United States Patent

[11] **3,634,789**

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[54] **GEOMETRICALLY DEPENDENT DISTRIBUTED-SECTION TRANSMISSION LINE ATTENUATOR**
11 Claims, 9 Drawing Figs.

[52] **U.S. Cl.** .... **333/81,** 333/84 M, 333/81 A
[51] **Int. Cl.** **H01p 1/22**
[50] **Field of Search** .... 333/81, 81 A, 84 M

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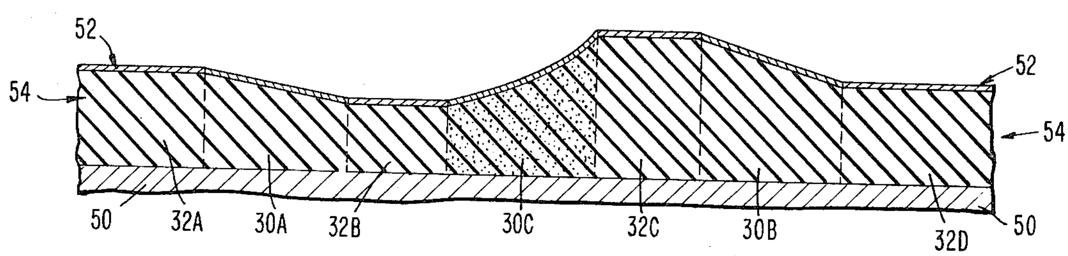
ABSTRACT: The attenuators include distributed series and distributed shunt resistance sections which attenuate signals propagating in one direction with minimal distortion and reflection, and attenuate signals propagating in a second direction with equivalent minimal distortion but with significant reflection. In the distributed series and shunt resistance sections, the characteristic impedance, which is determined by the geometry of the transmission structure, varies continuously. Disclosed transmission line attenuators include two distributed series resistance sections separated by a distributed shunt resistance section or two distributed shunt resistance sections separated by a distributed series resistance section. The distributed series resistance is a resistive section in the signal line and the distributed shunt resistance is a section of lossy dielectric between and interconnecting the signal line and the ground conductor.



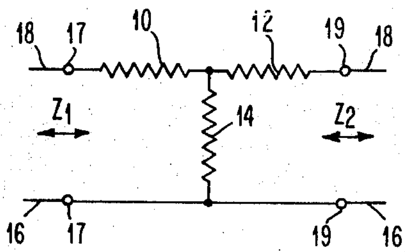


FIG. 1

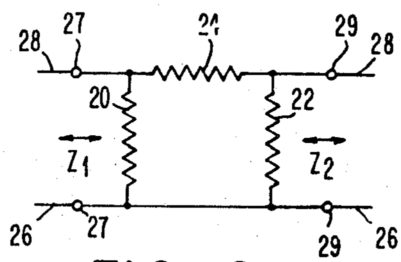


FIG. 2

FIG. 3

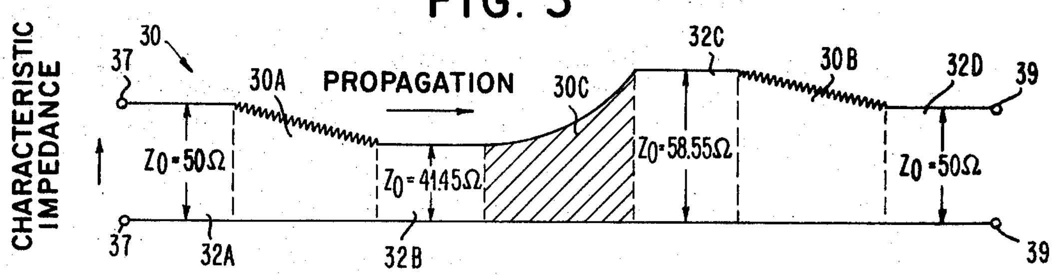
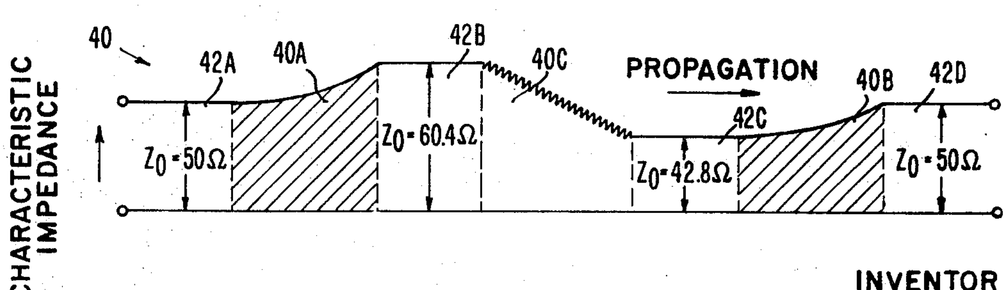


