

Dec. 28, 1954

F. W. BUBB

2,698,133

MECHANICAL SIMULATOR FOR DOWNHOLE PUMPING SYSTEMS

Filed Jan. 8, 1951

2 Sheets-Sheet 1

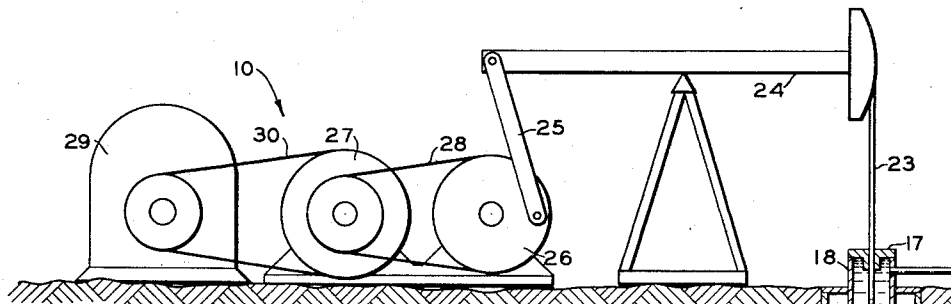


FIG. 1.

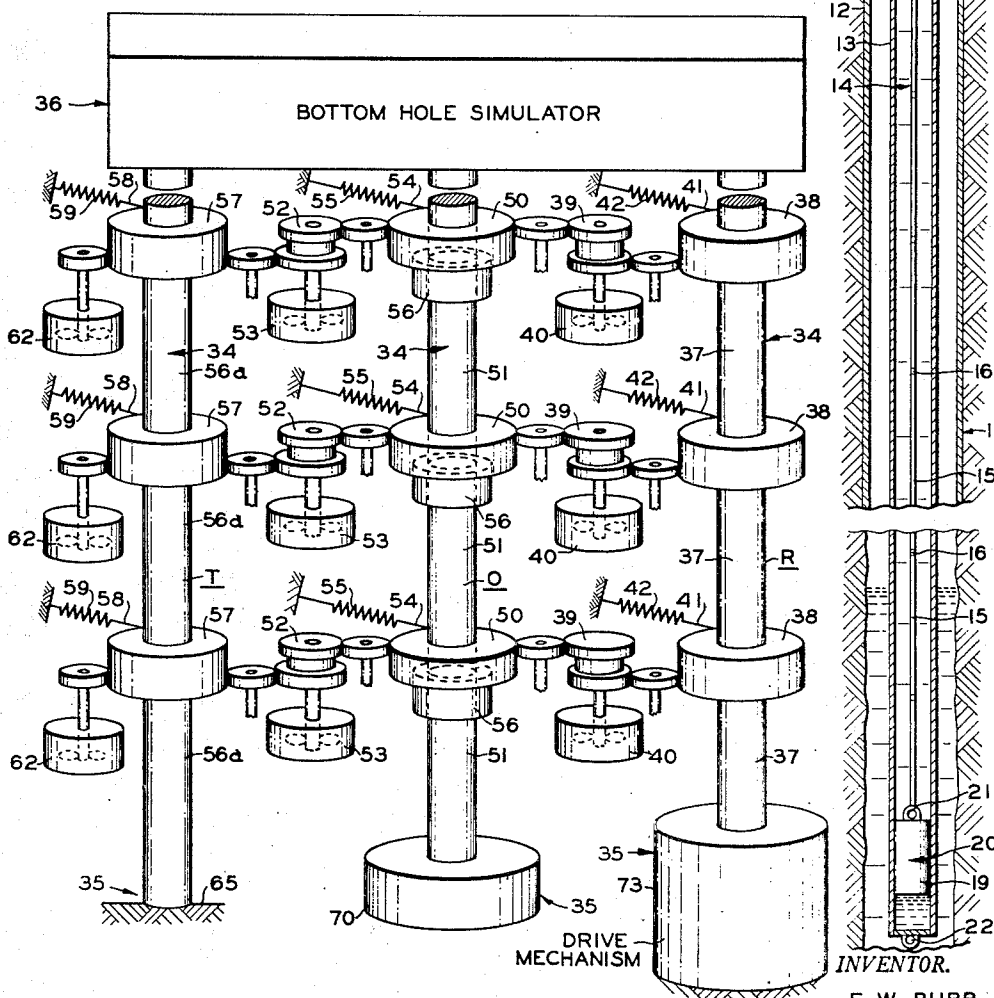


FIG. 2.

BY

Hudson & Young
ATTORNEYS

Dec. 28, 1954

F. W. BUBB

2,698,133

MECHANICAL SIMULATOR FOR DOWNHOLE PUMPING SYSTEMS

Filed Jan. 8, 1951

2 Sheets-Sheet 2

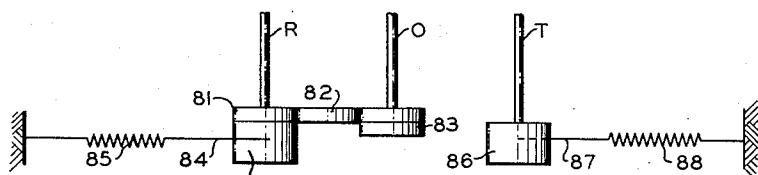


FIG. 3.

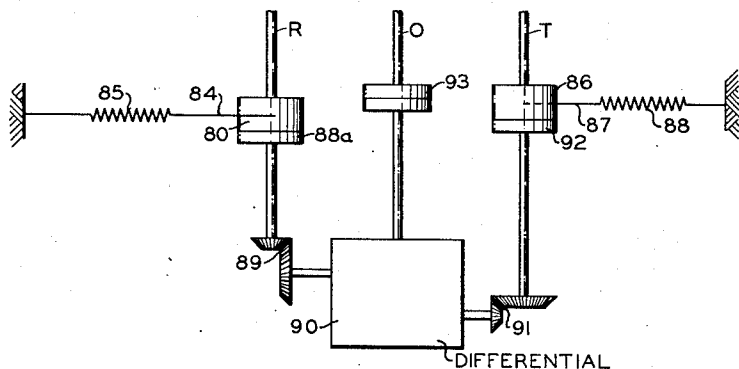


FIG. 4.

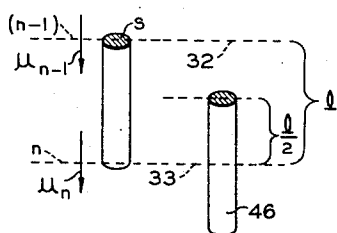


FIG. 5.

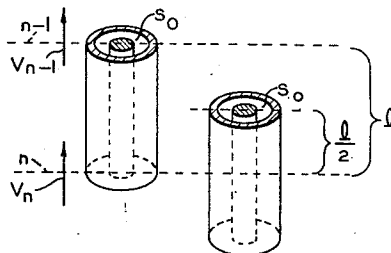


FIG. 6.

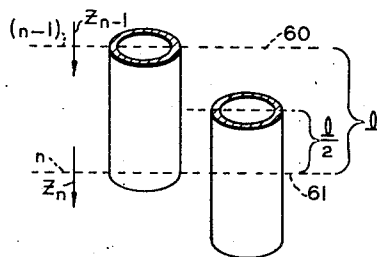


FIG. 7.

INVENTOR.

