

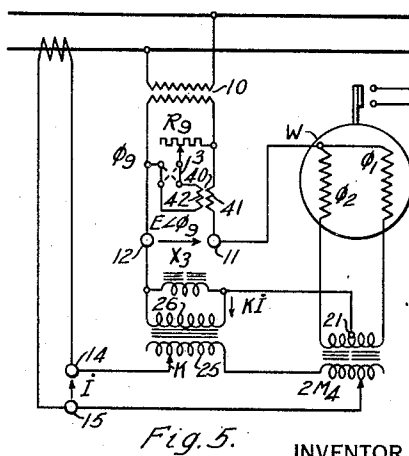
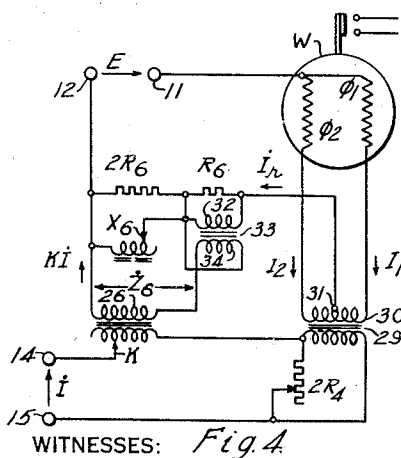
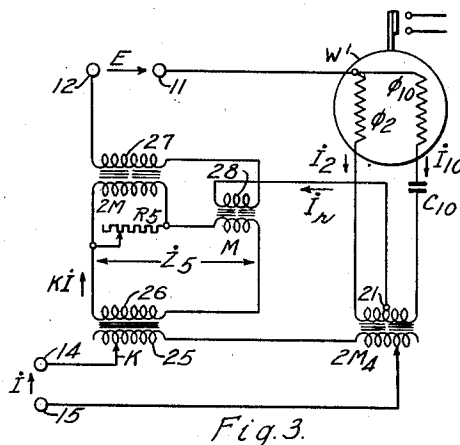
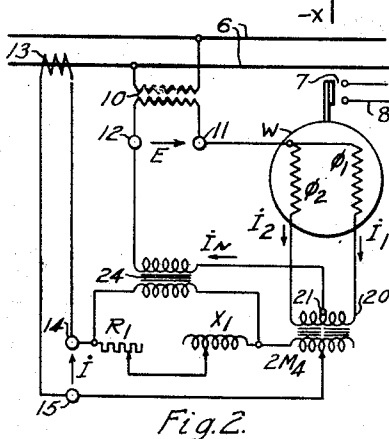
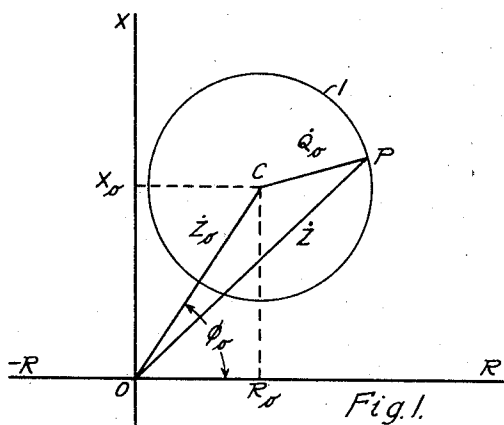
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B. E. LENEHAN

2,426,033

DISTANCE TYPE, PRODUCT RESPONSIVE RELAY.

Filed Feb. 16, 1945



H. F. Sussner.
New. C. Groome

Fig. 5. INVENTOR
Bernard E. Lenehan.
BY O. B. Buchanan
ATTORNEY

UNITED STATES PATENT OFFICE

2,426,033

DISTANCE TYPE PRODUCT RESPONSIVE
RELAYBernard E. Lenehan, Bloomfield, N. J., assignor to
Westinghouse Electric Corporation, East Pitts-
burgh, Pa., a corporation of Pennsylvania

Application February 16, 1945, Serial No. 578,161

9 Claims. (Cl. 175-294)

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My present invention relates to a new protec-
tive relay for alternating-current lines, the prin-
cipal novel characteristic feature of the relay
being that it has a distance-responsive charac-
teristic, or a modified impedance characteristic
which can be readily adjusted. My invention
relates more particularly to a novelly energized
wattmetric or product-responsive element which
has readily adjustable characteristics so that its
impedance-circle can be separately adjusted as to
center-placement and circle-radius.