FIG. 4



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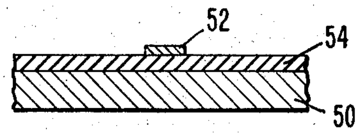

FIG. 5

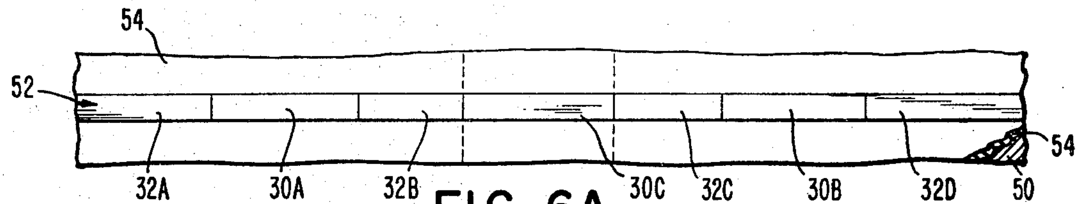

FIG. 6A

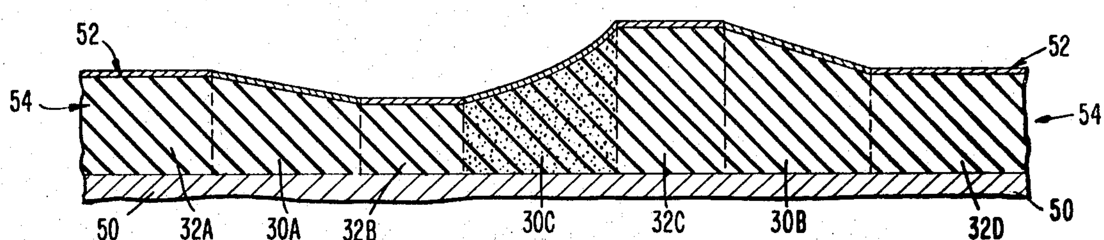

FIG. 6B

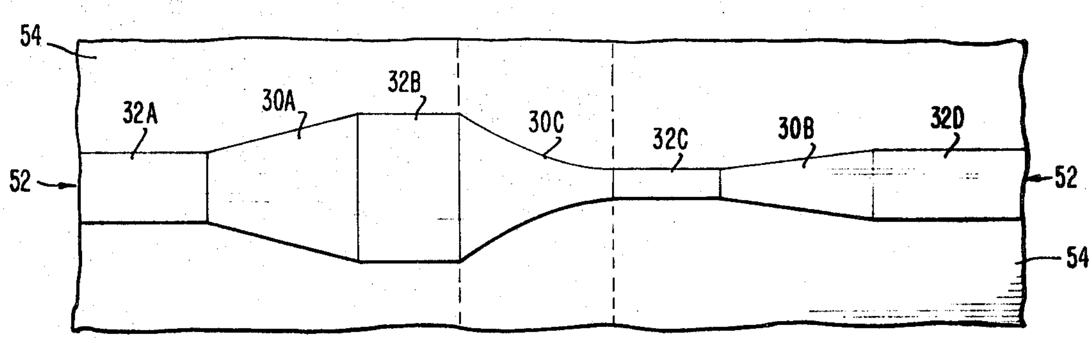

FIG. 7A

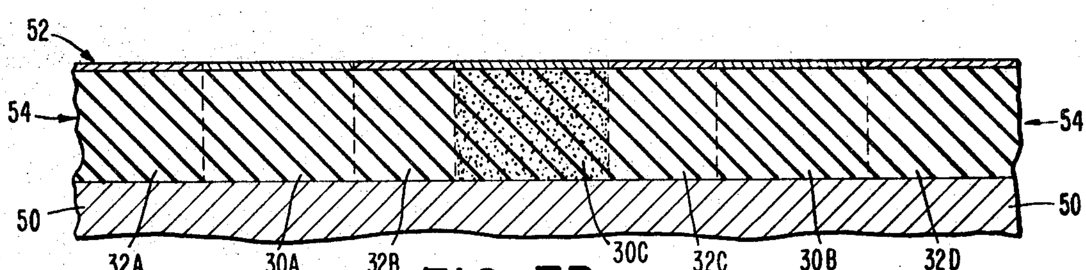

FIG. 7B

GEOMETRICALLY DEPENDENT DISTRIBUTED-SECTION TRANSMISSION LINE ATTENUATOR

BACKGROUND OF THE INVENTION

FIELD OF THE INVENTION

This invention relates to transmission line attenuators, symmetrical or unsymmetrical, designed for use in balanced or unbalanced transmission lines, which may be constructed in a variety of geometries (e.g. microstrip, stripline, coaxial, etc.), wherein attenuation is produced by distributed series and shunt resistive sections in which the characteristic impedance along these distributed sections is varied so that signals propagate in one direction with minimal distortion and reflection.

PRIOR ART

It is well known that the characteristic impedance of a transmission line is determined by the geometry of the line. Tapered transmission line sections have been used in driving a multiplicity of lumped memory loads and as terminations, as illustrated, for example, in U.S. Pat. No. 3,418,641 issued Oct. 29, 1964 to Thomas Fyfe and Paul Stuckert, and U.S. Pat. No. 2,778,887 issued to E. H. Bradly on Jan. 12, 1957. Further, the geometry of attenuator resistors has been varied to obtain desired characteristics as illustrated in U.S. Pat. No. 2,119,195. It is also well known that attenuators can be designed in a variety of connective forms using lumped elements.

SUMMARY

In accordance with the principles of the present invention improved transmission line attenuators are provided which equally attenuate signals propagating in either direction, but which accomplish the attenuation for signals propagating in one direction with minimal distortion and reflection. The attenuators include distributed series and shunt resistance sections which can be of any desired length so that the power dissipation attendant to the attenuation process can be distributed. The characteristic impedance