The present invention relates to roofs and specifically to a method and means for forming a roof structure wherein the structural framing may be constructed or fabricated in a shop remote from the site of use for the roof structure; a roof structure which economically spans large areas and eliminates the need for separate ceilings as well as permitting the use of various materials for its fabrication.

During recent years, the most prominent advance in the science of spanning roofs over large areas without the use of interior columns has been the development of thin shell construction, usually monolithic concrete.

I have found that a roof structure of the hyperbolic paraboloid type has striking architectural characteristics and offers many advantages both as to cost of such a structure and the use of a time saving space frame without sacrificing any of the desired architectural flexibility.

As a rule, hyperbolic paraboloid type roof structures have been expensive to produce as it required the forming of a roof slab of double curvature which characterizes this geometrical shape.

An object of my invention is to provide a roof system which solves the forming problem in a simple, easy and inexpensive manner and which places the roof structure of my invention in a favorable competitive position.

One of the basic concepts of my invention is the provision of a light lattice member support system wherein the individual lattice members in their final position in the structure conform closely to the lines of principal stress. Further the lattice members may be shop fabricated in a flat or single plane condition, collapsed for shipment to the site, and then expanded into a doubly curved shape between previously erected boundary members. Since the lattice members are designed to carry the principal stresses directly to the boundary members, the remaining roof elements supported from these lattice members are of secondary stress importance, thus permitting their selection from a wide variety of materials.

A further object of my invention is to provide a novel system for securing lattice work of the roof structure to boundary members.

A further object is to simplify the fabrication of a roof structure and the elimination of the influence of fixity on the structure when secured to boundary members.

A further object is the provision of a skeleton frame for an interior structure capable of having sheeting attached thereto and used as a base for waterproof roofing.

A further object is the provision of a structure which is adaptable to many uses and which is so formed and constructed as to eliminate the task of detailing and fabricating numerous special conventional connections thereto.

A further object is the provision of a roof structure requiring a minimum of sheathing.

A further object is the provision of a roof structure which is particularly adaptable to school classroom buildings, commercial buildings and industrial buildings and which may also be used for convention and sports halls and arenas which require roof systems spanning large distances so as to eliminate objectionable interior columns.

Primarily the present invention relates broadly to roof shells of double curvature which are generally known as an ellipse, circle, catenary, and parabola. I chose, however, for the present invention to designate my roof structure as substantially a hyperbolic paraboloid type of shell. It is a known fact that double curved concrete shells with edges stiffened by arches or ribs have great strength due to their ability to carry any continuous load principally by direct stresses, that is, by axial compression or tension. Stresses for thin shells are relatively small compared to the compressive strength of concrete and while localized bending may occur near the edges of a shell of this character due to displacement of the edge members, for the most part the shell is free of flexural forces. The direct forces acting in a doubly curved shell are easily determined by the cartesian system. An anticlastic shell such as a hyperbolic paraboloid may be considered either as a surface of translation or a warped parallelogram. Surface of translation is generated by translating or moving a vertical parabola having an upward curvature over another parabola having a downward curvature, the parabola of translation lying in a plane perpendicular to the first or vertical parabola but moving parallel to it. The surface may also be generated by moving along one boundary a straight line that remains parallel to the plane of the intersecting boundary member at all times but pivots so as to slide along the opposite boundary member. In other words, the paraboloid may be considered as generated by a principal parabola that moves parallel to itself along an inverse principal parabola. Stresses in such a structure are of easy determination for the reason that a hyperbolic paraboloid shell transfers loads to supports almost entirely by direct forces so that all material in the cross section of the shell is uniformly stressed. To those interested in a mathematical consideration of stresses in hyperbolic paraboloid concrete shells, reference is made to many excellent articles on the subject as well as in text books and specifically an article published by the Portland Cement Association entitled Elementary Analysis of Hyperbolic Paraboloid Concrete Shells; to the Journal of the American Concrete Institute, vol. 26, No. 5, January 1955, an article entitled Structural Application of Hyperbolic Paraboloidal Shells, page 397; and to an article appearing in vol. 82, No. ST 5, September 1956, of the Journal of the Structural Division of the American Society of Civil Engineers, entitled Hyperbolic Paraboloids and Other Shells of double Curvature.

