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APPLICATION FOR A STANDARD PATENT

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Melbourne

hereby apply for the grant of a standard patent for an invention entitled:

RELATIVE POSITION NAVIGATION SYSTEM WITH MEANS FOR COMPUTING SYNTHETIC AZIMUTH

which is described in the accompanying complete specification

Details of basic application(s):

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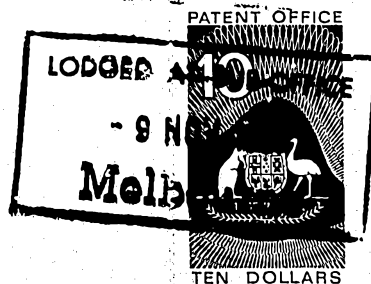
APPLICATION ACCEPTED AND AMENDMENTS

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**RELATIVE POSITION NAVIGATION SYSTEM WITH MEANS FOR COMPUTING SYNTHETIC AZIMUTH**
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- (57) Claim

1. A relative position navigation system installed in a first participant for computing the relative position of at least one participant, the navigation system comprising:

communication means, for receiving telemetry information from the other participant, including range means for determining the range, or distance, between the first participant and any other selected participant at at least three selected points in time,  $t_0$ ,  $t_1$ ,  $t_2$ ; and processor means having a plurality of computational functions;

a first function of said processor means being to compute the relative velocity vector for the first participant and the other participant for each of times  $t_0$ ,  $t_1$  and  $t_2$ ;

storage means for storing the ranges and relative velocity vectors at times,  $t_0$ ,  $t_1$  and  $t_2$ , over which time intervals, defined as  $(t_1 - t_0)$  and  $(t_2 - t_1)$ , one of the relative velocity vectors represents a change in direction from the other relative

velocity vector;

a second function of said processor means being to compute the azimuth, or, equivalently, the relative position in east-north coordinates, of the other participant relative to the first participant as a function of the range of the other participant from the first participant at each of the three selected times and of the change in position of the first participant relative to the other selected participant during the time intervals  $t_0$  to  $t_1$  and  $t_1$  to  $t_2$ ; and

a third function of said processor means being to compute the confidence level for the azimuth as a function of the statistical variances of the range means and the telemetered velocity data.

8. A method of finding the relative position between a first participant and at least one other participant at a selected point in time comprising the steps:

a) determining the range, or distance between the first participant and the other participant and receiving the velocity vector of the the participant from the other participant at each of at least three points in time,  $t_0$ ,  $t_1$ ,  $t_2$ , that define two time intervals, ( $t_0$  to  $t_1$ ) and ( $t_1$  to  $t_2$ ), during at least one of which intervals the direction of motion of the first participant relative to the other participant changes:

b) determining the change in position of the first participant relative to the other selected participant for time intervals ( $t_0$  to  $t_1$ ) and ( $t_1$  to  $t_2$ );

c) computing the non-ambiguous solution for the azimuth coordinate at time  $t_2$  as a function of the range between the first and other participant for each of the three selected points in item,  $t_0$ ,  $t_1$ ,  $t_2$  and the change in position of the first participant relative to the other participant during each of the time intervals ( $t_0$  to  $t_1$ )

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and ( $t_1$  to  $t_2$ ); and

d) computing the confidence level for the azimuth as a function of the statistical variances of the range-determining system and the velocity vector data.

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COMPLETE SPECIFICATION

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TO BE COMPLETED BY APPLICANT

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Complete Specification for the invention entitled:  
RELATIVE POSITION NAVIGATION SYSTEM WITH  
MEANS FOR COMPUTING SYNTHETIC AZIMUTH

The following statement is a full description of this invention  
including the best method of performing it known to me:-

RELATIVE POSITION NAVIGATION SYSTEM  
WITH MEANS FOR COMPUTING  
SYNTHETIC AZIMUTH

FIELD OF THE INVENTION

The present invention relates generally to navigation systems and more particularly to a relative position air navigation system having two or more participants and including means for computing the azimuth between any two participants.

BACKGROUND OF THE INVENTION

Various navigation systems are currently employed by aircraft for the purpose of ascertaining the position of the aircraft, or of other aircraft or landmarks relative to the aircraft. These systems typically produce two numbers to fix a participant's geodetic location on the ground ~~plane~~ <sup>plane</sup> --north and east, or range and bearing (azimuth). Systems such as TACAN, the Inertial Navigation System (INS), and True Air Speed (TAS)/Compass Systems are currently available and provide geodetic data in one or both of the above coordinate systems from which the relative position of two participants can be determined. However, the position data which can be shared by these systems is not accurate enough for relative navigation. For example, the INS error is measured in miles.

Furthermore, existing on-board relative navigation systems require more than two participants or necessitate the use of an angle-sensor to obtain bearing information, or both.

It is therefore one object of the present invention to provide an on-board relative navigation system which can be used in connection with the existing INS, Doppler, or TAS velocity



sensors to provide accurate relative position data between two or more participants with increased precision.

5 It is another object of the present invention to provide a relative navigation system which provides range and azimuth coordinates without the use of an angle-sensor.

10 It is yet another object of the present invention to provide a navigation system which automatically computes the azimuth coordinate of the relative position between two participants from range and velocity information shared by those participants.