An object of my invention is to energize the
two windings of a wattmetric relay with a cur-
rent-compensated voltage, with such adjustments
as to be able to completely adjust the character-
istic circle-response of the relay. The current-
compensated line-voltage is equal to the vectorial
sum of an alternating-current function of a line-
derived voltage, plus the product of an alternat-
ing-current function of a line-derived current
times an impedance. Throughout the specifica-
tion, when I refer to a wattmetric element or
relay, I mean any product-responsive element
which develops a torque equal to the product of
its two energized quantities times a function of
the angle between them, whether that angle-
function is a sine or a cosine or any intermediate
function.

In the accompanying drawing, Figure 1 is a
characteristic response-circle diagram which will
be referred to in the explanation of my invention,
and Figs. 2, 3, 4 and 5 are diagrammatic views
of circuits and apparatus illustrating my inven-
tion in four different forms of embodiment.

If a cosine-responsive product-type relay is
energized with two complex quantities, which
may be two diverse alternating magnetic fluxes

$$\phi_1 = E - \dot{a}\dot{I} \quad (1)$$

and

$$\phi_2 = \dot{c}E - \dot{b}\dot{I} \quad (2)$$

it will produce a torque which is the absolute
value of the algebraic sum of the products of the
respective terms, or

$$T = [\dot{c}E^2 - E\dot{I}(\dot{a}\dot{c} + \dot{b}) + \dot{a}\dot{b}\dot{I}^2] \quad (3)$$

If E is the line-voltage and $\dot{I}$ is the line-current
of an alternating-current line, the line-imped-
ance will be

$$\dot{Z} = \frac{E}{\dot{I}} \quad (4)$$

At the balance-point of the relay, the torque T
will be zero. Dividing Equation 3 by $\dot{I}^2$, and put-

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ting the equation equal to zero, the conditions
for the balance-point of the relay will be found
to be

$$0 = \dot{c}Z^2 - \dot{Z}(\dot{a}\dot{c} + \dot{b}) + \dot{a}\dot{b} \quad (5)$$

This is the equation of a circle, as will be seen
from Fig. 1, where the circle 1 represents any
circle, plotted on rectangular coordinates in terms
of the line-resistance R and the line-reactance
 X , with the center C of the circle at the imped-
ance $\overline{OC} = \dot{Z}_0$, and with a circle-radius $\overline{CP} = \dot{Q}_0$,
where P is any point in the circle. Then the
locus of the line-impedance $\overline{OP} = \dot{Z}$, or the locus
of all values of the line-impedance which termi-
nate in the circle, is the locus of the vector-sum

$$\dot{Z}_0 + \dot{Q}_0 = \dot{Z} \quad (6)$$

whence

$$(\dot{Z} - \dot{Z}_0)^2 = \dot{Q}_0^2 \quad (7)$$

whence

$$Z^2 - 2\dot{Z}\dot{Z}_0 + (Z_0^2 - Q_0^2) = 0 \quad (8)$$

Comparing Equations 5 and 8 and equating
equal powers of $\dot{Z}$,

$$\dot{c}Z^2 = Z^2; \dot{c} = 1 \quad (9)$$

$$-\dot{Z}(\dot{a}\dot{c} + \dot{b}) = -2\dot{Z}\dot{Z}_0; \dot{Z}_0 = \frac{\dot{a} + \dot{b}}{2} \quad (10)$$

$$\dot{a}\dot{b} = Z_0^2 - Q_0^2 = \frac{a^2 + 2\dot{a}\dot{b} + b^2}{4} - Q_0^2; Q_0 = \frac{1}{2}[\dot{a} - \dot{b}] \quad (11)$$

Equations 1 to 11 show that a cosine relay of
the wattmetric or product-responsive type, ener-
gized with $(E - \dot{a}\dot{I})$ and $(E - \dot{b}\dot{I})$, will have a cen-
ter-impedance $\dot{Z}_0$ or center-location at the aver-
age of the current-response coefficients $\dot{a}$ and $\dot{b}$,
and a diameter $2Q_0$ equal to the difference $(\dot{a} - \dot{b})$.

If we put

$$\dot{a} = R_a + jX_a = R_a + X_a \angle 90 \quad (12)$$

$$\dot{b} = R_b + jX_b = R_b + X_b \angle 90 \quad (13)$$

$$\dot{I} = I \angle (-\theta) = Ie^{-j\theta} = I \cos \theta - jI \sin \theta \quad (14)$$

$$\dot{Z} = R + jX = \frac{E}{\dot{I}} = \frac{E}{I} \angle \theta = Z \cos \theta + jZ \sin \theta \quad (15)$$

$$R = Z \cos \theta \quad (16)$$

and

$$X = Z \sin \theta \quad (17)$$

where θ is the angle by which the line-current $\dot{I}$
lags the line-voltage E , R is the line-resistance

