Feb. 19, 1963 J. HERMAN 3,078,462

ONE-TURN LOOP ANTENNA

Filed July 18, 1958 2 Sheets-Sheet 1

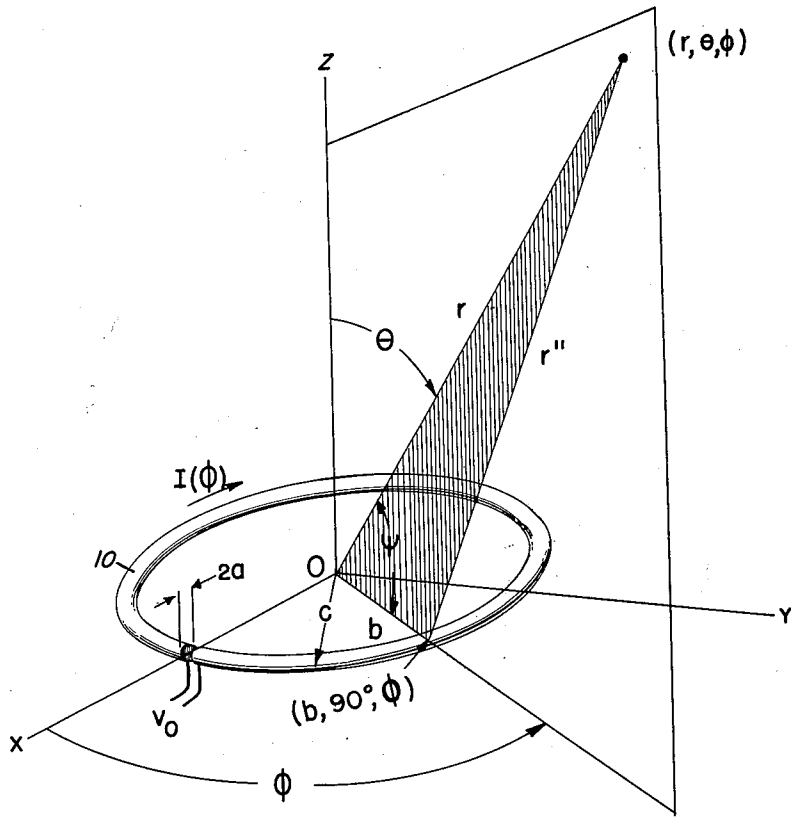



FIG. 1

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Filed July 18, 1958 2 Sheets-Sheet 2

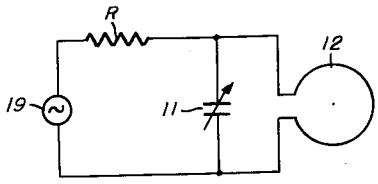



FIG. 2

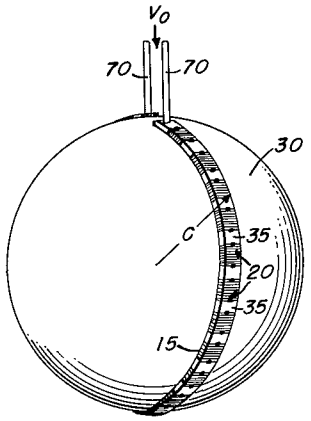



FIG. 3

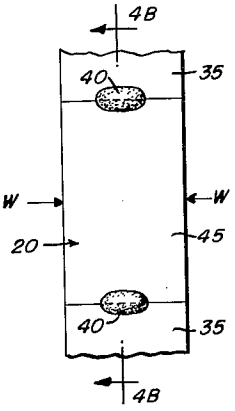



FIG. 4A

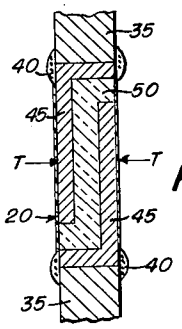



FIG. 4B

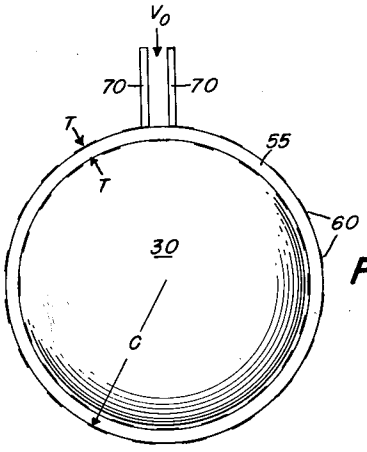



FIG. 5A

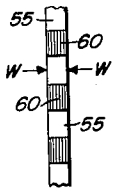



FIG. 5B

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3,078,462

# ONE-TURN LOOP ANTENNA

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Filed July 18, 1958, Ser. No. 749,570
2 Claims. (Cl. 343—744)
(Granted under Title 35, U.S. Code (1952), sec. 266)

The invention described herein may be manufactured and used by or for the Government for governmental purposes without the payment to me of any royalty thereon.

This invention relates to loop antennas in general, and more particularly to a one-turn loop antenna having specially chosen distributed and/or lumped impedances inserted in the loop.

Although loop antennas have been known for some time they have received relatively little attention in recent days. The chief reason for this is perhaps because of the absence of a usable theoretical analysis of the operational characteristics of the loop antenna. Without such a usable theoretical analysis, prediction of the operation of the loop, or the effects of various modifications thereof are well-nigh impossible. Any exploitation of the loop antenna thus never got off the ground. As a result, the development of loop antennas today is not very much more advanced than it was twenty years ago.

I have developed a complete mathematical theory for the characteristics of a one-turn, thin-wire loop antenna in air, and with a finite spherical core of material other than air. The theory holds for any size loop (as long as the loop can be considered a thin-wire loop) for any frequency, and for any core material, including air, whose characteristics are homogeneous and isotropic. Losses in the core are incorporated in the solution provided the volume current effect is negligible relative to the conduction current effect of the antenna, which is usually the case for practical cores.

Expressions have been derived from this theory for the input impedance, input power, radiated power, and radiation efficiency; and a relatively simple, straightforward technique has been developed for using these expressions to predict the effect of various modifications on the loop. As a result, I have discovered that very important new results can be obtained by incorporating in the loop specially chosen lumped and/or distributed impedances. These results include control of the antenna input impedance, control of the antenna radiation pattern, optimization of the radiation efficiency of a ferrite core loop antenna, and tailoring of the antenna characteristics.

As it is well known in the art such control of the characteristics of a loop antenna was heretofore not possible. This is because for a given loop geometry at a given frequency, the input impedance, radiation efficiency and radiation pattern were fixed and out of the control of the designer. Any attempted modifications of the loop were made on a substantially trial and error basis. These trial and error modifications met with little success because the chances of achieving the combination of values which would produce a desired result were very small.

It is the broad object of this invention, therefore, to provide means and methods for modifying a loop antenna so as to achieve predictable control over the antenna characteristics. This control is obtained where the loop antenna is used in air, or with a finite spherical core of material other than air.

It is another object to provide a one-turn loop antenna having predictable control of its radiation pattern.

A further object of this invention is to provide a one-turn loop antenna having a uniform radiation pattern in the plane of the loop, even where the loop perimeter approaches or exceeds a half wavelength.



Still another object is to provide a one-turn loop antenna having a controllable input impedance.

An additional object of this invention is to provide a one-turn loop antenna having a spherical ferrite core whose radiation efficiency is optimized.

A further object is to provide practical embodiments in accordance with the invention.

The above objects are accomplished by inserting distributed and/or lumped impedances in the antenna loop, and choosing their values in accordance with the derived equations to provide a desired set of antenna characteristics.

