May 1, 1923. **1,453,980**

R. S. HOYT

ATTENUATION EQUALIZER

Filed June 29, 1918 3 Sheets-Sheet 1

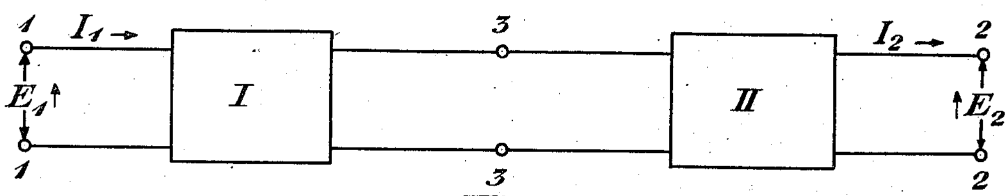

Fig. 1

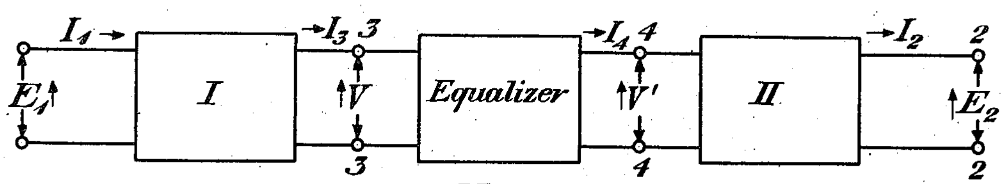

Fig. 2

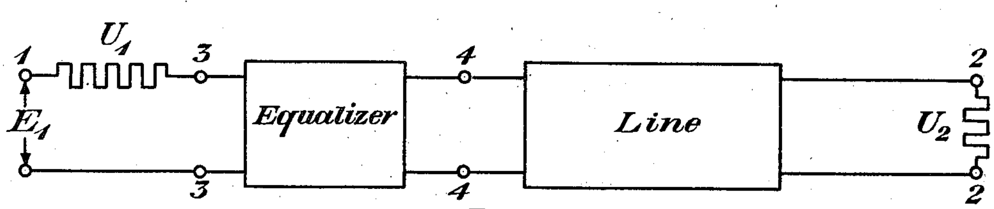

Fig. 3

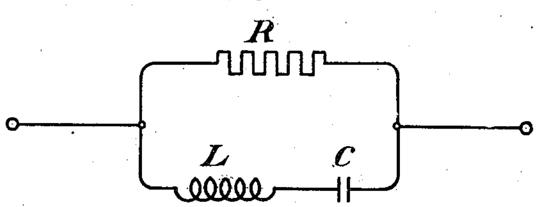

Fig. 4

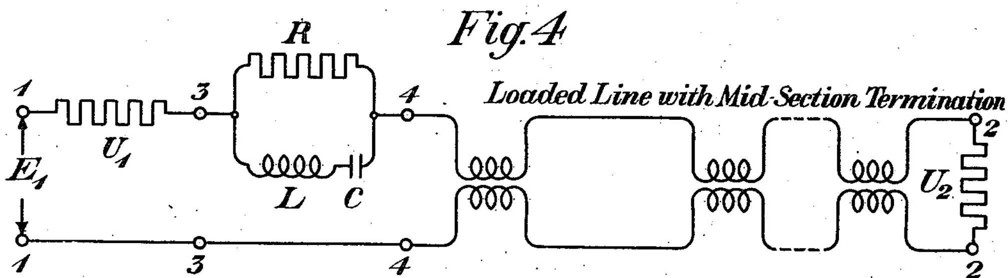

Fig. 5

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ATTENUATION EQUALIZER

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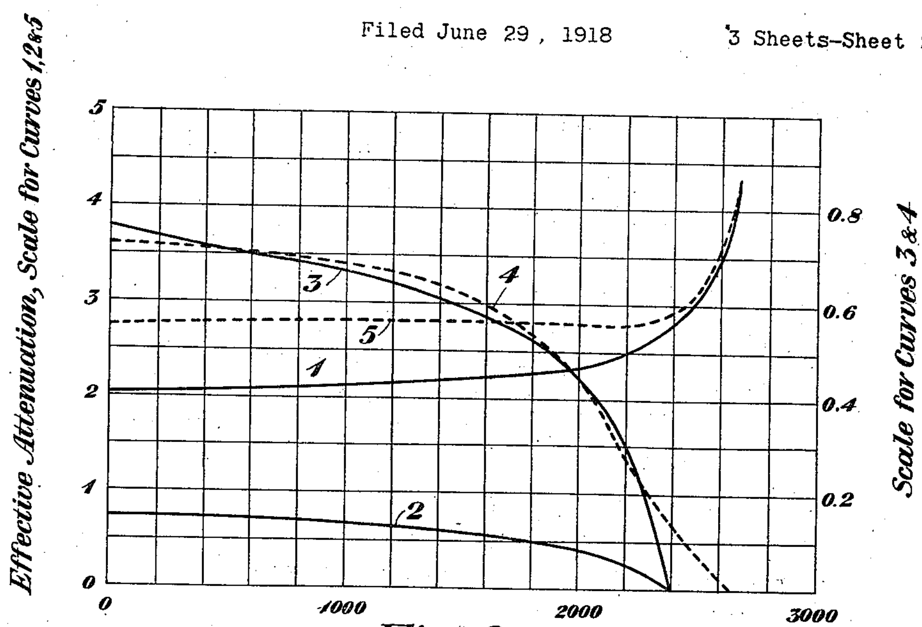