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up to the fault, and X is the inductive line-reactance up to the fault, then

$$\phi_1 = E - R_a I \angle (-\theta) - X_a I \angle (90 - \theta) \quad (18)$$

$$\phi_2 = E - R_b I \angle (-\theta) - X_b I \angle (90 - \theta) \quad (19)$$

$$\frac{T}{I^2} = 0 = R^2 + X^2 - R(R_a + R_b) - X(X_a + X_b) + R_a R_b + X_a X_b \quad (20)$$

$$\begin{aligned} & \left(R - \frac{R_a + R_b}{2} \right)^2 + \left(X - \frac{X_a + X_b}{2} \right)^2 \\ &= \frac{(R_a + R_b)^2 + (X_a + X_b)^2 - 4R_a R_b - 4X_a X_b}{4} \\ &= \frac{(R_a - R_b)^2 + (X_a - X_b)^2}{4} \end{aligned} \quad (21)$$

This is the equation for a circle with a radius

$$Q_0 = \frac{\sqrt{(R_a - R_b)^2 + (X_a - X_b)^2}}{2} \quad (22)$$

and with a center at

$$R_0 = \frac{R_a + R_b}{2} \quad (23)$$

$$X_0 = \frac{X_a + X_b}{2} \quad (24)$$

The slope of the circle-center impedance $\dot{Z}_0$ is

$$\phi_0 = \tan^{-1} \frac{X_0}{R_0} = \tan^{-1} \frac{X_a + X_b}{R_a + R_b} \quad (25)$$

The displacement Z_0 of the circle-center from the origin is

$$Z_0 = \frac{\sqrt{(R_a + R_b)^2 + (X_a + X_b)^2}}{2} \quad (26)$$

A cosine relay, responding to negative values of the torque T , and energized as set forth in Equations 1 to 25, will respond for all values of the line-impedance $\dot{Z}$ falling inside of the circle 1, and will hold back against its back-stop for all values of the line-impedance $\dot{Z}$ falling outside of the circle.

If the angle of maximum response of the relay has any value ϕ_r other than zero, so that response to the two energizing-quantities ϕ_1 and ϕ_2 is a maximum when ϕ_1 lags ϕ_2 by an angle ϕ_r , then if the lagging angle of ϕ_1 with respect to ϕ_2 is ϕ rather than ϕ_r the relay-torque will be

$$T = \phi_1 \phi_2 \cos (\phi - \phi_r) \quad (27)$$

However, the Torque-Equations 3 and 20 are valid only for a cosine-relay, or a relay having a critical angle $\phi_r = 0$. In order to make Equations 3 and 20 valid for a relay having a critical angle ϕ_r , it will be necessary to replace the energizing-quantity ϕ_1 of Equations 1 and 18 with another quantity ϕ_{10} which leads ϕ_1 by the response-angle ϕ_r of the relay, as shown by the equation

$$\phi_{10} = \phi_1 \angle \phi_r \quad (28)$$

For example, if the response-angle of the relay is $\phi_r = 90^\circ$, the relay will be a sine-responsive relay, responding to the product of its exciting quantities multiplied by the sine of the phase-angle ϕ between them, as shown by the torque-equation,

$$T = \phi_{10} \phi_2 \cos (\phi - 90) = \phi_{10} \phi_2 \sin \phi \quad (29)$$

Equation 21 shows that we are interested only in the sum and difference of R_a and R_b , and in

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the sum and difference of X_a and X_b , and that the difference-quantities $(R_a - R_b)$ and $(X_a - X_b)$ affect only the circle-radius Q_0 , and not the center-position (R_0, X_0) . It will sometimes be convenient, therefore, to make either $(R_a - R_b)$ or $(X_a - X_b)$ equal to zero.

Fig. 2 shows a form of embodiment of my invention in which

$$R_a = R_b = R_1 \quad (30)$$

$$X_a = X_1 + M_4 \quad (31)$$

and

$$X_b = X_1 - M_4 \quad (32)$$

Then the circle-radius will be

$$Q_0 = M_4 \quad (33)$$

the coordinates of the center will be at

$$R_0 = R_1 \quad (34)$$

$$X_0 = X_1 \quad (35)$$

the slope of the center-line will be

$$\phi_0 = \tan^{-1} \frac{X_1}{R_1} \quad (36)$$

and the center-displacement will be

$$Z_0 = \sqrt{R_1^2 + X_1^2} = Z_1 \quad (37)$$

Still more generically, we know that we are interested only in the vector value of $(\dot{a} + \dot{b})$ and the absolute value of $[\dot{a} - \dot{b}]$, as shown in Equations 10 and 11. We may write, therefore, the general equations

