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2,425,549

CALCULATION CHECKING DEVICE

Filed Aug. 31, 1944

2 Sheets-Sheet 1

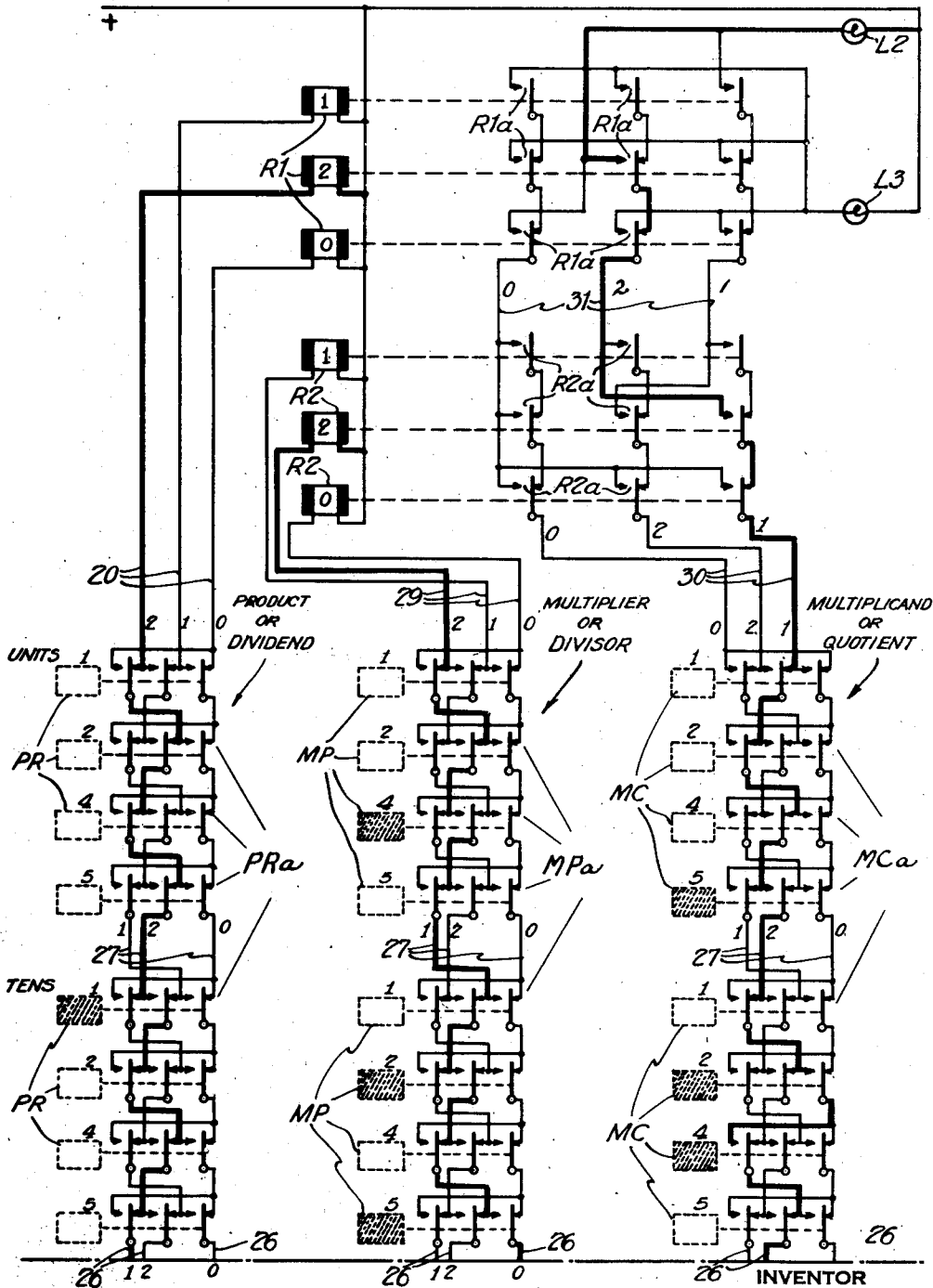


FIG. 1.