along the sections varies, decreasing along the series resistance section and increasing along the shunt resistance sections in the direction of distortionless and reflectionless attenuation, so that throughout the attenuator the normally expected reflections due to discontinuities and parasitic reactances associated with lumped resistive elements are avoided. Further, with the use of connected distributed series and shunt resistive sections, the characteristic impedances at the ends of the attenuator can be controlled to be the same, or different, and the attenuator can be inserted in a transmission line with minimal parasitic effects and minimal reduction of bandwidth at the high frequencies.

OBJECTS OF THE INVENTION

Therefore, it is an object of this invention to provide improved transmission line attenuators.

It is a further object to provide improved transmission line attenuators which can produce signal attenuation without significant signal reflection and distortion.

It is a further object of the invention to provide improved transmission line attenuators in which the power dissipation per unit length of the attenuator can be arbitrarily low without sacrifice of bandwidth.

Another object is to provide improved transmission line attenuators which can be employed in transmission lines without appreciably lowering the bandwidth of the signals which can be propagated through the structure.

These and other objects, features and advantages of the invention will be apparent from the following more particular description of preferred embodiments of the invention, as illustrated in the accompanying drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a circuit diagram of an unbalanced T-attenuator as conventionally drawn using lumped resistive elements.

FIG. 2 is a circuit diagram of an unbalanced π attenuator as conventionally drawn using lumped resistive elements.

FIG. 3 illustrates the construction of and the variation in the characteristic impedance along the length of a T-attenuator, according to the invention, which includes two distributed series resistance sections and a shunt resistance section.

