A technique for calculating traveltime of a seismic wave in three dimensional tilted transversely isotropic (3D TTI) media includes determining a wave vector, defining a unit vector, calculating an angle of the wave vector from an axis and performing a slowness determination. The technique may be practiced as a computer implemented set of instructions, and may be incorporated into measurement equipment.
FIG. 2

- I/O Bus
- Memory
- Storage
- Processor
- Power Supply
- Wavefront Data
- Sampling Tool
- Formation Data
- Power Supply
\textbf{FIG. 3}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Wave vector determination process.}
\end{figure}

\textbf{FIG. 4}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{Process flowchart for wave vector determination.}
\end{figure}
FIG. 7A

FIG. 7B

FIG. 8A

FIG. 8B
TRAVELTIME CALCULATION IN THREE DIMENSIONAL TRANSVERSELY ISOTROPIC (3D TTI) MEDIA BY THE FAST MARCHING METHOD

CROSS REFERENCE TO RELATED APPLICATIONS

[0001] This patent application claims priority from U.S. Provisional Patent Application Ser. No. 60/756,739 entitled “Traveltime Calculation in 3D TTI Media by the Fast Marching Method” filed on Jan. 9, 2006, the entire contents of which is incorporated by reference.

BACKGROUND OF THE INVENTION

[0002] 1. Field of the Invention

[0003] The invention herein relates to techniques for resolving imaging data collected during geophysical exploration.

[0004] 2. Background Information

[0005] A number of problems arise during geophysical exploration. For example, resolving seismic wave propagation data in isotropic and anisotropic formations (media) has required elaborate modeling. One model is that of the Kirchhoff migration model.

[0006] The traveltime calculation is the backbone of any Kirchhoff pre-stack depth migration. During the past decade, there have been numerous methods developed based upon the eikonal equation solver to calculate traveltimes in three dimensional (3D) isotropic media. Those methods are generally classified as either ray tracing or finite difference (FD) approaches.

[0007] Among them, one approach is the fast marching algorithm with first or higher order FD eikonal equation solver. This method has proven popular due to its computation efficiency, stability, and satisfactory accuracy (Popovici and Sethian 2002). It has been well recognized however, that most sedimentary rocks display transverse isotropy (TI) with a vertical symmetry axis (VTI) or a general tilted symmetric axis (TTI) to seismic waves. The phenomena can significantly affect focusing and imaging positions in seismic data migration. Recently, Atkinson (2002) presented a FD algorithm to solve first arrival traveltimes in 3D VTI media by a perturbation method. Jiao (2005) used a similar FD algorithm based on perturbation theory to calculate first arrival traveltimes in 3D TTI media. In addition, Zhang et. al. (2002) presented a FD scheme in the celerity domain to calculate first arrival traveltimes in 2D TTI media.

[0008] What are lacking are improvements to efficiency, accuracy and stability in order to reduce the costs associated with geological exploration.

SUMMARY OF THE INVENTION

[0009] Examples of certain features of the invention have been summarized here rather broadly in order that the detailed description thereof that follows may be better understood and in order that the contributions they represent to the art may be appreciated. There are, of course, additional features of the invention that will be described hereinafter and which will form the subject of the claims appended hereto.

[0010] Disclosed is a method for determining a traveltime of a seismic wave in three dimensional transversely isotropic (3D TTI) media, the method including: determining a vector for the wave; defining a unit vector for the wave; calculating an angle between the wave vector and an axis of symmetry of the media; and, using the calculated angle, determining a slowness of the wave to determine the travel time of the wave.

[0011] Also disclosed is a computer program product having computer readable instructions for determining a travel time of a seismic wave in three dimensional transversely isotropic (3D TTI) media, by: determining a vector for the wave; defining a unit vector for the wave; calculating an angle between the wave vector and an axis of symmetry of the media; and, using the calculated angle, determining a slowness of the wave to determine the travel time of the wave.