#### SUMMARY OF THE INVENTION

15 According to one aspect of the invention there is provided a relative position navigation system installed in a first participant for computing the relative position of at least one other participant, the navigation system comprising:

20 communication means, for receiving telemetry information from the other participant, including range means for determining the range, or distance, between the first participant and any other selected participant at at least three selected points in time,  $t_0$ ,  $t_1$ ,  $t_2$ ; and  
25 processor means having a plurality of computational functions;

30 a first function of said processor means being to compute the relative velocity vector for the first participant and the other participant for each of times  $t_0$ ,  $t_1$  and  $t_2$ ;

35 storage means for storing the ranges and relative velocity vectors at times,  $t_0$ ,  $t_1$  and  $t_2$ , over which time intervals, defined as  $(t_1 - t_0)$  and  $(t_2 - t_1)$ , one of the relative velocity vectors represents a change in direction from the other relative velocity vector;



a second function of said processor means being to compute the azimuth, or, equivalently, the relative position in east-north coordinates, of the other participant relative to the first participant as a function of the range of the other participant from the first participant at each of the three selected times and of the change in position of the first participant relative to the other selected participant during the time intervals  $t_0$  to  $t_1$  and  $t_1$  to  $t_2$ ; and

a third function of said processor means being to compute the confidence level for the azimuth as a function of the statistical variances of the range means and the telemetered velocity data.

According to a further aspect of the invention, there is provided a relative position navigation system installed in a first participant for computing and displaying the relative position of at least one other participant in the system comprising:

range means for determining the range, or distance,  $R_n$ , between the first participant and the other participant at any preselected point in time,  $t_n$ ;

velocity means for determining the velocity vector of the first participant at the preselected point in time,  $t_n$ ;

a data link between the first participant for communicating, from the other participant to the first participant, position data, including at least the velocity vector, at the preselected point in time,  $t_n$ ;

processor means having a plurality of computational functions;

a first function of said processor means being to compute the relative velocity vector for the first participant and any other selected participant for the selected point in time,  $t_n$ ;

storage means for storing the range and relative velocity vector for each of at least three points in time,  $t_n$ ,  $t_{n+1}$ ,  $t_{n+2}$ , the three points in time defining two time intervals therebetween, during which time at least one of the relative velocity vectors represents a change



in direction from one of the other relative velocity vectors; and

5 a second function of said processor means being to compute the azimuth, or equivalently, the relative position in east-north coordinates, of the other selected participant relative to the first participant as a function of the range and relative velocity data for the three selected points in time that define two time intervals, ( $t_n$ , to  $t_{n+1}$ ) and ( $t_{n+1}$  to  $t_{n+2}$ ); and

10 a third function of said processor means being to compute the confidence level for the azimuth as a function of the statistical variances of the range means and the telemetered velocity data.

15 According to yet a further aspect of the invention, there is provided a method of finding the relative position between a first participant and at least one other participant at a selected point in time comprising the steps:

20 a) determining the range, or distance, between the first participant and the other participant and receiving the velocity vector of the other participant from the other participant at each of at least three points in time,  $t_0$ ,  $t_1$ ,  $t_2$ , that define two time intervals, ( $t_0$  to  $t_1$ ) and ( $t_1$ , to  $t_2$ ), during at least  
25 one of which intervals the direction of motion of the first participant relative to the other participant changes;

30 b) determining the change in position of the first participant relative to the other selected participant for time intervals ( $t_0$  to  $t_1$ ) and ( $t_1$  to  $t_2$ );

35 c) computing the non-ambiguous solution for the azimuth coordinate at time  $t_2$  as a function of the range between the first and other participant for each of the three selected points in time,  $t_0$ ,  $t_1$ ,  $t_2$  and the change in position of the first participant relative to the other



participant during each of the time intervals ( $t_0$  to  $t_1$ ) and ( $t_1$  to  $t_2$ ); and

5 d) computing the confidence level for the azimuth as a function of the statistical variances of the range-determining system and the velocity vector data.

BRIEF DESCRIPTION OF THE DRAWINGS

10 In order that the invention can be more easily understood, a preferred embodiment will now be described with reference to the accompanying drawings, wherein:-

Figure 1 is a block diagram of the embodiment;

Figure 2 illustrates graphically the operation of the embodiment; and

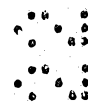
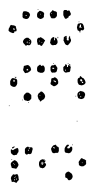
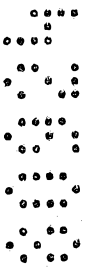
15 Figure 3 is a flow diagram of the operation of the embodiment.

DETAILED DESCRIPTION OF THE INVENTION

20 The specification uses the term "wingman" in the singular to refer to one participant whose location is being determined. It is understood, however, that, under normal conditions, the system will be determining the locations of a number of participants by performing the same operations described below for each participant. 25 Because the calculations are performed rapidly by computer, the operations appear to be performed simultaneously.

30 In addition, for the sake of clarity, the term "lead" will be used to refer to the participant taking the readings, receiving shared data and locating other participants. Either one of the participants, however, may be a stationary ground participant since the disclosed system and method require only relative motion.

35 Figure 1 illustrates, in block diagram form, the preferred embodiment of the system of the present



invention. The system is centered around a central processing unit 10 which has associated memory 12. The processor receives input from a data link 14, an on-board aircraft velocity sensor 16 and other on-board sensors 18 such as a baro-altimeter. Although the present invention is designed to be used by moving aircraft, it is adaptable for use by a ground station by fixing the velocity sensor input 16 at zero. The processor displays the results of its analysis graphically on a display 20 visible to the navigator. The display may be any known and commonly available display device, such as a Radar Electro-Optical display.

The details of the data link 14, the sensors 16 and 18, and the signals necessary to drive the display 20 are well known and will not be described in detail here. For the purposes of this disclosure, it is necessary only to understand that the data link 14 receives velocity-relative-to-ground information from, and determines distance (range) to, the selected wingman (whether a moving aircraft or a fixed ground station). The velocity information is readily available using devices commonly installed on aircraft, for example an INS.