FIG. 4 illustrates the construction of and the variation in the characteristic impedance along the length of a π attenuator, according to the invention, which includes tow distributed shunt resistance sections and a series resistance section.

FIG. 5 is a diagram illustrating the components of a conventional strip transmission line.

FIGS. 6A and 6B are top and side sectional views of one embodiment of the T-attenuator whose characteristics are depicted in FIG. 3.

FIGS. 7A and 7B are top and side sectional views of another embodiment of the T-attenuator whose characteristics are depicted in FIG. 3.

DESCRIPTION OF PREFERRED EMBODIMENTS

FIG. 1 is a drawing of a conventional unbalanced T-attenuator which employs two series resistors **10** and **12**, and a shunt resistor **14**. In this conventional representation it should be noted that impedance is defined at the input terminals **17** and output terminals **19** by Z_1 and Z_2, but that impedance is not defined between the input and output terminals **17** and **19**. The resistors, as shown here, are conventional lumped elements but the circuit diagram is generally representative of the disclosed type of attenuator, and as such, is helpful in explaining the present invention. The transmission line system in which the attenuator formed by resistors **10**, **12** and **14** is connected includes a ground element **16**, transmission elements **18** and a dielectric separating these elements. The geometric configuration of **16**, **18** and the dielectric determine the characteristic impedance of the line. Assuming a characteristic impedance of 50 ohms for the line, (i.e., Z_1=Z_2=50 ohms) values of 8.55 ohms for each of the series resistance elements **10** and **12**, and a value of 141.9 ohms for the shunt resistance element **14**, the attenuator produces 3 db. of attenuation. When lumped elements are used, the attenuation produced as well as the distortions and reflections due to discontinuities and parasitic reactances associated with the lumped resistors **10**, **12** and **14** are the same regardless of the direction of signal propagation through the attenuator.

The same is true for the unbalanced π attenuator shown in FIG. 2 which includes two shunt lumped resistive elements **20** and **22** and a lumped series resistive element **24** connected in a line formed of a ground element **26** and transmission elements **28**. Assuming again line characteristic impedances of 50 ohms, a value of 292 ohms for each of the shunt resistive elements **20** and **22**, and a value of 17.6 ohms for the series resistance element, the attenuator produces 3 db. of attenuation. As with the attenuator of FIG. 1, the π attenuator of FIG. 2 using lumped elements is insensitive to the direction of signal propagations in terms of not only the attenuation it produces, but also in terms of the distortions and reflections due to discontinuities and parasitic reactances associated with the lumped resistors **20**, **22** and **24**. Again, it is pointed out that the impedance is defined at the input terminals **27** and output terminals **29** by Z_1 and Z_2, but that the impedance is not defined between the input and output terminals **27** and **29**.

FIG. 3 is a diagrammatic representation of the characteristics of a T-attenuator in accordance with the principles of the present invention. In this figure, distances along the transmission line structure is the abscissa, and the value of characteristic impedance, Z_o, is the ordinate. The line **30** generally represents the value of characteristic impedance along the length of the line which includes the attenuator. The small fluctuation in the line 30, at 30A and 30B, does not indicate fluctuation in characteristic impedance but is rather a symbolic representation of the two sections of the line which include the series-distributed resistors of the attenuator. Similarly, the shaded portion 30C under curve 30 indicates the section of the line which includes the distributed shunt resistor of the attenuator. In addition to the three sections 30A, 30B and 30C, the drawing is divided to illustrate four other sections of the line designated 32A, 32B, 32C, and 32D. Each of these four sections is a transmission line section of constant characteristic impedance. Section 32A and 32D represent the 50-ohm transmission line in which the attenuator is connected, and sections 32B and 32C are sections of minimum and maximum constant impedance which separate the shunt resistance section 30C from the series resistance sections 30A and 30B. Sections 32B and 32C can be of any length equal to or greater than zero and are not necessary to the practice of the invention since the resistive sections 30A, 30C and 30B, can be connected directly to each other. The separating sections 32B and 32C may be used and serve in the illustrative embodiment to more graphically represent the impedance characteristics along the length of the attenuator.