Fig. 6

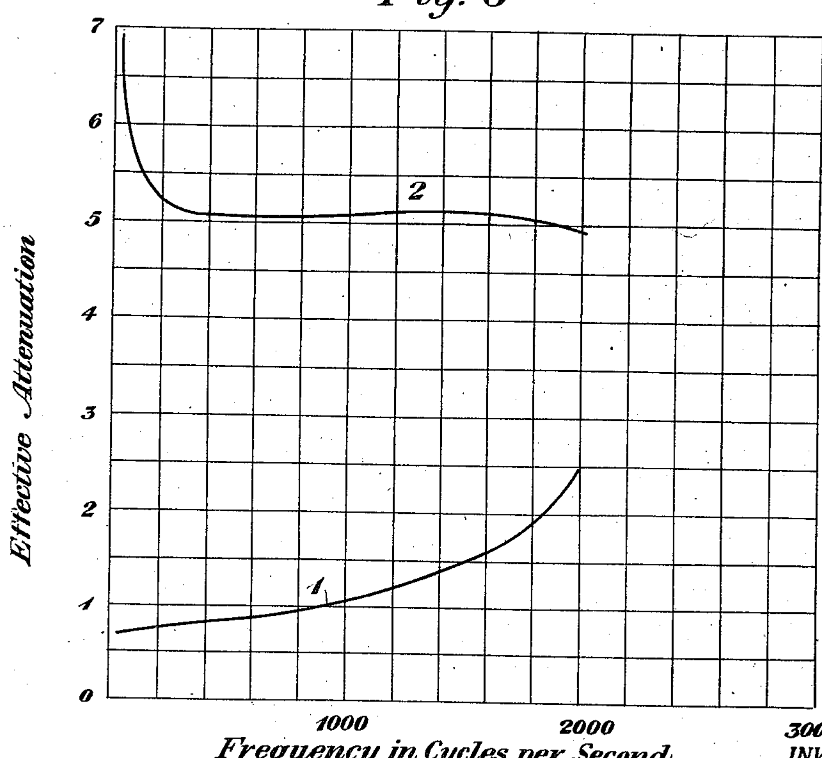

Fig. 12

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R. S. HOYT

ATTENUATION EQUALIZER

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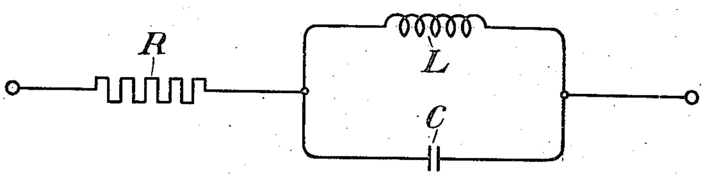

Fig. 7

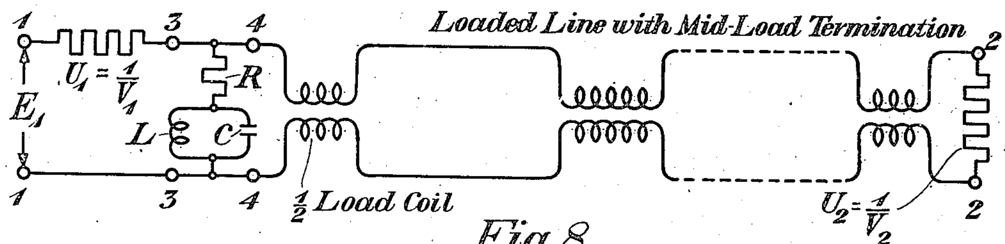

Fig. 8

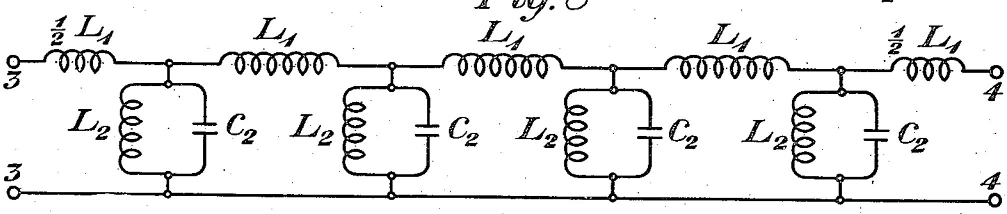

Fig. 9

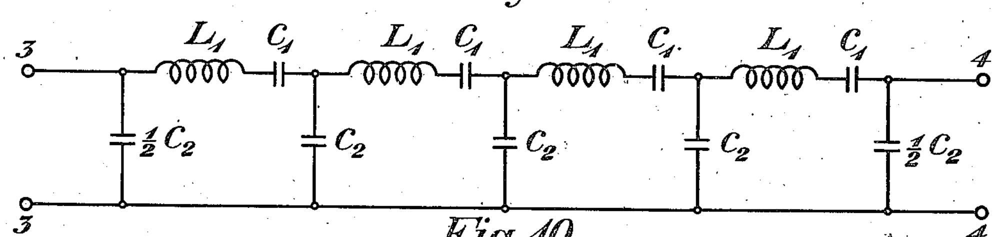

Fig. 10

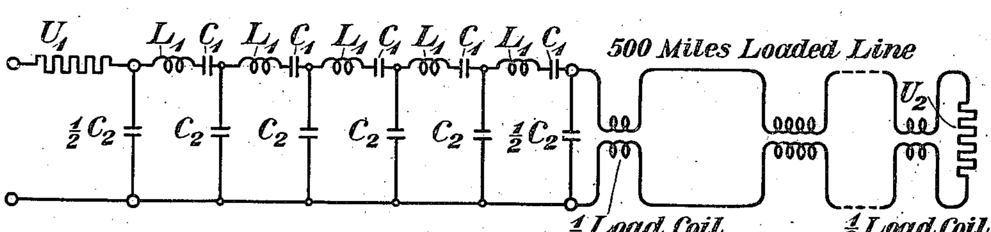

Fig. 11

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BY G. E. Folk,
ATTORNEY.

Patented May 1, 1923. 1,453,980

UNITED STATES PATENT OFFICE.

RAY S. HOYT, OF BROOKLYN, NEW YORK, ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

ATTENUATION EQUALIZER.

Application filed June 29, 1918. Serial No. 242,567.

To all whom it may concern:

Be it known that I, RAY S. HOYT, residing at Brooklyn, in the county of Kings and State of New York, have invented certain Improvements in Attenuation Equalizers, of which the following is a specification.

This invention relates to transmission systems and more particularly to system for the telephonic transmission of speech. Its object is to provide means whereby the distortion of the received signals, present in all types of signaling systems, may be minimized to the end that the received signals shall be a faithful copy of the transmitted signals. The object of the invention is attained by the cooperative combination of a signaling system and an auxiliary system so designed and proportioned that the resultant distortion of the system is substantially zero.

It is well known to those acquainted with the art that the telephonic waves which represent articulate speech comprise component waves of a large number of frequencies the more important of which range from roughly 200 cycles per second, to 2,500 cycles per second. It is evident that for clear articulation of the received speech it is desirable that the relative amplitudes of these components or harmonics shall be the same in the received wave as in the transmitted wave. When such a condition obtains, the transmitting system is called distortionless. In actual transmission systems, however, it is well known that the distortion with respect to frequency is considerable, and, in particular, when the transmission system includes long lines, the distortion may be very large. The usual character of this distortion is a diminution of the amplitudes of the high frequency component currents with respect to the low frequency currents, whereby the character of the received speech may be so modified as to seriously obscure the articulation and clearness of speech.

From the foregoing it will be clear that ordinarily the required function of the attenuation equalizer of this invention is to discriminate against the low frequency component currents to the end that the discrimination of the actual system against the high frequency component currents shall be substantially neutralized. It will be understood, however, that this invention is not limited to such a function since it may be used also to equalize transmission over a system which discriminates against its low frequency components.

Moreover, it will be understood that this invention is not limited to the function of attaining exact equalization, but the equalizer may be so constructed as to produce for the resultant system a desired variation of amplitude with respect to frequency, which departs by a predetermined amount from the exact equalization heretofore discussed. This may be accomplished by merely designing the equalizer with respect to an assumed variation in the transmission system proper, which differs from the actual variation by the desired amount.