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2 Sheets-Sheet 2

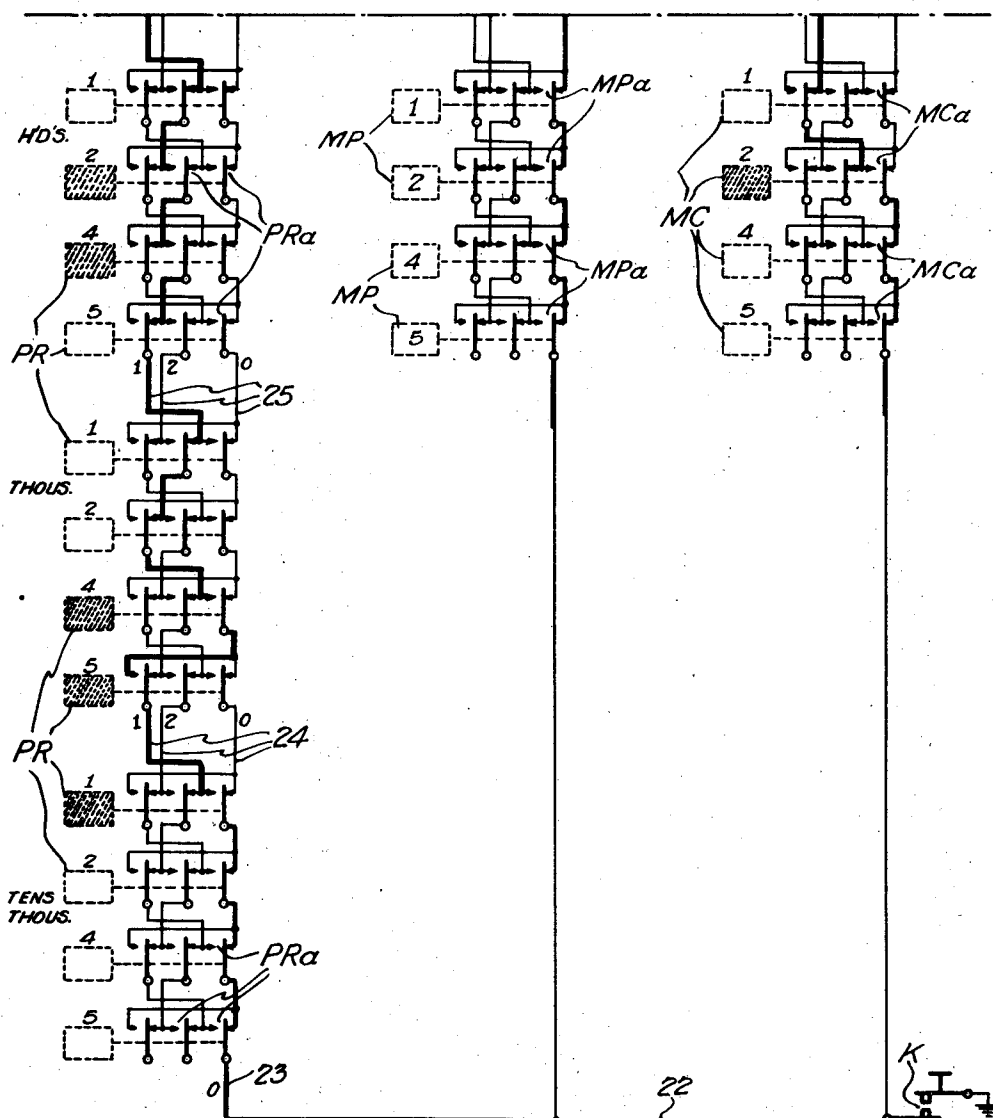


FIG. 2.

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CALCULATION CHECKING DEVICE

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5 Claims. (Cl. 235—61)

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This invention relates to calculating machines and more particularly to apparatus for checking or verifying the results of computations.

The principal object of the invention is to provide a simple, speedy mechanism in which the factors and the obtained result of a multiplying or dividing operation may be set up and the correctness of the result ascertained.