F. W. BUBB

BY

Hudson & Young
ATTORNEYS

1

2,698,133

MECHANICAL SIMULATOR FOR DOWNHOLE PUMPING SYSTEMS**Frank W. Bubbs, Webster Groves, Mo., assignor to Phillips Petroleum Company, a corporation of Delaware****Application January 8, 1951, Serial No. 204,927****13 Claims. (Cl. 235—61)**

This invention relates to a simulator which is a mechanical analogue of a pumping system. In one specific aspect, it relates to a simulator which is a mechanical analogue of the tubing, sucker rod, oil column, and pump in a deep well pumping unit.

A typical deep well system includes a series or "string" of metal rods, referred to in the art as sucker rods, these rods extending through tubing positioned in the well, the lower end of the tubing carrying a pumping unit. The tubing, in turn, is suspended within a well casing and, under some conditions, the lower portion of the interspace between the tubing and casing may contain a column of oil. At the top of the well, the sucker rod string extends through a stuffing box and is driven by a prime mover, such as an electric motor or an internal combustion engine, through a flywheel or "bull wheel" driven by the prime mover. This flywheel, in turn, drives a crank or counterbalance which is coupled through a walking beam to the top end of the sucker rod string.

As the pumping operation is carried out, a column of oil rises in the region between the tubing and the sucker rod string, thus producing a viscous drag both upon the tubing and sucker rod string. The weight and elasticity of the sucker rod string, as well as the tubing and oil column, produce elastic strains in the pumping unit so that the sucker rod string and tubing behave in a manner somewhat analogous to elongated springs.

This elastic movement of the sucker rod string and tubing oftentimes results in failure of one or more sucker rods and, more often, in inefficient operation of the pumping system. It has not been possible, in the past, to predict the effect of changes in various operating variables of the system with any degree of accuracy, except by purely empirical or "cut and try" methods, for the obvious reason that the movement of the sucker rod string several miles below the surface of the earth cannot be observed nor is it possible to observe the manner of operation of the downhole pump.

It has been proposed to construct a mechanical model of the pumping system so that adjustments of various operating variables of the scale model would produce effects thereon similar to the effects of changes in corresponding variables upon the actual pumping system. However, in order to obtain any useful results by such a system, it can be demonstrated mathematically that the scale model itself would be of such large size and prohibitive cost as to render construction thereof entirely impractical. It has also been proposed to provide a system of weights and springs to behave in the same manner as the pumping system but, here again, the difficulties of adjustment of various parts of the system and the difficulties in making the model itself can be demonstrated to be so great as to eliminate the practical possibility of constructing such apparatus.

I have discovered that a mechanical analogue of a pumping system can be set up on a practical scale to eliminate the difficulties of providing a scale model or a system of weights and springs. In this system, a series of flywheels connected by shafts are provided to represent the oil column, the tubing, and the sucker rod string, the rotary inertia of the flywheels representing the inertia of corresponding elements of the oil column, sucker rod string or tubing. The torque impressed upon the flywheels represents stress imposed upon corresponding elements of the sucker rod string, tubing and oil column, while the angular velocity and displacement of the flywheels represents the velocity and displacement of corresponding parts of the

2

pumping unit. In corresponding fashion, other parameters of the analogue or mechanical system represent variables and constants of the actual pumping unit. It is a feature of the invention that a clutch arrangement is provided to prevent simulation of tension upon the oil column so that the physical fact that oil cannot be subjected to tension is taken into account in constructing the analogue system.

It is an object of the invention to provide an improved mechanical system which is an analogue of an actual well pumping unit, which system may be utilized to predict the effect of changing various operating conditions of the pumping system.

It is a still further object to provide apparatus which is reliable in operation, which utilizes standard mechanical components, and is of the lowest cost commensurate with the complexity of the data obtained therefrom.

Various other objects, advantages and features of the invention will become apparent from the following detailed description taken in conjunction with the accompanying drawings, in which:

Figure 1 is a vertical sectional view, partially in elevation, of a typical deep well pumping system;

Figure 2 is a view of the analogue system simulating the operation of the deep well pumping unit;

Figure 3 is a view of the pump-simulating unit representing conditions during the upstroke;

Figure 4 is a view of the pump-simulating unit representing conditions during the downstroke; and

Figures 5, 6, and 7 are diagrammatic views illustrating features of the invention.

Referring now to Figure 1, I have illustrated, in a schematic manner, a typical deep well pumping system to which the simulator of this invention is applicable, this system including driving mechanism 10 located at the surface and a downhole pumping unit 11. The unit 11 includes the usual well casing 12 within which is mounted tubing 13 having a sucker rod string 14 mounted therein. The string 14 includes a number of sucker rods 15 connected together by joints 16, the upper end of the string extending through a stuffing box 17 in a casing head 18 and the lower end being attached to a plunger 19 forming a part of a pump 20 which includes an upper or traveling valve 21 and a bottom or stationary valve 22. The uppermost sucker rod or "polished" rod 23 is driven by the mechanism 10 which includes a walking beam 24 having one end secured to the polished rod and its other end secured to a pitman 25 which is driven by a crank or counterbalance 26. The crank is driven by a flywheel 27, as through a chain drive 28, and the flywheel, in turn, is driven by a prime-mover 29, as by a second chain drive 30. The prime mover 29 may be an electric motor, an internal combustion engine, or any other suitable type of engine.

In accordance with the invention, the operation of the downhole unit 11 is mechanically simulated by the apparatus of my invention. The apparatus of Figure 2 simulates the behavior of the downhole unit 11. The torque upon various parts of the analogue system represents stress at corresponding parts of the pumping unit, the angular displacement at various points of the analogue system represents displacement of corresponding parts of the actual pumping system, and angular velocity in the analogue system represents the velocity of corresponding parts of the mechanical system. In order for the two systems to be analogues of each other, it is necessary that the differential equations describing the behavior of the analogue system be of similar form to the differential equations describing the behavior of the actual pumping unit. Thereupon, by suitably adjusting the coefficients of various terms of the analogue system, as by changing the parameters of various parts of the analogue system, a condition is obtained whereby the torque, angular displacement, angular velocity and other parameters of the analogue system correspond to the stress, displacement, velocity and other corresponding variables of the pumping unit. At this time, the two systems are analogues of each other and further changes in the parameters of the analogue system produce effects thereon which enable accurate prediction of the effects of corresponding changes in the pumping unit.

In the simulator, a line T represents the tubing 13, a line

O represents the oil column and a line R represents the sucker rod string 15. The subscripts of the symbols represent the line to which they are applicable such as the tubing, oil or rod line. The following notation is used in the differential equations just referred to, each equation being given both in its dimensionless and regular form, the dimensionless variables being indicated by placing a bar over the symbol for the variable. It will be understood that each variable is thus represented by a pure number, indicated by the barred letter, multiplied by an appropriate combination of fundamental units which, for the pumping unit, are time, length and force, and which, for the analogue system, are time, angle or angular displacement, and torque, convenient units for these variables being selected as set forth in the first part of the following table, the remainder of the table indicating the notation to be used hereinafter.

ν —prime mover frequency or revolutions per second.
 l —length of segment of rod, tube or oil.