The attenuation is produced in the attenuator represented in FIG. 3 by the two series resistance sections 30A and 30B and the shunt resistance section 30C, which form a distributed T-attenuator in which signals are attenuated when propagated from left to right without distortion or reflection. For the 3 db. attenuator described with reference to FIG. 1, resistive sections 30A and 30C have values of 8.55 ohms and shunt section 30C has a value of 141.9 ohms but the resistance in each of these sections is distributed over the length of section of the line in which it is incorporated. Further, the characteristic impedance of the line within the attenuator is decreased continuously along the distributed series resistance sections 30A and 30B and increased continuously over the distributed shunt resistance section 30C so that signals can be propagated in the one direction without distortion or reflection. The design, in this case, is such that the terminal sections 32A and 32D of the attenuator both have the same characteristic impedance of 50 ohms. Assuming a strip transmission line, the geometric details of which are illustrated in FIGS. 6A, 6B, 7A and 7B, the series resistance is introduced by a resistive section in the strip and the shunt resistance by a lossy dielectric, that is, a dielectric between the strip and the ground plane which allows a predetermined degree of conduction between these two elements which is constant as a function of frequency.

As is indicated in FIG. 3, the characteristic impedance of the line decreases linearly along the two resistive sections 30A and 30B. This linear decrease in characteristic impedance is the optimum for a distributed series resistance section in which the series resistance introduced has a constant value per unit length over the length of the distributed section. The length of the section and the rate of change of characteristic impedance depends upon the resistivity of the material used and can be varied to distribute the power dissipated over sections of line of arbitrary length. The characteristic impedance of the parallel shunt section 30C, as shown in FIG. 3, increases at an increasing rate, and the plot of characteristic impedance versus distance is a section of an equilateral hyperbola, if the shunt resistance introduced has a constant value per unit length over the length of the distributed section.

Thus, and in contrast to the conventional attenuator shown in FIG. 1 in which the characteristic impedance of the structure is not defined in the region between the input and output terminals 17 and 19, in the attenuator of the present invention the characteristic impedance of the structure is preferably defined at essentially every point between the input terminals 37 and the output terminals 39 of FIG. 3.

The attenuator whose characteristics are represented in FIG. 3 is symmetrical in the sense that signals propagating in either direction are attenuated equally and without distortion. The attenuator whose characteristics are represented in FIG. 3 in unsymmetrical in the sense that signals propagating from left to right are transmitted without reflections while signals propagating from right to left produce significant reflections. This lack of distortion regardless of direction in the idealized case and elimination of reflections in one direction, is achieved by using distributed resistance sections in which the characteristic impedance is continuously varied in the manner illustrated as a function of the series or shunt resistance which is introduced. At the same time, the use of the distributed elements in this manner eliminates the parasitic reactances associated with lumped elements and, therefore, the attenuation is achieved without lowering the bandwidth or otherwise altering the frequency response of the attenuator.

As is illustrated in FIG. 4, an unbalanced π attenuator is fabricated using the same principles. In this figure, the characteristic impedance along the attenuator is represented by curve 40. The line includes two shunt resistance sections 40A and 40B and a series resistance section 40C located between the shunt resistive sections. As in FIG. 3, the resistance sections are separated by maximum and minimum constant impedance sections, here designated 42B and 43C, and the end sections 42A and 42D have equal characteristic impedances of 50 ohms. Following the example above given for FIG. 2, the total series resistance introduced by distributed series resistance section 40C is 17.6 ohms, and the total shunt resistance introduced by each of the distributed-shunt resistance sections 40A and 40B is 292 ohms. This design provides 3 db. of attenuation and since the characteristic impedance along the shunt sections is increased continuously and along the series section is decreased continuously as a function of the resistance introduced, reflections and distortion are eliminated for signals propagating from left to right.

The actual values of characteristic impedance required along shunt and series resistance sections can be calculated with the following equations.

For a series resistance section where the distributed series resistance has a constant value per unit length, the relationship is:

$$Z_x = Z_o - R_K x$$

where Z_x = the characteristic impedance at any distance x along the section in the desired direction of propagation (x=0 at the beginning of the series resistance section);

Z_0 = the characteristic impedance at the beginning of the section;

R_K = the constant series resistance per unit length, and

x = any point along the section where x=0 at the beginning of the section and increases positively from left to right.