For the particular case of a uniformly distributed reactance inserted in the loop, it has been discovered that the theory can be applied in an amazingly simple and straightforward manner so that the solution of actual antenna problems becomes a practical reality. This simplicity results because the loop antenna has been found to act as a transmission line which supports an infinite number of modes. By inserting a properly chosen uniformly distributed reactance in the loop, a selected mode is made predominant, the antenna characteristics for the selected mode then becoming the characteristics of the antenna. This leads to a very great simplification in the design of loop antennas using the derived equations.

The specific nature of the invention, as well as other objects, uses and advantages thereof, will clearly appear from the following description and from the accompanying drawing, in which:

FIGURE 1 is a pictorial representation of the spherical geometry employed in formulating the theory of the one-turn thin-wire loop antenna.

FIGURE 2 is a circuit diagram of a circuit for adjusting the resistive input impedance of a one-turn loop antenna having a uniformly distributed reactance in the loop so as to match the output resistance of a generator.

FIGURE 3 is a perspective view of a practical embodiment of the invention wherein closely spaced lumped reactances around the loop are used to simulate a uniformly distributed reactance.

FIGURE 4A is an enlarged view of one portion of the loop showing a lumped capacitance in detail. FIGURE 4B is a cross-sectional view taken along 4B—4B in FIGURE 4A.

FIGURE 5A is a front view of another practical embodiment of the invention. FIGURE 5B is an end view of a portion of the embodiment of FIGURE 5A.

To explain how to make and use the invention, it is desirable that the solution and formulation of the basic theory first be described. In so doing, the equations needed for the construction of practical embodiments in accordance with the invention will be presented, and actual examples of their use to solve practical antenna problems will be demonstrated.

## BASIC SOLUTION AND FORMULATION OF THE THEORY FOR A THIN-WIRE, ONE-TURN LOOP IN AIR

In formulating the theory of the one-turn thin-wire loop antenna, the spherical geometry shown in FIGURE 1 was used. The wire loop **10** lies in the X—Y plane and the source of power of peak voltage $V_0$ is applied at $\phi=0°$. The radius to the extreme edge of the loop **10** is $c$, the wire radius is $a$, and $b$ is the radius from the origin O to the center of the wire.

In deriving the mathematical solution, the current distribution $I(\phi)$ on the loop **10** was assumed to consist of

a Fourier series in the variable $\phi$ with unknown current coefficients $a_0$, $a_k$ and $b_k$ as given below:

$$I(\phi)=a_0+\sum_{k=1}^{\infty}a_k\cos k\phi+b_k\sin k\phi \quad (1)$$

The solution for the fields induced by this current flow in the region outside of a sphere containing the loop **10** was then formulated using Maxwell's equations and the general spherical wave equation. It was found that these fields consist of three independent families of waves, one TE family, and two TM families, TM$^a$ and TM$^b$. These TE and TM families have the following form:

TE

$$\sum_{\substack{n\geq 1=k-1,\\ k+1,\ldots}}^{\infty}\ \sum_{\substack{k=0,1,\\ 2,\ldots}}^{\infty}$$

$$rH_{r(nk)}=-jA_{nk}n(n+1)P_n^k(\cos\theta)\frac{\hat{H}_n(\beta r)}{br}[a_k\cos k\phi+b_k\sin k\phi]$$

$$rE_{\theta(nk)}=n^*A_{nk}\frac{kP_n^k(\cos\theta)}{\sin\theta}\hat{H}_n(\beta r)[a_k\sin k\phi+b_k\cos k\phi]$$

$$rH_{\phi(nk)}=jA_{nk}\frac{kP_n^k(\cos\theta)}{\sin\theta}\hat{H}'_{n1}(\beta r)[a_k\sin k\phi-b_k\cos k\phi]$$

$$rE_{\phi(nk)}=n^*A_{nk}\frac{d}{d\theta}[P_n^k(\cos\theta)]\hat{H}_n(\beta r)[a_k\cos k\phi+b_k\sin k\phi]$$

$$rH_{\theta(nk)}=-jA_{nk}\frac{d}{d\theta}[P_n^k(\cos\theta)]\hat{H}_n(\beta r)[a_k\cos k\phi+b_k\sin k\phi]$$

TM$^a$

$$\sum_{\substack{n\geq 0=k-1,\\ k+1,\ldots}}^{\infty}\ \sum_{\substack{k=1,2,\\ 3,\ldots}}^{\infty}$$

$$rE_{r(nk)}=n^*F_{nk}(n+1)(n+2)P_{n+1}^k(\cos\theta)\frac{\hat{H}_{n+1}(\beta r)}{\beta r}[a_k\sin k\phi-b_k\cos k\phi]$$

$$rE_{\theta(nk)}=n^*F_{nk}\frac{d}{d\theta}[P_{n+1}^k(\cos\theta)]H_{n+1}^1(\beta r)[a_k\sin k\phi-b_k\cos k\phi]$$

$$rH_{\phi(nk)}=-jF_{nk}\frac{d}{d\theta}[P_{n+1}^k(\cos\theta)]\hat{H}_{n+1}(\beta r)[a_k\sin k\phi-b_k\cos k\phi]$$

$$rE_{\phi(nk)}=*F_{nk}k\frac{P_{n+1}^k(\cos\theta)}{\sin\theta}H'_{n+1}(\beta r)[a_k\cos k\phi+b_k\sin k\phi]$$

$$rH_{\theta(nk)}=jF_{nk}k\frac{P_{n+1}^k(\cos\theta)}{\sin\theta}\hat{H}_{n+1}(\beta r)[a_k\cos k\phi+b_k\sin k\phi]$$

TM$^b$

$$\sum_{\substack{n\geq k-1\\ k+1,\ldots}}^{\infty}\ \sum_{\substack{k=1,2,\\ 3,\ldots}}^{\infty}$$

$$rE_{r(nk)}=n^*G_{nk}n(n-1)P_{n-1}^k(\cos\theta)\frac{\hat{H}_{n-1}(\beta r)}{\beta r}[a_k\sin k\phi-b_k\cos k\phi]$$



$$rE_{\theta(nk)}=n^*G_{nk}\frac{d}{d\theta}[P_{n-1}^k(\cos\theta)]\hat{H}'_{n-1}(\beta r)[a_k\sin k\phi-b_k\cos k\phi]$$

$$rH_{\phi(nk)}=-jG_{nk}\frac{d}{d\theta}[P_{n-1}^k(\cos\theta)]\hat{H}_{n-1}(\beta r)[a_k\sin k\phi-b_k\cos k\phi]$$

$$rE_{\phi(nk)}=n^*G_{nk}k\frac{P_{n-1}^k\cos\theta)}{\sin\theta}\hat{H}'_{n-1}(\beta r)[a_k\cos k\phi+b_k\sin k\phi]$$

$$rH_{\theta(nk)}=jG_{nk}k\frac{P_{n-1}^k\cos\theta)}{\sin\theta}\hat{H}_{n-1}(\beta r)[a_k\cos k\phi+b_k\sin k\phi]$$

It can be seen that the expressions are in the forms of a product of functions of the three coordinates $\theta$, $\phi$, and $r$. The $\theta$ dependence is that of associated Legendre polynomials $P_n^m$ with argument $\cos\theta$, the $r$ dependence is in the form of spherical Hankel functions of the second kind $\hat{H}_n$ with argument $\beta r$, and the $\phi$ dependence is in the form of trigonometric functions $\cos k\phi$ and $\sin k\phi$. The $A_{nk}$, $F_{nk}$, and $G_{nk}$ coefficients are constants while $n^*$ is the complex intrinsic impedance of the medium. The electric field component is represented by E and the magnetic field component by H, the subscripts corresponding to the geometry of FIGURE 1. For any field component, the total field is the sum of the contribution from each mode of each wave family.