$$\dot{a} = \dot{Z}_3 + \dot{Z}_4 \quad (38)$$

$$\dot{b} = \dot{Z}_3 - \dot{Z}_4 \quad (39)$$

where $\dot{Z}_3$ and $\dot{Z}_4$ are any impedances. These relations yield a circle-center impedance $\dot{Z}_0$ expressed by

$$\dot{Z}_0 = \frac{\dot{a} + \dot{b}}{2} = \dot{Z}_3 \quad (40)$$

and a circle-radius Q_0 expressed by the absolute value of Z_4 as shown by the equation

$$Q_0 = \frac{1}{2} [\dot{a} - \dot{b}] = Z_4 \quad (41)$$

The absolute value of the impedance-component Z_3 in Equation 40 expresses the ohms (or distance) displacement Z_0 of the circle-center C from the origin, while the phase-angle of the impedance-component $\dot{Z}_3$ expresses the slope ϕ_0 of the circle-center line $\dot{Z}_0$.

Equation 41 shows that the phase-angle of the impedance-component $\dot{Z}_4$ is quite immaterial.

This impedance-component $\dot{Z}_4$ may be pure resistance, pure reactance, or any combination of the two.

There are many forms in which these principles may be embodied.

Fig. 3 shows a form of embodiment of my invention in which the center-placement component $\dot{Z}_3$ is separated into two factors, a variable scalar quantity or absolute value Z_3 , and a variable phase-angle ϕ_3 , and the illustration is intended to be symbolic of any means to that end. The angle-changing element is shown in the form of a compensator-impedance $\dot{Z}_5$ which is variable in phase-angle without changing its magnitude; and the displacement-changing element is shown in the form of a variable ratio or factor K , which affects the magnitude of the current $K\dot{I}$ which is

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circulated through the compensator-impedance $\dot{Z}_5$ from the source of line-derived current $\dot{I}$.

In Fig. 3, therefore,

$$\dot{a} = K\dot{Z}_5 + jM_4 \quad (42)$$

$$\dot{b} = K\dot{Z}_5 - jM_4 \quad (43)$$

$$\begin{aligned} \dot{Z}_5 &= \frac{j2MR_5}{R_5 + j2M} - jM = \frac{j2MR_5 - jMR_5 + 2M^2}{R_5 + j2M} \\ &= jM \frac{(R_5 - j2M)^2}{R_5^2 + 4M^2} \\ &= M \angle (90^\circ - 2 \tan^{-1} \frac{2M}{R_5}) \quad (44) \end{aligned}$$

Hence

$$\dot{a} = KM \angle (90^\circ - 2 \tan^{-1} \frac{2M}{R_5}) + M_4 \angle 90^\circ \quad (45)$$

$$\dot{b} = KM \angle (90^\circ - 2 \tan^{-1} \frac{2M}{R_5}) - M_4 \angle 90^\circ \quad (46)$$

$$\dot{Z}_0 = KM \angle (90^\circ - 2 \tan^{-1} \frac{2M}{R_5}) \quad (47)$$

$$Z_0 = KM \quad (48)$$

$$\phi_0 = 90^\circ - 2 \tan^{-1} \frac{2M}{R_5} \quad (49)$$

$$Q_0 = M_4 \quad (50)$$

The particular form of impedance $\dot{Z}_5$ in Fig. 3 is convenient for changing the center-line angle ϕ_0 of the response-circle between an upper limit of 90° , or coincidence with the +X axis, corresponding to $R_5 = \infty$ or open-circuited, or an upper limit of 85° if the maximum value of R_5 is $(45.8)M$, and a lower limit dependent upon the minimum practicable limit of R_5 commensurate with an acceptable burden on the source of line-derived current $\dot{I}$, or a lower limit of 50° if the minimum value of R_5 is $(4.3)M$, for example.

In Fig. 3, I have also illustrated the application of my invention to a sine-type relay W' having two force-reacting elements ϕ_{10} and ϕ_2 , and producing a force proportional to $\phi_{10}\phi_2 \sin \phi$, as discussed in connection with Equations 26 to 29. In order to make my invention applicable to such a relay, it is necessary to make the exciting-current in ϕ_{10} lead the exciting-current in ϕ_2 by 90° when the same voltage is impressed on both ϕ_{10} and ϕ_2 . This may be approximated by including a capacitor C_{10} in series with the winding ϕ_{10} and properly choosing the relative numbers of turns in ϕ_{10} and ϕ_2 to produce substantially the same flux in each, when the impressed voltage is the same.