An anticlastic concrete shell with stiffened edges carries any continuous load by direct stresses, that is, by axial compression or tension and for the most part the shell is free of flexural forces. Hence, it is that the edge members need not be capable of resisting lateral forces and the direct forces acting on the anticlastic shell are obtained directly from a consideration of statics alone.

In the drawings:

FIGURE 1 is a fragmentary perspective view showing four quadrants of a roof structure and embodying the invention;

FIGURE 2 is a plan view of lattice work in expanded position for one quadrant of a roof structure;
FIGURE 3 shows the lattice work in FIGURE 2 in collapsed condition; FIGURE 4 is a plan view of a quadrant of the roof skeleton structure showing boundary members enclosing the lattice members such as shown in FIGURE 2; FIGURE 5 is a view taken substantially on the line 5--5 of FIGURE 4; FIGURE 6 is a view looking in the direction of the arrow 6 of FIGURE 4; FIGURE 7 is an enlarged fragmentary, sectional view on the line 7--7 of FIGURE 1, of a connection which may be used between the boundary members and the lattice members; FIGURE 8 is a fragmentary sectional view taken on the line 8--8 of FIGURE 7; FIGURE 9 is an enlarged elevation of one of the members used in the connection shown in FIGURES 7 and 8; FIGURE 10 is a fragmentary, partially sectional view showing a pin connection between two channel type lattice members; FIGURE 11 is an enlarged fragmentary, sectional view taken on the line 11--11 of FIGURE 1 showing the lattice construction provided with a ceiling and with roofing material; FIGURE 12 is a diagrammatic view of a single quadrant of a roof structure embodying the invention, and showing mathematical notations for use in the design of said quadrant; FIGURE 13 is a further diagrammatic view on line 13--13 of FIGURE 15, the view containing notations; FIGURE 14 is an outline plan view of one of the quadrants of the roof structure; and, FIGURE 15 is a section on the line 15--15 of FIGURE 14, the said view having mathematical notations thereon.

Referring now to the drawings and specifically to FIGURES 2 and 3, I have shown lattice members designated as 1 and 2, the lattice members, of which there are a plurality, are in overlapped or in crossed relationship and pinned together for movement at spaced points as shown at 3. The pins may take the form of rivets, see FIGURE 10, and the lattice members 1 and 2 may be of channel form as illustrated in FIGURE 10, the web portions of said members being in juxtaposition. While the lattice members may be collapsed as shown in FIGURE 3 so as to occupy a small space, the lattice members when expanded, as shown in FIGURE 2 may assume various configurations. Thus the spacing between the lattice members may substantially form squares or diamonds, as is obvious. The lattice members are adapted to be confined by boundary members such as shown in FIGURE 4, the members of which are designated as 4, 5, 6 and 7. In FIGURE 4 the boundary or edge beams, or members may take the sectional form shown in FIGURE 7 which is to say, beams of channel form.

Assuming that I have chosen a quadrant for a roof structure of square form so far as the boundary members are concerned, the lattice members 1 and 2 are placed within said boundary members in such a manner that the lattice members assume, together with the boundary members, the form of a hyperbolic paraboloid or substantially so. The number of lattice members and their points of connection with the boundary members must be determined from a consideration of the stresses to be encountered and the size of the roof structure. The boundary members are laid out to suit the desired architectural concept and the boundary members transfer the load or forces from the lattice work to the columns or other load support elements at the vertices.

In determining the shape of the lattice members, a circular arc is chosen such as is shown at 8 in FIGURE 4 which passes through the points 9, 10, and 11. Where the points 9 and 10 are intersections or vertices between boundary members 6 and 7, and 4 and 5 respectively, the point 11 on said circular arc lies in the plane bisecting the angle between the planes determined by 9, 12 and 10; and 9, 10 and 13 respectively, so the point 11 lies midway between the diagonal 9--10 and the diagonal 12--13. This arc is then moved parallel to itself toward points 10 and 12, the arc resting at all times on the boundary members generated is substantially that of a hyperbolic paraboloid since a relatively flat circular arc differs minutely from a paraboloid of the same span. A selected module is then laid off along the generating arc and where these points intersect the boundary members, the opposing lattice members 2 are located.

If now the two elevations shown in FIGURES 5 and 6 are examined, it will be seen that the lattice members 2 are in compression and bow upwardly while the tension members 1 bow downwardly thus satisfying the hyperbolic paraboloid stress requirements, with the lattice members 1 and 2 lying along the line of principal stresses.