The invention may now be more fully understood by reference to the following description when read in connection with the accompanying drawings. In the drawings,

Figure 1 is a schematic diagram of a transmission system comprising two sections, said diagram being provided in order to assist in understanding the development of certain general formulæ pertaining to transmission;

Fig. 2 is a schematic diagram of the system of Fig. 1 with an attenuation equalizer inserted;

Fig. 3 is a schematic diagram of a line circuit with terminating impedances and having an attenuation equalizer inserted therein;

Fig. 4 is a diagram of a series impedance type of equalizer;

Fig. 5 is a diagram showing the equalizer of Fig. 4 applied to a loaded line with mid-section termination, that is, to a loaded line which begins with a line section equal to a half loading-section, a loading-section being the portion of line existing between any two succeeding loading coils;

Fig. 6 is a diagram showing a series of curves illustrating the attenuation of the various parts of the system of Fig. 5;

Fig. 7 is a diagram of a shunt impedance type of equalizer;

Fig. 8 is a diagram showing the equalizer of Fig. 7 applied to a loaded line with mid-load termination, that is, to a loaded line which begins with a half load coil, a half load coil being one whose impedance is half the impedance of a normal or whole coil;

Figs. 9 and 10 illustrate two types of periodic structures which may be used as attenuation equalizers;

Fig. 11 is a diagram showing the attenuation equalizer of Fig. 10 applied to a loaded line with mid-load termination; and

Fig. 12 is a diagram showing attenuation curves for the system of Fig. 11.

The general theory underlying the attenuation equalizer of this invention will now be developed, after which the specific types will be disclosed and the appropriate design formulæ will be derived.

Referring to Fig. 1, a transmission system is schematically represented, which is shown as consisting of two parts I and II connected together at terminals 3, 3. An E. M. F. E_1 is illustrated as impressed between terminals 1, 1 and for generality an E. M. F. E_2 between terminals 2, 2. If the current entering terminals 1 is designated by I_1 and the current in terminals 2 by I_2, then I_1 and I_2 are related to the impressed E. M. F.'s. by equations of the form:

$$\begin{aligned} I_1 &= T_{11}E_1 - T_{12}E_2 \\ I_2 &= T_{21}E_1 - T_{22}E_2 \end{aligned} \qquad (1)$$

In the above expressions T_{11}, T_{12}, T_{21} and T_{22} are the coefficients of admittance of the system. T_{11} is equal to the current flowing into terminal 1 of the system when unit E. M. F. is applied between terminals 1, 1 and terminals 2, 2 are short circuited. Similarly T_{21} is equal to the current flowing through terminals 2, 2 under the same conditions. T_{22} is equal to the current flowing through terminals 2, 2 when a unit E. M. F. is applied to terminals 2, 2 and terminals 1, 1 are short circuited.

When we are considering transmission from 1 to 2, E_2 is set equal to zero, whence:

$$\begin{aligned} I_1 &= T_{11}E_1 \\ I_2 &= T_{21}E_1 \end{aligned} \qquad (2)$$

The coefficient T_{21} is the transfer admittance of the system, that is the ratio of the current received at terminals 2 to the E. M. F. impressed at terminals 1. This coefficient may be theoretically determined when the system is specified or it may be experimentally measured. In general, it will be found that T_{21} is a function of the frequency of the impressed E. M. F.; it is the variation of T_{21} with respect to frequency which causes the distortion, which it is the object of this invention to eliminate.

Fig. 2 is a diagram of the system of Fig. 1 with an equalizer, schematically represented, inserted at terminals 3, 3. If we designate by V and V' the voltages between terminals 3, 3 and 4, 4 respectively (the arrows associated with the voltages indicating the directions from lower to higher potentials), by A_{11}, A_{13}, A_{31}, A_{33} the admittances of part I, by B_{44}, B_{42}, B_{24}, B_{22} the admittances of part II, and by C_{33}, C_{34}, C_{43}, C_{44} the admittances of the equalizer, the equations of the system are:

$$\begin{aligned} I_1 &= A_{11}E_1 - A_{13}V \\ I_3 &= A_{31}E_1 - A_{33}V \\ I_3 &= C_{33}V - C_{34}V' \\ I_4 &= C_{43}V - C_{44}V' \\ I_4 &= B_{44}V' - B_{42}E_2 \\ I_2 &= B_{24}V' - B_{22}E_2 \end{aligned} \qquad (3)$$

Confining our attention to transmission from 1 to 2 and consequently setting E_2 equal to zero, the solution of equations (3) gives:

$$I_2 = E_1\frac{A_{31}B_{24}C_{43}}{(A_{33}+C_{33})(B_{44}+C_{44}) - C_{34}C_{43}} \qquad (4)$$

Therefore the transfer admittance $T'_{21} = I_2/E_1$ of the system of Fig. 2 is:

$$T'_{21} = \frac{A_{31}B_{24}C_{43}}{(A_{33}+C_{33})(B_{44}+C_{44}) - C_{34}C_{43}} \qquad (5)$$

The significance of the admittance is easily seen. Thus, referring to Fig. 2, C_{33} is equal to the current flowing into terminal 3 of the equalizer when a unit E. M. F. is applied between terminals 3, 3 and terminals 4, 4 are short-circuited. Similarly C_{43} is equal to the current flowing thru terminals 4, 4 under the same conditions. C_{44} is equal to the current flowing thru terminals 4, 4 when a unit E. M. F. is applied across terminals 4, 4, and terminals 3, 3 are short-circuited. From these definitions the meaning of the other admittances is self evident.

If the equalizer is removed in Fig. 2 the equations of the system for transmission from 1 to 2 are:

$$\begin{aligned} I_1 &= A_{11}E_1 - A_{13}V \\ I_3 &= A_{31}E_1 - A_{33}V \\ I_4 &= B_{44}V' \\ I_2 &= B_{24}V' \\ I_3 &= I_4 \\ V &= V' \end{aligned} \qquad (6)$$

Solving the above equations we obtain the following as the transfer admittance of Fig. 2 with the equalizer omitted:

$$T_{21} = \frac{B_{24}A_{31}}{A_{33}+B_{44}} \qquad (7)$$

Now in the actual system the transfer admittance T_{21} varies with the frequency; in the equalized system the requirement is that T'_{21} as given by (5) shall be substantially constant over the frequency range required for the telephonic transmission of speech. This result is attained when equalizer, characterized by the parameters C_{33}, C_{44}, C_{34}, C_{43}, is so proportioned that the absolute value of T'_{21} as given by (5) is substantially independent of the frequency.