As in any antenna problem, the solution is now obtained by determining the coefficients, $a_0$, $a_k$, and $b_k$ in terms of known values, such as the applied voltage $V_a$, the frequency $f$, the internal impedance function $Z^i(\phi)$ of the loop, and the loop geometry shown in FIGURE 1. In formulating the solution, the induced tangential electric field $E_\phi{}^i$ at the extreme outer edge of the loop **10** ($r=c$, $\theta=90°$) is first calculated from the TM and TE field equations. This relationship for $E_\phi{}^i$ is as follows:

$rE_{\theta(nk)}$
$rH_{\theta(nk)}$
TM$^a$

$$[a_k\sin k\phi-b_k\cos k\phi]$$

$rE_{\theta(nk)}$

$$E_\phi{}^i=-\frac{n^*K_0}{2c}a_0-\sum_{k=1}^{\infty}\frac{n^*K_k}{c}[a_k\cos k\phi+b_k\sin k\phi] \quad (2)$$

$$K_0=\sum_{n=1,3,5\ldots}^{\infty}R_{n0}\hat{H}_n(\beta c) \quad (3)$$

$$R_{n0}=\frac{2n+1}{n(n+1)}[P_n{}^1(0)]^2\hat{J}_n(\beta b) \quad (4)$$

$$K_k=\sum_{n=k+1,k+3,\ldots}^{\infty}R_{nk}\hat{H}_n(\beta c)+\sum_{n=k-1,k+1}^{\infty}S_{nk}H_{n+1}^1(\beta c)+\sum_{n=k+1,k+3}^{\infty}T_{nk}\hat{H}_{n-1}^1(\beta c) \quad (5)$$

$$R_{nk}=\frac{(2n+1)(n+k)(n-k+1)}{2n(n+1)}\frac{(n-k+1)!}{(n+k-1)!}[P_n^{k-1}(0)]^2\hat{J}_n(\beta b) \quad (6)$$

$$S_{nk}=\frac{k^2(n+k)}{2(n+1)(n+k+1)}\frac{(n-k+1)!}{(n+k-1)!}[P_n^{k-1}(0)]^2\hat{J}_n(\beta b) \quad (7)$$

$$T_{nk}=-\frac{k^2(n-k+1)}{2n(n-k)}\frac{(n-k+1)!}{(n+k-1)!}[P_n^{k-1}(0)]^2\hat{J}_n(\beta b) \quad (8)$$

The terms used in the above equations may be defined as follows:

$>n^*$=complex intrinsic impedance of the medium

$$=n_0\sqrt{\frac{K_u'-jK_u''}{K_\epsilon'-jK_\epsilon''}} \quad (8a)$$

where:

$n_0$=intrinsic impedance of free space=120 ohms.

$K_u'$ and $K_\epsilon'$ are the non-dimensional real components of the permeability and dielectric constant, respectively (for free space their value is unity).

$K_u''$ and $K_\epsilon''$ are the non-dimensional imaginary components of the permeability and dielectric loss factor of the medium (for air, they are zero).

$\beta$=complex phase constant of the medium

$$=\beta_0\sqrt{(K_u'-jK_u'')(K_\epsilon'-jK_\epsilon'')} \quad (8b)$$

where:

$\beta_0$=complex phase constant of free space

$$=2\pi f\sqrt{u_0\epsilon_0}=\frac{2\pi}{\lambda_0}$$

$f$=frequency

$u_0$=permeability of free space

$\epsilon_0$=dielectric constant of free space

$\lambda_0$=wavelength in free space

$H_n(\beta c)$=spherical Hankel function of the second kind with argument $(\beta c)=J_n(\beta c)-j\hat{N}_n(\beta c)$

where:

$\hat{J}_n(\beta c)$=spherical Bessel function with argument $(\beta c)$

$N_n(\beta c)$=spherical Bessel function of the second kind with argument $(\beta c)$

$\hat{H}_n'(\beta c)$=derivative of $H_n(\beta c)$

$P_n^k(0)$=associated Legendre polynomial with argument $\cos\theta=0$

A known boundary condition is now applied. This condition is that total tangential electric field $E_\phi^T$ at each point of the extreme outer edge of the loop ($r=c$, $\theta=90°$), is equal to the product of the current $I(\phi)$ and the internal impedance function $Z^i(\phi)$ around the loop. Mathematically this general relationship may be expressed as:

$$E_\phi^T=E_\phi^i+E_\phi^a=Z^i(\phi)I(\phi) \quad (9)$$

where $E_\phi^a$ is the applied electric field resulting from the applied voltage $V_0$. This relationship of Equation 9 is justified in the case of a thin wire where the electrical length $\beta_0 a<<1$.

The terminal zone effect for the gap in the loop at which a voltage source $V_a$ is applied may be eliminated by the use of the well-known mathematical concept of a slice or delta function generator so that the applied electric field $E_\phi^a$ will exist only in the gap. This delta function generator is defined as an infinite electric field in a region of infinitesimal length having a line integral across the region which is finite and equal to the applied voltage $V_a$. When so defined the relation between the applied electric field $E_\phi^a$ and the applied voltage $V_a$ is as follows:

$$E_\phi^a=\frac{1}{c}V_a(\phi)\delta(\phi-\phi_a) \quad (10)$$

where $\delta(\phi-\phi_a)$ is the translated delta function, $\phi_a$ is the point in the loop **10** at which the voltage source $V_a$ is applied, and $V_a(\phi)$ is the applied voltage source $V_a$ expressed as the sum of an infinite number of continuous functions. The property of the translated delta function $\delta(\phi-\phi_a)$ is such that:

$$\int_{-\infty}^{\infty}V_a(\phi)\delta(\phi-\phi_a)=V_a(\phi_a) \quad (11)$$



Since in FIGURE 1 the voltage source $V_0$ is applied at $\phi_a=0$, Equation 10 becomes:

$$E_\phi^a=\frac{1}{c}V_0(\phi)\delta(\phi) \quad (12)$$

and Equation 11 becomes:

$$\int_{-\infty}^{\infty}V_0(\phi)\delta(\phi)d\phi=V_0(o)=V_0 \quad (13)$$

Now substituting Equation 2 for $E\phi^i$ and Equation 12 for $E\phi^a$ in Equation 9, the following equation is obtained:

$$V_0(\phi)\delta(\phi)=cZ^i(\phi)I(\phi)+\frac{\eta^*K_0}{2}a_0+\sum_{k=1}^{\infty}\eta^*K_k[a_k\cos k\phi+b_k\sin k\phi] \quad (14)$$

Equation 1 for the current distribution $I(\phi)$ is repeated below:

$$I(\phi)=a_0+\sum_{k=1}^{\infty}[a_k\cos k\phi+b_k\sin k\phi] \quad (1)$$

Using Equations 1 and 14 and well known mathematical procedures, such as the use of orthogonality relations, the coefficients $a_0$, $a_k$, and $b_k$ can now be calculated in terms of known values for any internal impedance function $Z^i(\phi)$ around the loop.