Unit of force—force required to stretch a sucker rod segment by an amount l .

e_r, e_o, e_t —elastic modulus of the sucker rod, oil and tube, respectively.

s_r, s_o, s_t —cross sectional area of sucker rod, oil and tube, respectively.

m —equivalent inertia of pump engine.

f —frictional coefficient of engine.

h —positional constant for engine.

w_r, w_o, w_t —weight per unit length of sucker rod, oil and tube, respectively.

w_p, w_v —weight of plunger and valve, respectively.

k, k_1, k_2 —viscous coefficient in expressions $k(u+v)$, $k_1(v+w)$, k_2w giving viscous drag per unit length between sucker rod and oil, between oil column and tube, and between tube and oil surrounding tube, respectively.

q_n —upward pressure on bottom of tube due to oil between tube and casing. ($q = e\bar{q}$).

t —time ($t = \frac{\bar{t}}{\nu}$).

x_n, y_n, z_n —displacement of tube, rod, and oil, respectively.
 ($x = \bar{x}l$; $y = \bar{y}l$; $z = \bar{z}l$).

u_n, v_n, w_n —velocity of rod, oil, and tube, respectively.
 ($u = \bar{u}\nu l$; $v = \bar{v}\nu l$; $w = \bar{w}\nu l$).

$a_n, b_n, c_n = \dot{u}_n, \dot{v}_n, \dot{w}_n$, respectively, time derivatives of velocity, or acceleration.
 ($a = \bar{a}\nu^2 l$; $b = \bar{b}\nu^2 l$; $c = \bar{c}\nu^2 l$).

p_n —tension in rod line ($p = e\bar{p}$).

q_n —compression of oil line ($q = e\bar{q}$).

r_n —tension of tubing line ($r = e\bar{r}$).

Ω —equivalent driving torque simulating the engine.

μ —number of torsional oscillations per second imposed by driving unit.

D —amplitude of angular oscillation imposed by driving unit.

Unit of torque—torque required to twist segment of rod line through an angle of one radian.

$$\left(E = \frac{\pi G r^4}{2L} \right)$$

E_r, E_o, E_t —torque to twist segment of rod, oil or tubing line through an angle of one radian.

M —equivalent moment of inertia of driving unit.

F —equivalent drag coefficient for simulator driving unit.

H —equivalent positional coefficient for driving unit.

J_r, J_o, J_t —constant torque imposed on flywheels in rod, oil and tubing lines, respectively, simulating the weight of corresponding segments of these lines.

I_r, I_o, I_t —moment of inertia of flywheels in rod, oil and tubing lines, respectively, simulating inertia of corresponding segments of rod, oil and tube.

J_p, J_v —constant torque imposed on flywheels simulating weight of plunger and valve, respectively.

K, K_1, K_2 —viscous coefficients of expressions simulating viscous drag, as corresponding to the expressions at the left.

Q_n —constant torque imposed on flywheel simulating upward pressure of oil on bottom of tube ($Q = E\bar{Q}$).

I_p —moment of inertia of flywheel simulating inertia of plunger.

I_v —moment of inertia of flywheel simulating inertia of valve end and tubing.

T —time ($T = \frac{1}{\mu} \bar{T}$).

X_n, Y_n, Z_n —angular displacement of flywheels simulating displacement of oil, rod, and tube, respectively.
 ($X_n = \bar{X}_n$; $Y_n = \bar{Y}_n$; $Z_n = \bar{Z}_n$).

U_n, V_n, W_n —angular velocity of flywheels in rod, oil and tubing lines simulating velocity of rod, oil and tube.
 ($U_n = \mu \bar{U}_n$; $V_n = \mu \bar{V}_n$; $W_n = \mu \bar{W}_n$).

$A_n, B_n, C_n = \dot{U}_n, \dot{V}_n, \dot{W}_n$, respectively, time derivatives of velocity, or acceleration.
 ($A_n = \mu^2 \bar{A}_n$; $B_n = \mu^2 \bar{B}_n$; $C_n = \mu^2 \bar{C}_n$).

P_n —torque in shaft segment of rod line simulating tension ($P = E\bar{P}$).

Q_n —torque in shaft segment simulating compression in oil line ($Q = E\bar{Q}$).

R_n —torque in shaft segment simulating tension of tubing line ($R = E\bar{R}$).

G, r, L —shear modulus radius and length, respectively, of shaft in rod, oil or tubing line.

g —gravity constant.

In the mathematical analysis of the pumping unit and analogue system which follows, the respective differential equations are first derived by the method of finite differences and, from each such equation, the corresponding dimensionless equation is derived and follows the basic equation. In accordance with the method of finite differences, the casing, tubing and sucker rod string are divided into a large number N of segments. During operation of the pumping unit, each segment of the sucker rod moves upward or downward in accordance with the mechanical laws governing the system.

In particular, considering the n th segment of the sucker rod, as defined by the dotted lines 32, 33, Figure 5, in an interval of time Δt , a volume of rod $s u_n \Delta t$ passes out at the bottom of the segment, and a volume of rod $s u_{n-1} \Delta t$ passes into the top of the segment with the result that a volume $s(u_n - u_{n-1}) \Delta t$ is removed from the segment. Accordingly, in this interval of time Δt , there is an increment Δp_n in tensile stress in accordance with the following equation:

$$\Delta p_n = \frac{s_r e_r (u_n - u_{n-1}) \Delta t}{l}; \quad \bar{\Delta p}_n = (u_n - u_{n-1}) \bar{\Delta t} \quad \text{Eq. (1)}$$

This equation has its counterpart in the analogue system of Figure 2 wherein shaft unit 34 represents the n th segment of the pumping unit, unit 35 represents the top of the pumping unit, and unit 36 represents the pump at the bottom of the well. The segment 34 of the rod line or shaft 37 includes a flywheel 38 having a moment of inertia I_r simulating the inertia of the corresponding segment of the sucker rod. The flywheel 38 is attached to one input of a differential 39, the other input of which is attached to a corresponding flywheel of the "O" line or shaft to simulate viscous drag between the oil column and rod, the driven member of the differential being attached to a damper 40 which produces a resistive torque $K(U_n + V_n)$. A constant torque J_r representing the weight of the rod segment is produced by attaching a string 41 to the flywheel, the string in turn, being attached to a long spring 42 whose tension is not appreciably changed by movement of the flywheel. Alternatively, the flywheel may be the armature of a small electric motor, which is subjected to a constant drag or torque by current and field adjustments to the motor. The inevitable rotational inertia of the differential may be apportioned between the two adjacent flywheels. In a time interval ΔT the n th flywheel 38 turns through an angle $U_n \Delta T$, and the $(n-1)$ th flywheel 38 turns through an angle $U_{n-1} \Delta T$. Accordingly, there is an increment of torque upon shaft 37 as set forth by the following equation:

$$\Delta P_n = E_r (U_n - U_{n-1}) \Delta T; \quad \bar{\Delta P}_n = (\bar{U}_n - \bar{U}_{n-1}) \bar{\Delta T} \quad \text{Eq. (2)}$$

It will be noted that Equations 1 and 2 are of identical form and, upon comparing the dimensionless forms of these equations, the following relationships are obtained between the variables p_n, u_n and t of the pumping unit and the variables P_n, U_n and T of the analogue system, the symbol σ denoting the time ratio

$$\frac{\bar{p}}{\mu}$$

and the symbol ρ denoting the stiffness ratio

$$\frac{E}{e_s}$$

$$p_n = \frac{P_n}{\rho} \quad \text{Eq. (3)}$$

$$u_n = \sigma l U_n \quad \text{Eq. (4)}$$

$$t = \frac{T}{\sigma} \quad \text{Eq. (5)}$$

In the pumping unit of Figure 1, the downward velocity of the rod at the n th station is u_n and the upward velocity of the oil is v_n with the result that there is an upward viscous drag upon the rod element of magnitude $k(u_n + v_n)$. From these data, the equation of motion of a rod segment 46 which straddles the segment between lines 32, 33 may be deduced, this segment extending distance

$$\frac{l}{2}$$

5

above and below the n th station point. This segment has acting upon it a total tension p_{n+1} on its bottom end, a total tension p_n on its top end, the aforementioned upward viscous drag $kl(u_n + v_n)$ and a downward weight w_n . The resulting force produces a downward acceleration a_n at the mid-point of this segment in accordance with the equation

$$p_{n+1} - p_n = \frac{w_n l a_n}{g} + kl(u_n + v_n) - w_n l$$

$$\frac{es}{w_n l} (\bar{p}_{n+1} - \bar{p}_n) = \frac{v^2 l}{g} \bar{a}_n + \frac{klv}{w_n} (\bar{u}_n + \bar{v}_n) - 1$$