If the equalizer consists merely of an impedance Z (which may be a single element or may be a combination of elements) in series with the line the equations for transmission from 1 to 2 in Fig. 2 are as follows:

$$
\begin{aligned}
I_1 &= A_{11}E_1 - A_{13}V \\
I_3 &= A_{31}E_1 - A_{33}V \\
I_3 &= C_{33}V - C_{34}V' \\
I_4 &= C_{43}V - C_{44}V' \\
I_4 &= B_{44}V' \\
I_2 &= B_{24}V' \\
I_3 &= I_4 \\
V &= V' + ZI_3
\end{aligned}
\qquad (8)
$$

Solving these equations and noting that $C_{33}=C_{44}=C_{34}=C_{43}=1/Z$ we obtain as the transfer admittance from 1 to 2 the following:

$$T'_{21} = \frac{A_{31}B_{24}}{A_{33}(1+ZB_{44})+B_{44}} \qquad (9)$$

which can also be obtained from (5) by substituting $C_{33}=C_{44}=C_{34}=C_{43}=1/Z$ therein. If on the other hand the equalizer consists of an admittance Y in shunt across the line the equations of the systems for transmission from 1 to 2 are:

$$
\begin{aligned}
I_1 &= A_{11}E_1 - A_{13}V \\
I_3 &= A_{31}E_1 - A_{33}V \\
I_3 &= C_{33}V - C_{34}V' \\
I_4 &= C_{43}V - C_{44}V' \\
I_4 &= B_{44}V' \\
I_2 &= B_{24}V' \\
V &= V' \\
I_3 &= I_4 + YV
\end{aligned}
\qquad (10)
$$

From these equations we may obtain the following as the expression for the transfer admittance from 1 to 2:

$$T'_{21} = \frac{A_{31}B_{24}}{A_{33}+B_{44}+Y} \qquad (11)$$

If the system which is equalized consists of a transmission line of characteristic impedance K and propagation constant Γ closed by the impedances U_1 and U_2 and if the equalizer be inserted between the line and the impedance U_1 as shown in Fig. 3, then

$$
\begin{aligned}
A_{11} &= A_{13} = A_{31} = A_{33} = \frac{1}{U_1} = V_1 \\
B_{44} &= \frac{1}{K} = H;\ 1/B_{22} = K + U_2 \qquad (12) \\
B_{24} &= B_{42} = \frac{2}{K+U_2}e^{-\Gamma} = \frac{2HV_2}{H+V_2}e^{-\Gamma}
\end{aligned}
$$

where H, V_1, V_2 are the reciprocals of K, U_1, U_2, respectively, and are therefore admittances.

Equations (12) in so far as these relate to the transmission line constants, follow from well known formulæ, on the assumption that the line is so long that the sending-end current is independent of the receiving end terminal impedance. B_{44} is therefore 1/K, that is equal to the current entering an infinitely long line due to unit E. M. F.; while B_{42} is the corresponding received current.

If the equalizer consists merely of an impedance Z (which may be a single element or may be a combination of elements) in series with the line in Fig. 3, then by (9) and (12)

$$T'_{21} = \frac{2Ke^{-\Gamma}}{(K+U_2)(K+U_1+Z)} \qquad (13)$$

If, on the other hand, the equalizer consists of an admittance in shunt across the line, then by (11) and (12)

$$T'_{21} = \frac{2HV_1V_2}{(H+V_2)(H+V_1+Y)}e^{-\Gamma} \qquad (14)$$

If the equalizer consists of a periodic structure of n sections having a characteristic impedance K′, and propagation constant Γ′ per section, it may be readily shown that

$$
\begin{aligned}
C_{33} &= C_{44} = \frac{1}{K'}\,\frac{1+e^{-2n\Gamma'}}{1-e^{-2n\Gamma'}} \\
& \qquad\qquad\qquad\qquad (15) \\
C_{34} &- C_{43} = \frac{2}{K'}\,\frac{e^{-n\Gamma'}}{1-e^{-2n\Gamma'}}
\end{aligned}
$$

If further this equalizer is inserted between a resistance U_1 and a long line (K, Γ), (see Fig. 3) whose A and B admittances are given by (12), then by reference to (5), (12), and (15) it will be seen that the transfer admittance of this system is

$$T'_{21} = \frac{4KK'e^{-(\Gamma+n\Gamma')}}{(K+K')(K'+U_1)(K+U_2)}\;\frac{1}{1-\dfrac{K-K'}{K+K'}e^{-2n\Gamma'}} \qquad (16)$$

Neglecting the term which contains the factor $e^{-2n\Gamma'}$, which is commonly small, (16) reduces to

$$T'_{21}=\frac{4KK'e^{-(\Gamma+n\Gamma')}}{(K+K')(K'+U_1)(K+U_2)} \quad (17)$$

Series impedance type of equalizer.

Equation (13) may be written

$$T'_{21}=\frac{e^{-\Gamma}}{\sqrt{4U_1U_2}}\,\frac{\sqrt{4KU_1}}{K+U_1}\,\frac{\sqrt{4KU_2}}{K+U_2}\,\frac{K+U_1}{K+U_1+Z} \quad (18)$$

It is now convenient to introduce a set of parameters defined by the following equations:

$$\begin{aligned} A+iA' &= \Gamma \\ a+ia' &= \log_e\left[\frac{K+U_1+Z}{K+U_1}\right] \\ b+ib' &= \log_e\left[\frac{K+U_1}{\sqrt{4KU_1}}\right] \\ c+ic' &= \log_e\left[\frac{K+U_2}{\sqrt{4KU_2}}\right] \end{aligned} \quad (19)$$

In the above equations the two elements of each of the expressions on the left hand side of the equality sign are respectively the real and imaginary components of the corresponding expressions on the right hand side, which latter expressions are in general complex functions. Thus, for any transmission system, the attenuation coefficient A is the real part of the propagation coefficient Γ of the system, while iA' is the imaginary component, of which A' is a real expression and i denotes the operator $\sqrt{-1}$.

From (19) it follows that

$$\begin{aligned} e^{-A} &= \left|e^{-\Gamma}\right| \\ e^{a} &= \left|\frac{K+U_1+Z}{K+U_1}\right| \\ e^{b} &= \left|\frac{K+U_1}{\sqrt{4KU_1}}\right| \\ e^{c} &= \left|\frac{K+U_2}{\sqrt{4KU_2}}\right| \end{aligned} \quad (20)$$

wherein a pair of vertical lines enclosing an expression indicates that the absolute value of such expression is meant.

From equation (18) it follows that the absolute value of the transfer admittance of the system may be expressed as

$$\left|T'_{21}\right|=\left|\frac{e^{-\Gamma}}{\sqrt{4U_1U_2}}\right|\left|\frac{\sqrt{4KU_1}}{K+U_1}\right|\left|\frac{\sqrt{4KU_2}}{K+U_2}\right|\left|\frac{K+U_1}{K+U_1+Z}\right| \quad (21)$$

whence, by equations (20),

$$\left|T'_{21}\right|=\frac{e^{-A}e^{-b}e^{-c}e^{-a}}{\left|\sqrt{4U_1U_2}\right|}$$

that is

$$\left|T'_{21}\right|=\frac{e^{-(A+a+b+c)}}{\left|\sqrt{4U_1U_2}\right|} \quad (22)$$

In the practically important case considered below, the terminal impedances U_1 and U_2 are pure resistances and hence constants whose values are independent of frequency. For such case it follows from (22) that distortion with respect to frequency will be eliminated if the sum $A+a+b+c$ can be made constant over the range of frequencies necessary for the telephonic transmission of speech, since all frequencies within that range will then be transmitted with the same attenuation. Although, in practice, U_1 and U_2 are not always independent of frequency, the effect of their variation with frequency is ordinarily negligible; and when not negligible it can be equalized by slightly modifying the equalizer, or by introducing an auxiliary equalizer.