Range may be determined by any of a number of conventional techniques. In the preferred embodiment, a predetermined coded signal is sent by the lead on the base frequency and received by the wingman. After a fixed time delay the wingman re-transmits the coded signal to the lead. Upon receipt, the lead is able to determine the range of the wingman as a function of the time delay between the lead's transmission and receipt of the coded signal.

By way of example, the operation of the invention will be described with reference to Fig. 2. Fig. 2 illustrates graphically the information acquired by the lead and the results of computations performed on the



data.

At time  $t_0$ , the lead receives data giving the range  $R$  of the wingman and its current velocity vector (speed and direction) relative to ground,  $V_0$ . The value  
5  $R_0$  may be viewed as the radius of a circle  $C_0$  centered at the lead and on which the wingman is somewhere located. At time  $t_1$ , the lead again acquires information comprising range  $R_1$  and velocity  $V_1$ . The range  $R_1$  defines a second circle  $C_1$  centered at the lead. The  
10 second circle is offset in space from the first by the change in relative position (integrated relative velocity vector) between the lead and the wingman between times  $t_0$  and  $t_1$ . The relative change is also called the apparent integrated velocity vector (delta).

15 The intersection of the two circles defines two points, designated  $(e_1 -, n_1 -)$  and  $(e_1 +, n_1 +)$ . One of these points is the true relative location of the wingman. The other is a false solution. Any algorithm that computes two-dimensional positions (range and  
20 azimuth) from one-dimensional data (ranges) gives rise to two ambiguous solutions at a single point in time (e.g.,  $t_1$ ). One will be correct and one will be false.

The ambiguity can be eliminated by taking a third reading at time  $t_2$  with a different relative  
25 directional heading, preferably at least on the order of one degree. To obtain a different directional heading, either the lead or the wingman must change direction relative to the other. After taking the third reading, two new points  $(e_2 +, n_2 +)$  and  $(e_2 -, n_2 -)$  can be  
30 computed as the intersection of the second and third circles  $C_1$  and  $C_2$  respectively. If there was a change in relative direction, one of the two new points will always be different from both of the original points and, under ideal conditions, the other of the new points will be the  
35 same as one of the original points. This point where three circles intersect resolves the ambiguity and



establishes the true relative position of the wingman. Although the algorithm is titled "Synthetic Azimuth" the (range, azimuth) coordinate system is typically not the most convenient choice for computation, thus the true and false solutions are computed as  $(e_{n^+}, n_{n^+})$  and  $(e_{n^-}, n_{n^-})$ . In (range, azimuth) coordinates, the solutions are  $(R_n, \tan^{-1}(e_{n^+} / n_{n^+}))$  and  $R_n, \tan^{-1}(e_{n^-} / n_{n^-})$ .

The points  $(e^+, n^+)$  and  $(e^-, n^-)$  are computed using the following formulae:

$$e_n^+ = u^+ \cos \psi + v \sin \psi \quad e_n^- = u^- \cos \psi + v \sin \psi$$

$$n_n^+ = -u^+ \sin \psi + v \cos \psi \quad n_n^- = -u^- \sin \psi + v \cos \psi$$

Psi is the angle clockwise from true north of the vector change in relative position over the time interval  $t_{n-1}$  to  $t_n$ . The vector change in relative position is the integral from  $t_{n-1}$  to  $t_n$  of the relative velocity vector.

$u^+$ ,  $u^-$  and  $v$  are computed from the range information and change in integrated relative velocity (Delta) between two readings:

$$v = \frac{R_0^2 - R_1^2 - \Delta^2}{2\Delta}$$

$$u^+ = \sqrt{R_1^2 - v^2}$$

$$u^- = -\sqrt{R_1^2 - v^2}$$



Points  $(e_1^+, n_1^+)$  and  $(e_1^-, n_1^-)$  represent the true and false positions of the other participant relative to the first participant relative to the first participant at time  $t_n$  in an east-north coordinate system, the true position being either the "=" or the "-" solution, depending on the relative geometry of the participants. Points  $(U_n^+, V_n^+)$  and  $(U_n^-, V_n^-)$  represent the same two positions in a coordinate system defined by the v-axis being rotated by "psi" clockwise from the north.  $R_0$  and  $R_1$  represent the ranges between the first and second participants at times  $t_{n-1}$  and  $t_n$ .

Delta is the magnitude of the vector change in relative position over the time interval  $t_{n-1}$  to  $t_n$ . that is,  $\sqrt{(\int V_e)^2 + (\int V_n)^2}$

Although, under ideal conditions, only three readings need be taken to determine the true location of the wingman, noise and minor variations in data values make this difficult in practice. Consequently, in practice, a number of readings are taken before a position estimate is displayed to the navigator. The number of readings taken depends on the reliability of the data as determined using a covariance analysis. When the data are found to be reliable, only a few readings need to be taken to pinpoint the wingman's relative location. When the data are less reliable, more readings are taken.