[0012] Further disclosed is a sampling tool including: equipment for sampling within a wellbore, the sampling tool further having a coupling to an electronics unit, the electronics unit including a computer program product having computer readable instructions for determining a travel time of a seismic wave in three dimensional transversely isotropic (3D TTI) media, by determining a vector for the wave; defining a unit vector for the wave; calculating an angle between the wave vector and an axis of symmetry of the media; and, using the calculated angle, determining a slowness of the wave to determine the travel time of the wave.

BRIEF DESCRIPTION OF THE FIGURES

[0013] For detailed understanding of the present invention, references should be made to the following detailed description of the embodiment, taken in conjunction with the accompanying drawings, in which like elements have been given like numerals, wherein:

[0014] FIG. 1 depicts a sampling tool within a wellbore;

[0015] FIG. 2 depicts aspects of an electronics unit;

[0016] FIG. 3 depicts a relationship between a phase angle for a wavefront and a group angle for a ray;

[0017] FIG. 4 is a flow chart depicting exemplary aspects of a method for calculating traveltime;

[0018] FIG. 5 depicts aspects of a 3D traveltime cube;

[0019] FIG. 6A and FIG. 6B, collectively referred to as FIG. 6, depict vertical slices (z-x) and (z-y) respectively of the traveltime cube in FIG. 5 through a source position;

[0020] FIG. 7A and FIG. 7B, collectively referred to as FIG. 7, depict a horizontal slice (x-y) of the traveltime cube of FIG. 5, and a relative traveltime error (%) distribution between algorithm results and analytical results for the horizontal slice, respectively;

[0021] FIG. 8A and FIG. 8B, collectively referred to as FIG. 8, depict aspects of a four-layer 3D velocity model and a traveltime cube generated by a point source at (1500,1500,10), respectively;

[0022] FIG. 9A and FIG. 9B, collectively referred to as FIG. 9, depict a horizontal (x-y) slice and a vertical (x-z) slice through the source point of the traveltime cube of FIG. 6B; and,
FIG. 10A and FIG. 10B, collectively referred to as FIG. 10, depict a vertical (y-z) slice through the source point of the traveltimes cube in FIG. 6B, and a comparison with computation results from assumption of vertical symmetry axis (VTA), respectively.

Detailed Description of the Invention

Disclosed herein is a method for calculation of first arrival traveltimes in three-dimensional transversely isotropic (3D TTI) media that is based on the fast marching method. The method disclosed is comparatively more accurate than other prior art techniques. Further, the method provides advantages in that certain beneficial aspects of the fast marching method are not perturbed. For example, the method preserves computational efficiency and substantial stability for any 3D TTI velocity model applied to isotropic media, wherein the model includes large velocity gradients and arbitrary orientation of symmetry axis. In addition, the method disclosed can be advantageously applied to a Kirchhoff pre-stack depth migration for 3D TTI media and to estimate TTI parameters for Vertical Seismic Profiling (VSP) data.

As depicted in FIG. 1, in typical embodiments, a sampling tool 20 is disposed within a wellbore 11. The sampling tool 20 is suspended by a wireline 12, typically from a derrick 14 using a pulley system 13 and a service vehicle 15. The sampling tool 20 transmits and receives a series of wavefronts 19 using equipment for sampling within the wellbore 11. Relying upon wavefront data (such as knowledge of the character of the each transmitted wavefront 19 and received wavefront 19), the character of the surrounding earth 10 and formations 4 therein may be determined. Resolving the wavefront data may be completed in accordance with the teachings herein.

One non-limiting example of the tool 10 is the 3DExplorer™ tool, which is an induction logging instrument produced by Baker Hughes of Houston, Tex. As discussed herein, reference to the tool 10 and aspects thereof generally refer to the exemplary and non-limiting embodiment, the 3DExplorer™ tool 10.

Referring to FIG. 2, and in regard to the sampling tool 20, the sampling tool 20 is typically coupled to an electronics unit 200. The electronics unit 200 typically includes, without limitation, at least one power supply 201, an input/output bus 202, a processor 203, storage 204, memory 205 and other components (not shown) such as an input device and an output device. Other components may be included as deemed suitable.

In typical embodiments, the electronics unit 200 receives the wavefront data 210 from the sampling tool 20 and processes the wavefront data 210 to produce formation data 220.

To order place the teachings into context, a review of the prior art is now presented.