Eq. (6)

In Figure 2, a torque $-P_n$ acts on one end of the n th segment and a torque P_{n+1} acts on the other end of the segment. Spring 42 produces a constant torque J_r upon flywheel 38 and differential 39 together with damper 40 produce a torque $K(U_n + V_n)$. These torques accelerate the segment 34 in accordance with the following equation:

$$P_{n+1} - P_n = \frac{I_r A_n}{g} + K(U_n + V_n) - J_r$$

$$\frac{E}{J_r} (\bar{P}_{n+1} - \bar{P}_n) = \frac{I_r \mu^2}{g J_r} \bar{A}_n + \frac{K \mu}{J_r} (\bar{U}_n + \bar{V}_n) - 1$$

Eq. (7)

It will be noted that Equations 6 and 7 are of similar form and, upon comparing the coefficients and variables of the dimensionless forms of the equations, the following relationships are obtained between the coefficients and variables of the equations representing the analogue system and the equations representing the pumping unit of Figure 1.

$$J_r = w_r l p \quad \text{Eq. (8)}$$

$$I_r = w_r l^2 \sigma^2 p \quad \text{Eq. (9)}$$

$$K = k l^2 \sigma p \quad \text{Eq. (10)}$$

$$v_n = \sigma l V_n \quad \text{Eq. (11)}$$

$$a_n = A_n \sigma^2 l \quad \text{Eq. (12)}$$

Similar equations may be derived with respect to the compressive strain of the segments of the oil column and the motion of a general segment of the oil column. Thus, in a time interval Δt , a volume $s_o v_n \Delta t$, Figure 6, of oil flows upwardly into the bottom of the n th segment, a volume $s_o v_{n-1} \Delta t$ flows out at the top of the n th segment and an increment $s_o (v_n - v_{n-1}) \Delta t$ is accumulated within the segment. This increment is compressed and produces an increase Δq_n in the total oil pressure of an amount determined by the following equation:

$$\Delta q_n = s_o e_o (v_n - v_{n-1}) \Delta t / l$$

$$\Delta \bar{q}_n = \frac{s_o e_o}{se} (\bar{v}_n - \bar{v}_{n-1}) \Delta \bar{t}$$

Eq. (13)

The segment 34 of the oil line or shaft includes a flywheel 50 fixed on shaft 51, the flywheel being connected, as previously noted, to one side of differential 39. The flywheel is also connected to one side of a differential 52, the other side of which is connected to a flywheel on the tubing line. The differential 52 has a damper 53 which produces a viscous drag $K_1(V_n + W_n)$ upon the oil line. A line 53 and a spring 55 produce a constant resistive torque J_o upon the flywheel 50. The following equation is derived similarly to Equation 2 and expresses the torque in shaft segment 51.

$$\Delta Q_n = E_o (V_n - V_{n-1}) \Delta T$$

$$\Delta \bar{Q}_n = \frac{E_o}{E} (\bar{V}_n - \bar{V}_{n-1}) \Delta \bar{T}$$

Eq. (14)

It will be noted that Equations 13 and 14 are of similar form and, upon comparing the coefficients of the dimensionless forms of the equations, the following relationships are obtained between the coefficients and variables of the equations representing the analogue system and the equations representing the pumping unit.

$$q_n = \frac{Q_n}{p} \quad \text{Eq. (15)}$$

$$E_o = s_o e_o p \quad \text{Eq. (16)}$$

6

The equation of motion of the oil segment which extends a distance

$$\frac{l}{2}$$

above and below the lower end of the n th oil segment, as is shown by Figure 6, is governed by the viscous drag on the oil due to the sucker rod movement which, as previously indicated, is $k(u_n + v_n)$ and the viscous drag on the oil due to the friction thereof against the tube, which is $k_1(v_n + w_n)$ both expressions representing drag per unit length. The bottom of this segment is subjected to the compressive force q_{n+1} and the top of this segment is subjected to the compressive force $-q_n$. Finally, the weight of oil per unit length w_o multiplied by the length l must be considered in the equation of motion which, therefore, becomes

$$q_{n+1} - q_n = w_o l b_n / g + w_o l + kl(u_n + v_n) + k_1 l (v_n + w_n)$$

$$\frac{es}{w_o l} (\bar{q}_{n+1} - \bar{q}_n) = \frac{v^2 l}{g} \bar{b}_n + \frac{klv}{w_o} (\bar{u}_n + \bar{v}_n) + \frac{k_1 v l}{w_o} (\bar{v}_n + \bar{w}_n) + 1$$

Eq. (17)

By analogy to the derivation of Equation 7, the equation of motion of segment 34 of the oil line is as follows:

$$Q_{n+1} - Q_n = \frac{I_o B_n}{g} + K(U_n + V_n) + K_1(V_n + W_n) + J_o$$

$$\frac{E}{J_o} (\bar{Q}_{n+1} - \bar{Q}_n) = \frac{I_o \mu^2}{g J_o} \bar{B}_n + \frac{K \mu}{J_o} (\bar{U}_n + \bar{V}_n) + \frac{K_1 \mu}{J_o} (\bar{V}_n + \bar{W}_n) + 1$$

Eq. (18)

It will be noted that Equations 17 and 18 are of similar form and, equating the coefficients of the dimensionless forms of these equations, the following relationships are obtained between the coefficients and variables.

$$J_o = w_o l p \quad \text{Eq. (19)}$$

$$I_o = w_o l^2 \sigma^2 p \quad \text{Eq. (20)}$$

$$K_1 = k_1 l^2 \sigma p \quad \text{Eq. (21)}$$

$$w_n = \sigma l W_n \quad \text{Eq. (22)}$$

It is known that the oil column cannot support a tensile stress. That is, although the oil can be and is compressed, it cannot be subjected to tension. Therefore, Equation 17 is subject to the restriction that q , the stress on the oil column, cannot become negative which would represent a tensile stress. This important limitation has not been taken into account prior to my invention, insofar as I am aware, and this limitation destroys the continuity of differential Equation 17 which practically makes impossible a mathematical solution of the equation. However, this limitation may be readily imposed upon the apparatus of Figure 2 and, to this end, I have provided a clutch 56 at each segment where a shaft 51 joins a flywheel 50. Accordingly, the connection between the flywheel and shaft is released whenever the torque in these parts is in a sense corresponding to tension upon the oil column. This limitation is expressed mathematically by the following restrictions upon Equations 17 and 18.

$$q_n \leq 0; \bar{q}_n \leq 0; Q_n \leq 0; \bar{Q}_n \leq 0 \quad \text{Eq. (23)}$$