In general, if the characteristic phase-angle of the relay is ϕ_r so that its maximum torque is produced when the angle ϕ between its two cooperating fluxes ϕ_1 and ϕ_2 is ϕ_r , then the relay-torque T is $\phi_1\phi_2 \cos (\phi - \phi_r)$, and it is necessary to cause the voltage-responsive excitation-components of the two fluxes to be dephased by ϕ_r , and to cause the corresponding current-responsive excitation-components of the two fluxes to be likewise dephased by ϕ_r . In Fig. 3, a sine-responsive relay W' is presupposed, and hence the angle ϕ_r is 90° . In the other figures, a cosine-responsive relay W is presupposed, and hence the characteristic relay-angle ϕ_r is zero.

In Fig. 4, I have shown a further form of the phase-changing means, in the form of an imped-

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ance $\dot{Z}_6$ which has different practical limits of angle-variation.

In Fig. 4,

$$\begin{aligned} 5 \quad \dot{Z}_6 &= \frac{2jX_6R_6}{2R_6 + jX_6} - R_6 = -R_6 \frac{(2R_6 - jX_6)^2}{4R_6^2 + X_6^2} \\ &= R_6 \angle \left(-180^\circ - 2 \tan^{-1} \frac{X_6}{2R_6} \right) \quad (51) \end{aligned}$$

$$10 \quad \dot{a} = K\dot{Z}_6 + R_4 \quad (52)$$

$$\dot{b} = K\dot{Z}_6 - R_4 \quad (53)$$

$$Z_0 = KR_6 \quad (54)$$

$$15 \quad \phi_0 = -180^\circ - 2 \tan^{-1} \frac{X_6}{2R_6} \quad (55)$$

$$Q_0 = R_4 \quad (56)$$

Thus, in Fig. 4, if X_6 is varied and R_6 held constant, the circle-center angle ϕ_0 may be changed, for example, from 120° , which is -240° , corresponding to $X_6 = (1.15)R_6$, to 60° , or -300° , corresponding to $X_6 = (3.46)R_6$.

In all of the foregoing derivations, it has been assumed that the relay-current, $\dot{I}_r = \dot{I}_1 + \dot{I}_2$, or $\dot{I}_r = \dot{I}_{10} + \dot{I}_2$, is negligibly small with respect to the compensator-current $\dot{I}$, within acceptable limits of error.

It will further be noted, from Equation 10, that the center-line angle ϕ_0 of the response-circle 1 of the relay is the same as the phase-angle of the average value, $\dot{Z}_0 = (\dot{a} + \dot{b})/2$, of the coefficients $\dot{a}$ and $\dot{b}$ of the line-current response of the relay. Furthermore, the voltage-responsive coefficient $\dot{c}$ in Equation 2 must be unity, as shown in Equation 9. Taking into consideration, also, the relations shown in Equations 41 and 47 to 49, we may write the quantities to be multiplied, thus:

$$40 \quad \phi_1 = E - (K\dot{Z}_6 + \dot{Z}_7) \dot{I} = E - K_6 Z_6 \dot{I} \angle \phi_6 - \dot{Z}_7 \dot{I} \quad (57)$$

$$\phi_2 = E - (K\dot{Z}_6 - \dot{Z}_7) \dot{I} = E - K_6 Z_6 \dot{I} \angle \phi_6 + \dot{Z}_7 \dot{I} \quad (58)$$

Then the circle-center displacement will be

$$45 \quad Z_0 = KZ_6 \quad (59)$$

and the circle-center angle will be

$$\phi_0 = \angle \phi_6 \quad (60)$$

and the circle-radius will be

$$50 \quad Q_0 = Z_7 \quad (61)$$

or the absolute value of $\dot{Z}_7$, regardless of its phase-angle.

Since the phase-angle of $\dot{Z}_7$ is immaterial, the impedance-component $\dot{Z}_7$ may be replaced by

$$\dot{Z}_7 = \dot{Z}_8 \angle \phi_6 \quad (62)$$

yielding, from Equations 57 and 58,

$$60 \quad \phi_1 = E - (KZ_6 + \dot{Z}_8) \dot{I} \angle \phi_6 \quad (63)$$

$$\phi_2 = E - (KZ_6 - \dot{Z}_8) \dot{I} \angle \phi_6 \quad (64)$$

Multiplying both E and $\dot{I}$ by the same phase-angle $(-\phi_6)$ will not change the torque, as such an operation would be equivalent to introducing a phase-shifting impedance into the current-response and the voltage-response, or equivalent to choosing a point in the sine-wave in which the instantaneous value of the voltage is

$$70 \quad e = \sin (wt - \phi_6)$$

instead of $e = \sin wt$. In other words, the angle ϕ_6 in Equations 63 and 64 is in effect the relative angle between the current-responsive excitation-component and the voltage-responsive excitation-