Referring to the teachings of Thomsen (see “Weak Elastic Anisotropy” by Thomsen, L., Geophysics., Vol. 51, No. 10, October 1986 pp. 1954-1966), a linearly elastic material is defined as one in which each component of stress \( \sigma_{ij} \) is linearly dependent upon every component of strain \( e_{ij} \). Since each directional index may assume values of 1, 2, 3 (representing directions x, y, z), there are nine relations, each one involving one component of stress and nine components of strain. These nine equations are conventionally expressed in Equation 1:

\[
\sigma_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} C_{ijkl} e_{kl}, \quad i, j = 1, 2, 3
\]

where the 3x3x3 elastic modulus tensor \( C_{ijkl} \) characterizes the elasticity of the medium.

Referring to FIG. 3, a phase angle \( \theta \) for the wavefront 19 is depicted in relation to the group angle \( \phi \) for a ray. It is important to clarify the distinction between the phase angle \( \theta \) and the ray angle \( \phi \) (along which the energy propagates). Referring to FIG. 1, the wavefront 19 is locally perpendicular to the propagation vector \( \mathbf{k} \), since \( \mathbf{k} \) points to the direction the maximum rate of increase in phase. The phase velocity \( v(\theta) \) is also called the wavefront velocity, since it measures the velocity of the wavefront 19 along \( \mathbf{k} \). Since the wavefront 19 is non-spherical, it is clear that \( \theta \) (also called the wavefront normal angle) is different from \( \phi \), the ray angle from the source point to the wavefront 19.

The velocities of three possible seismic wavefronts (P, SV, and SH) may therefore be given respectively as:

\[
v_{p}(\theta) = \left( \frac{1}{2\pi} \left[ C_{tt} + C_{ee} + (C_{tt} - C_{ee}) \sin^{2} \theta + D(\theta) \right] \right)^{1/2}
\]

\[
v_{sv}(\theta) = \left( \frac{1}{2\pi} \left[ C_{tt} + C_{ee} + (C_{tt} - C_{ee}) \sin^{2} \theta - D(\theta) \right] \right)^{1/2}
\]

\[
v_{sh}(\theta) = \left( \frac{1}{2\pi} \left[ C_{tt} \sin^{2} \theta + C_{ee} \cos^{2} \theta \right] \right)^{1/2}
\]

where

\[
D(\theta) = \frac{(C_{tt} - C_{ee})^{2} + 2(2C_{tt} + C_{ee} - C_{tt} - C_{ee})}{\sin^{2} \theta + (C_{tt} + C_{ee} - 2C_{tt} - 4C_{tt} + 4C_{ee}) \sin^{2} \theta}
\]

As noted by Thomsen, some suitable combinations of components of the stress and strain are suggested to describe aspects of the anisotropy within the formation. These combinations (known also as Thomsen’s parameters) are:

\[
\varepsilon = \frac{(C_{tt} - C_{ee})}{2C_{tt}}, \quad \gamma = \frac{(C_{tt} - C_{ee})}{2C_{tt}}; \quad \text{and,}
\]

\[
\delta = \frac{(C_{tt} + C_{ee})^{2} - (C_{tt} - C_{ee})^{2}}{2C_{tt}(C_{tt} - C_{ee})};
\]

Accordingly, a general eikonal equation for describing a local grid isotropic or transverse isotropic (TI) medium can be written as:
where \( \tau(x,y,z) \) is a travel time derivative component for each axis of the model and \( s(x,y,z) \) is the phase slowness for a 3D velocity model. In isotropic media, \( s(x,y,z) \) is a function of coordinates \((x,y,z)\) only, while in 3D TI media, \( s(x,y,z) \) is a function of the coordinates \((x,y,z)\), \( \gamma \), and \( \delta \), and the wave vector \( \mathbf{k} \) relative to the TI symmetry axis.