For a shunt resistance section with a constant value of shunt resistance per unit length, the relationship is:

$$Y_x = Y_0 - G_K x$$

where

G_K = the constant shunt conductance per unit length which is equal to reciprocal of the constant shunt resistance per unit length;

Y_x = the characteristic admittance at any distance x along the section in the desired direction of propagation (x=0 at the beginning of the shunt resistance section);

Y_0 = the characteristic admittance at the beginning of the section; and

x is as defined above.

In actual practice in the design and fabrication of attenuators, it is possible to fabricate lines with constant values of series and shunt resistance per unit length, but this is not always the simplest procedure, particularly for the shunt resistance. Therefore, a more general formula for determining the characteristic impedance at any point along a series resistive section is:

$$Z_x = Z_o - \int_0^x r\,dx \text{ for } x \geq 0$$

where $r=f(x)$ = resistance per unit length which may be a variable function of x, the distance along the section.

For a shunt resistance section, the relationship in terms of admittances $Y(=1/Z)$ is:

$$Y_x = Y_o - \int_0^x g dx \text{ for } x \geq 0$$

where $g=f(x)$ = the conductance per unit length which may be a variable function of x, the distance along the section.

These design equations assume, as is common in transmission line analysis, that the series resistance and shunt conductance along the normal sections of the line (i.e., 32A, 32B, 32C, and 32D in FIG. 3 and 42A, 42B, 42C and 42D in FIG. 4) are zero.

FIGS. 6A, 6B, 7A and 7B illustrate the actual structure of embodiments of a strip transmission line attenuator of the type whose characteristics are shown in FIG. 3. The components of a conventional strip transmission line are shown in FIG. 5 to include a ground plane **50**, a strip or signal conductor **52** and a separating dielectric **54**. FIGS. **6A** and **6B** are top and side sectional views of a distributed T-attenuator in strip transmission line form in which the changes in characteristic impedance along the series and shunt resistance sections are accomplished by varying the distance between the ground conductor **50** and strip conductor **52**. The line is divided into sections corresponding to the sections shown in FIG. 3 and identified with the same reference characters **32A**, **30A**, **32B**, **30C**, **32C**, **30B**, **32D**. The design equations for the characteristic impedance of strip transmission line are, of course, well known. Along the constant impedance sections **32A**, **32B**, **32C**, **32D**, a good dielectric (e.g., Teflon, polystyrene, foamed polystyrene, etc.) essentially a perfect insulator, and similarly good conductors (e.g., copper for the strip **52**) are employed. In the distributed resistance sections **30A** and **30B**, the dielectric may be the same but the strip **52** in these sections is formed of a resistive strip (e.g., a strip of resistive alloy or a thin film strip of conductive material which yields the desired resistance as determined by $R=\rho(L/A)$).

In the shunt resistive section, the strip line conductor **52** is formed of the same highly conductive material as in the constant impedance sections but the dielectric is here a lossy dielectric. The dielectric is connected between the strip and the ground plane and allows a predetermined degree of conduction between these two elements which is constant as a function of frequency, and which introduces the shunt resistance along the section **30C** between strip **52** and ground plane **50**. This lossy dielectric may be a composite material such as that used in the construction of composition resistors.

In order to decrease the characteristic impedance in series resistance sections **30A** and **30B**, the thickness of the dielectric **54** is decreased and similarly the spacing between conductor **52** and ground plane **50**. Even with a constant value of resistance per unit length introduced in conductor **52** over these sections, the linear decrease in characteristic impedance is achieved by a variation in spacing which, though it appears so at the scale in which FIG. **6B** is drawn, is not exactly linear. This is due to the fact that the characteristic impedance of such a strip transmission line is not a linear function of the distance between the strip and the ground plane. In the shunt resistance section **30C**, assuming the same dielectric constant throughout the section, the difference in spacing between conductor **52** and ground plane **50**, of course, causes the length of the conduction path through the lossy dielectric to increase from left to right. Therefore, the conductance per unit length is not constant. This factor, in addition to the nonlinear relationship between spacing and characteristic impedance, causes the curvature of conductor strip **52** in section **30C**, though it appears hyperbolic at the scale to which FIG. **6B** is drawn, to differ from a true hyperbolic contour.