In carrying out the object of the invention the mathematical concept of congruence is employed, which concept may be briefly stated by saying that two integers a and b are "congruent modulo 3" if they leave the same remainder on division by 3. Thus, 1, 4, 7, 10, 13, 16, 19, etc., are all congruent modulo 3 since they leave the remainder 1. Also 2, 5, 8, 11, 14, 17, 20, etc., are congruent modulo 3 since they leave the remainder 2 and 3, 6, 9, 12, 15, 18, etc., are congruent modulo 3 since they leave the remainder 0.

It is also known that a number is congruent modulo 3 if the sum of its digits is likewise congruent to the same modulus. Thus, 785 is divisible by 3 with a remainder 2 and the sum of the remainders (1 for 7, 2 for 8 and 2 for 5), which is 5 is divisible by 3 with the same remainder 2.

In multiplying or adding congruences with respect to a fixed modulus 3 comparison for checking purposes may be carried out by handling the remainder of a digit instead of the digit itself and by following such procedure the numbers involved remain small i. e. for modulus 3 only the digits 1, 2 and 0 become involved.

By always replacing any number a by the number from the set 0, 1, 2 to which it is congruent, we need only use the following addition and multiplication tables:

	$a \times b$				$a \times b$		
	0	1	2		0	1	2
0	0	0	0	0	0	0	0
1	0	1	2	1	0	1	2
2	0	2	1	2	0	2	1

In devising a structure embodying the above principles, for checking a multiplying computation there is provided a relay chain which obtains the remainder of the sum of the digits of a product divided by 3, a second relay chain which obtains the remainder of the sum of the digits of the multiplier divided by 3 and a third relay chain which obtains the remainder of the sum of the digits of the multiplicand divided by 3.

A relay multiplying device simplified to multiply only the digits 1 and 2 is controlled by the two-factor chains and the product is compared with the result of the addition of the digits of the initial product. Thus, where it is desired to check the multiplication $74 \times 265 = 19610$, the ap-

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paratus performs the additions to restate the problem as

$$2 \times 1 = 2$$

and then multiplies 2 by 1 to obtain $2 = 2$. Comparison of the opposite sides of the equation indicate correctness or incorrectness.

For checking division, for example,

$$19610 \div 74 = 265$$

the quotient is handled as the multiplicand and the dividend is handled as the product. For addition a multiplier of 1 is used. Thus, to compare a sum 265 with another sum 265, the operations performed are as though the problem were

$$1 \times 265 = 265,$$

resulting in $1 \times 1 = 1$ or $1 = 1$.

Other objects of the invention will be pointed out in the following description and claims and illustrated in the accompanying drawings, which disclose, by way of example, the principle of the invention and the best mode, which has been contemplated, of applying that principle.

In the drawings:

Figs. 1 and 2 taken together constitute a wiring diagram of the apparatus.

In the drawings, magnets designated PR represent relay magnets that may be energized from some suitable source such as keys or record cards, in accordance with the value of a product. Each denominationally ordered digit is represented by combinational energization of four magnets PR having the values 1, 2, 4 and 5, so that for a product such as 19610, magnets PR will be energized as emphasized by crosshatching in the left hand vertical column of magnets.

Each magnet PR controls a set of three contact arms of contacts generally designated PRa and these are interconnected to form an adding chain extending through three wires 20 to three magnets R1. The wiring arrangement of this chain is such that it transmits to the wires 20 (valued 0, 1 and 2) the sum of the digits represented on magnets PR, less the greatest multiple of 3 in such sum. In other words, the chain obtains the remainder resulting from the sum of the digits divided by 3. Viewed in another manner, it may be said that the chain adds the digits represented on magnets PR and in adding them "casts out" or ignores 3's.

The operation may perhaps best be understood by tracing the circuits for a specific problem. In the left hand chain for magnets PR the circuit emphasized by heavy lines represents the current path involved in the summation of the digits of the product 19610 with 3's "cast out." The sum of these digits is 17 and this divided by 3 leaves a remainder of 2, in accordance with which the 2 magnet R1 is energized through the 2 wire 20.