For the techniques disclosed herein, each space derivative \( \tau_x, \tau_y, \) and \( \tau_z \) can be calculated by a fast marching FD scheme:

\[
\left[ \frac{\max(D_{\tau_x} - D_{\tau_x})}{\max(D_{\tau_x} + D_{\tau_x})} \right]^{1/2} = s_{ijk},
\]

where \( D^+ \) and \( D^- \) are forward and backward FD operators and \( s_{ijk} \) is the slowness at grid point \((i,j,k)\). An important part in this algorithm is the determination of \( s_{ijk} \) for each grid location in the 3D TI media. An exemplary embodiment of aspects of an algorithm 400 for the traveltime determination is provided in FIG. 4.

Referring to FIG. 4, in a first stage, wave vector determination 401 is completed. Typically, determination of the wave vector (normal to wavefront), denoted as \( (\tau_x, \tau_y, \tau_z) \), is determined using a recursive loop from each previous traveltime calculation for each grid location. It should be noted that wave vector determination 401 inherently calls for wave identification or detection. In one embodiment, identification occurs by generation of the wave. Thus, initial aspects of the wave may be known.

Next, unit vector definition 402 is completed. Typically, the unit vector of the symmetry axis of the TI media is defined as \( \left( \cos \theta, \sin \theta, \cos \phi \right) \), where \( \theta \) is the azimuth of the symmetry axis measured from the \( x \) direction and \( \phi \) is the dip angle of the symmetry axis measured from the \( z \) direction.

In a third stage of angle calculation 403, the angle \( \alpha \) between the wave vector \( \mathbf{k} \) and the symmetry axis in each local TI medium grid is typically calculated as:

\[
\alpha = \cos^{-1} \left( \frac{\cos \phi \sin \theta \tau_x + \sin \phi \sin \theta \tau_y + \cos \theta \tau_z}{\tau_z} \right),
\]

where \( \delta \) and \( \phi \) correlate to Equations (3) and (5) above, \( v_{\rho_0} \) and \( v_{\phi_0} \) are vertical velocities for \( P \) and \( SV \) waves in each local TTI medium grid and \( D(\epsilon, \delta, \alpha, v_{\rho_0}, v_{\phi_0}) \) is defined as:

\[
D(\epsilon, \delta, \alpha, v_{\rho_0}, v_{\phi_0}) = \frac{1}{2} \left[ \frac{\tau_z^2}{\tau_x^2} \right].
\]

Similarly, the slowness \( S_{ijk} \) (SV) of the SV wave and the slowness \( S_{ijk} \) (SH) of the SH wave in each local TTI medium is, respectively, determined as:

\[
S_{ijk}(SV) = \left[ \frac{1}{\max(\|\mathbf{y}_1(\alpha\cos \theta + \delta \sin \theta, \alpha \sin \theta - \delta \cos \theta, \delta)\|)} \right]^{1/2},
\]

where \( \| \cdot \| \) denotes the Euclidean norm. It should be noted that Equations 9a to 9d provide accurate determinations of the slowness \( S_{ijk} \) for each of three waves (\( P, SV, SH \)) for substantially all strong 3D TI media.

The algorithm 402 was first tested using a constant TTI medium with parameters \( v_{\rho_0} = 2500 \text{ m/s}, v_{\phi_0} = 1250 \text{ m/s}, \epsilon = 0.15 \), and \( \delta = 0.10 \). The tilted angle of the symmetry axis was \( 30^\circ \) and \( 0^\circ \). FIG. 5 shows the traveltime cube of the \( P \) wave on a \( 201 \times 201 \times 201 \) grid points and the center point source \( 500 \text{ m}, 500 \text{ m}, 10 \text{ m} \). The 3D traveltime cube \( (201 \times 201 \times 201) \) depicted in FIG. 5 was generated by a point source at \( 500 \text{ m}, 500 \text{ m}, 10 \text{ m} \) and a tilted symmetry axis \( \alpha = 30^\circ \) and \( 0^\circ \). The vertical \( P \)-wave velocities for the four layers were respectively \( 1500 \text{ m/s}, 2500 \text{ m/s}, 3500 \text{ m/s}, \) and \( 4500 \text{ m/s} \). The grid points for the model were selected as \( 201 \times 201 \times 301 \) with \( 15 \text{ m} \) grid spacing. FIG. 8B depicts the traveltime cube of the \( P \) wave generated from a point source at \( 1500 \text{ m}, 1500 \text{ m}, 10 \text{ m} \).