The movement of the tubing is governed generally by the same equations as the movement of the sucker rod. Referring to Figure 7, and considering the n th segment of the tubing, as defined by dotted lines 60 and 61, a volume of tubing $s_t w_n \Delta t$ passes out of the bottom of the segment during an interval Δt , a volume of tubing $s_t w_{n-1} \Delta t$ passes into the top of the segment and, accordingly, in this interval, there is an increment in tensile stress in accordance with the following equation.

$$\Delta r_n = s_t e_t (w_n - w_{n-1}) \Delta t / l$$

Eq. (24)

$$\Delta \bar{r}_n = \frac{s_t e_t}{se} (\bar{w}_n - \bar{w}_{n-1}) \Delta \bar{t}$$

This equation has its counterpart in the analogue system of Figure 2 in the segment 34 of the tubing line which includes a shaft 56a carrying a flywheel 57 connected as previously stated to one side of differential 52. A constant torque J_t is impressed upon flywheel 57 by a

line 58 connected to a spring 59. A damper 62 produces an addition variable torque upon the flywheel representing viscous drag $K_2 W_n$ upon those segments having oil between the tubing and casing. The torque upon shaft 56a is as follows:

$$\Delta R_n = E_t (W_n - W_{n-1}) \Delta T \quad \text{Eq. (25)}$$

$$\Delta \bar{R}_n = \frac{E_t}{E} (\bar{W}_n - \bar{W}_{n-1}) \Delta \bar{T}$$

This equation being of similar form to Equation 24, the following relationships hold between the coefficients and variables of the dimensionless forms of the equations, as previously set forth:

$$r_n = \frac{R_n}{\rho} \quad \text{Eq. (26)}$$

$$E_t = s_t e_t \rho \quad \text{Eq. (27)}$$

The equation of motion of the n th tube segment which extends a distance

$$\frac{l}{2}$$

above and below the segment just mentioned is determined by the total tension r_{n+1} at the bottom of the segment, the tension r_n at the top of the segment, the viscous drag $k_1 l (v_n + w_n)$ produced by friction of the oil column against the tubing and an upward viscous drag $k_2 l w_n$ produced by oil or other fluid positioned between the casing and the tubing. This latter term need only be considered for the particular segments wherein oil is included between the casing and tubing. Accordingly, the equation of motion of this segment is as follows:

$$r_{n+1} - r_n = w l c_n / g + k_1 l (v_n + w_n) + k_2 l w_n - w l \quad \text{Eq. (28)}$$

$$\frac{es}{w l} (\bar{r}_{n+1} - \bar{r}_n) = \frac{v^2 l}{g} \bar{c}_n + \frac{k_1 v l}{w l} (\bar{v}_n + \bar{w}_n) + \frac{k_2 v l}{w l} \bar{w}_{n-1}$$

In the analogue system of Figure 2, the equation of motion is as follows, as will be evident upon comparison with Equation 7.

$$R_{n+1} - R_n = \frac{I_t C_n}{g} - J_t + K_1 (V_n + W_n) + K_2 W_n \quad \text{Eq. (29)}$$

$$\frac{E}{J_t} (\bar{R}_{n+1} - \bar{R}_n) = \frac{I_t \mu^2}{g J_t} \bar{C}_n + \frac{K_1 \mu}{J_t} (\bar{V}_n + \bar{W}_n) + \frac{K_2 \mu}{J_t} \bar{W}_{n-1}$$

The torque unit 62 is included only for those segments of the tubing wherein a stationary body of fluid is positioned between the casing and tubing. The following relationships between the pumping unit and analogue system follow from comparison of the dimensionless forms of Equations 28 and 29.

$$J_t = w l^2 \rho \quad \text{Eq. (30)}$$

$$I_t = w l^2 \sigma \rho \quad \text{Eq. (31)}$$

$$c_n = C_n \sigma^2 l \quad \text{Eq. (32)}$$

$$K_2 = k_2 l^2 \rho \sigma \quad \text{Eq. (33)}$$

From the foregoing description, it will be evident that the simulator unit 34 is an analogue of any typical segment of the tubing, sucker rod string, and oil column. The actual pumping unit may be divided into as many segments as desired, each such segment being represented by a corresponding unit 34 in the system of Figure 2, the accuracy becoming greater with an increasing number of segments. In order to compensate for the fact that a finite difference analysis is accurate only up to an interval of one-half the element of extension, half-flywheels

$$\frac{I_r}{2}, \frac{I_o}{2}, \text{ and } \frac{I_t}{2}$$

not shown, may be inserted at the ends of the respective rod, oil and tubing lines.

As stated, the simulator as thus far described simulates the sucker rod string together with its associated oil column and tubing. The simulator also includes the unit 35 representing conditions at the top of the well and

the unit 36 representing conditions at the bottom of the well.

The top of the tubing string is anchored to the casing and to the ground so that it must remain stationary, that is, its displacement must be zero at all times. This condition is represented in Figure 2 by anchoring the tubing line, as indicated at 65, at its top. This condition is represented by the following equations:

$$z_0 = 0; Z_0 = 0 \quad \text{Eq. (34)}$$

With respect to the oil line, the oil is discharged at the top of the tubing into a storage tank against a substantially constant head. That is, the pressure at the top of the oil line is constant. This condition is realized in the system of Figure 2 by inserting a half flywheel 70 together with spring and spring as shown by the parts 54 and 55 at the top of the oil line to simulate this end condition. This condition is represented by the following equations.

$$q_0 = k_0; Q_0 = K_0 \quad \text{Eq. (35)}$$

The sucker rod string is driven at its top end by the drive mechanism which includes the prime mover, flywheel, crank and walking beam of Figure 1. Although I have disclosed a simulator unit in my copending application, Serial No. 204,926, filed January 8, 1951, now U. S. Patent No. 2,661,898, entitled "Simulator," which takes into account the reaction of the sucker rod string upon the drive mechanism, which unit will be connected through an electro-mechanical transducer to the top of the rod line, for present purposes, the driving system may be assumed to be a positively acting mechanical system of one degree of freedom. The action of the drive mechanism upon the top of the sucker rod may, therefore, be represented by the following function, the terms of which represent, respectively, the resultant force applied to the top of the sucker rod, the lumped kinetic energy of the drive mechanism, the lumped potential energy of the engine and the force or reaction which the sucker rod exerts upon the drive mechanism.

$$q(t) = m a_0 + h y_0 + f u_0 + p_0(t) \\ \bar{g} = \frac{m v^2 l}{es} \bar{a}_0 + \frac{f v l}{es} \bar{u}_0 + \frac{h l}{es} \bar{y}_0 + \bar{p}_0 \quad \text{Eq. (36)}$$

For present purposes, it is assumed that the reaction of the sucker rod system upon the drive mechanism is negligible, which is true in certain practical applications and where results are desired only to a predetermined degree of accuracy, so that the motion at the top end of the sucker rod string may be expressed as a sinusoidal function. As indicated, the reaction of the sucker rod system upon the drive mechanism is taken into account when the simulator for the drive mechanism herein before referred to is connected at the top end of the sucker rod line.