Tracing the circuit in detail, when key contacts

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K are closed wire 22 is grounded to complete a circuit to wire 23 having a zero value, which circuit continues through the normally closed right hand contacts PRa of the 5, 4 and 2 magnets PR in the tens of thousands order. Since the 1 magnet PR is energized and its contacts PRa shifted, the circuit switches over to the 1 valued wire of a group of three designated 24. From here it continues through the shifted contacts of the 5 and 4 magnets PR and the normal contacts of the 2 and 1 magnets PR in the thousands order to the 1 valued wire of the group designated 25. This 1 represents the remainder of the sum of 1 and 9 divided by 3.

From the 1 wire 25 the circuit continues through the contacts of the hundreds order to the 1 wire of a group 26, representing the remainder of $1+9+6$ divided by 3. The circuit then continues through the tens order contacts PRa where the setting of a 1 directs the circuit to the 2 wire of a group designated 27 and since there is no significant setting in the units order the circuit extends to the 2 wire 20 and the 2 magnet R1, and wire 28 to positive side of a suitable current source.

In an exactly similar manner, the vertical column of magnets MP is energized in accordance with the value of a multiplicand and in this case a setting of 74 is indicated with the circuit through its contacts emphasized from wire 22 to the 2 wire 29. The remainder of $7+4$ divided by 3 is 2 and therefore, through the 2 wire 29 there is an energization of the 2 magnet R2.

The third column of magnets MC is energized in accordance with a multiplicand amount 265 and the contacts MCa direct a circuit from wire 22 through the emphasized path to the 1 wire 30 representing the remainder of $2+6+5$ divided by 3. It follows, therefore, that with a product (or a dividend), a multiplier (or a divisor) and a multiplicand (or a quotient) set on the magnets PR, MP and MC, the closure of contacts K will cause concurrent completion of circuits to wires 20, 29 and 30, in accordance with the related remainders.

Magnets R2 shift contacts $R2a$ to direct the circuit from wires 30 to wires 31 in accordance with the remainder of the product of the digits represented on wires 29 and 30, divided by 3. For the example chosen, these digits are 2 and 1 and therefore their product 2 is represented by continuation of the circuit through the 2 wire 31. Inspection will show that through the simple set of nine contacts $R2a$ all possible combinations of 0, 1 and 2 are taken care of. The following table will indicate the several possible conditions.

Wire 30	Magnet R2	Wire 31
0	X	0
0	X	1
0	X	2
1	X	0
1	X	1
1	X	2
2	X	0
2	X	1
2	X	2

It will be noted that only in the last case of 2×2 is there a "casting out" of a 3 to reduce 4 to 1.

Magnets R1 shift contacts $R1a$ which serve to compare the computed product representation on wires 31 with the product representation on wires 20. The contacts $R1a$ are interconnected so that when current is on the same wires 20 and 31 a lamp L2 will light as for the example traced. For any dissimilarity the circuit from wire 31 will extend to a lamp L3 to indicate non-comparison of

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the two result representations. As an example, assume current in the 1 wire 31 and the 2 wire 20. Accordingly, the 2 magnet R1 will be energized and the circuit from 1 wire 31 extends through the right hand contacts $R1a$ of the 0 magnet R1 (normal), through the right hand contacts $R1a$ of the 2 magnet R1 (shifted), lamp L3 to positive wire 28.

From the foregoing it is seen that upon setting of magnets PR, MP and MC and closure of key contacts K, there is a practically instantaneous checking of the computation and an indication given of the correctness or incorrectness by lamp L2 and L3. It will be particularly noted that the right hand circuit chain carries out three distinct functions at one and the same time, namely, 1—summation of the digits of the multiplicand through contacts MCa , 2—multiplication of the MC and MP digits through contacts $R2a$, and 3—comparison of the PR and $MC \times MP$ digits through contacts $R1a$.