Since the compression members of lattice 2 intersect the tension members of the lattice 1 at equally spaced distances and are pin connected as by rivets 3 shown in FIGURE 10, the lattice members may be prefabricated in flat condition and collapsed for shipment, as shown in FIGURE 5.

In erecting the structure, the boundary members 4, 5, 6 and 7 of one quadrant are assembled and shored either in their final position or at a temporarily lower level to facilitate the placing of the lattice in position. In FIGURE 1 the lattice has been expanded into position and placed on top of the boundary members 4 to 7 inclusive for one quadrant, with the boundary members secured at the vertices to column supports 14. It will be noted in FIGURE 1 that the four quadrants are so arranged as to be symmetrical about any two intersecting boundary members. Since the members of the lattice work are relatively flat, the lattice may be sprung downwardly for connection with the boundary members. The connections to the boundary members may take the form shown in FIGURES 7, 8 and 9 and wherein the web 15 of each boundary member is provided with a series of spaced transverse bores 16 formed to receive a flanged disk 17, the said disk having a central slot 18, the edges bonding the said slot being on an arc or lip shaped as shown in FIGURE 7. A clip 19 of U-shape is passed through the slot 18, the lip portion 20 of which is adapted to confine a pin 21 with the pin engaging one or more washers or shims 22 on the inner surface of disk 17. The legs of the said clip are riveted to and embrace overlapped ends of the lattice members 1 and 2. As shown in FIGURE 5, the slot 18 is of elongated form conforming in size to the width of the clip. The construction is such that the clip and the disk may be rotated in a vertical plane so that the angle of intersection of the lattice members with the boundary members may be locked in final position by driving the pin 21 into the bight portion 20 of the clip. This connection provides an unlimited variation in angularity and eliminates the task of detailing and fabricating numerous special conventional connections, each differing slightly from each other.

One of the advantages of this invention is that a wide variety of roof constructions can be fabricated economically. Once the skeleton framework is in place, such conventional roof constructions as light steel deck, wood, or plywood sheathing can be attached and used as a base for the waterproof roofing; or permanent or temporary forming can be suspended below the lattice work and monolithic roof of gypsum or Portland cement concrete can be poured in place without the necessity of complicated shoring.

Furthermore, one advantage of the permanent forming is illustrated by using an expanded metal lath below the lattice, plastering the ceiling with at least a scratch coat and using this for the form of a poured gypsum concrete.
roof. This gives a light structure, provides good insulation and results in economy. This construction is illustrated in FIGURE 11, wherein the roof coat is shown at 23, the metal lath at 24 and plaster at 25.

The operation, use and advantages of the invention are as follows.

In order to understand the invention and the particular design by which quadrants of the roof structure are fabricated and the stresses determined, I might say initially that the number of lattice members are determined much in the same manner that the spacing of rafters in a building are determined, in view of the material to be used; i.e., plywood, concrete, etc. Also, it is convenient to assume that the rise of the roof structure is usually about one-third of the dimension of a boundary member. Further, the lattice work members lie on or close to the line of principal stress.

My roof structure and its fabrication approximates a true hyperbolic paraboloid calculation by simple trigonometry as may be readily demonstrated for a given plan, dimensions and height, the radius and length of generating arc. In the general case which I have given in FIGURE 14, I show the basic quadrant as square in plane, or, the scheme may be applied equally well to plans which are not square; i.e., rectangular. For instance, referring to FIGURES 13, 14 and 15, if the sides a and b and the rise h are given, then from the diagrams and the figure then given: sides a, b and rise h.

\[ e = a \sin 45^\circ = b \sin 45^\circ \]

\[ \tan \alpha = h/2c = \tan \beta = h/c = : \alpha \text{ and } \beta \text{ are known} \]

\[ \theta = 180^\circ - \beta \]

\[ \theta /2 = 90 - \beta /2 \]

\[ \phi = 180 - \phi /2 - \alpha \]

\[ x = \sin \alpha \]

\[ x /2 = C \sin \alpha \]

\[ 2 \sin \phi \]

\[ \text{rad. } R = (x/2)^2 + C^2 \]

\[ 2(x/2) \]

\[ \sin \phi /2 = C /R \]

\[ \text{Length of arc } L = 2 \pi R \frac{\phi}{360} \]

In the general equations, it may be shown that for a specific case, the rise at any point along the vertical projection of the general generating arc for the hyperbolic paraboloid shape very closely approximates the values just given in the equations.