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component at unity power-factor. Hence we may write

$$\phi_1 = E \angle (-\phi_e) - (KZ_6 + \dot{Z}_8) \dot{I} \angle (\phi_6 - \phi_e) \quad (65)$$

$$\phi_2 = E \angle (-\phi_e) - (KZ_6 - \dot{Z}_8) \dot{I} \angle (\phi_6 - \phi_e) \quad (66)$$

It will be seen, therefore, that the circle-center angle ϕ_0 is the difference between the phase-angle $(\phi_6 - \phi_e)$ of the current-responsive coefficient, and the phase-angle $(-\phi_e)$ of the voltage-responsive coefficient, at unity power-factor. Thus, instead of rotating the current-responsive terms through an angle ϕ_6 to obtain a center-line angle $\phi_0 = \phi_6$, it would be possible to leave the phase-angle of the current-responsive term equal to zero, by putting $\phi_6 = \phi_e$, and to obtain the desired center-line angle $\phi_0 = -\phi_e$ by rotating the response of the relay to the line-voltage E through a leading angle $(-\phi_e)$ while keeping the current-responsive terms in phase with the line-current $\dot{I}$. Or both the current and voltage responses may be rotated, yielding $\phi_0 = (\phi_6 - \phi_e)$.

Many phase-shifting networks are known.

In Fig. 5, I have shown a voltage-shifting network ϕ_9 of a type which is described and claimed in an application of H. J. Carlin, Serial No. 583,926, filed March 21, 1945, and assigned to the Westinghouse Electric Corporation. By a decrease of a secondary resistance R_9 from infinity to a certain large value, with either position of the reversing-switch 3, the voltage E may be shifted through any angle ϕ_9 up to 30° , in either direction, without materially changing its magnitude, within acceptable limits of error.

In Fig. 5, the circuit is such that

$$\phi_1 = E \angle (\pm \phi_9) - jX_3 \dot{I} - jM_4 \dot{I} \quad (67)$$

$$\phi_2 = E \angle (\pm \phi_9) - jX_3 \dot{I} + jM_4 \dot{I} \quad (68)$$

$$\dot{Z}_0 = X_3 \angle (90^\circ \mp \phi_9) \quad (69)$$

$$Z_0 = X_3 \quad (70)$$

$$\phi_0 = 90^\circ \mp \phi_9 \quad (71)$$

$$Q_0 = M_4 \quad (72)$$

It is believed that the various circuits and parts shown in Figs. 2 to 5 will be clear from the foregoing description. These figures may be briefly described as follows:

In Fig. 2, one phase of an alternating-current line is shown at 6, and a cosine-responsive wattmetric relay is shown at W, having two voltage-type windings ϕ_1 and ϕ_2 , and having a relay-contact 7 which closes when the relay responds. The details of the relay circuit 8 which is controlled by the contact 7 are not indicated, as my invention relates to the means for obtaining the contact-closing response at 7, rather than the use which is made of that response in the controlled relaying circuits 8.

In Fig. 2, a potential transformer 10 serves as a source of the line-voltage E , which is applied across the terminals 11 and 12 of the relay. A line-current transformer 13 serves as a source of line-current $\dot{I}$, which is applied across the relay-terminals 14 and 15. The relaying current $\dot{I}$ is circulated through a circuit including the compensator-resistance R_1 , the compensator inductance X_1 , and the primary winding of a mutual reactor $2M_4$, all of these values being adjustable. The mutual reactor $2M_4$ is provided with a secondary winding 20 having a mid-tap 21.

In Fig. 2, the current-responsive voltage-drop which appears across the resistor R_1 and the reactor X_1 is added to the line-derived voltage E , either directly, or through a suitable coupling transformer 24 having any appropriate turn-

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ratio. The relay-terminal 11 is connected to one end of each of the relay-windings ϕ_1 and ϕ_2 . The other ends of the windings ϕ_1 and ϕ_2 are connected to the terminals of the secondary winding 20 of the mutual impedance element $2M_4$. The mid-tap 21 of this secondary winding 20 is connected to the other voltage-terminal 12 of the relay through the secondary winding of the compensator-transformer 24. The response of the apparatus shown in Fig. 2 has already been discussed, and need not be further reviewed.