While there have been shown and described and pointed out the fundamental novel features of the invention as applied to a single modification, it will be understood that various omissions and substitutions and changes in the form and details of the device illustrated and in its operation may be made by those skilled in the art without departing from the spirit of the invention. It is the intention, therefore, to be limited only as indicated by the scope of the following claims:

What is claimed is:

1. In an apparatus of the class described, a set of product relay contacts and connections therethrough, a set of multiplier relay contacts and connections therethrough, a set of multiplicand relay contacts and connections therethrough, each set being adjustable to represent a term of a computation to be checked and the connections therethrough being in accordance with the table of addition with 3's cast out, a set of product summation magnets operated by the product relay contacts in accordance with the remainder of the sum of the digits set therein, divided by 3, a set of multiplier summation magnets operated by the multiplier relay contacts in accordance with the remainder of the sum of the digits set therein, divided by 3, contacts adjusted by each set of magnets, multiplying circuit connections interconnecting the contacts related to the multiplier summation magnets and the multiplicand relay contacts in accordance with the table of multiplication with 3's cast out, comparison connections between said multiplying connections and the contacts related to the product summation magnets in accordance with the conditions of comparison, and current responsive devices included in said circuit connections to indicate the result of the comparison.

2. Apparatus for checking the approximate correctness of a multiplying calculation comprising a set of relay contacts for each of the multiplier, multiplicand and product terms, circuit connections for each set of contacts controlled thereby to establish a current path representative of the remainder of the sum of the digits of the related term divided by 3, a set of magnets in each of the multiplier and product circuit connections for signifying such remainders, means controlled by the multiplier magnets and through the multiplicand current paths for establishing a current path representative of the remainder of the sum of the digits of the said multiplier times the remainder of the sum of the digits of the multiplicand divided by 3, and

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means controlled by the product magnets and through the last named current path for indicating agreement between the product remainder and the remainder of the product of the multiplicand and multiplier remainders.

3. In an apparatus of the class described, a set of contacts settable to represent the remainder of the sum of the digits of a product, divided by 3, a second set of contacts settable to represent the remainder of the sum of the digits of a multiplier, divided by 3, a set of contacts settable to represent the digits of a multiplicand, circuits serially interconnecting the contacts of the last set in accordance with the table of addition with 3's cast out, said circuits continuing in series through the contacts of the second set in accordance with the table of multiplication with 3's cast out, and further continuing serially through the contacts of the first set so as to establish a continuous current path through all of said sets of contacts when the set product representation equals the product of the set multiplier representation times the sum of the set multiplicand digits with 3's cast out.

4. In an apparatus of the class described, a set of contacts settable to represent the remainder of the sum of the digits of a product divided by a predetermined digit, a second set of contacts settable to represent the remainder of the sum of the digits of a multiplier, divided by said predetermined digit, a set of contacts settable to represent the digits of a multiplicand, circuits serially interconnecting the contacts of the last set in accordance with the table of addition with said predetermined digit and multiples thereof cast out, said circuits continuing in series through the contacts of the second set in accordance with the table of multiplication with said predetermined digit and multiples thereof cast out, and further continuing serially through the

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contacts of the first set so as to establish a continuous current path through all of the said sets of contacts when the set product representation equals the product of the set multiplier representation times the sum of the set multiplicand with said predetermined digit and multiples thereof cast out.

5. In an apparatus of the class described, a set of contacts settable to represent the remainder of the sum of the digits of a product, divided by 3, a second set of contacts settable to represent the remainder of the sum of the digits of a multiplier, divided by 3, a set of contacts settable to represent the digits of a multiplicand, circuits serially interconnecting the contacts of the last set in accordance with the table of addition with 3's cast out, said circuits continuing in series through the contacts of the second set in accordance with the table of multiplication with 3's cast out, and further continuing serially through the contacts of the first set so as to establish a continuous current path through all of said sets of contacts when the set product representation fails to equal the product of the set multiplier representation times the sum of the set multiplicand digits with 3's cast out.

HANS P. LUHN.

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Certificate of Correction

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August 12, 1947.

HANS P. LUHN

It is hereby certified that error appears in the printed specification of the above numbered patent requiring correction as follows: Column 1, line 38, in the table, for the heading " $a \times b$ " first occurrence, read $a + b$; and that the said Letters Patent should be read with this correction therein that the same may conform to the record of the case in the Patent Office.

Signed and sealed this 11th day of November, A. D. 1947.

[SEAL]

THOMAS F. MURPHY,
Assistant Commissioner of Patents.