In FIGS. **7A** and **7B**, the same reference numerals are employed to designate the component sections of another embodiment of the attenuator whose characteristics are shown in FIG. 3. In this embodiment, the materials of the dielectric **54** and the conductive strip **52** are the same as for the embodiment of FIGS. **6A** and **6B**. However, in the embodiment of FIGS. **7A** and **7B**, the distance between strip **52** and ground plane **50** is constant throughout, and the variation in characteristic impedance is achieved by varying the width of the strip **52** in sections **30A**, **30B** and **30C**. Assuming that the dielectric **54** has the same dielectric constant throughout, it can be seen that in this embodiment, the series resistance per unit length in sections **30A** and **30B** is not linear, nor is the shunt resistance per unit length in shunt resistance section **30C** constant. The desired relationship between resistance per unit length introduced and the appropriate characteristic impedance at any point is controlled by appropriate changes in the width of conductor **52**. Thus, again, though it appears so at the scale in which FIG. **7A** is drawn, the edges of the strip **52** in the regions **30A** and **30B** are not truly linear and the edges of the strip **52** in the region **30C** are not truly hyperbolic.

The structure of a π attenuator of the type shown in FIG. **4** is similar to those in FIGS. **6A**, **6B**, **7A** and **7B** for the T-type attenuator with the exception that in the π attenuator there are two distributed shunt resistance sections separated by a distributed series resistance section. Further, it should be apparent that the principles of the subject invention are not restricted to π- and T-type attenuators but that distributed series and shunt resistive sections can be combined in the same way to form the many other types of attenuators known in the art. It is also equally clear that the principles of the invention may be equally applied to transmission lines which are not of the strip transmission line form of FIGS. **6A**, **6B**, **7A** and **7B**. Coaxial lines and other unbalanced, as well as balanced lines, may incorporate distributed series and shunt resistive sections. The geometry of the line is controlled to achieve the necessary value of characteristic impedance so that the advantages of the invention in terms of distortionless and reflectionless attenuation are achieved. Further, it should be clear that attenuators built in accordance with the principles of the invention need not be symmetrical. Further, in many applications propagation in one direction is all that may be necessary, and a single series and single shunt section are all that are required for certain of these applications. It is further clear that by properly designing the resistance to be introduced in each of the sections employed as well as the resistivity of the material used in the conductive strip and the lossy dielectric used in the shunt resistance sections, the line can be so designed that the characteristic impedance at one end is different from the characteristic impedance at the other end of the attenuator.

It is also possible to combine a distributed series and a distributed shunt section in a single section of the line. In this case, the changes in characteristic impedance required, an increase for the shunt resistance and a decrease for the series resistance, can be balanced by the appropriate choice of materials and an essentially constant impedance along the single section may be possible.

While the invention has been particularly shown and described with reference to preferred embodiments thereof, it will be understood by those skilled in the art that the foregoing and other changes in form and details may be made therein without departing from the spirit and scope of the invention.

What is claimed is:

1. An unsymmetrical transmission line attenuator for attenuating a signal propagating in at least the direction along the length of the transmission line attenuator comprising:

a. a distributed-series resistance extending along a first section of the transmission line attenuator;

b. a distributed-shunt resistance being formed by a lossy dielectric of uniform resistivity extending along a second section of the transmission line attenuator;

c. the characteristic impedance of the attenuator decreasing in said one direction along said first section with the decrease in the characteristic impedance being a function of the series resistance of the first section and;

d. the characteristic impedance of the attenuator increasing in said one direction along said second section with the increase in characteristic impedance being a function of the shunt resistance of the said second section;

e. a conductor for propagating said signal;

f. a ground plane.

2. The transmission line attenuator of claim 1 wherein said attenuator includes another distributed series resistance extending along a third section of the attenuator, the characteristic impedance of said third section decreasing in said one direction along that section with the decrease in characteristic impedance being a function of the series resistance of the third section and, said second section being connected between said first and third sections in said transmission line.