The specific type of the attenuation equalizer now to be considered is shown in Fig. 4, and consists of a resistance R in parallel with the serial combination of an inductance L and a capacity C. If p denotes $2\pi f$ when f is the frequency, and i denotes the imaginary operator $\sqrt{-1}$, the expression for the impedance Z is

$$\begin{aligned} Z &= \frac{R(iLp+1/iCp)}{R+iLp+1/iCp} \\ &= \frac{R(1-LCp^2)}{(1-LCp^2)+iRCp} \end{aligned} \quad (23)$$

The general design procedure for this type of equalizer will now be laid down. Referring to formula (22) it is evident that the resultant effective attenuation of the system is $A+b+c+a$, and that the equalizer affects only one coefficient, namely a. The coefficients A, b, and c are given by formulæ (20) and the line constants, whence the sum $A+b+c$ may be taken as the data of the problem. Further, if $A_2+b_2+c_2$ is the value of $A+b+c$ at the highest frequency f_2 of the range over which equalization is desired, it is evident that the resultant attenuation of the system will have a uniform constant value equal to $A_2+b_2+c_2$ if

$$a=(A_2+b_2+c_2)-(A+b+c) \quad (24)$$

for all frequencies less than f_2, and further that no attenuation will be introduced by the equalizer at frequency f_2. (If the attenuation $A+b+c$ does not increase with frequency throughout the entire frequency-range contemplated, then f_2 should be taken as the frequency corresponding to maximum attenuation.)

After computing A, b, c from the fundamental data by aid of formulæ (20), the first step toward the design is to compute by (24) the values of the attenuation a which the ideal equalizer should furnish over the frequency-range contemplated. Since the equalizer here considered (Fig. 4) is characterized by three independent constants (R, L, C,) these constants can be so evaluated that the equalizer attenuation will have its ideal values at three different frequencies. One of these frequencies, namely f_2, has already been assigned; for the other two it is convenient to choose the frequency O, and a selected intermediate frequency f_1. The ideal values of a for the three frequencies O, f_1, f_2 will be denoted by a_0, a_1, a_2. Since, by (24), the attenuation a_2 furnished by the equalizer should be zero at the frequency f_2, the equalizer impedance should be zero at that same frequency. If the equalizer were to be proportioned in accordance with these values, the equalization would be exact at frequencies O, f_1 and f_2. However, the frequency zero is unimportant, and exact equalization at the frequency f_2 is also of minor practical importance. It has been found, therefore, that more satisfactory equalization may be secured by choosing a value of a_0 slightly different from that for exact equalization at frequency O, and also making the impedance of the equalizer zero at a frequency f_3 which is nearly but not exactly equal to f_2. Having therefore decided on appropriate values of a_0, a_1 and f_3, the values of the equalizer elements R, L, C, are determined as follows: By formula (23) the equalizer impedance, and therefore a, is zero if

$$1-LCp^2=o$$

Since this is to be zero at frequency f_3 we have:

$$LC=1/(2\pi f_3)^2 \quad (25)$$

At zero frequency the impedance of the equalizer is simply R; therefore, by (20)

$$\left|1+\frac{R}{(K+U_1)_0}\right|=e^{a}{}_0$$

when $(K+U_1)_0$ is the value of $(K+U_1)$ at frequency zero. For the practically important case considered below, in which $(K+U_1)_0$ is pure resistance, the solution of this equation gives

$$R=(e^{a}{}_0-1)\ (K+U_1)_0 \quad (26)$$

By formulæ (23) and (25) the impedance Z_1 of the equalizer at frequency $f_1=p_1/2\pi$ is

$$Z_1=\frac{R[1-(f_1/f_3)^2]}{1-(f_1/f_3)^2+iRCp_1}$$

Hence, by formulæ (20),

$$\left|1+\frac{R[1-(f_1/f_3)^2]}{[1-(f_1/f_3)^2+iRCp_1](K+U_1)_1}\right|=e^{a}{}_1$$

where $(K+U_1)_1$ is the value of $(K+U_1)$ at frequency f_1. Solving this equation for C, in the practically important case for which $(K+U_1)_1$ is pure resistance, we have

$$C=\frac{[1-(f_1/f_3)^2]}{2\pi f_1 R}\sqrt{\frac{[1+R/(K+U_1)_1]^2-e^{2a}{}_1}{e^{2a1}-1}} \quad (27)$$

wherein R is known from (26). L is then determined from (25), which gives

$$L=1/C(2\pi f_3)^2 \quad (28)$$

Design of series impedance type of equalizer for a specific case.

The equalizer of Fig. 4 will now be designed in accordance with the foregoing formula to equalize transmission over the system shown in Fig. 5, the system consisting of a periodically loaded transmission line with terminal impedances U_1 and U_2 which are pure resistances of 1540 ohms each. The line is terminated at mid shunt (that is, mid-section) position and has the following specifications:

Wire	#19 B. & S. gauge.
Cap. per mile	$.064\times10^{-6}$ farads.
Resistance per mile	86 ohms.
Leakage per mile	Proportional to frequency and equal to 0.896×10^{-6} at frequency 800.
Load coil inductance	.175 henry.
Length of line	60 loading sections, 69.6 miles.
Length of loading section	1.16 miles.

The required range of frequencies is from zero to about 2400 cycles per second. We have then

Inductance per section, $L_0=.175$.
Capacity per section, $C_0=.0742\times10^{-6}$.
Cut-off frequency, $f_c=1/\pi\sqrt{L_0C_0}=2800$.
Further, $\sqrt{L_0/C_0}=1540$.

Since the line is terminated at mid-section, the characteristic impedance is given by:

$$K=\frac{\sqrt{L_0/C_0}}{\sqrt{1-(f/f_c)^2}}=\frac{1540}{\sqrt{1-(f/2800)^2}}$$

(See equation (5) of patent to Hoyt No. 1,167,693 of Jan. 11, 1916).

The results of a computation of the attenuation of 60 sections of the line by means of the known formulæ for periodically loaded lines is illustrated in curve 1 of Fig. (6). Curve (2) of Fig. (6) is a plot of the required valve of a as given by equation (24), while curve (3) is a plot of curve (2) on a larger scale.

A study of curve (3) has led to the adoption of the following values:

$$a_0 = .720$$
$$f_1 = 1920$$
$$a_1 = .484$$
$$f_3 = 2650$$

We have therefore

$$(H+V_1)_1 = 3655$$
$$(H+V_1)_0 = 3080$$
$$e^{a}{}_0 = 2.0544$$
$$e^{a}{}_1 = 1.6226$$
$$e^{2a}{}_1 = 2.6327$$

Hence by formula (26)

$$R = (2.0544-1)\ (3080) = 3250 \text{ ohms}$$

By formula (27)

$$C = \frac{1-(1920/2650)^2}{(2\pi)\ (1920)\ (3250)}\sqrt{\frac{(1+3250/3655)^2-2.6327}{2.6327-1}}$$
$$= .0092\times10^{-6} \text{ farads.}$$

and finally by formula (28)

$$L = 10^6/.0092(2\pi2650)^2 = .391 \text{ henry.}$$

Curve (4) of Fig. (6) shows the computed attenuation actually furnished by the equalizer having the above given values while curve (5) is a plot of the resultant attenuation of the system. It will be seen that the attenuation is substantially constant over the required range of frequencies.