In Figure 2, the driving unit is represented by mechanism 73, the behavior of which is represented by the following equation:

$$\Omega(T) = M A_0 + F U_0 + H Y_0 + P_0(T) \\ \bar{\Omega} = \frac{\mu^2 M}{E} \bar{A}_0 + \frac{\mu F}{E} \bar{U}_0 + \frac{H}{E} \bar{Y}_0 + \bar{P}_0 \quad \text{Eq. (37)}$$

Comparison between the Equations 36 and 37 yields the following relationships between the parameters of the analogue system and pumping unit:

$$M = m l \sigma^2 \rho \quad \text{Eq. (38)}$$

$$F = f l \sigma \rho \quad \text{Eq. (39)}$$

$$H = h l \rho \quad \text{Eq. (40)}$$

The end conditions for the bottom end of the pumping unit are determined by the nature of the pump used, its dimensions, and the material from which it is made. This downhole pumping unit is represented in the system of Figure 2 by the unit 36.

This unit is provided with suitable clutches to provide the arrangement of Figure 3 during the upstroke and the arrangement of Figure 4 during the downstroke.

Referring now to Figure 1, during the upstroke, the working valve 21 in the plunger is closed and the plunger lifts the oil column upwardly at its own velocity so that $v_N = -u_N$. In Figure 3, a flywheel 80 simulates the in-

ertia I_p of the plunger, this flywheel being secured to the oil line by a clutch 81 and a gear 82 secured to a flywheel 83 at the end of the oil line. When flywheel 80 rotates in a clockwise direction, corresponding to the upstroke of the pump, clutch 81 is engaged with the result that the rod line is mechanically connected to the oil line. That is

$$V_N = U_N \quad \text{Eq. (41)}$$

Furthermore, during the upstroke, the oil flowing into the working barrel 19, Figure 1, does not follow the plunger fast enough to exert appreciable pressure on its underside. The forces acting upon the plunger are, therefore, the weight of the plunger w_p , the upward tension p_N in the rod and the downward pressure q_N of the oil above the plunger. The following equation, accordingly describes the motion of the plunger.

$$w_p + q_N - p_N = w_p a_N / g$$

$$1 + \frac{es}{w_p} (\bar{q}_N - \bar{p}_N) = \frac{v^2 l}{g} \bar{a}_N \quad \text{Eq. (42)}$$

In Figure 3, when clutch 81 is engaged, a torque $-P_N$ is exerted upon flywheel 80 by the rod line and a torque Q_N is transmitted to this flywheel by gear 82. In addition a line 84 and spring 85 produce a constant torque J_p upon flywheel 80, the equation of motion of which is, therefore:

$$J_p + Q_N - P_N = \frac{I_p A_N}{g}$$

$$1 + \frac{E}{J_p} (\bar{Q}_N - \bar{P}_N) = \frac{I_p \mu^2}{g J_p} \bar{A}_N \quad \text{Eq. (43)}$$

The following relationships follow from comparison of the dimensionless forms of Equations 42 and 43.

$$J_p = w_p \rho p \quad \text{Eq. (44)}$$

$$I_p = w_p l \sigma^2 \rho \quad \text{Eq. (45)}$$

The forces acting upon valve 22, Figure 1, are its weight w_b , the upward tension r_N in the metal of the tube and total upward pressure q_b on the bottom of the valve due to the head of oil between the head and casing. The equation of motion of the valve is, therefore, as follows:

$$w_b - q_b - r_N = w_b c_N / g$$

$$1 - q_b / w_b - \frac{es}{w_b} \bar{r}_N = \frac{v^2 l}{g} \bar{c}_N \quad \text{Eq. (46)}$$

A flywheel 86 of inertia I_b is secured to the end of the tubing line, the flywheel being subjected to a torque R_N by the line. A constant torque $(J_b - Q_b)$ is also applied to the flywheel by a string or line 87 and a spring 88. The equation of motion of flywheel 86 is, therefore:

$$J_b - Q_b - R_N = \frac{I_b C_N}{g}$$

$$1 - \frac{\bar{Q}_b}{J_b} - \frac{E}{J_b} \bar{R}_N = \frac{I_b \mu^2}{g J_b} \bar{C}_N \quad \text{Eq. (47)}$$

Comparison of the dimensionless forms of these equations yields the following relationships:

$$J_b = w_b \rho p \quad \text{Eq. (48)}$$

$$I_b = w_b l \rho \sigma^2 \quad \text{Eq. (49)}$$

$$Q_b = q_b \rho \quad \text{Eq. (50)}$$

Thus, during the upstroke, the assembly of Figure 3 produces a condition simulating the motion of and forces acting upon the plunger, traveling valve 21, and fixed valve 22 of Figure 1.

During the interval of time representing the downstroke, unit 36 is arranged as shown in Figure 4. The downward velocities of plunger 19 and the end of tube 13 being u_N and w_N , respectively, a volume of oil $(s_r + s_o)(u_N - w_N)\Delta t$ is displaced upwardly through valve 21 during an interval of time Δt . This upwardly displaced oil occupies an equal volume $s_o(v_N + u_N)\Delta t$ in the column above the plunger. Accordingly,

$$u_N s / s_o = v_N + w_N (s + s_o) / s_o \quad \text{Eq. (51)}$$

In Figure 4, flywheel 80 is connected through a clutch 88a and gears 89 to one side of a differential 90, the

other side of which is connected through gearing 91 and a clutch 92 to flywheel 86. The driven element of differential 90 is connected to the oil line by a clutch 93. The gear ratio at 91 is such that a velocity of w_N at clutch 92 drives the corresponding input to the differential at a speed

$$W_N \frac{(s + s_o)}{s_o}$$

This speed is added to the speed V_N at clutch 93 to produce a rotation of gear unit 89. However, the gear ratio at 89 is such that the adjacent input to the differential is

$$U_N \frac{s}{s_o}$$

Accordingly, the following relationships exist at the unit of Figure 4 when the shafts rotate in a counterclockwise direction, corresponding to the downstroke of the pump, the clutches 88a, 92 and 93 all being engaged during this period.

$$U_N \frac{s}{s_o} = V_N + W_N \frac{(s_r + s_o)}{s_o} \quad \text{Eq. (52)}$$

If the dimensionless forms of Equations 51 and 52 are derived and their coefficients compared, certain of the same relationships already derived will be obtained between the coefficients and variables of the two sets of equations. It will be further noted that the two equations are of similar form so that the unit of Figure 4 accurately simulates the aforesaid end condition during the downstroke period.

Since oil passes freely through the valve 21 during the downstroke, the pressure intensity in the oil below and above plunger 19 is nearly equal. The cross-section of oil in the barrel, however, exceeds that above the plunger in the ratio

$$\frac{(s_r + s_o)}{s_o}$$

and, consequently, the upward force on the lower end of the plunger exceeds that on the upper end of the plunger by

$$\frac{q_N s_r}{s_o}$$

Further, the plunger is subjected to the downward force of its weight w_p and the tension p_N in the sucker rod string. The equation of motion of the plunger during the downstroke is, therefore,

$$w_p - q_N s / s_o - p_N = w_p a_N / g \quad \text{Eq. (53)}$$

Similarly, in Figure 4, assuming that the inertia of the differential 90 is distributed rotatably among the flywheels 80 and 86, the torque at the lower side of flywheel 80 is

$$\frac{s_r Q_N}{s_o}$$

and at the other side of flywheel 80, the torque is $-P_N$. Further, string 84 and spring 85 produce a constant torque J_p . Thus, during the downstroke period, the equation of motion of flywheel 80 is as follows:

$$J_p - Q_N \frac{s_r}{s_o} - P_N = \frac{I_p A_N}{g} \quad \text{Eq. (54)}$$

It will be noted that Equations 53 and 54 are of the same form so that the simulator portrays the movement of the plunger during the downstroke. If the dimensionless forms of these equations are derived and their coefficients are compared, certain of the relationships already obtained between the coefficients and variables of the two systems will be found to hold.