3. The transmission line attenuator of claim 1 wherein said attenuator includes another distributed parallel resistance being formed by a lossy dielectric of uniform resistivity extending along a third section of the attenuator, the characteristic impedance of said third section increasing in said one direction along the third section with the increase in characteristic impedance being a function of the shunt resistance of the third section and, said first section being connected in said transmission line attenuator between said second and third sections.

4. The transmission line attenuator of claim 1 wherein the distributed-series resistance of said first section comprises a resistive element having a constant resistance per unit length and the characteristic impedance of said first section is decreased linearly along the length of the section.

5. The transmission line attenuator of claim 2 wherein the dielectric constant of the transmission line attenuator is the same throughout said first and third sections and said increases and decreases in the characteristic impedance of the line are provided by changes in geometry of the transmission line attenuator.

6. The transmission line attenuator of claim 1 wherein the distributed shunt resistance of said second section comprises a dielectric element having a constant resistance per unit length and the characteristic impedance of said first section is increased at an increasing rate along the length of the section.

7. An unsymmetrical transmission line attenuator comprising:

a. a first section of said transmission line attenuator including a distributed-series resistance;

b. a second section of said transmission line attenuator including a distributed-shunt resistance being formed by a lossy dielectric of uniform resistivity;

c. a third section of said transmission line attenuator including a distributed-series resistance;

d. said second section being connected between said first and third sections;

e. the characteristic impedance of said transmission line attenuator being equal to a predetermined value at the beginning of said first section and equal to the same predetermined value at the end of said third section; and

f. the characteristic impedance varying between a minimum value of characteristic impedance at the end of said first section and a maximum value of characteristic impedance at the end of said section;

g. a conductor for propagating an electrical signal;

h. a ground plane.

8. The transmission line attenuator of claim 7 wherein the characteristic impedance of said transmission line attenuator decreases continuously in one direction along the length of said first and third sections and increases continuously in said one direction along the length of said second section.

9. An unsymmetrical transmission line attenuator comprising:

a. a first section of said transmission line attenuator including a distributed-shunt resistance being formed by a lossy dielectric of uniform resistivity;

b. a second section of said transmission line attenuator including a distributed-series resistance;

c. a third section of said transmission line attenuator including a distributed-shunt resistance being formed by a lossy dielectric of uniform resistivity;

d. said second section being connected between said first and third sections;

e. the characteristic impedance of said transmission line attenuator being equal to a predetermined value at the beginning of said first section and equal to the same predetermined value at the end of said third section; and

f. the characteristic impedance varying between a maximum value at the end of said first section and a maximum value of characteristic impedance at the end of said second section;

g. a conductor for propagating an electrical signal;

h. a ground plane.

10. The transmission line attenuator of claim 9 wherein the characteristic impedance of said transmission line attenuator increases continuously in one direction along the length of said first and third sections and decreases continuously in said one direction along the length of said second section.

11. An unsymmetrical transmission line attenuator including:

a. first and second sections;

b. said first section including a distributed-series resistance and said second section including a distributed-parallel resistance;

c. said first section of said transmission line attenuator including a length of line having a conducting ground element, a resistive transmission element, and a dielectric separating the ground element and the transmission element along the length of the section; and

d. the geometric configuration of said first section being such that

$$Z_x = Z_o - \int_0^x r\,dl \text{ for } x \geq 0$$

where

Z_x = the characteristic impedance of the line at any point along the length of the section

Z_0 = the characteristic impedance of the line at the beginning of the section

$r=f(x)$ = the resistance per unit length of the resistive transmission element;

e. said second section of said transmission line including a conducting ground element, a conducting transmission element and a conducting dielectric separating said ground element and said transmission element and connected to both elements along the length of said section; and

the geometric configuration of said second section being such that

$$Y_x = Y_o - \int_0^x g\,dl \text{ for } x > 0$$

where

Y_x = the characteristic admittance of the line at any point along the length of the section

Y_0 = the characteristic admittance of the line at the beginning of the section

$g=f(x)$ = the conductance per unit length of the conducting dielectric.

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