The covariances is computed as a matrix of variance and covariance values:

$$\text{cov} = \begin{bmatrix} \sigma_{ee}^2 & \sigma_{en}^2 \\ \sigma_{ne}^2 & \sigma_{nn}^2 \end{bmatrix}$$



The covariance values are computed as follows:

$$\sigma_{EE}^2 = \frac{\partial E}{\partial R\phi}^2 \sigma_{R\phi}^2 + \frac{\partial E}{\partial R1}^2 \sigma_{R1}^2 + \frac{\partial E}{\partial \Delta}^2 \sigma_{\Delta}^2 + \frac{\partial E}{\partial \psi}^2 \sigma_{\psi}^2$$

5

$$\sigma_{NN}^2 = \frac{\partial N}{\partial R\phi}^2 \sigma_{R\phi}^2 + \frac{\partial N}{\partial R1}^2 \sigma_{R1}^2 + \frac{\partial N}{\partial \Delta}^2 \sigma_{\Delta}^2 + \frac{\partial N}{\partial \psi}^2 \sigma_{\psi}^2$$

10

$$\sigma_{EN}^2 = \frac{\partial E}{\partial R\phi} \frac{\partial N}{\partial R\phi} \sigma_{R\phi}^2 + \frac{\partial E}{\partial R1} \frac{\partial N}{\partial R1} \sigma_{R1}^2 + \frac{\partial E}{\partial \Delta} \frac{\partial N}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial E}{\partial \psi} \frac{\partial N}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{NE}^2 = \sigma_{EN}^2$$

15

WHERE:

$$\sigma_{R\phi}^2, \sigma_{R1}^2, \sigma_{\Delta}^2, \sigma_{\psi}^2$$

are (standard statistical characteristics) of the measurement errors associated with the ranging system and navigation sensors and include those associated with, and already "known" by the lead apparatus and those associated with and "known" by, the wingman apparatus, and which are telemetered to the lead apparatus.

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Fig. 3 illustrates the steps involved in the operation of the invention. In step 28, the lead acquires the variances of the wingman's navigation sensors from the wingman. The lead's own variance and the variance of



the ranging system are stored with the program in the memory 12 of FIG. 1. In step 30, the lead acquires telemetry data from the wingman via the data link using standard techniques. The telemetry information contains the velocity vector information needed for the computations and the telemetry signal is used to measure the range data. The lead's own velocity vector is acquired to step 32.

If this is the first set of data, i.e., at time  $t_0$ , then, at step 34, the program branches to step 35 where the data are stored in memory 12 (Fig.1) for future use. The system then waits a predetermined amount of time, typically on the order of 10 seconds, before acquiring a new set of data from the wingman. This time delay gives the lead and wingman time to change their relative positions enough to provide a meaningful second reading. Also, during this time information may be obtained from other wingman for use in identical, but separate, calculations.

At step 30, a second set of data are acquired from the wingman and, at step 32, the lead's own velocity vector is again noted. Since this is the second set of data, i.e., time  $t_1$ , the program passes through step 34 to step 36 where the apparent velocity vector is computed by subtracting the relative velocity vector at  $t_0$  from that at  $t_1$ . The relative velocity vectors are determined by subtracting the absolute velocity vector of the lead from the absolute velocity vector of the wingman.

Next, at step 38, the solutions to equations 1 and 2 above are calculated to produce the two points  $(e_1^+, n_1^+)$  and  $(e_1^-, n_1^-)$ . The confidence level is then computed at step 40 using the covariance equations.

At step 42, the system branches back to step 30 and again retrieves telemetry data because a minimum of three data points are needed. Consequently, steps 30 through 40 are repeated for time  $t_2$  to produce points  $(e_2^+, n_2^+)$  and  $(e_2^-, n_2^-)$ .



At step 44, the data points are compared in an attempt to resolve the ambiguous point and determine the true relative location of the wingman. At step 46, the system reviews the covariance values to determine whether the reliability of the data is high enough to display the wingman's relative position.

5

If the covariance is low, the data were reliable and the result can be displayed. If the covariance was high, the data were unreliable and more data points need to be taken. If more data points are taken, steps 30 through 44 are repeated until the reliability is high enough to permit display of the data.

10

Once the decision to display data is made the data points are filtered using standard techniques (step 48) to produce a single point. The point is then displayed on the display screen 20 (Fig. 1) at step 50. the pilot is thus given a visual indication of the wingman's present relative location.

15

Although this process has been described with respect to a single wingman/lead pair, it is apparent that virtually any number of participants could be involved in the process. Each participant will be interrogated separately from its range and velocity vectors and the computations would be done for each participant individually. The results could then be displayed virtually simultaneously on a display screen.

20

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THE CLAIMS DEFINING THE INVENTION ARE AS FOLLOWS:

1. A relative position navigation system installed in a first participant for computing the relative position of at least one participant, the navigation system comprising:

communication means, for receiving telemetry information from the other participant, including range means for determining the range, or distance, between the first participant and any other selected participant at at least three selected points in time,  $t_0$ ,  $t_1$ ,  $t_2$ ; and

processor means having a plurality of computational functions;

a first function of said processor means being to compute the relative velocity vector for the first participant and the other participant for each of times  $t_0$ ,  $t_1$  and  $t_2$ ;

storage means for storing the ranges and relative velocity vectors at times,  $t_0$ ,  $t_1$  and  $t_2$ , over which time intervals, defined as  $(t_1 - t_0)$  and  $(t_2 - t_1)$ , one of the relative velocity vectors represents a change in direction from the other relative velocity vector;

a second function of said processor means being to compute the azimuth, or, equivalently, the relative position in east-north coordinates, of the other participant relative to the first participant as a function of the range of the other participant from the first participant at each of the three selected times and of the change in position of the first participant relative to the other selected participant during the time intervals  $t_0$  to  $t_1$  and  $t_1$  to  $t_2$ ; and

a third function of said processor means being to compute the confidence level for the azimuth as a function of the statistical variances of the range means



and the telemetered velocity data.