Further during the downstroke period, the forces acting upon stationary valve 22 are the weight of the valve w_b , the downward pressure q_N of the oil in the barrel of the plunger, an upward pressure q_b on the bottom of the valve head, and the tension r_N in the metal at the end of the tube. This valve, accordingly, has the following equation of motion.

$$w_b - q_b + q_N - r_N = w_b c_N / g \quad \text{Eq. (55)}$$

In Figure 4, the torque at the upper end of flywheel 86 is R_N , the torque at the lower end of flywheel 86 is $Q_N(s_r + s_o)$, and a constant torque $(J_b - Q_b)$ is produced by spring 88 and line 87. The equation of motion of flywheel 86 is, therefore:

$$J_b - Q_b + Q_N \frac{(s_r + s_o)}{s_o} - R_N = \frac{I_b C_N}{g} \quad \text{Eq. (56)}$$

It will be noted that Equations 55 and 56 are of the same form and, hence, the unit accurately simulates the operation of valve 22 during the downstroke period. If the dimensionless forms of these equations are derived and their coefficients compared, certain of their relationships already derived will be found.

In the operation of the circuit, the simulator of Figure 2 is used to simulate conditions existing in an actual pumping installation. In this actual installation, most of the constants are known and these constants are utilized by substituting them in the equations previously derived to determine the corresponding parameters of similar parts of the analogues of these constants. Before making these substitutions, the value of μ , v , and E are assumed from which data σ and ρ may be readily calculated. In this connection it will be noted

$$\rho = \frac{E}{es}$$

and that the cross sectional areas of various parts of the actual pumping system as well as their elastic modulus are known from which it follows that ρ is readily calculated once a value for E is assumed. It is further necessary to assume a value for the segment length l which may conveniently be the length of one section of the sucker rod string and, in the particular unit shown by Figure 2, the equivalent driving force Ω , and the maximum amplitude D of the driving force are assumed quantities. When the simulator of my aforementioned copending application is utilized to produce the driving force, the quantities Ω , μ , and D are obtained by measurements of the simulator output. As an alternative, an actual model of the engine shown in Figure 1 as driving the sucker rod may be used. A rack and pinion is then used to convert the linear motion of polished rod into the rotation necessary for the rest of the simulator. Once these constants have been assumed and calculated, as stated, the known constants of the actual pumping system are substituted in the equations to provide corresponding parameters for the constants of the analogue system. As an example of such substitution, referring to Equation 8, the weight w_r of a unit length of sucker rod is known and the actual pumping system is constructed. The quantities l and ρ are assumed or calculated as just indicated and, upon substitution of these values in Equation 8, the quantity I_r , that is, the constants of springs 42 in the rod segments are readily calculated. Similarly, with respect to Equation 9, since the weight of a rod segment w_r is known and the quantities l , σ , and ρ are known or calculated, the value I_r of the inertia of the flywheels 38 in the rod line are readily calculated from Equation 9. In an entirely similar fashion, the parameters of the analogue system set forth in Equations 10, 16, 19, 20, 21, 26, 27, 30, 31, 33, 34, 35, 38, 39, 40, 44, 45, 48, 49, and 50 are readily calculated since the righthand side of each of these equations contains only the known or assumed quantities l , v , Ω , D , μ , σ , ρ , and g which are assumed or calculated in the manner previously set forth together with a constant which is readily measurable or known on the actual pumping system. This enables the entire unit of Figures 2, 3 and 4 to be constructed, each component having a value as determined by the respective foregoing equations.

When this is done, the relationships set forth in Equations 3, 4, 5, 11, 12, 15, 22 and 32 hold between the variables of the analogue system and the variables of the actual pumping system. The values of the assumed quantities are then changed until the analogue system behaves in a manner analogous to the actual pumping system. Thereupon, the effect of any changes desired in the parameters of the mechanical system may be accurately predicted by noting the effect of corresponding changes on the analogue system. This enables operating conditions to be determined so that the dimensions and physical conditions of the actual pumping sys-

tem are at an optimum to provide most efficient operation with resultant elimination of difficulties due to sucker rod breakage and malfunctioning of other parts of the apparatus.

Where all the necessary constants are not known in the actual system so that the corresponding parameters of the analogue system are correspondingly unknown, the known values are used in the manner previously set forth to determine the constants of the corresponding parts of the analogue circuit. Reasonable values are assumed for the other known constants and corresponding values are set in the simulator by use of the aforementioned equation.

The simulator and its operation are then studied and conclusions are drawn as to the extent to which the performance depends on the unknown constants for which values had been assumed. Usually it will be found that some of these are critical. Others may lie anywhere within a fairly wide range without appreciable effect on the performance of the simulator. Also, operation of the simulator suggests measurements to be taken on the prototype which either permit computation of those critical constants or serve as indices to the performance of the prototype.

Finally, I wish to point out that the driving unit 73 of Figure 2 can be the electrical driving mechanism simulator of my copending application, Serial No. 204,926, now Patent No. 2,661,898, a suitable mechano-electrical transducer being provided to convert the output of the drive mechanism simulator to a mechanical impulse for driving the end of the rod line.

While the invention has been described in connection with a present, preferred embodiment thereof, it is to be understood that this description is illustrative only and is not intended to limit the invention, the scope of which is defined by the appended claims.

I claim:

1. In apparatus for mechanically simulating a down-hole pumping system including a sucker rod string, tubing surrounding said sucker rod string and an oil column between said rod and said tube, in combination; a plurality of units, one for each segment of the pumping system, each unit including a shaft representing each of a tubing segment, a rod segment, and an oil column segment, each shaft having a flywheel coupled thereto whose moment of inertia represents the inertia of the segment and a drag device coupled thereto representing the weight of the segment, means coupling the oil shaft and the tubing shaft to simulate viscous drag between the tubing segment and the oil column segment, means coupling the oil shaft and the rod shaft to simulate viscous drag between the rod segment and the oil column segment, and a clutch attached to each segment of the oil shaft to prevent the oil shaft torque from becoming negative; means coupled to each of said shafts for simulating the driving mechanism and end conditions at the top of the pumping system; and means coupled to each of said shafts simulating the operation of the pump at the bottom of said system.
2. In apparatus for mechanically simulating a down-hole pumping system including a sucker rod string, tubing surrounding said sucker rod string and an oil column between said rod and said tube, in combination; a plurality of units, one for each segment of the pumping system, each unit including a shaft representing each of a tubing segment, a rod segment, and an oil column segment, each shaft having a flywheel coupled thereto whose moment of inertia represents the inertia of the segment and a drag device coupled thereto representing the weight of the segment, a differential coupling the oil shaft and the tubing shaft to simulate viscous drag between the tubing segment and the oil column segment, and a differential coupling the oil shaft and the rod shaft to simulate viscous drag between the rod segment and the oil column segment; means for simulating the driving mechanism at the top of the pumping system including a drive device coupled to said rod shaft for producing a driving force representative of the force of the driving mechanism at the top of the pumping system, a constant torque device connected to the oil shaft to simulate the pressure at which oil is discharged from the tubing, and means secured to the top of the tubing shaft to prevent rotation thereof; and means coupled to each of said shafts for simulating the operation of the pump at the bottom of said system.