In order to give a concrete example of the design for a single quadrant of a roof structure, reference is made to FIGURE 12 and to the notations appearing thereon, this being a single quadrant and the design to be considered is a 30' x 30' roof; i.e., the roof would have four of the quadrants of the type shown in FIGURE 12, each boundary member being substantially 15' in length. The following defines the constants, roof loading, the boundary member loading, as well as other design factors. The following notes are given.

\[ DL = \text{Dead load (in weight of structure in lbs./sq. ft. or } 7.5') \]

\[ LL = \text{Live load (such as snow)} \]

\[ W = \text{Total load per foot of length on lattice member (spaced at 2')} \]

\[ H = \text{Axial force on single lattice member} \]

\[ V = \text{Increment of axial force transferred to boundary member at one point of intersection of the two lattice members} \]

\[ A = \text{Cross sectional area of lattice member selected (in square inches)} \]

\[ F_a = \text{Allowable compression stress on lattice member for ratio of length=spacing between members } \times 0.9 \] (for effect of continuity) to least radius of gyration (\( a/b \)).

For this ratio

\[ \frac{I}{r_a} = \frac{24 \times 9}{0.134} = 161. \]

\[ \text{Allowable compression stress } = 5870\# / \text{sq. in.} \]

\[ \square' = \text{Square foot} \]

\[ \square'' = \text{Square inches} \]

\[ H = \text{Axial force on horizontal boundary member} \]

\[ T = \text{Axial force on sloping boundary member} \]

\[ P = \text{Total load (LL+DL) on quadrant} \]

\[ L = \text{Length of corner column (assumed at 10 ft.)} \]

\[ f_a = \text{Actual compression stress in pounds/sq. inch } = f_c \]

\[ 5' C \]

\[ 5'' Channel \]

\[ 6.7' \text{ lbs./lin. ft.} \]

Constants:

\[ \tan \alpha = 0.333 \]

\[ \sin \alpha = 0.316 \]

\[ \cos \alpha = 0.949 \]

\[ a/b = 15' \]

\[ h = 5' \]

\[ K = h/ab = 1/45 \]

Roof Loading:

\[ \text{Roofing} = 6 \text{ lbs./} \square'' \]

\[ 21/2'' Py. \]

\[ \text{Ac. Plaster} = 11 \]

\[ \text{Struct. steel} = 8 \]

DL = 30

\[ \text{LL} = 20 \]

DL+LL = 50

Boundary member loading:

\[ \text{Contrib. area} = 56 \]

\[ \text{DL} = 30 \]

\[ \text{LL} = 20 \]

\[ \text{DL+LL} = 50 \]

FOR LATTICE MEMBERS @ 2'-0'' O.C.—USE BAR SIZE CHANNEL 2 x 4 x 9/16 @ 1.68''

\[ W = 50 \times 2 = 100\# /'' \]

\[ H = 100 \times 45 \]

\[ 2K = 2 \]

\[ 2250\# \]

\[ V = 2 \times 2250 \times \sqrt{2} = 3180\# \]

\[ 2 \times 2250 \times \sqrt{2} = 3180\# \]

\[ 1 = 21.6 \]

\[ r_2 = 0.134' \]

\[ l = 24 \times 0.9 = 21.6' \]

\[ f_a = 5870\# / \square'' \]

\[ 2250 \# \]

\[ 4600\# / \square'' \]

FOR BOUNDARY MEMBERS IN COMPRESSION (CORNER COLUMNS)

\[ N = 15 \times 125 = 1800\# \text{ horiz.} \]

\[ T = 16,900 \]

\[ 0.949 = 17,800\# \text{ on slope} \]

Check:

\[ \text{Total load } = \text{15'x15'x50 = 11,250} \]

\[ P = 2T \times \sin \alpha = 2 \times 17,800 \times 0.316 = 11,250\# \]