2. A system as claimed in Claim 1 wherein the second function of said processor means further includes solving the following equations to determine possible locations of the other participant at points  $(e_n^+, n_n^+)$  and  $(e_n^-, n_n^-)$  at each of times  $t_1$  and  $t_2$  :

$$e_n^+ = u^+ \cos \psi + v \sin \psi \quad e_n^- = u^- \cos \psi + v \sin \psi$$

$$n_n^+ = -u^+ \sin \psi + v \cos \psi \quad n_n^- = -u^- \sin \psi + v \cos \psi$$

$$v = \frac{R_0^2 - R_1^2 - \Delta^2}{2\Delta}$$

$$u^+ = \sqrt{R_1^2 - v^2}$$

$$u^- = \sqrt{R_1^2 - v^2}$$

and further includes means for comparing the points  $(e_1^+, n_1^+)$  and  $(e_1^-, n_1^-)$  with the points  $(e_2^+, n_2^+)$  and  $(e_2^-, n_2^-)$  to find two points which coincide and means for displaying the results of the comparison, points  $(e_1^+, n_1^+)$  and  $(e_1^-, n_1^-)$  being the true and false positions of the other participant relative to the first participant at time  $t_n$  in an east-north coordinate system, the true position being either the "+" or the "-" solution, depending on the relative geometry of the participants, points  $(u_n^+, v_n^+)$  and  $(u_n^-, v_n^-)$  being the same two positions in the coordinate system defined by the v-axis being rotated by "psi" clockwise from the north,  $R_0$  and  $R_1$  being the ranges between the first another participants at times  $t_{n-1}$  and  $t_n$  respectively, and "delta" being the magnitude of the vector change in relative position of the two participants over the time interval defined by  $t_n - t_{n-1}$ .



3. A system as claimed in Claim 2 wherein the third function of said processor means further includes determining the confidence level for the points  $(e_n^+, n_n^+)$  and  $(e_n^-, n_n^-)$  using the following formulae:

$$\text{cov} = \begin{bmatrix} \sigma_{ee}^2 & \sigma_{en}^2 \\ \sigma_{ne}^2 & \sigma_{nn}^2 \end{bmatrix}$$

$$\sigma_{EE}^2 = \frac{\partial E}{\partial R\emptyset}^2 \sigma_{R\emptyset}^2 + \frac{\partial E}{\partial R1}^2 \sigma_{R1}^2 + \frac{\partial E^2}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial E^2}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{NN}^2 = \frac{\partial N^2}{\partial R\emptyset} \sigma_{R\emptyset}^2 + \frac{\partial N^2}{\partial R1} \sigma_{R1}^2 + \frac{\partial N^2}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial N^2}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{EN}^2 = \frac{\partial E}{\partial R\emptyset} \frac{\partial N}{\partial R\emptyset} \sigma_{R\emptyset}^2 + \frac{\partial E}{\partial R1} \frac{\partial N}{\partial R1} \sigma_{R1}^2 + \frac{\partial E}{\partial \Delta} \frac{\partial N}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial E}{\partial \psi} \frac{\partial N}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{NE}^2 = \sigma_{EN}^2$$

where:

$$\sigma_{R\emptyset}^2, \sigma_{R1}^2, \sigma_{\Delta}^2, \sigma_{\psi}^2$$

are variances (standard statistical characteristics) of the measurement errors associated with the ranging system and navigation sensors and include those associated with, and already "known" by, the first participant and those associated with, and "known" by, the other participant, and which are telemetered to the first participant, and wherein the number of selected points in time is a function of the confidence level.

4. A relative position navigation system installed in a first participant for computing and displaying the relative position of at least one other participant in the system comprising:

range means for determining the range, or



distance,  $R_n$ , between the first participant and the other participant at any preselected point in time,  $t_n$ ;

velocity means for determining the velocity vector of the first participant at the preselected point in time,  $t_n$ ;

a data link between the first participant for communicating, from the other participant to the first participant, position data, including at least the velocity vector, at the preselected point in time,  $t_n$ ;

processor means having a plurality of computational functions;

a first function of said processor means being to compute the relative velocity vector for the first participant and one other selected participant for the selected point in time,  $t_n$ ;

storage means for storing the range and relative velocity vector for each of at least three points in time,  $t_n$ ,  $t_{n+1}$ ,  $t_{n+2}$ , the three points in time defining two time intervals therebetween, during which time at least one of the relative velocity vectors represents a change in direction from one of the other relative velocity vectors; and

a second function of said processor means being to compute the azimuth, or, equivalently, the relative position in east-north coordinates, of the other selected participant relative to the first participant as a function of the range and relative velocity data for the three selected points in time that define two time intervals, ( $t_n$  to  $t_{n+1}$ ) and ( $t_{n+1}$  to  $t_{n+2}$ ); and

a third function of said processor means being compute the confidence level for the azimuth as a function of the statistical variances of the range means and the telemetered velocity data.

5. A system as claimed in Claim 4 wherein the second function of said processor means determines the azimuth position by computing the point of intersection of



three circles,  $C_0$ ,  $C_1$ ,  $C_2$ ,

circle  $C_0$ , having a radius equal to the range  $R_0$  between the first participant and the other selected participant at a selected time  $t_n$ ;

circle  $C_1$  having a center translated from the center of circle  $C_0$  by an amount equal to the apparent change in integrated velocity between the participants for the time interval ( $t_n$  to  $t_{n+1}$ ), and a radius equal to the range  $R_1$  between the first participant and the selected other participant at time  $t_{n+1}$ , and

circle  $C_2$  having its center translated from the center of circle  $C_1$  by an amount equal to the apparent change in integrated velocity between the participants during the time interval ( $t_{n+1}$  to  $t_{n+2}$ ), and a radius equal to the range  $R_2$  between the first participant and the other participant at time  $t_{n+2}$ .