In the above specific case the object was to equalize the transmission; that is, to make the resultant attenuation conform to a preassigned horizontal straight line graph, curve 2 representing the attenuation which the equalizer must furnish to accomplish this object. If, instead, the object were to make the resultant attenuation conform to a preassigned curved graph (instead of to a preassigned horizontal straight line graph) the design procedure would be exactly the same as above if curve 2 is drawn to represent the attenuation which the equalizer must furnish to make the resultant attenuation conform to the preassigned curved graph. Similar remarks apply, of course, to the other types of equalizers described below.

Shunt admittance type of equalizer.

If the equalizer consists of an admittance Y, bridged across the line between the sending end admittance V_1 and the line (K, Γ) closed through an admittance V_2 at the receiving end, the transfer admittance is given by (14), which may be written

$$T'_{21} = e^{-\Gamma}\sqrt{\frac{V_1V_2}{4}}\ \frac{\sqrt{4HV_1}}{H+V_1}\ \frac{\sqrt{4HV_2}}{H+V_2}\ \frac{H+V_1}{H+V_1+Y} \quad (29)$$

H being the characteristic admittance of the line, or the reciprocal of the characteristic impedance.

If we now introduce a set of parameters, defined by the following equations

$$A+iA' = \Gamma$$
$$a+ia' = \log_e\left[\frac{H+V_1+Y}{H+V_1}\right]$$
$$b+ib' = \log_e\left[\frac{H+V_1}{\sqrt{4HV_1}}\right] \quad (30)$$
$$c+ic' = \log_e\left[\frac{H+V_2}{\sqrt{4HV_2}}\right]$$

it follows that

$$|e^{-\Gamma}| = e^{-A}$$
$$\left|\frac{H+V_1+Y}{H+V_1}\right| = e^{a}$$
$$\left|\frac{H+V_1}{\sqrt{4HV_1}}\right| = e^{b} \quad (31)$$
$$\left|\frac{H+V_2}{\sqrt{4HV_2}}\right| = e^{c}$$

substituting these values in equation (29) we have

$$\left|T'_{21}\right| = \left|\sqrt{\frac{V_1V_2}{4}}\right| e^{-(A+a+b+c)} \quad (32)$$

as the expression for the absolute value of the transfer admittance.

Now let the bridged equalizer be as shown in Fig. 7 and consist of a resistance element R in series with the parallel combination of an inductance L and a capacity C. The admittance Y of this combination is

$$Y = \frac{1-LCp^2}{R(1-LCp^2)+iLp} \quad (33)$$

The design procedure is now similar to that for the series type equalizer. Having selected appropriate values of a_0, f_1, a_1, and f_3, we have

$$LC = 1/(2\pi f_3)^2 \quad (34)$$

and, for the practically important case in which the admittances H and V_1 are real,

$$R = 1/(e^{a}{}_0-1)\ (H+V_1)_0 \quad (35)$$

whence,

$$L = R\,\frac{[1-(f_1/f_3)^2]}{2\pi f_1}\sqrt{\frac{[1+1/R(H+V_1)_1]^2-e^{2a}{}_1}{e^{2a}{}_1-1}} \quad (36)$$

and

$$C = 1/L(2\pi f_3)^2 \quad (37)$$

These follow from (31) and (33), the admittance of the equalizer at zero frequency being simply 1/R.

The design of the equalizer of Fig. 7 will now be worked out for the system shown in Fig. 8. This system is identical with that of Fig. 5 except that the loaded line is terminated at mid-load instead of mid-section position. The formula for the characteristic impedance K is then

$$K=\sqrt{L_0/C_0}\sqrt{1-(f/f_c)^2}$$
$$=1540\sqrt{1-(f/2800)^2}$$

(see equation (4) of Patent No. 1,167,693). Hence the characteristic admittance H is

$$H=\frac{1}{1540\sqrt{1-(f/2800)^2}}$$

If we select the same values of the parameters as before, namely

$$a_0=.720$$
$$f_1=1920$$
$$a_1=.484$$
$$f_3=2650$$

then, by substitution of these values in formulæ (35)—(37) we have:

R=728 ohms
L=.0218 henry
$C=.1656\times10^{-6}$ farads

Owing to the exact mathematical relation obtaining between the equalizers of Figs. (4) and (7), curves 4 and 5 of Fig. (5) are also valid for the particular design given above.

Wave-filter type of equalizer.

A third type of attenuation equalizer may be obtained by a special design of the periodic structure or wave filter which is disclosed in patent to Campbell No. 1,227,113 of May 22, 1917. The distinguishing property of the structure, as fully set forth in the above mentioned specification, is that of transmitting freely or without attenuation all currents whose frequencies lie within a preassigned range or ranges of frequency while attenuating currents of all frequencies lying outside said range or ranges. In the present invention the use made of this characteristic property is quite distinct from that set forth in the above mentioned specification, in that in the present invention, the wave filter is so proportioned that the attenuation introduced by said filter within the range of telephonic frequencies is complementary to the attenuation introduced by the transmission system with which it is cooperatively combined to the end that the resultant attenuation shall be substantially constant over the desired range of frequencies.

In other words use is made of the fact that in a periodic structure of the type disclosed in the above patent the attenuation does not increase sharply at the cut off frequency of the band of frequencies transmitted, but increases gradually to a large value, and by properly proportioning the structure this gradual increase may be extended over the telephonic range in inverse order to the line attenuation to add sufficient attenuation to the line attenuation to secure a substantially uniform resultant attenuation.

Any or all of the specific types of wave filters disclosed in U. S. Patent No. 1,227,113 may be employed as attenuation equalizers when designed in accordance with the principles and formulæ hereinafter set forth; but it has been found that the specific types illustrated in Figs. (9) and (10) are particularly adapted, when properly proportioned, to render loaded line systems substantially distortionless.

The propagation constant of the wave filter, or periodic structure, is determined by equation (2) of Patent No. 1,227,113:

$$\operatorname{Cosh}\Gamma'=1+Z_1/2Z_2$$
$$=1+\frac{\gamma^2}{2} \qquad (38)$$

In this equation Z_1 and Z_2 are the impedances in series with and in shunt across the line, respectively; and $\gamma^2=Z_1/Z_2$. Furthermore the limiting frequencies of free transmission, here designated by f_1 and f_2, as stated in the above mentioned patent are determined by:

$$\frac{Z_1}{Z_2}=0$$
$$\qquad (39)$$
$$\frac{Z_1}{Z_2}=-4$$

Letting $p=2\pi f$ when f is any frequency and referring to the type of wave filter in which each section consists of an inductance in series with the lines, and an inductance and capacity in parallel in shunt across the line as shown in Fig. 9,

$$Z_1=ipL_1$$
$$Z_2=\frac{ipL_2}{1-p^2L_2C_2} \qquad (40)$$
$$\gamma^2=\frac{L_1}{L_2}(1-p^2L_2C_2)$$