To obtain Equation 14 in a more practical form the impedance function of $Z^i(\phi)$ will now be expressed as the sum of a continuous distributed impedance function $Z_D{}^i(\phi)$ around the loop and a lumped impedance function $Z_L{}^i(\phi)$ as follows:

$$Z^i(\phi)=Z_D{}^i(\phi)+Z_L{}^i(\phi) \quad (15)$$

The impedance function $Z^i(\phi)_D$ is continuous and can readily be handled in the solution of Equation 14. This function $Z_D{}^i(\phi)$ represents the presence of a continuous impedance function around the loop. The function $Z_L{}^i(\phi)$, on the other hand, represents the presence of lumped impedances in the loop so that periodic impedance discontinuities exist. To conveniently handle these discontinuities in Equation 14 they will be approximated in a manner similar to that used to represent the driving source; that is, as impulse sinks utilizing the translated delta function as shown below:

$$cZ_L{}^i(\phi)=\sum_m\frac{V(\phi)_m\delta(\phi-\phi_m)}{I(\phi)_m} \quad (16)$$

where

$$V(\phi)_m=I(\phi)_mZ_m \quad (17)$$

$I(\phi)_m$ is the current at the lumped impedance $Z_m$. The $m$ index of summation refers to the number of inserted impedances.

Substituting Equation 15, 16, and 1 into Equation 14 gives

$$V_0(\phi)\delta(\phi)-\sum_m V(\phi)_m\delta(\phi-\phi_m)=\left[cZ_D{}^i(\phi)+n^*\frac{K_0}{2}\right]a_0+\sum_{k=1}^{\infty}[cZ_L{}^i(\phi)+n^*K_k][a_k\cos k\phi+b_k\sin k\phi] \quad (18)$$

Using orthogonality relations Equation 18 can now readily be solved to give the values of $a_0$, $a_k$, and $b_k$ in terms of known values for any distributed impedance function $Z_D{}^i(\phi)$ and/or lumped impedance $Z_m$ inserted in the loop. Each $V(\phi)_m$ can be determined by substituting for each $I(\phi)_{min}$ terms of the known impedances (perfect conductor impedances) and applied voltage. Once the values of $a_0$, $a_k$, and $b_k$ are known, the input admittance

$$Y_{in}=\frac{I(\phi)}{V_0}$$

(and the iput impedance $Z_{in}$, which is the reciprocal thereof) may then be obtained as follows:

$$Y_{in}=Y_0+\sum_{k=1}^{\infty}Y_k=\frac{1}{Z_m} \tag{19}$$

where:

$$Y_0=\frac{a_0}{V_0}=\frac{1}{Z_0} \tag{20}$$

$$Y_k=\frac{a_k}{V_0}=\frac{1}{Z_k} \tag{21}$$

The input power $P_{in}$ then follows as:

$$P_{in}=\frac{V_0^2}{2}\left[G_0+\sum_{k=1}^{\infty}G_k\right]=P_{0_{in}}+\sum_{k=1}^{\infty}P_{k_{in}} \tag{22}$$

where:

$G_0$ is the real part of $Y_0$
$G_k$ is the real part of $Y_k$

Using the complex Poynting vector method, the radiated power $P_{RAD}$ then follows as: (assuming free space for air)

$$P_{RAD}P_{0_{RAD}}+\sum_{k=1}^{\infty}P_{k_{RAD}} \tag{23}$$

where:

$$P_{0_{RAD}}=\pi n_0\frac{|a_0^2|}{2}\sum_{n=1,3\ldots}^{\infty}R_{n0}\hat{J}_n(\beta_0 b) \tag{24}$$

$$P_{k_{RAD}}=\pi n_0\frac{(|a_k|^2+|b_k|^2)}{2}\left[\sum_{\substack{n\geqq=k-1,\\ k+1\ldots}}^{\infty}R_{nk}\hat{J}_n(\beta_0 b)+\sum_{\substack{n\geqq 0=k-1,\\ k+1,\ldots}}^{\infty}\frac{(n+2)}{(2n+3)}S_{nk}\hat{J}_n(\beta_0 b)-\sum_{\substack{n\geqq 1,=k-1,\\ k+1,\ldots}}^{\infty}\frac{(n-1)}{(2n-1)}T_{nk}\hat{J}_n(\beta_0 b)\right] \tag{25}$$

The radiation efficiency E now follows as:

$$E=\frac{P_{in}}{P_{RAD}}=\frac{P_{0_{in}}}{P_{0_{RAD}}}+\sum_{k=1}^{\infty}\frac{P_{k_{in}}}{P_{k_{RAD}}} \tag{26}$$

## EXTENSION OF THE THEORY TO THE CASE OF A THIN-WIRE, ONE-TURN LOOP WITH A SPHERICAL CORE OF MATERIAL OTHER THAN AIR

Once the solution for the loop in air is determined, the solution for the multimedium problem follows from well known mathematical procedures which can be found in standard texts on electromagnetic theory. The spherical core is assumed to be a material other than air where the outer edge of the loop coincides with the surface of the core; that is, the radius $r_c$ of the core is equal to the outer radius $c$ of the loop shown in FIGURE 1. Upon solution it is found that the only equations previously presented which are affected and modified are Equation 3 for $K_0$, Equation 5 for $K_K$, Equation 24 for $P_{0RAD}$ and Equation 25 for $P_{kRAD}$. These modified equations are presented below. The superscript $c$ will be used to indicate that these equations are for the case of a spherical core other than air.

$$K_0{}^c=\sum_{n=1,3,5\ldots}^{\infty}R_{n0}\hat{H}_n(\beta c)M_n \tag{27}$$

$$K_k{}^c=\sum_{\substack{n=k+1,\\ k+3,\ldots.}}^{\infty}R_{nk}\hat{H}_n(\beta c)M_n\sum_{\substack{n=k-1,\\ k+1,\ldots}}^{\infty}S_{nk}\hat{H}^1_{n+1}(\beta c)M_{n+1}+\sum_{n=k+1,\,k+3,\ldots}^{\infty}T_{nk}\hat{H}^1_{n-1}(\beta c)M_{n-1} \tag{28}$$



$$P_{0_{RAD}}=\frac{\pi|n^*|^2}{n_0}\frac{V_0^2}{2}|Y_0|^2\sum_{n=1,3,5\ldots}^{\infty}|R_{n0}||\hat{J}_n(\beta b)|\frac{|M_n\hat{H}_n(\beta c)|^2}{|\hat{H}_n(\beta_0 c)|^2} \tag{29}$$

$$P^c_{k_{RAD}}=\frac{\pi|n^*|^2}{n_0}\frac{(|a_k|^2+|b_k|^2)}{2}\left[\sum_{\substack{n\geqq 1=k-1,\\ k+1,\ldots}}^{\infty}|R_{nk}||\hat{J}_n(\beta b)|\frac{|M_n\hat{H}_n(\beta c)|^2}{|\hat{H}_n(\beta_0 c)|^2}+\sum_{\substack{n\geqq 0=k-1,\\ k+1,\ldots}}^{\infty}\frac{(n+2)}{(2n+3)}|S_{nk}||\hat{J}_n(\beta b)|\frac{|M_{n+1}\hat{H}^1_{n+1}(\beta c)|^2}{|\hat{H}^1_{n+1}(\beta_0 c)|^2}+\sum_{n\geqq 1=k-1,\,k+1,\ldots}^{\infty}\frac{(n-1)}{(2n-1)}|T_{nk}||\hat{J}_n(\beta b)|\frac{|M_{n+1}\hat{H}^1_{n+1}(\beta c)|^2}{|\hat{H}^1_{n+1}(\beta_0 c)|^2}+\sum_{n\geqq 1=k-1,\,k+1,}\frac{(n-1)}{(2n-1)}\left|T_{nk}\right|\left|\hat{J}(\beta b)\right|\frac{|M_{n-1}\hat{H}^1_{n-1}(\beta c)|^2}{|\hat{H}^1_{n-1}(b_0 c)|^2}\right] \tag{30}$$