Use 5 C 6.7:

\[ L = 15.8'' \]

\[ A = 1.95\# / \square '' \]

\[ r_2 = 1.95'' \]

\[ l = 15.8 \times 12 = 189\# / \square '' \]

\[ f_a = 12,440\# / \square '' \]

\[ 17,800 \]

\[ 1.95 \]

\[ 9130\# / \square'' \]

Tension member:

\[ \text{Min. } A_x = 16,900 = 0.84\# /'' \]
10' corner column—Use 3" φ std. pipe

\[
L = 10' \quad A = 2.23\text{"} \quad r = 1.16\text{"}
\]
\[
\frac{L}{r} = \frac{10 \times 12}{1.16} = 103, \quad F_c = 11,860\text{#/"}^2 \quad \text{O.K.}
\]
\[
st = \frac{11,250}{2.23} = 5050\text{#/"}^2 \quad \text{O.K.}
\]

FOR BOUNDARY MEMBERS IN TENSION (RIDGE COLUMNS)

Say minimum boundary member 5 C 6.7 or same as for compression member—Use 5 C 6.7

Ridge columns—Use 4" φ std. pipe

\[
L = 15' \quad A = 3.17\text{"} \quad r = 1.51\text{"}
\]
\[
\frac{L}{r} = \frac{15 \times 12}{1.51} = 119, \quad F_c = 10,130\text{#/"}^2 \quad \text{O.K.}
\]
\[
st = \frac{11,250}{3.17} = 3550\text{#/"}^2 \quad \text{O.K.}
\]

10' corner columns (for unbalanced LL only)—Use 3" φ std. pipe

\[
L = 10' \quad r = 1.16\text{"}
\]
\[
\frac{L}{r} = \frac{10 \times 12}{1.16} = 103, \quad \text{O.K.}
\]

MATERIAL TAKE OFF FOR 30' x 30' ROOF

(a) For Boundary Members in Compression (Corner Columns)

Lbs.

Lattice members, 1.68×900 1510
Boundary members, 6.7×60×4 1610
Tension members, 2.9×30×4 350
Corner columns, 7.6×10×4 300

Sub total 3770
Details 15% 570

Total wt. 4340
Unit wt. 4340=4.8#/"^2
900

(b) For Boundary Members in Tension (Ridge Columns)

Lbs.

Lattice members, 1.68×900 1510
Boundary members, 6.7×60×4 1610
Ridge columns, 10.8×15×4 650
Corner columns, 7.6×10×4 300

Sub total 4070
Details 15% 610

Total wt. 4680
Unit wt. 4680=5.2#/"^2
900

It is thought that those skilled in the art to which this invention pertains, will fully understand the design features of the present roof structure as well as the mathematics involved in its design. The present roof structure has been thoroughly tested in actual use and has been found to perform in a satisfactory manner.

I claim:

1. A skeleton frame structure of hyperbolic paraboloid configuration including: four straight, rigid boundary members, two of which are stressed axially in compression and the other two are stressed in tension, said boundary members being joined at their ends to enclose an area which when projected onto a horizontal plane assumes approximately the shape of a parallelogram, two of said boundary members lying in one plane and the other two of said boundary members lying in a second plane which intersects the first plane along a diagonal of the area enclosed, and a lattice secured to all four of the boundary members, and formed of structural tension members and structural compression members lying within the confines of the boundary members; the actual lengths of each of said tension members and of said compression members being greater than its respective projected lengths between the boundary members, each of the tension members being arranged approximately parallel to said diagonal and being curved in one plane and positioned with the concave side facing upward, each of said compression members being approximately parallel to the second diagonal of the area enclosed and being curved in one plane and positioned with the convex side facing upward, each of the tension members and the compression members being so located that the respective lines of force carried through them intersect approximately at the lines of force acting through the axes of the boundary members, and pivot means joining said tension members and said compression members at substantially all points of intersection within the confines of the four boundary members so that the lattice formed by said tension and compression members may be collapsed and later expanded into position between said boundary members, the tension and the compression members each being subject to equal direct stresses with no components normal to each other at points of intersection, whereby neither of these members tends to impose a stress on the other member at such points of intersection.

2. The structure as set forth in claim 1, said lattice being joined to the boundary members by rotative connections thus eliminating the task of detailing and fabricating numerous special conventional connections, each differing slightly from each other being a pivotal connection whereby the lattice formed by said tension and compression members may be collapsed and later expanded into position between said boundary members.

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