6. A system as claimed in Claim 4 wherein the second function of said processor means further includes solving the following equations to determined possible locations of the other participants at points ( $e_n^+$ ,  $n_n^+$ ) and ( $e_n^-$ ,  $n_n^-$ ) at each time  $t_n$ :

$$e_n^+ = u^+ \cos \psi + v \sin \psi \quad e_n^- = u^- \cos \psi + v \sin \psi$$

$$n_n^+ = -u^+ \sin \psi + v \cos \psi \quad n_n^- = -u^- \sin \psi + v \cos \psi$$

$$v = \frac{R_0^2 - R_1^2 - \Delta^2}{2\Delta}$$

$$u^+ = \sqrt{R_1^2 - v^2}$$

$$u^- = \sqrt{R_1^2 - v^2}$$



and further includes means for comparing the points  $(e_n^+, n_n^+)$  and  $(e_n^-, n_n^-)$  at each point in time to find two points which coincide, points  $(e_1^+, n_1^+)$  and  $(e_1^-, n_1^-)$  being the true and false positions of the other participant relative to the first participant at time  $t_n$  in an east-north coordinate system, the true position being either the "+" or the "-" solution, depending on the relative geometry of the participants, points  $(u_n^+, v_n^+)$  and  $(u_n^-, v_n^-)$  being the same two positions in the coordinate system defined by the v-axis being rotated by "psi" clockwise from the north,  $R_0$  and  $R_1$  being the ranges between the first and other participants at times  $t_{n-1}$  and  $t_n$  respectively, and "delta" being the magnitude of the vector change in relative position of the two participants over the time interval defined by  $t_n - t_{n-1}$ .

7. A system as claimed in Claim 6 wherein the third function of said processor means further includes means for determining the confidence level for the points  $(e_n^+, n_n^+)$  and  $(e_n^-, n_n^-)$  using the following formulae:

$$\text{cov} = \begin{bmatrix} \sigma_{ee}^2 & \sigma_{en}^2 \\ \sigma_{ne}^2 & \sigma_{nn}^2 \end{bmatrix}$$

$$\sigma_{EE}^2 = \frac{\partial E}{\partial R_0}^2 \sigma_{R_0}^2 + \frac{\partial E}{\partial R_1}^2 \sigma_{R_1}^2 + \frac{\partial E^2}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial E^2}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{NN}^2 = \frac{\partial N^2}{\partial R_0} \sigma_{R_0}^2 + \frac{\partial N^2}{\partial R_1} \sigma_{R_1}^2 + \frac{\partial N^2}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial N^2}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{EN}^2 = \frac{\partial E}{\partial R_0} \frac{\partial N}{\partial R_0} \sigma_{R_0}^2 + \frac{\partial E}{\partial R_1} \frac{\partial N}{\partial R_1} \sigma_{R_1}^2 + \frac{\partial E}{\partial \Delta} \frac{\partial N}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial E}{\partial \psi} \frac{\partial N}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{NE}^2 = \sigma_{EN}^2$$



where:

$$\sigma_{R\emptyset}^2, \sigma_{R1}^2, \sigma_{\Delta}^2, \sigma_{\psi}^2$$

are variances (standard statistical characteristics) of the measurement errors associated with the ranging system and navigation sensors and include those associated with, and already "known" by, the first participant and those associated with, and "known" by, the other participant and which are telemetered to the first participant, and wherein the number of selected points in time is a function of the confidence level.

8. A method of finding the relative position between a first participant and at least one other participant at a selected point in time comprising the steps:

a) determining the range, or distance between the first participant and the other participant and receiving the velocity vector of the the participant from the other participant at each of at least three points in time,  $t_0$ ,  $t_1$ ,  $t_2$ , that define two time intervals, ( $t_0$  to  $t_1$ ) and ( $t_1$  to  $t_2$ ), during at least one of which intervals the direction of motion of the first participant relative to the other participant changes:

b) determining the change in position of the first participant relative to the other selected participant for time intervals ( $t_0$  to  $t_1$ ) and ( $t_1$  to  $t_2$ );

c) computing the non-ambiguous solution for the azimuth coordinate at time  $t_2$  as a function of the range between the first and other participant for each of the three selected points in item,  $t_0$ ,  $t_1$ ,  $t_2$  and the change in position of the first participant relative to the other participant during each of the time intervals ( $t_0$  to  $t_1$ ) and ( $t_1$  to  $t_2$ ); and

d) computing the confidence level for the azimuth as a function of the statistical variances of the



range-determining system and the velocity vector data.

9. A method as claimed in Claim 8 wherein the computation of step (c) is performed by defining the point of interaction of three circles,  $(C_0, C_1, C_2)$ , circle  $C_0$  having a radius equal to the range  $R_0$  between the first participant and the other selected participant at time  $t_0$  ;

circle  $C_1$  having a center translated from the center of circle  $C_0$  by an amount equal to the apparent change in integrated velocity between the participants for the time interval  $(t_n$  to  $t_1)$ , and a radius equal to the range  $R$  between the first participant and the other participant at time  $t_1$ , and

circle  $C_2$  having its center translated from the center of circle  $C_1$  by an amount equal to the apparent change in integrated velocity between the participants during the time interval  $(t_1$  to  $t_2)$ , and a radius equal to the range  $R_2$  between the first participant and the other participant at time  $t_2$ .