The modifying factors $M_n$, $M_{n+1}$, $M_{n-1}$ are defined by the following equations:

$$M_n=\frac{j}{\hat{J}_n(\beta c)\hat{H}_n(\beta c)\left[\frac{\hat{J}_n{}^1(\beta c)}{\hat{J}_n(\beta c)}-\frac{n^*}{n_0}\frac{\hat{H}_n{}^1(\beta_0 c)}{\hat{H}_n(\beta_0 c)}\right]} \tag{31}$$

$$M_{n+1}=\frac{j}{\hat{J}^1_{n+1}(\beta c)\hat{H}^1_{n+1}(\beta c)\left[\frac{n^*}{n_0}\frac{\hat{H}_{n+1}(\beta_0 c)}{\hat{H}^1_{n+1}(\beta_0 c)}-\frac{\hat{J}_{n+1}(\beta c)}{\hat{J}^1_{n+1}(\beta c)}\right]} \tag{32}$$

$$M_{n-1}=\frac{j}{\hat{J}^1_{n-1}(\beta c)\hat{H}^1_{n-1}(\beta c)\left[\frac{n^*}{n_0}\frac{\hat{H}_{n-1}(\beta_0 c)}{\hat{H}^1_{n-1}(\beta_0 c)}-\frac{\hat{J}_{n-1}(\beta c)}{\hat{J}^1_{n-1}(\beta c)}\right]} \tag{33}$$

In the above solution of the multimedium problem it has been assumed that the characteristics of the core medium are homogeneous and isotropic. Losses in the core are incorporated in the solution as indicated by the appearance of the complex intrinsic impedance $n^*$ in the equations. In this connection it is assumed that the volume current in the core is negligible relative to the conduction current effect of the antenna. For practical core mediums (ferrite for example), the above assumptions are valid.

## PHYSICAL INTERPRETATION OF THE BASIC THEORY

From Equations 19 through 25 it can be seen that the input admittance $Y_{in}$, the input power $P_{in}$, the radiated power $P_{RAD}$, and the radiation efficiency E are expressed as the sum of individual contributions from each component of the current $I(\phi)$. The loop, therefore, may be considered to act as a transmission line which supports an infinite number of $k$ modes of transmission with each mode having its own set of characteristics and each presenting to the driving source an impedance in parallel with those of all other modes. Furthermore, the Equations 23, 25 and 29—30 for radiated power indicate that the radiation may be considered as being comprised of an infinite number of $n$ modes of spherical wave front radiation emanating from the center of the loop. These $k$ and $n$ modes are interdependent as shown by the summation limits in the radiated power equations.

## USING THE DERIVED THEORY FOR LOOP ANTENNA DESIGN

From the derived equations the effects of inserting in the loop various types and combinations of distributed and/or lumped impedances may be calculated. While some of the equations appear quite fearsome at first glance, it has been found that numerical answers can be obtained therefrom using standard mathematical procedures, even though the calculations are somewhat laborious. By programming the more tedious calculations on a computer, the time required can be considerably reduced.

An analysis of the terms involved in the pertinent equations shows that the major difficulty is introduced by the spherical Bessel $\hat{J}_n$ and Hankel $\hat{H}_n$ functions of complex argument such as appear in Equations 3–5. Such functions have never been tabulated because of the obvious infinity of values for the argument. However, they can be calculated by two well known methods: the first is the use of recursion formulas, and the second is the expansion of each order in polynomial form.

The calculation of $K_k$ which is the sum of three summations in Equation 5 might appear particularly difficult, but is readily calculable by assigning values to $k$, beginning with zero, and summing each component of $K_k$ over $n$ for each $k$. The order of $n$ is of course taken sufficiently high to yield a close approximation to the limit. The same criterion also applies to $k$ for the

$$\frac{1}{K_k}$$

summation.

It can be seen, therefore, that although the necessary calculations are somewhat laborious, they are readily performed and may thus be employed for loop antenna design.

Using the above-described calculation procedures, I have calculated the effect of various combinations and types of distributed and lumped impedances inserted in the loop. These calculations showed that very considerable changes can be produced in the characteristics of the antenna, even by the insertion of a single lumped impedance. It is possible, therefore, by theoretically calculating the effects of various combinations and types of inserted impedances, and constructing a one-turn loop antenna in accordance therewith, to arrive at a set of antenna characteristics which will be most favorable for a given application. The previous rigidity of performance which was heretofore characteristic of the loop antenna is thus at an end.

While the above-described equations permit the characteristics of a one-turn loop antenna to be predicted for any type and combination of distributed and lumped impedances, the reverse procedure of determining the inserted impedance combination which will give a desired antenna characteristic is not readily calculable. In some instances this may pose a serious practical problem in the use of the derived equations where a particular antenna characteristic is desired.

I have discovered that this difficulty is overcome to a very considerable extent by using a uniformly distributed reactance as the impedance inserted in the loop. The use of this uniformly distributed reactance leads to a tremendous simplification in the use of the derived equations, whereby the inserted reactance which will achieve a desired antenna characteristic can be determined directly from the derived equations. This will be brought out by the following discussion.

## APPLICATION OF THE THEORY TO THE PARTICULAR CASE OF UNIFORMLY DISTRIBUTED REACTANCE INSERTED IN THE LOOP

### (1) *Perfect Conductor Values*

To apply the theory to the case of uniformly distributed reactance inserted in the loop, Equation 14 will first be solved for the case where the loop is assumed to be a perfect conductor, that is $Z^1(\phi)=0$. Using orthogonality relations, the solution of Equation 14 for $a_0$, $a_k$, and $b_k$ becomes:

$$a_{0s}=\frac{V_0}{\pi\eta^* K_0} \tag{34}$$

$$a_{ks}=\frac{V_0}{\pi\eta^* K_k} \tag{35}$$

$$b_{ks}=0 \tag{36}$$

The additional subscript $s$ is used to indicate that the values are for the perfect conductor case, and will be so used hereafter. The equations for $Y_{in-s}$, $P_{RAD-s}$, and $E_s$ then are determined by substituting Equations 34, 35, and 36, in the previously derived Equations 15–26 or 27–30 for a core other than air.

### (2) *Derivation of the Equations in Terms of Perfect Conductor Values for the Case of Uniformly Distributed Reactance Inserted in the Loop*