Recognizing that a horizontal slice \((x,y)\) at a depth of \( 2250 \text{ m} \), FIG. 9B displays a vertical \((x,z)\) slice through the source point of the traveltime cube displayed in FIG. 8B. Examples provided in FIG. 8B, FIG. 9, and other TTI models are indicative of advantages that the algorithm 400 provides. These advantages include, among other things, efficiency, stability, and accuracy for calculating first arrival traveltimes for a 3D TTI model having large velocity gradients and arbitrary orientation of symmetry axis.

In addition, FIG. 10A shows another vertical \((y,z)\) slice through the source point of the traveltime cube (FIG. 8B).
with a comparison with the computation results using the assumption of strata having a vertical symmetry axis (VTI) (dashed lines) instead of a tilted symmetry axis (as depicted in FIG. 10B). The results show that if TTI medium is assumed to be VTI, for this model, the difference in traveltimes can be up to about 50 ms.

The numerical examples provided demonstrate that the algorithm 400 is efficient, stable, and accurate for calculating first arrival traveltimes in 3D TTI media. The examples also indicate that treating a TTI medium as VTI could result in traveltime errors, however, this is not conclusive. Advantageously, the algorithm 400 can be applied to a Kirchhoff pre-stack depth migration for 3D TTI media with comparatively little difficulty, as well as to estimation of TTI parameters for vertical seismic profile (VSP) data.

The algorithm 400 may be implemented as a method of the present invention and also may be implemented as a set computer executable of instructions on a computer readable medium, comprising ROM, RAM, CD ROM, Flash or any other computer readable medium, now known or unknown that when executed cause a computer to implement the method of the present invention.

While the foregoing disclosure is directed to the exemplary embodiments of the invention various modifications will be apparent to those skilled in the art. It is intended that all variations within the scope of the appended claims be embraced by the foregoing disclosure. Examples of the more important features of the invention have been summarized rather broadly in order that the detailed description thereof that follows may be better understood, and in order that the contributions to the art may be appreciated. There are, of course, additional features of the invention that will be described hereinafter and which will form the subject of the claims appended hereto.

What is claimed is:

1. A method for determining a travel time of a seismic wave in three dimensional transversely isotropic (3D TTI) media, the method comprising:
   - determining a vector for the wave;
   - defining a unit vector for the wave;
   - calculating an angle between the wave vector and an axis of symmetry of the media; and,
   - using the calculated angle, determining a slowness of the wave to determine the travel time of the wave.

2. The method as in claim 1, further comprising at least one of identifying and generating the wave.

3. The method as in claim 1, wherein determining the wave vector comprises using a recursive loop from each previous traveltime determination.

4. The method as in claim 1, wherein a unit vector for a symmetry axis is defined as:

   \((\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \phi)\);

   where

   \(\phi\) represents the azimuth of the symmetry axis measured from the x direction; and,

   \(\theta\) represents the dip angle of the symmetry axis measured from the z direction.

5. The method as in claim 1, wherein calculating the angle \(\alpha\) comprises solving the relationship:

   \(\alpha = \cos^{-1}\left[\left(\frac{\tau_x \cos \phi - \sin \phi \sin \theta \tau_y + \tau_z \cos \phi}{\sqrt{\tau_x^2 + \tau_z^2}}\right)\right]\)

   where

   \(\phi\) represents the azimuth of the symmetry axis measured from the x direction;

   \(\theta\) represents the dip angle of the symmetry axis measured from the z direction;

   \(\tau_x\) represents a traveltime derivative component for an x-axis;

   \(\tau_y\) represents the traveltime derivative component for a y-axis; and,

   \(\tau_z\) represents the traveltime derivative component for a z-axis.