10. A method as claimed in Claim 8 wherein the calculation of step (c) solves the following equations to determine possible locations of the other participant at points  $(e_n^+, n_n^+)$  and  $(e_n^-, n_n^-)$  at each of times  $t_1$  and  $t_2$  :

$$e_n^+ = u^+ \cos \psi + v \sin \psi \quad e_n^- = u^- \cos \psi + v \sin \psi$$

$$n_n^+ = -u^+ \sin \psi + v \cos \psi \quad n_n^- = -u^- \sin \psi + v \cos \psi$$

$$v = \frac{R_0^2 - R_1^2 - \Delta^2}{2\Delta}$$

$$u^+ = \sqrt{R_1^2 - v^2}$$

$$u^- = \sqrt{R_1^2 - v^2}$$



and compares the points  $(e_1^+, n_1^+)$  and  $(e_1^-, n_1^-)$  with the points  $(e_2^+, n_2^+)$  and  $(e_2^-, n_2^-)$  to find two points which coincide, points  $(e_1^+, n_1^+)$  and  $(e_1^-, n_1^-)$  being the true and false positions of the other participant relative to the first participant at time  $t_n$  in an east-north coordinate system, the true position being either the "+" or the "-" solution, depending on the relative geometry of the participants, points  $(u_n^+, v_n^+)$  and  $(u_n^-, v_n^-)$  being the same two positions in the coordinate system defined by the v-axis being rotated by "psi" clockwise from the north,  $R_0$  and  $R_1$  being the ranges between the first and other participants at times  $t_{n-1}$  and  $t_n$  respectively, and "delta" being the magnitude of the vector change in relative position of the two participants over the time interval defined by  $t_n - t_{n-1}$ .

11. A method as claimed in Claim 10 wherein the confidence level for the points  $(e_1^+, n_1^+)$  and  $(e_1^-, n_1^-)$  is determined using the following formulae:

$$\text{cov} = \begin{bmatrix} \sigma_{ee}^2 & \sigma_{en}^2 \\ \sigma_{ne}^2 & \sigma_{nn}^2 \end{bmatrix}$$

$$\sigma_{EE}^2 = \frac{\partial E}{\partial R_0}^2 \sigma_{R_0}^2 + \frac{\partial E}{\partial R_1}^2 \sigma_{R_1}^2 + \frac{\partial E^2}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial E^2}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{NN}^2 = \frac{\partial N^2}{\partial R_0} \sigma_{R_0}^2 + \frac{\partial N^2}{\partial R_1} \sigma_{R_1}^2 + \frac{\partial N^2}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial N^2}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{EN}^2 = \frac{\partial E}{\partial R_0} \frac{\partial N}{\partial R_0} \sigma_{R_0}^2 + \frac{\partial E}{\partial R_1} \frac{\partial N}{\partial R_1} \sigma_{R_1}^2 + \frac{\partial E}{\partial \Delta} \frac{\partial N}{\partial \Delta} \sigma_{\Delta}^2 + \frac{\partial E}{\partial \psi} \frac{\partial N}{\partial \psi} \sigma_{\psi}^2$$

$$\sigma_{NE}^2 = \sigma_{EN}^2$$

where:

$$\sigma_{R_0}^2, \sigma_{R_1}^2, \sigma_{\Delta}^2, \sigma_{\psi}^2$$





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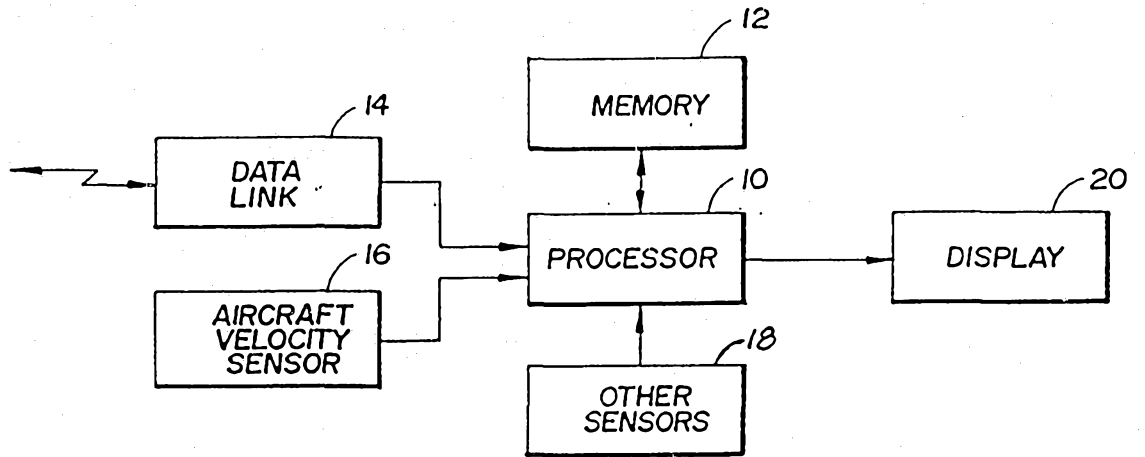