In Fig. 3, a sine-responsive wattmetric element W' is utilized, as already discussed, necessitating the inclusion of the capacitor C_{10} in series with the relay-winding ϕ_{10} , as already explained. The current-responsive circuit, which is connected to the relay-terminals 14 and 15 in Fig. 3, includes the primary winding 25 of a variable-ratio transformer K, as well as the primary winding of the previously described mutual reactor $2M_4$. The secondary winding of the transformer K circulates the current $K\dot{I}$ through the impedance $\dot{Z}_5$, which consists of a variable resistance R_5 in series with the primary winding of a fixed mutual reactance M. The resistance R_5 is shunted by the primary winding of a fixed mutual reactance $2M$. The two mutual reactors $2M$ and M have secondary windings 27 and 28 which are serially connected, in opposite polarities, between the voltage-terminal 12 and the mid-point 21. The characteristics of this circuit, as shown in Fig. 3, have already been discussed.

In Fig. 4, the same variable transformer K is utilized, but the mutual reactance $2M_4$ is replaced by a variable resistance $2R_4$, the voltage of which is applied to the relay-windings ϕ_1 and ϕ_2 through a transformer 29 having a secondary winding 30 which is provided with a mid-tap 31. The mid-tap 31 is connected to the voltage-terminal 12 through two resistors R_6 and $2R_6$. The resistor R_6 is shunted by the secondary winding 32 of a transformer 33, which also has a primary winding 34. The resistor $2R_6$ is shunted by a variable inductance X_6 . The secondary winding 26 of the variable transformer K supplies the current $K\dot{I}$ to the parallel-circuit combination $2R_6$ and X_6 , in series with the transformer-winding 34, the latter being connected in reversed polarity so as to send the current $K\dot{I}$ in the reversed direction through the resistor R_6 . These circuits of Fig. 4 have also been analyzed in the preceding mathematical discussion, and need no further comment.

In Fig. 5, the phase-changing network ϕ_9 is connected between the potential transformer 10 and the voltage-terminals 11 and 12, so as to supply the voltage $E \angle \phi_9$ to the voltage-terminals 11 and 12 of the relay. The phase-changing network ϕ_9 consists of a transformer 40 having a primary winding 41 which is connected in series circuit relation between the potential transformer 10 and the relay-terminal 11. The transformer 40 also has a secondary winding 42 which is connected, through the reversing switch 3, between the relay-terminal 12 and the variable resistor R_9 . The other terminal of the variable resistor R_9 is connected to the same potential-transformer terminal which is connected to the primary winding 41.

The current-responsive circuit 14-15 in Fig. 5 contains the primary winding 25 of the variable transformer K, and the primary winding of the mutual reactor $2M_4$, the same as in Fig. 3. The

secondary winding 26 of the variable transformer K circulates the current Ki through the inductance X_3 which is connected between the voltage-terminal 12 and the mid-tap 21. The effect of the circuit-connection shown in Fig. 5 has already been discussed and needs no further explanation.

In all of the embodiments of my invention, it will be observed that I have provided means for independently adjusting the radius Q_0 and the position of the center C of the response-circle 1. The center-position C can be fixed either by separately adjusting its coordinates R_0 and X_0 , or by separately adjusting its center-line slope ϕ_0 and its displacement Z_0 .

While I have illustrated and explained my invention in connection with four different forms of embodiment, I wish it to be understood that there are many different forms of embodiment, as many variations may be made, without departing from the essential features of my invention. I desire, therefore, that the appended claims shall be accorded the broadest construction consistent with their language and the prior art.

I claim as my invention:

1. An adjustable distance-type product-responsive relay having two diverse cooperating alternating magnetic fluxes, means for so utilizing said fluxes as to produce a torque in response to the product of said fluxes, multiplied by $\cos(\phi - \phi_r)$ where ϕ is the angle between said two cooperating fluxes and ϕ_r is the characteristic phase-angle of the relay, and two diverse flux-producing means for producing the respective fluxes, characterized by each of said flux-producing means comprising means for producing a voltage-responsive excitation-component in response to a line-derived voltage, and means for producing two current-responsive excitation-components in response to a line-derived current, said two current-responsive excitation-components being added, one to the other, in one flux, and subtracted, one from the other, in the other flux, a first adjustment-means for jointly varying a first one of the two current-responsive excitation-components of both fluxes, and a second adjustment-means for jointly varying the second one of the two current-responsive excitation-components of both fluxes, said two flux-producing means including circuit-connection means for causing a phase-angle of substantially ϕ_r to exist between each of the corresponding excitation-components of the two cooperating fluxes.