6. The method as in claim 1, wherein determining the slowness \(S_{\theta\phi}\) comprises solving the relationships:

   \(S_{\theta\phi}(P) = \frac{1}{\tau_v} \sqrt{\left(\frac{1 + \sin^2 \alpha \delta}{\sin^2 \phi \sin^2 \theta \sin^2 \phi \cos^2 \theta} + \frac{1}{\sin^2 \phi \sin^2 \theta \sin^2 \phi \cos^2 \theta}\right)}\); and,

   \(S_{\theta\phi}(S) = \frac{1}{\tau_v} \sqrt{\left(\frac{1 + \sin^2 \alpha \delta}{\sin^2 \phi \sin^2 \theta \sin^2 \phi \cos^2 \theta} + \frac{1}{\sin^2 \phi \sin^2 \theta \sin^2 \phi \cos^2 \theta}\right)}\);

   where

   \(v_v\), \(v_\phi\), represent vertical velocities for P and SV waves, respectively;

   \(\alpha\) represents an angle between the wave vector and an axis of symmetry of the media; and,

   \(\epsilon\), \(\delta\), \(\gamma\), and \(D\) comprises relationships of components of stress and strain for the media.

7. The method as in claim 1, wherein the media comprises features having at least one of a transverse isotropy (TI) and a tilted symmetric axis isotropy (TTI).

8. The method as in claim 1, wherein determining the travel time comprises determining the travel time for a Kirchhoff pre-stack depth migration.

9. A computer program product comprising computer readable instructions for determining a travel time of a seismic wave in three dimensional transversely isotropic (3D TTI) media, by:

   - determining a vector for the wave;
   - defining a unit vector for the wave;
   - calculating an angle between the wave vector and an axis of symmetry of the media; and,
   - using the calculated angle, determining a slowness of the wave to determine the travel time of the wave.

10. The computer program product as in claim 9, further comprising at least one of identifying and generating the wave.

11. The computer program product as in claim 9, wherein determining the wave vector comprises using a recursive loop from each previous traveltime determination.

12. The computer program product as in claim 9, wherein a unit vector for a symmetry axis is defined as:

   \((\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \phi)\);

   where

   \(\phi\) represents the azimuth of the symmetry axis measured from the x direction; and,
θ represents the dip angle of the symmetry axis measured from the z direction.

13. The computer program product as in claim 9, wherein calculating the angle α comprises solving the relationship:
\[
\cos^{-1}\left(\frac{\tau_x \cos \phi \sin \theta + \tau_y \sin \phi \sin \theta + \tau_z \cos \theta}{\tau_x^2 + \tau_y^2 + \tau_z^2}\right)
\]

where

φ represents the azimuth of the symmetry axis measured from the x direction;

θ represents the dip angle of the symmetry axis measured from the z direction;

τx represents a travelt ime derivative component for an x-axis;

τy represents the travelt ime derivative component for a y-axis; and,

τz represents the travelt ime derivative component for a z-axis.

14. The computer program product as in claim 9, wherein determining the slowness Sjkl comprises solving the relationships:

\[
S_{jkl}(P) = \frac{1}{V_{p0}} \left\{\frac{1}{1+\alpha D} \right\}^{1/2}
\]

\[
S_{jkl}(S) = \frac{1}{V_{s0}} \left\{\frac{1}{1+\alpha D} \right\}^{1/2}
\]

\[
S_{jkl}(SH) = \frac{1}{V_{s0}} \left\{\frac{1}{1+2\gamma} \right\}^{1/2}
\]

where

Vp0, Vs0 represent vertical velocities for P and SV waves, respectively;

α represents an angle between the wave vector and an axis of symmetry of the media; and,

δ, γ and D comprises relationships of components of stress and strain for the media.

15. The computer program product as in claim 9, wherein the media comprises features having at least one of a transverse isotropy (TI) and a tilted symmetric axis isotropy (TTI).

16. The computer program product as in claim 9, wherein determining the travel time comprises determining the travel time for a Kirchhoff pre-stack depth migration.

17. A sampling tool comprising:

equipment for sampling within a wellbore, the sampling tool further comprising a coupling to an electronics unit, the electronics unit comprising a computer program product comprising computer readable instructions for determining a travel time of a seismic wave in three dimensional transversely isotropic (3D TTI) media, by:

determining a vector for the wave;

defining a unit vector for the wave;

calculating an angle between the wave vector and an axis of symmetry of the media; and,

using the calculated angle, determining a slowness of the wave to determine the travel time of the wave.

* * * * *