3. In apparatus for mechanically simulating a downhole pumping system including a sucker rod string, tubing surrounding said sucker rod string and an oil column between said rod and said tube, in combination; a plurality of units, one for each segment of the pumping system, each unit including a shaft representing each of a tubing segment, a rod segment, and an oil column segment, each shaft having a flywheel coupled thereto whose moment of inertia represents the inertia of the segment and a drag device coupled thereto representing the weight of the segment, a differential coupling the oil shaft and the tubing shaft to simulate viscous drag between the tubing segment and the oil column segment, and a differential coupling the oil shaft and the rod shaft to simulate viscous drag between the rod segment and the oil column segment; a unit for simulating the driving mechanism at the top of the pumping system including a drive device coupled to said rod shaft for producing a driving force representative of the force of the driving mechanism at the top of the pumping system, a constant torque device connected to the oil shaft to simulate the pressure at which oil is discharged from the tubing, and means secured to the top of the tubing shaft to prevent rotation thereof; and a unit for simulating the operation of the pump at the bottom of said system, said last-mentioned unit including a plurality of clutches, a flywheel at the end of each of the rod, oil and tubing shafts, a differential, a constant drag device attached to each of the last-mentioned flywheels in the rod and tubing shafts, said clutches coupling the rod shaft directly to the oil shaft during the period representing the upstroke of the pump, and said clutches coupling one side of the differential to the rod shaft, the other side of the differential to the tubing shaft, and the driven differential element to the oil shaft during the period representing the upstroke of the pump.

4. In apparatus for simulating the downstroke of a downhole pump including a plunger, a traveling valve, and a stationary valve, in combination, a shaft mechanically simulating each of the oil column, tubing and sucker rod string; a unit for simulating the operation of the pump at the bottom of said system, said last-mentioned unit including a plurality of clutches, a flywheel attached to the end of each of the rod, oil and tubing shafts, a differential, a constant drag device connected to each of the last-mentioned flywheels in the rod and tubing shafts, said clutches coupling the rod shaft directly to the oil shaft during the period representing the upstroke of the pump, and said clutches coupling one side of the differential to the rod shaft, the other side of the differential to the tubing shaft, and the driven differential element to the oil shaft during the period representing the upstroke of the pump.

5. Apparatus for simulating a downhole pumping unit comprising a shaft simulating a column of oil pumped by the unit; means for applying a plurality of torques to said shaft at spaced intervals thereon, said intervals representing individual segments of the oil column, said torques simulating stress on the oil column at corresponding segments thereof whereby said torques tend to cause rotation of said shaft simulating displacement of corresponding segments of the oil column; and means to disengage any one of the torques applied to said shaft whenever the value of said torque is such as to tend to cause rotation of said shaft in a direction which represents tension in the oil column.

6. The combination in accordance with claim 5 wherein said means to disengage the torques from said shaft comprises a plurality of clutches coupling respective said means for applying torques to said shaft, said individual torques being connected to said shaft whenever the values thereof are such as to cause rotation of said shaft in a first direction and being disconnected from said shaft whenever the values thereof are such as to tend to cause rotation of said shaft in a second direction.

7. Apparatus for simulating a segment of a downhole pumping unit including a tubing segment, a rod segment, and an oil column segment comprising, in combination; first, second, and third shafts simulating, respectively, said tubing, rod, and oil column segments; first, second, and third flywheels attached, respectively, to said first, second, and third shafts, the inertia of said flywheels simulating the inertia of corresponding segments; first, second, and third constant drag means attached, respectively, to said first, second, and third shafts, said drag means simulating the

weights of corresponding segments; means coupling said rod shaft to said oil column shaft whereby rotation of said rod shaft imparts rotation to said oil column shaft simulating viscous drag between the rod segment and the oil column segment; and means coupling said oil column shaft to said tubing shaft whereby rotation of said oil column shaft imparts rotation to said tubing shaft simulating viscous drag between the oil column segment and the tubing segment.

8. The combination in accordance with claim 7 wherein said means coupling said rod shaft to said oil column shaft and said means coupling said oil column shaft to said tubing shaft each comprises a differential, the two driving members of said differential being connected, respectively, to said two shafts being coupled, and means coupled to the driven member of said differential to impart a drag thereto to simulate the viscous drag between corresponding segments in the pumping unit.

9. The combination in accordance with claim 7 further comprising means coupled to said tubing shaft to impart a viscous drag thereto tending to prevent rotation of said tubing shaft, said viscous drag representing the friction exerted by a stationary body of oil outside the tubing segment.

10. Apparatus for simulating a downhole pumping system including a sucker rod string, tubing surrounding said sucker rod string, an oil column between said rod and said tubing, drive mechanism positioned at the surface to impart reciprocating motion to the upper end of the sucker rod string, and a pump attached to the lower end of the sucker rod string to force said column of oil upward inside said tubing comprising, in combination; first, second, and third shafts simulating, respectively, said rod, tubing, and oil column; a plurality of first, second, and third flywheels attached, respectively, to each of said first, second, and third shafts, the inertia of said flywheels simulating the inertia of corresponding segments of said rod, tubing, and oil column; a plurality of first, second and third constant drag devices attached, respectively, to said first, second, and third shafts, said drag devices simulating the weight of corresponding segments of said rod, tubing, and oil column; means coupled to corresponding first ends of said rod, tubing, and oil shafts simulating the driving means and surface conditions of the pumping system, said last mentioned means imparting reciprocating rotary motion to said first end of said rod shaft; a plurality of means coupling said rod shaft to said oil column shaft whereby rotation of said rod shaft imparts rotation to said oil column shaft simulating viscous drag between respective rod segments and oil column segments; a plurality of means coupling said oil column shaft and said tubing shaft whereby rotation of said oil column shaft imparts rotation to said tubing shaft simulating viscous drag between respective oil column segments and tubing segments; and means coupled to corresponding second ends of said rod, tubing, and oil column shafts simulating the operation of the pump at the bottom of the system.

11. The combination in accordance with claim 10 further comprising means coupled to said tubing shaft to impart a viscous drag thereto tending to prevent rotation of said tubing shaft, said viscous drag representing the friction exerted by a stationary body of oil outside the tubing segment.

12. The combination in accordance with claim 10 wherein said means coupling the first ends of said shafts comprises a driving unit for imparting a reciprocating rotary motion to said first end of the rod shaft simulating the action of the driving unit attached to the uppermost end of the sucker rod string, means coupled to said first end of the oil column shaft to impart a constant torque thereto simulating the constant pressure at which the oil is discharged from the tubing, and means attached to said first end of the tubing shaft to prevent rotation thereof simulating rigid attachment of the tubing at the surface.

13. The combination in accordance with claim 10 wherein said means simulating operation of the pump positioned at the bottom of the system comprises a flywheel connected to said second ends of each of the rod, tubing, and oil column shafts; means coupled to each of said last mentioned flywheels attached to the rod and tubing shafts to impart a constant torque thereto; a differential; and a series of clutches, said clutches coupling

15

said rod shaft to said oil column shaft whenever the three shafts rotate in a first direction simulating the upstroke of the pump, and said clutches coupling one side of said differential to said rod shaft, the other side of said differential to said tubing shaft, and the movable differential element to said oil column shaft whenever said three shafts rotate in a second direction simulating the downstroke of the pump.

16

References Cited in the file of this patent

UNITED STATES PATENTS

	Number	Name	Date
5.	1,228,392	Beighlee	June 5, 1917
	1,662,272	Klopstig	Mar. 13, 1928
	1,799,134	Hardy	Mar. 31, 1931
	2,551,502	Montrose-Oster	May 1, 1951