2. An adjustable distance-type product-responsive relay having two diverse cooperating alternating magnetic fluxes, means for so utilizing said fluxes as to produce a torque in response to the product of said fluxes, multiplied by $\cos(\phi - \phi_r)$ where ϕ is the angle between said two cooperating fluxes, and ϕ_r is the characteristic phase-angle of the relay, and two diverse flux-producing means for producing the respective fluxes, characterized by each of said flux-producing means comprising means for producing a voltage-responsive excitation-component in response to a line-derived voltage, and means for producing two current-responsive excitation-components which are out of phase with each other in response to a line-derived current, and means for separately adjusting said current-responsive excitation-components, said two flux-producing means including circuit-connection means for causing a phase-angle of substantially

ϕ_r to exist between each of the corresponding excitation-components of the two cooperating fluxes.

3. The invention as defined in claim 1, characterized by said first adjustment-means being a compensator-means comprising a variable resistance and a variable reactance traversed by a line-derived current.

4. The invention as defined in claim 1, characterized by said first adjustment-means comprising means for varying the magnitude, without substantially varying the phase-angle, of said first one of the two current-responsive excitation-components of both fluxes, in combination with an adjustable phase-shifter means for varying the relative phase-angle between the voltage-responsive excitation-component and the first current-responsive excitation-component in each flux without substantially varying their magnitudes.

5. An adjustable distance-type product-responsive relay having two diverse cooperating alternating magnetic fluxes, means for so utilizing said fluxes as to produce a torque in response to the product of said fluxes, multiplied by $\cos(\phi - \phi_r)$, where ϕ is the angle between said two cooperating fluxes, and ϕ_r is the characteristic phase-angle of the relay, and two diverse flux-producing means for producing the respective fluxes, characterized by each of said flux-producing means comprising means for producing a voltage-responsive excitation-component in response to a line-derived voltage, and means for producing a current-responsive excitation-component in response to a line-derived current, a first adjustment-means for causing substantially identical quantities to be added to the two current-responsive excitation-components of the two fluxes whereby the sum of the two current-responses of the two fluxes is varied without substantially changing their difference, and a second adjustment-means for causing substantially identical quantities to be added, in one case, and subtracted, in the other case, from the respective current-responsive excitation-components of the two fluxes whereby the magnitude of the difference of the two current-responses of the two fluxes is varied without substantially changing their sum, said two flux-producing means including circuit-connection means for causing a phase-angle of substantially ϕ_r to exist between each of the corresponding excitation-components of the two cooperating fluxes.

6. The invention as defined in claim 5, characterized by said first adjustment-means being a compensator-means comprising a variable resistance and a variable reactance traversed by a line-derived current.

7. An adjustable distance-type product-responsive relay having two diverse cooperating alternating magnetic fluxes, means for so utilizing said fluxes as to produce a torque in response to the product of said fluxes, multiplied by $\cos(\phi - \phi_r)$, where ϕ is the angle between said two cooperating fluxes, and ϕ_r is the characteristic phase-angle of the relay, a first flux-producing means for producing a first one of said cooperating fluxes in response to the vectorial sum of a voltage-responsive excitation-component in response to a line-derived voltage, plus a first current-responsive excitation-component in response to a line-derived current, plus a second current-responsive excitation-component in response to a line-derived current, a second flux-producing means for producing the second one of said co-

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operating fluxes in response to the vectorial sum of the same voltage-responsive excitation-component, plus the same first current-responsive excitation-component, minus the second current-responsive excitation-component, and separate adjustment-means for separately varying the first and second current-responsive excitation-components.

8. The invention as defined in claim 7, characterized by the means for adjusting said first current-responsive excitation-component being a compensator-means comprising a variable resistance and a variable reactance traversed by a line-derived current.

9. An adjustable distance-type product-responsive relay having two diverse cooperating alternating magnetic fluxes, means for so utilizing said fluxes as to produce a torque in response to the product of said fluxes, multiplied by $\cos(\phi - \phi_r)$, where ϕ is the angle between said two cooperating fluxes, and ϕ_r is the characteristic phase-angle of the relay, a first flux-producing means for producing a first one of said cooperating fluxes in response to the vectorial sum of a voltage-responsive excitation-component in response to a line-derived voltage, plus a first current-responsive excitation-component in response to a line-derived current, plus a second current-responsive excitation-component in

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response to a line-derived current, a second flux-producing means for producing the second one of said cooperating fluxes in response to the vectorial sum of the same voltage-responsive excitation-component, plus the same first current-responsive excitation-component, minus the second current-responsive excitation-component, a first adjustment-means for varying the relative phase-angle between the voltage-responsive excitation-component and the first current-responsive excitation-component without substantially varying their magnitudes, a second adjustment-means for varying the magnitude of the first current-responsive excitation-component without substantially varying its phase-angle, and a third adjustment-means for varying the magnitude of the second current-responsive excitation-component.

BERNARD E. LENEHAN.

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