If p_1 and p_2 denote $2\pi f_1$ and $2\pi f_2$ respectively, then, (39) and (40),

$$p_1=1/\sqrt{L_2C_2}$$
$$p_2=p_1\sqrt{1+4L_2/L_1} \qquad (41)$$

If w denotes the ratio $f/f_1=p/p_1$, then by substituting the value given by the first equation of (41) in the third equation of (40), we find that

$$\gamma^2=\frac{L_1}{L_2}1-w^2) \qquad (42)$$

If r denotes the ratio $f_2/f_1=p_2/p_1$, the last equation of (41) gives

$$\frac{L_1}{L_2}=\frac{4}{r^2-1}$$

and substituting this value in (42) we have

$$\gamma^2=4\frac{1-w^2}{r^2-1} \quad (43)$$

The corresponding equations for the type of filter shown in Fig. 10 are:

$$Z_1=ipL_1+\frac{1}{ipC_1}$$

$$Z_2=\frac{1}{ipC_2} \quad (44)$$

$$\gamma^2=\frac{C_2}{C_1}(1-p^2L_1C_1)$$

$$p_1=1/\sqrt{L_1C_1}$$

$$p_2=p_1\sqrt{1+4C_1/C_2} \quad (45)$$

and

$$\gamma^2=4\frac{1-w^2}{r^2-1} \quad (46)$$

(46) being identical with (43).

It may be shown that the characteristic impedance of a wave filter when terminated at mid-series (that is, mid-load) is

$$K'_a=\sqrt{Z_1Z_2}(1+\gamma^2/4)^{1/2} \quad (47)$$

and when terminated at mid-shunt (that is, mid-section)

$$K'_b=\sqrt{Z_1Z_2}(1+\gamma^2/4)^{-1/2} \quad (48)$$

Formula (47) for K'_a can be derived in a simple manner by considering in an infinitely long wave filter, terminating at mid-series, the first periodic interval extending from mid-series to mid-series position. If the impedance of this filter is denoted by K'_a the impedance of the remaining portion (which is also infinitely long) is equal to K'_a; and thus the distant end of the first periodic interval is closed through an impedance equal to K'_a. Now the impedance K'_a of the system consisting of one periodic interval ($Z_1/2$, Z_2, $Z_1/2$) having its distant end closed through an impedance K'_a is clearly some function of this closing impedance K'_a and of the preceding elements Z_1 and Z_2. In fact, as is now evident,

$$K'_a=\frac{Z_1}{2}+\frac{Z_2(K'_a+Z_1/2)}{Z_2+K'_a+Z_1/2}$$

On clearing of fractions, this yields a quadratic equation for determining K'_a, and the solution is the value, given by (47). Formula (48) for K'_b can be established in an analogous manner.

If the values derived above for Z_1, Z_2 and γ^2 are substituted in (47) and (48) the mid-series and mid-shunt characteristic impedances of the filter of Fig. 9 are respectively:

$$\left.\begin{aligned}K'_a&=i\sqrt{\frac{L_1}{C_2}}\sqrt{\frac{w^2}{1-w^2}\,\frac{r^2-w^2}{r^2-1}}\\ K'_b&=i\sqrt{\frac{L_1}{C_2}}\sqrt{\frac{w^2}{1-w^2}\,\frac{r^2-1}{r^2-w^2}}\end{aligned}\right\} \quad (49)$$

Similarly for the type of Fig. 10,

$$K'_a=-i\sqrt{\frac{L_1}{C_2}}\sqrt{\frac{1-w^2}{w^2}\,\frac{r^2-w^2}{r^2-1}} \quad (50)$$

$$K'_b=-i\sqrt{\frac{L_1}{C_2}}\sqrt{\frac{1-w^2}{w^2}\,\frac{r^2-1}{r^2-w^2}} \quad (51)$$

If the propagation constant Γ' of the periodic structure is denoted by $\Gamma'=B+iB'$ when B and B' are both real, it may be shown, from equations (38) and (43); or (38) and (46), that

$$\cosh\Gamma'=\frac{1+r^2-2w^2}{r^2-1}$$

But

$$\begin{aligned}\cosh\Gamma'&=\cosh(B+iB')\\&=\cosh B\cos B'+i\sinh B\sin B'\end{aligned}$$

Now, if we consider frequencies below the range of free transmission (that is, $f<f_1$, so that $w<1$), the expression $(1+r^2-2w^2)/(r^2-1)$ in the first equation for $\cosh\Gamma'$ is real; and consequently, in the second equation for $\cosh\Gamma'$, the term $i\sinh B\sin B'$ must vanish whence

$$\sinh B\sin B'=0$$

Since B is finite $\sin B'$ must equal zero and $\cos B'$ therefore equals unity. Consequently

$$\left.\begin{aligned}\cosh B&=\frac{1+r^2-2w^2}{r^2-1}\\ \sinh\left(\frac{B}{2}\right)&=\sqrt{\frac{1-w^2}{r^2-1}}\end{aligned}\right\} \quad (52)$$

where B is the attenuation per section.

If now the filter is to be applied, as in Fig. 3, to the case of a transmission line (K,Γ) with terminal impedances U_1 and U_2, the appropriate formula is (17); which, for the present purpose, may conveniently be written in the form

$$T'_{21}=\frac{e^{-(\Gamma+n\Gamma')}}{\sqrt{4U_1U_2}}\,\frac{\sqrt{4U_1K'}}{U_1+K'}\,\frac{\sqrt{4K'K}}{K'+K}\,\frac{\sqrt{4KU_2}}{K+U_2} \quad (53)$$

Introducing a set of parameters defined by the equations

$$\left.\begin{aligned}a+ia'&=\log_e\left[\frac{K+U_2}{\sqrt{4KU_2}}\right]\\ b+ib'&=\log_e\left[\frac{K'+K}{\sqrt{4KK'}}\right]\\ c+ic'&=\log_e\left[\frac{U_1+K'}{\sqrt{4U_1K'}}\right]\end{aligned}\right\} \quad (54)$$

whence

$$\left.\begin{aligned}\left|\frac{K+U_2}{\sqrt{4KU_2}}\right|&=e^a\\ \left|\frac{K'+K}{\sqrt{4KK'}}\right|&=e^b\\ \left|\frac{U_1+K'}{\sqrt{4U_1K'}}\right|&=e^c\end{aligned}\right\} \quad (55)$$

and remembering that $\Gamma = A + iA'$ and $\Gamma' = B + iB'$ so that

$$e^{-A} = |e^{-\Gamma}| \qquad (56)$$
$$e^{-B} = |e^{-\Gamma'}|$$

we see, from equation (53), that

$$\left|T'_{21}\right| = \frac{e^{-(A+nB+a+b+c)}}{|\sqrt{4U_1U_2}|} \qquad (57)$$

The application of these formulæ in designing the equalizer will be illustrated in connection with the specific case worked out below.