In Equation 14 the internal impedance function $Z^1(\phi)$ is now assumed to be a uniformly distributed reactance X per unit length around the loop. The total internal distributed reactance $X_T$ may then be expressed as:

$$X_T=2\pi X \tag{37}$$

The insertion of the total distributed reactance $X_T$ in the loop carries with it the practical fact that a loss component is associated therewith. The total uniformly distributed impedance inserted will thus be represented as:

$$Z_T=R_T+jX_T \tag{38}$$

where $R_T$ is the total loss component associated with $X_T$. The Q is then

$$Q_T=\frac{X_T}{R_T}$$

Again solving Equation 14 using orthogonality relations, the equations for $a_{0D}$, $a_{kD}$, and $b_{kD}$ for the uniformly distributed reactance case then become:

$$a_{0D}=\frac{V_0}{Z_T+Z_{0s}}=a_{0s}\left[\frac{Z_{0s}}{Z_{0s}+Z_T}\right] \tag{39}$$

$$a_{kD}=\frac{V_0}{\frac{Z_T}{2}+Z_{ks}}=a_{ks}\left[\frac{Z_{ks}}{Z_{ks}+\frac{Z_T}{2}}\right] \tag{40}$$

$$b_k=0 \tag{41}$$

The equations for $Y_{in-D}$, $P_{in-D}$, $P_{RAD-D}$ and $E_D$ then follows as:

$$Y_{in-D}=Y_{0D}+\sum_{k=1}^{\infty}Y_{kD}=\frac{1}{Z_{in-D}} \tag{42}$$

where:

$$Y_{0D}=\frac{1}{Z_T+Z_{0s}}=\frac{1}{Z_{0D}} \tag{43}$$

$$Y_{kD}=\frac{1}{\frac{Z_T}{2}+Z_{ks}}=\frac{1}{Z_{kD}} \tag{44}$$

$$P_{in-D}=\frac{V_0^2}{2}\left[G_{0D}+\sum_{k=1}^{\infty}G_{kD}\right]=P_{0D_{in}}+\sum_{k=1}^{\infty}P_{kD_{in}} \tag{45}$$

where:

$G_{0D}$ is the real part of $Y_{0D}$
$G_{kD}$ is the real part of $Y_{kD}$

$$P_{RAD-D}=P_{0D_{RAD}}+\sum_{k=1}^{\infty}P_{kD_{RAD}} \tag{46}$$

where:

$$P_{0D_{RAD}}=P_{0s_{RAD}}\left|\frac{Z_{0s}}{Z_{0s}+Z_T}\right|^2 \quad (47)$$

$$P_{kD_{RAD}}=P_{ks_{RAD}}\left|\frac{Z_{ks}}{Z_{ks}+\frac{Z_T}{2}}\right|^2 \quad (48)$$

$$E_{0D}=\frac{P_{0D_{RAD}}}{P_{0D_{in}}} \quad (49)$$

$$E_{kD}=\frac{P_{kD_{RAD}}}{P_{kD_{in}}} \quad (50)$$

In the above equations the subscript D is used to indicate that these values are for the case of a uniformly distributed reactance inserted in the loop.

After performing a considerable number of calculations for the case of a uniformly distributed reactance $X_T$ inserted in the loop, I discovered that when the reactance $X_T$ is chosen so that a particular mode impedance [$Z_{0D}$ or $Z_{kD}$ in Equations 43 and 44], is approximately a pure resistance, the characteristics for that mode then become predominant, the effects of all the other modes becoming negligible. This has been found to occur because the reactive portions of the mode impedances are quite large relative to the resistive portions, and in addition, are widely different from one another. It is this mode predominance effect, which occurs when a uniformly distributed reactance is inserted in the loop, which brings about a great simplification in the use of the derived equations.

As can be seen from Equations 43 and 44, a selected mode impedance may be made predominant merely by making $X_T$ equal and opposite to the reactive component of the selected mode impedance. For a given antenna the perfect conductor values $Z_{0s}$ and $Z_{ks}$ are known so that the value of $X_T$ which will make a selected mode predominant is readily ascertainable. Thus, from Equations 38–50 the antenna characteristics that would result if each mode were made predominant are also readily ascertainable. It is apparent, therefore, that it is now possible to choose the mode that will have the most favorable characteristics for a desired application. Since these characteristics vary widely for the different modes of a given loop, a wide number of possible characteristics are available for selection by the designer. And, since the characteristics of each mode are a function of known values, such as $n^*$, $\beta$, $K_0$, and $K_k$, all of which are known constants, these values may be chosen so that a selected mode will have practically any desired set of characteristics. Some solutions to practical loop antenna problems which are possible by inserting a properly chosen uniformly distributed reactance in the loop will now be presented.

## SOLVING PRACTICAL LOOP ANTENNA PROBLEMS BY INSERTING A PROPERLY CHOSEN UNIFORMLY DISTRIBUTED REACTANCE IN THE LOOP

### (1) *Input Impedance Control*

One of the serious problems attendant with antennas in general is the inability to adjust or control the input impedance presented by the antenna. It is usually necessary, therefore, to employ impedance matching networks to provide proper matching. Using a uniformly distributed reactance X inserted in a one-turn loop antenna, this input impedance problem is now solvable. For a given antenna, the mode which gives the most favorable input impedance may be made predominant. Further, the constants $n^*$, $\beta$, $K_0$, and $K_k$ may be adjusted so that the selected mode will have the desired impedance.

It is also possible to choose the total distributed reactance $X_T$ so that the selected mode provides a net reactance instead of a pure resistance where such is desired for matching purposes. This is possible because in most cases the reactances of the non-selected modes will be sufficiently larger than the desired net reactance so as to still make their effect negligible relative to the selected mode.



### (2) *Radiation Pattern Control*

Where the perimeter of a loop antenna approaches or exceeds a half wavelength, it has been heretofore difficult to obtain a radiation pattern which is uniform (circular) in the plane of the loop, and at the same time, horizontally polarized (that is, $E_\phi$ is uniform in the plane of the loop). This has been accomplished in the prior art only by means of a variety of complex arrangements such as the well known Alford loop. (See Electronic and Radio Engineering, 4th ed., by Terman, p. 908). This difficult problem is now readily solvable by means of the present invention.

As can be seen from the TE and TM families, the E and H fields are proportional to the current coefficients, $a_0$, $a_k$, and $b_k$ in Equation 1 for the current $I(\phi)$. For the condition where the current is uniform around the loop (that is, $I(\phi)=a_0$), a uniform $E_\phi$ field will be obtained in the $\phi$ direction (that is, in the plane of the loop). In the $\theta$ direction uniform current $I(\phi)$ will result in an $E_\phi$ field having approximately a figure eight pattern. The field pattern for the $E_\phi$ field will thus have the form of a doughnut with its axis coinciding with the loop axis.

It is evident that the condition for uniform current $I(\phi)=a_0$ is readily accomplished for the case of a uniformly distributed reactance inserted in the loop. The current coefficient $b_k$ is already zero [Equation 41], and the coefficients $a_k$ can be made negligible by making the zero mode $a_0$ predominant; that is, by making $X_T$ equal and opposite to the reactive component of $Z_{0s}$ in Equation 43. Since the theory holds for any size loop, this method of obtaining uniform current around the loop also holds for any size loop, including the case where the loop perimeter approaches or exceeds a half wavelength.

At this time it is interesting to note the type of radiation pattern obtained for the case where the $a_1$ current mode ($k=1$) is made predominant by making

$$\frac{X_T}{2}$$

equal and opposite to the reactive component of $Z_{1s}$ [see Equation 44]. For this condition the $E_\phi$ field is uniform in the $\theta$ direction and approximately a figure eight in the $\phi$ direction. The $E_\phi$ pattern will then have the form of a doughnut with its axis perpendicular to the loop axis and lying symmetrically along the 0 to 180° line in the plane of the loop. For the $E_\theta$ field, the pattern in the $\theta$ and $\phi$ directions are both approximately figure eights resulting in a two-lobe $E_\theta$ field pattern also lying symmetrically along the 0 to 180° line in the plane of the loop. Such patterns for the $E_\theta$ and $E_\phi$ fields have never before been obtained with a loop antenna. A wide variety of other radiation patterns are also possible by making other modes predominant. Using the derived equations, therefore, those skilled in the art will readily be able to provide a variety of desirable radiation patterns.