Fig. 1

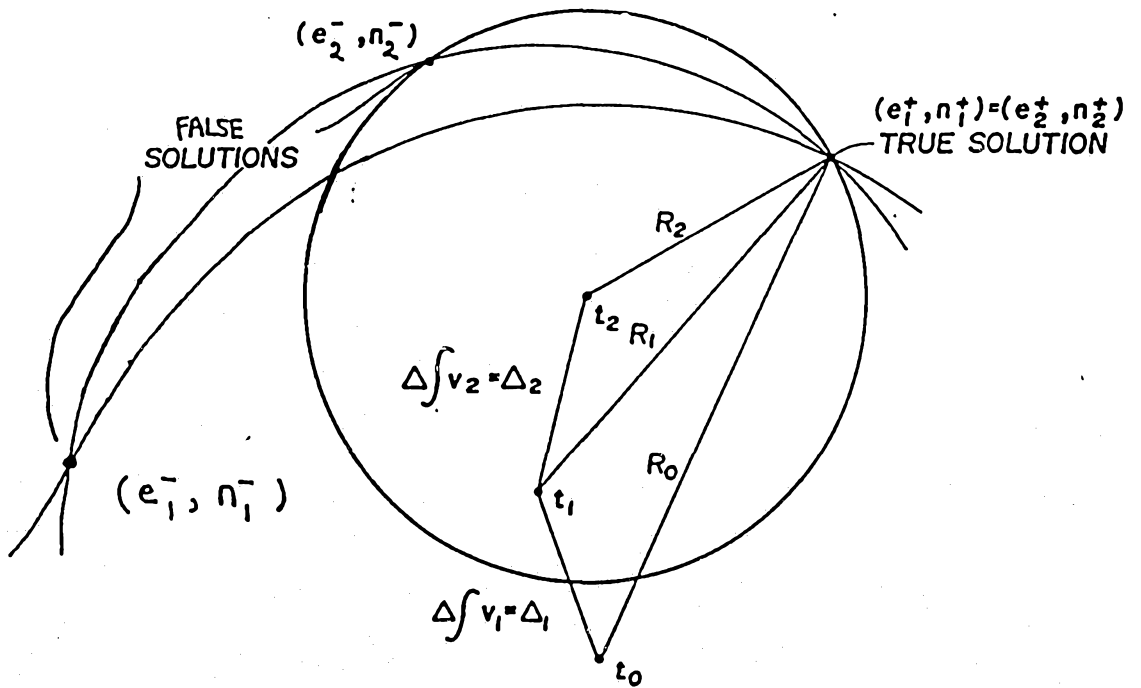


Fig. 2

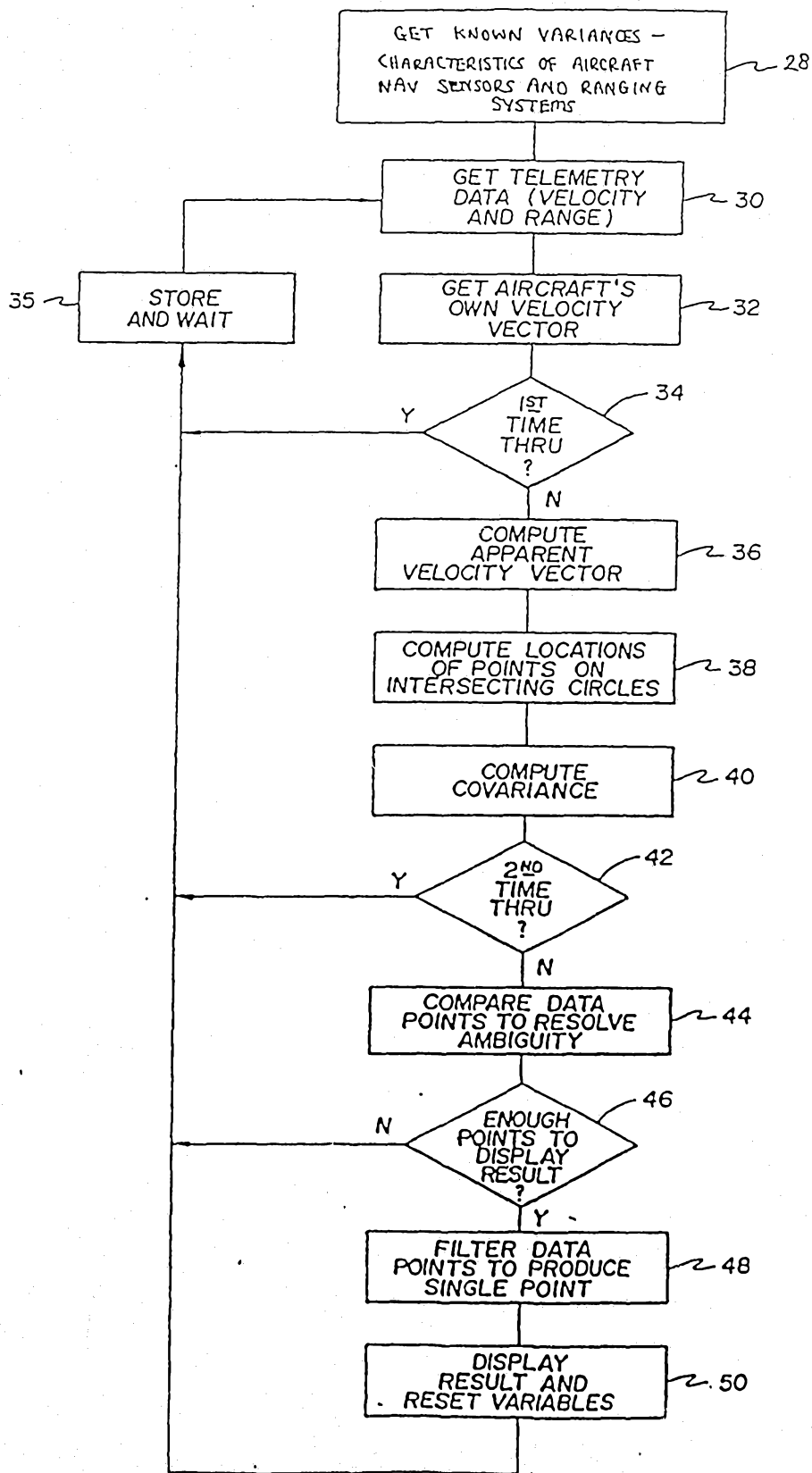


Fig. 3