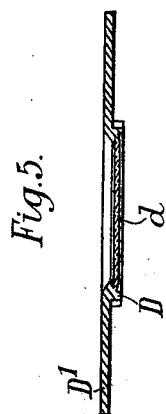


Fig. 6.

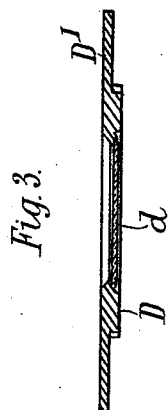
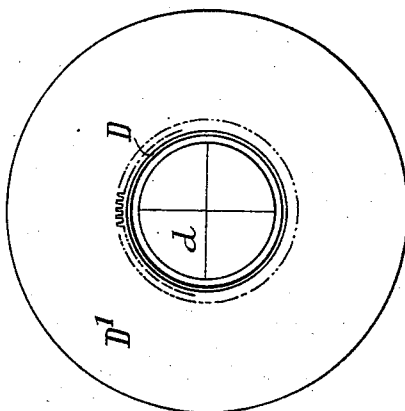
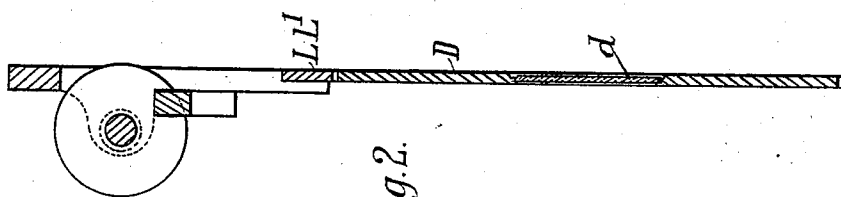
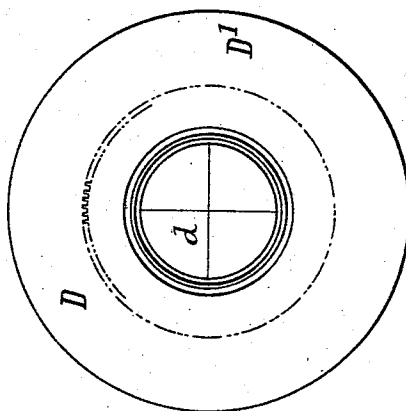


Fig. 4.



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UNITED STATES PATENT OFFICE.

GEORGE UDNY YULE, OF LONDON, ENGLAND.

APPARATUS FOR HARMONIC ANALYSIS.

SPECIFICATION forming part of Letters Patent No. 532,625, dated January 15, 1895.

Application filed September 8, 1894. Serial No. 522,498. (No model.)

To all whom it may concern:

Be it known that I, GEORGE UDNY YULE, a subject of the Queen of Great Britain, residing at 40 Argyll Road, Kensington, in the city of London, England, have invented an Improved Arrangement and Combination of Apparatus to be Employed in Harmonic Analysis, of which the following is a specification.

My invention is illustrated by the accompanying drawings, in which—

Figure 1 is a plan of the combined apparatus in a position it might assume in practice. Fig. 2 is a cross section of the rolling parallel ruler showing a rolling disk in use therewith, and Figs. 3 and 4 and Figs. 5 and 6 are respectively under side views and cross sections of other forms of rolling disks.

The theorem upon which my invention is based is that known as "Fourier's," by which it is known that if y be a function of x , y may be expanded in terms of sines and cosines of multiples of x , so that if x range from $+\pi$ to $-\pi$, we may write:

$$y = A_0 + A_1 \sin x + A_2 \sin 2x + A_3 \sin 3x + \dots + B_1 \cos x + B_2 \cos 2x + B_3 \cos 3x + \dots$$

where A_0 is the mean ordinate of the curve,

$$A_n = \frac{1}{\pi} \int_0^{2\pi} y \sin nx \, dx$$

$$B_n = \frac{1}{\pi} \int_0^{2\pi} y \cos nx \, dx$$

The following is a description of my improved apparatus and of the mode of using the same.

Let O P Q, Fig. 1, be the curve required to be analyzed. Let D be a disk whose circumference is equal to $\frac{1}{n}$ of O P (where $n=1, 2, 3$, or 4, &c.) which is capable of rolling without slip along the edge L L' of a parallel ruler or straight edge, the said ruler being necessarily of the character of a rolling parallel ruler, that is to say, a ruler only capable of such motion that every point in it describes a perpendicular to the line O P. Bring the center of the disk D over the point O and then carry it completely round the curve by moving the parallel ruler in the aforesaid manner and by rolling the disk D along its edge. Then it

may be shown that any point A of the disk, initially on its horizontal diameter at a distance r from the center, will describe a curve whose area is

$$\text{area of O Q P O} \pm nr \pi A_n$$

where π signifies as usual the ratio of the circumference of a circle to its diameter, and similarly a point A', initially on the vertical diameter at a distance r from the center of the disk, will describe a curve whose area is

$$\text{area of O Q P O} \pm nr \pi B_n.$$

The sign $+$ or $-$ to be taken depends on the initial position of A or A' to the right or left, above or below, the center of the disk. r may conveniently be made a multiple of $\frac{1}{\pi}$.

The area of O Q P O, the original curve, will have been already determined in order to obtain A_0 . Consequently if we can determine the areas of the curves described by the points A A' of the disk, we shall have the coefficients A_n and B_n .

The following is a method of carrying the invention into effect:

L L' is a bar of sheet brass or other metal with a fine rack cut along its fore edge, and attached to the front of a rolling parallel ruler.

D is a flat toothed wheel gearing into the rack. Its center may be given by an inserted disk d of glass marked on the under side with two lines at right angles, the continuations of the marked lines passing through or perpendicularly under the points A A'. As many such wheels or disks will be required as terms in the Fourier series, the numbers of teeth in them being in the ratios one, one-half, one-third, one-fourth, &c., and the largest wheel having as many teeth as the length O P of the rack.

To obtain the area of the curve described by A or A', slight depressions may be formed at these points on the upper side of the wheel or disk. If now the tracer X' of a planimeter X be allowed to rest in one of these depressions A A', while the center of the disk is carried completely round the curve O Q P, the difference between the initial and final planimeter readings will be proportioned to the area of O Q P O $\pm A_n$ (or B_n as the case may be). As the tracer X' of the planimeter

will be raised above the surface of the paper on which the curve is drawn by this proceeding, it may be made adjustable vertically within its socket.

5 It may be desirable, especially in obtaining the coefficients of the higher terms of the series, to bring the points A A' outside the toothed wheel D. In this case a metal disk D' is attached above the wheel D, as shown at
10 Figs. 3 to 6, so as to sweep over the rack bar L L' when the wheel D gears into it, and the depressions at A A' may be formed in the upper disk D'.

I would here remark that the form of planimeter indicated in the drawings is that
15 known as "Amsler's." I do not however confine myself to the use of such form of planimeter as other forms may also be employed.

Having fully described my invention, what
20 I desire to claim and secure by Letters Patent is—

1. The improved apparatus to be employed in harmonic analysis consisting in the ar-

25 rangement and combination of a parallel moving rack, a set of toothed disks, having centrally inserted glass disks marked with lines crossing each other at right angles to indicate the center, and having depressions in the continuations of such cross lines to receive the
30 tracer of the planimeter, and a planimeter, substantially as described.

2. The combination with a planimeter and a parallel ruler having a rack on its fore edge, of a set of toothed disks having centrally inserted glass disks marked with lines crossing
35 each other at right angles to indicate the center, and having depressions in the continuations of such cross lines to receive the tracer of the planimeter, substantially as herein shown and described and for the purpose stated.

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