### (3) *Optimization of the Radiation Efficiency of a Ferrite Core Loop Antenna*

Another practical utilization in accordance with this invention is the optimization of the radiation efficiency of a loop antenna having a ferrite spherical core. An analysis of the derived equations shows that the radiation efficiency obtained for the condition of uniform current around the loop, $I(\phi)=a_0$, not only provides a uniform radiation pattern in the plane of the loop but also provides maximum radiation efficiency for the case of a spherical core. Thus, to maximize the radiation ef-

ficiency of a ferrite loop antenna the zero mode is made predominant. Since low radiation efficiency has been a serious drawback to the use of ferrite cores, this method of optimizing radiation efficiency is of considerable importance.

## APPLICATION OF THE EQUATIONS FOR A ONE-TURN LOOP HAVING A UNIFORMLY DISTRIBUTED REACTANCE TO A SPECIFIC NUMERICAL EXAMPLE

To illustrate how the uniformly distributed reactance loop antenna can be advantageously applied in antenna design, a specific numerical example will now be given. It is to be understood that this will merely be a single example of many possibilities, and in no way is intended to limit the scope of the invention. For this example it will be assumed that the antenna comprises a one-turn loop made of silver-coated bronze wire having a wire radius $a$=0.021 inch, an outer loop radius $c$=1.6485 inches, and a Stackpole 2285 ferrite core with a core radius of $c$=1.6485 inches.

For the above-identified ferrite medium the values at 170 megacycles of the parameters $K_u'$, $K_u''$, $K_\epsilon'$ and $K_\epsilon''$ (which are the previously defined non-dimensional real and imaginary components of the permeability and dielectric constant, respectively) have been obtained from actual experimental data by well known methods. The values of the complex intrinsic impedance $n^*$ and the complex phase constant $\beta c$ may then be calculated as shown in Equations 8($a$) and 8($b$). These values of $K_u'$, $K_u''$, $K_\epsilon'$, $K_\epsilon''$, $n^*$ and $\beta c$ at 170 megacycles are given in Table I below:

TABLE I

| Frequency | $K_u'$ | $K_u''$ | $K_\epsilon'$ | $K_\epsilon''$ | $n^*$ | $\beta c$ |
|---|---|---|---|---|---|---|
| 170 megacycles.... | 10.75 | 0.227 | 9.5 | 0.077 | $127.66\pi/-0.4222°$ | $1.5068/-0.836°$ |

Using the above table, the perfect conductor values $Z_{in-s}$, $P_{in-s}$, $P_{RAD-s}$, and $E_s$ are calculated. Since the loop necessarily has a self internal impedance due to skin effect and ohmic resistance, this should be taken into account. The self internal impedance is of course uniformly distributed and may be designated as $Z_T'=R_T+jX_T'$. Calculating $Z_T'$ by well known means and substituting the calculated values therefor in Equations 38–41, initial loop values of $Z'_{in-s}$, $P'_{in-s}$, $P'_{RAD-s}$, and $E'_s$ are obtained. These primed values are now used in Equations 38–41 instead of the perfect conductor values. The calculated initial values at 170 megacycles are shown in Table II below.

TABLE II

| $Z'_{in-s}$, ohms | $P'_{in-s}$, microwatts | $P'_{RAD-s}$, microwatts | $E'_s$, percent |
|---|---|---|---|
| $1154+j6003$ | $15.520\ V_0^2$ | $1.522\ V_s^2$ | 9.81 |

In Table II and hereinafter, it is to be understood that the source voltage $V_0$ is in peak volts.

The mode values for $Z'_{in-s}$, $P'_{in-s}$, $P'_{RAD-s}$, and $E'_s$ up to $k$=3 are given in Table III, A, B, C, and D below.

TABLE III

A

| $Z'_{in-s}$ | $Z'_{0s}$ | $Z_{1s}$ | $Z'_{2s}$ | $Z'_{3s}$ |
|---|---|---|---|---|
| $1154+j6003$ | $2.384+j494.4$ | $10.848-j859.8$ | $27.643-j3250$ | $49.181-j6549$ |

B

| $P'_{in-s}$ | $P'_{0_{in-s}}$ | $P'_{1_{in-s}}$ | $P'_{2_{in-s}}$ | $P'_{3_{in-s}}$ |
|---|---|---|---|---|
| $15.520\ V_0^2$ | $4.877\ V_0^2$ | $7.336\ V_0^2$ | $1.308\ V_0^2$ | $0.573\ V_0^2$ |

C

| $P'_{RAD-s}$ | $P'_{0_{RAD-s}}$ | $P'_{1_{RAD-s}}$ | $P'_{2_{RAD-s}}$ | $P'_{3_{RAD-s}}$ |
|---|---|---|---|---|
| $1.522\ V_0^2$ | $1.363\ V_0^2$ | $0.158\ V_0^2$ | $0.00003\ V_0^2$ | -------- |

D

| $E'_s$, percent | $E'_{0s}$, percent | $E'_{1s}$, percent | $E'_{2s}$, percent | $E'_{3s}$, percent |
|---|---|---|---|---|
| 9.81 | 27.95 | 2.15 | ------ | ------ |

It will now be shown how the $k$=0 mode is made predominant and the results obtained thereby. As described previously, the zero mode predominance condition achieves a desirable uniform radiation pattern. And, where the losses consumed in the loss component $R_T$ associated with the inserted uniformly distributed reactance $X_T$ are not too large, zero mode predominance also achieves an optimized radiation efficiency $E_D$.

To achieve zero mode predominance the inserted uniformly distributed reactance $X_T$ is chosen so that the zero mode impedance $Z_{0D}$ is a pure resistance. From Table IIIA, $Z_{0s}'=2.384+j494.4$ so that to make $Z_{0D}$ a pure resistance, Equation 43 shows that $X_T$ should be a capacitive reactance equal to −494.4 ohms. It can be seen from Table III and Equations 38–50 that for this value of $X_T$ inserted the effects of all other modes are negligible, the zero mode thus being predominant. The uniform radiation pattern is now the radiation pattern of the antenna and the resulting zero mode characteristics with $X_T$ inserted are now the characteristics of the loop antenna. Assuming a reasonable value of $Q_T$=400 for the inserted capacitive reactance $X_T$=494.4 ohms, the associated dissipative component $R_T$ is substantially 1.2 ohms. Using these values of $R_T$ and $X_T$ in Equations 38–50, the following zero mode values (which are now those for the antenna) are obtained as shown in Table IV below:

TABLE IV

| $Z_{0D}=Z_{in-D}$, ohms | $P0_{in-D}=P_{in-d}$, microwatts | $P0_{RAD-D}=P_{RAD-D}$, microwatts | $E_{0D}=E_D$, percent |
|---|---|---|---|
| $3.585+j0$ | $0.1395\ V_0^2$ | $0.0260\ V_0^2$ | 18.6 |

From Tables I and IV it can be seen that the radiation

efficiency has increased from 9.81% to 18.6%. Also, the input impedance has been radically changed from 1154+$j$6003 ohms to the very much smaller value of 3.584 ohms.

It is also possible to provide any other higher antenna resistive input impedance than 3.584 ohms by means of the circuit shown in FIG. 2. The numeral **11** represents the one-turn loop antenna with the total uniformly distributed capacitance $X_T$ inserted in the loop. A variable capacitor **11** is in parallel with the one-turn loop antenna **12**, and the parallel combination thereof is fed by a generator **19** having an output resistance R. To match the generator output resistance R, the value of the total inserted distributed capacitance is chosen so that the input impedance of the one-turn loop antenna **12** is the resistance R in parallel with an inductance. The variable capacitance **11** is then adjusted to tune out this inductance, the input impedance presented by the parallel combination of the one-turn loop antenna **12** and the variable capacitance **11** then being equal to the generator output resistance R. As explained previously, the zero mode net reactance required for matching purposes will ordinarily be sufficiently small as compared to the reactances of the other modes so as not to effect the zero mode predominance condition.

For example, if an input resistance of say 100 ohms were desired instead of 3.584 ohms, this value can be obtained by inserting in the loop a capacitive reactance of $X_T$=479.2 ohms instead of 494.4 ohms. The input impedance $Z_{in-D}$ then becomes 3.584+$j$15.2 ohms which is equivalent to a resistance of 100 ohms in parallel with an inductance of 15.7 ohms. This parallel inductance is now tuned out by the variable capacitance **11** so that the remaining antenna input impedance is a resistive value of 100 ohms as desired. From Table IIIA it will be seen that the zero mode is still predominant for this value of $X_T$=479.2 ohms inserted in the loop.

The above-described illustrative example thus shows how a one-turn loop antenna can be tailored to provide a uniform radiation pattern in the plane of the loop, optimum radiation efficiency, and practically any desired resistive input impedance merely by properly choosing the total uniformly distributed reactance $X_T$ inserted in the loop.

## PRACTICAL EMBODIMENTS OF A LOOP ANTENNA HAVING UNIFORMLY DISTRIBUTED REACTANCE INSERTED IN THE LOOP

Practical embodiments of a one-turn loop antenna having uniformly distributed reactance in the loop have been built and tested. In building and testing these embodiments it has been determined that various practical assumptions are possible without upsetting the validity of the derived theory. First, it has been determined that a uniformly distributed reactance may be approximated by using closely spaced lumped reactances around the loop as is sometimes done in building an artificial transmission line.

Secondly, it has been found that the radius of the ferrite core used is not critical, and the calculated results will not be significantly altered as long as the wire loop is in close proximity to the core. Resting the loop on the surface of the core has been found to be entirely adequate.

Thirdly, as regards the value of the self internal impedance of the loop $Z_T'=jX_T'+R_T'$ to be used in Equations 38–41 to obtain the initial values of the mode characteristics with no inserted uniformly distributed impedance, it has been found that the value of $X_T'$ is maintained as long as the perimeter at each cross section of the actual loop is approximately equal to that of the round wire. That is, the $X_T'$ value is the same as that for a round wire as long as the perimeter at each cross section is approximately equal to $2\pi a$, where $a$ is the radius of the round wire. Each reactance of any set of lumped reactances inserted in the loop to approximate a uniformly distributed impedance, therefore, should have a perimeter substantially equal to $2\pi a$. The value of $X_T'$ in the equations will then be the same as for a round wire having the same perimeter, and the reactance value of the inserted impedance (excluding leads) will then serve as the inserted reactance. The value of the ohmic resistance $R_T'$ can readily be calculated, measured, or estimated by well known means. The value of $R_T'$ ordinarily has only a minor effect on the resultant initial values.

FIGURE 3 shows a number of closely lumped capacitances **20** inserted in a one-turn loop **15** to approximate a uniformly distributed reactance. FIG. 4A is an enlarged view of one portion of the loop **15** showing one capacitor in detail, and FIG. 4B is a cross-sectional view along 4B—4B in FIG. 4A. To achieve approximately the value of $X_T$=494.4 ohms total internal distributed reactance at 170 megacycles required to make the zero mode predominant in the numerical example, twenty-five capacitors each having a value of 50 micromicrofarads at 170 megacycles are employed as shown. As in the numerical example, the radius $c$ to the extreme edge of the loop is chosen to be approximately 1.6485 inches, the perimeter $2\omega+2t$ is chosen to be approximately equal to

$$2\pi a=2\pi\times 0.021''$$

and the core **30** is made of Stackpole 2285 ferrite with a core radius such that the loop **15** rests snugly on the core **30**. The applied voltage $V_0$ is applied to terminals **70** forming about a 1/16 inch gap in the loop **15**.

As shown in FIGS. 4A and 4B, the plates **45** of the capacitor **20** are connected to the connecting wires **35** by suitable means such as solder joints **40**. The dielectric of the capacitor is represented by **50**. The value of the capacitor **20** used in determining the distributed reactance is the capacitance of each unit **20** only, without the connecting leads **35**.

With the embodiment of FIG. 3 constructed as described above, experimental data shows that the zero mode is predominant and the antenna characteristics obtained are remarkably close to those predicted by the numerical example.

FIGS. 5A and 5B show another embodiment of the invention for achieving a uniformly distributed capacitance in the loop. A dielectric loop **55** of rectangular cross section is snugly fit around a spherical core **30**. Electrodes **60** are formed on the dielectric loop **55** so as to be symmetrically and alternatively located around the inner and outer peripheries of the loop, thereby providing in effect a uniformly distributed series capacitance. As was done for the embodiment of FIG. 3, the radius $c$ to the extreme outer edge, the perimeter $2\omega+2t$, and the ferrite core **30** may be chosen to provide the initial values shown in Tables III and IV, and the value of the inserted uniformly distributed capacitances may likewise be adjusted to provide zero mode predominance. The size and spacing of the electrodes **60** in cooperation with the dielectric loop **55** which are necessary to provide a given uniformly distributed reactance may readily be determined by well known means.

The spherical ferrite core employed in the above two embodiments was constructed from seven ferrite disks about three and a half inches in diameter and about one-half inch thick. These seven disks were ground to optical flatness and then mechanically clamped to form a cylinder about three and a half inches long. The cylinder was immersed in heated melted paraffin and after cooling the excess paraffin was wiped off. The cylinder was then gripped with a U clamp on its end surfaces and ground to a sphere using a diamond wheel grinder.

It will be apparent that the embodiments shown are only exemplary and that various modifications can be made in construction and arrangement within the scope of the invention as defined in the appended claims.

From the desirable results obtained with the one-turn

loop antenna by inserting distributed and/or lumped impedances in the loop, it is felt that the proper use of inserted impedances in other types of antenna configurations will also lead to new and desirable results.

I claim as my invention:

1. In combination with a single-turn loop antenna, means for supplying current at a predetermined frequency to said loop and impedance means substantially uniformly distributed around, and forming a part of, said loop for causing a predetermined $k$ current distribution mode to predominate at said frequency independently of the electrical length of said loop, where $k$ is any integer equal to or greater than one, wherein said impedance means comprises a reactance $X_T$ inserted in said loop having a value equal and opposite to the reactive portion of $Z_{ks}$, the mode impedance, where

$$Z_{ks} \text{ equals } \frac{V_0}{I(\phi)}$$

where $V_0$ is the voltage impressed at the input terminals of said antenna and $I(\phi)$, the loop current in the $\phi$ direction, $=a_0+\Sigma(a_k \cos k\phi+b_k \sin k\phi)$ where $a_0$, $a_k$ and $b_k$ are the co-efficients of a Fourier series representation of the current wave $I(\phi)$, and $\phi$ is the angle between two lines lying in the plane of the antenna and passing through the geometrical center of said loop, where the first of said lines also passes through the point on said antenna where the input voltage is applied and the second of said lines passes through any other arbitrary point on said loop.



2. The invention in accordance with claim 1, wherein said reactance $X_T$ is approximately equal and opposite to twice the reactive portion of the initial first mode impedance $Z_{1s}'$ of said loop, where $Z_{1s}'$ represents the first mode impedance defined in claim 1 with a correction for the fact that said loop antenna is not a perfect conductor, thereby making the $k=1$ mode predominant.


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