For the practically important case in which U_1 and U_2 are pure and constant resistances it follows that the problem involved in the design of the filter type equalizer is to make the sum $A+nB+a+b+c$ substantially constant over a specified range of frequencies, since then by equation (57) the transfer admittance of the equalized system is constant. The line attenuation A and the line impedance K are data of the problem. The choice of the type of filter, its termination (mid-series or mid-shunt, ordinarily), the number of sections n and the parameters p_1, r and

$$\sqrt{L_1/C_2}$$

are at our disposal; as are also, in many cases, the absolute values of the terminal impedances U_1 and U_2, since their absolute values can be varied by the choice of suitable transformers, to connect them to the line and filter respectively. The choice of these parameters is a matter of engineering study and trial and error by aid of the formulæ already developed.

The design of the type of filter shown in Fig. 10 to equalize transmission over the system shown in Fig. 11 will now be worked out. It is required to equalize transmission between the limiting frequencies 200 and 2,000 cycles per second, for a system consisting of 500 miles of open wire No. 8 B. W. G. loaded line. The line has a critical or cut-off frequency f_c of 2350 cycles per second, and the value of

$$\sqrt{L_0/C_0}$$

of the line is 1900. The line is terminated at midload position whence.

$$K = 1900\sqrt{1-(f/2350)^2}$$

(See equation (4) of Patent No. 1,167,693.)

The line attenuation A for the 500 miles is shown by curve (1) of Fig. 12. This attenuation curve may be theoretically calculated when the line constants are specified or may be experimentally determined. In any case it is a datum of the problem. After considering several tentative values, the following values of the design parameters were adopted:

$$f_1 = 2150$$
$$r = 2.24$$
$$n = 5$$
$$U_1 = 270 \text{ ohms}$$
$$U_2 = 1800 \text{ ''}$$
$$\sqrt{L_1/C_2} = 135 \text{ ''}$$

It was also decided to terminate the filter at mid-shunt position.

By reference to formulæ previously derived we have then

$$K' = -i(135)\sqrt{\frac{1-(f/2150)^2(2.24)^2-1}{(f/2150)^2(2.24)^2-(f/2150)^2}} \text{ by (51)}$$

$$\cosh B = \frac{1+(2.24)^2-2(f/2150)^2}{(2.24)^2-1} \text{ by (52)}$$

$$L_1C_1 = 1/(2\pi 2150)^2$$

$$1+4\frac{C_1}{C_2} = (2.24)^2 \text{ by (45)}$$

$$\frac{L_1}{C_2} = (135)^2$$

Solving these last equations the constants of the filter are:

$$L_1 = 10\times10^{-3} \text{ henry}$$
$$C_1 = 0.55\times10^{-6} \text{ farad}$$
$$C_2 = 0.55\times10^{-6} \text{ farad}$$

By equations (55) and the above given values, the attenuation coefficients a, b and c may be computed, while B is computed from the above given formula (52) for cosh B. The resultant attenuation.

$$A+nB+a+b+c$$

is thus computed for various frequencies. Its graph as a function of the frequency is shown in curve (2) of Fig. 12. It will be observed that its value is substantially constant over the preassigned range from 200 to 2,000 cycles per second, and therefore the required equalization of transmission is accomplished. The absolute value of the resultant attenuation is considerably increased but the loss which is thus introduced by the equalizer may be compensated for by employing a repeater.

It will be seen that by means of this invention, the distortion due to the increase in attenuation with increase in frequency in a given transmission system, may be substantially eliminated by inserting in the system an impedance arrangement or network so designed as to increase the attenuation for the lower frequencies to such extent that the resultant attenuation of the system will be substantially constant over the range of frequencies equalized. While this necessarily involves an increase in the total transmission loss, the loss may be made up by repeaters.

It will also be obvious that the general principles herein disclosed may be embodied in many other organizations widely differing from those illustrated without departing from the spirit of the invention as defined in the following claims.

What is claimed is:

1. The combination with a transmission line in which the attenuation varies for different frequencies, of an auxiliary localized network associated with said transmission line at a point along its length and so constructed and proportioned that the attenuation therein varies with the frequency in such a manner that the resultant attenuation of the combined system will be substantially equal over a desired range of frequencies.

2. The combination with a transmission line in which the attenuation varies for different frequencies, of an auxiliary localized network associated with said transmission line at a point along its length and so constructed and proportioned as to increase the attenuation of the less attenuated frequencies to such an extent that all frequencies over a desired range will be transmitted over the combined system with substantially the same attenuation.

3. The combination with a transmission line in which higher frequencies are attenuated more than lower frequencies, of an auxiliary localized network associated with said transmission line at a point along its length and so constructed and proportioned that lower frequencies are attenuated more than higher frequencies to such extent that the resultant attenuation of the combined system is substantially equal over a desired range of frequencies.

4. The combination of a transmission line over which different frequencies are transmitted with different attenuations, and an attenuation equalizer for the line comprising a localized network associated with said transmission line at a point along its length and whose elements are so related to each other and to the line, and so proportioned, that all frequencies within a desired range are transmitted over the line and attenuation equalizer with substantially equal attenuation.

5. The combination of a transmission line over which higher frequencies are transmitted with greater attenuation than lower frequencies, and an attenuation equalizer for the line comprising a localized network associated with said transmission line at a point along its length and whose elements are so related to each other and to the line, and so proportioned, that lower frequencies are attenuated more than high frequencies to such extent that the resultant attenuation of the system is substantially equal over a desired range of frequencies.

6. In a transmission system, a transmission line over which different frequencies are transmitted with different attenuations, and an attenuation equalizer, said attenuation equalizer comprising a localized network associated with said transmission line at a point along its length and consisting of a plurality of elements, so proportioned and so related to each other and to the line, that all frequencies within a desired range will be transmitted over the system with substantially equal attenuation.

7. In a transmission system, a transmission line over which higher frequencies are transmitted with greater attenuation than lower frequencies and a localized attenuation equalizer associated with the line at a point along its length, said attenuation equalizer being so constructed and proportioned with reference to the line as to increase the attenuation of lower frequencies to such extent that all frequencies within a desired range will be transmitted over the system with substantially equal attenuation.

8. In a transmission system, a transmission line over which different frequencies are transmitted with different attenuation, and an attenuation equalizer serially connected with the line, said attenuation equalizer comprising resistance, inductance and capacity elements so proportioned and so related to each other and to the line, that all frequencies within a desired range will be transmitted over the system with substantially equal attenuation.

9. In a transmission system, a transmission line over which higher frequencies are transmitted with greater attenuation than lower frequencies, and an attenuation equalizer serially connected with the line, said attenuation equalizer comprising resistance, inductance and capacity elements so proportioned and related to each other and the line as to increase the attenuation of lower frequencies to such extent that all frequencies within a desired range will be transmitted over the system with substantially equal attenuation.

10. In a transmission system, a transmission line, and an attenuation equalizer, said attenuation equalizer comprising a localized network associated with said transmission line at a point along its length and whose elements are so proportioned and related to

each other and the line that all frequencies within a desired range will be transmitted over the system with substantially equal attenuation.

11. In a transmission system, a transmission line, and an attenuation equalizer serially connected with said line said attenuation equalizer comprising resistance and inductance elements so proportioned and related to each other and the line that all frequencies within a desired range will be transmitted over the system with substantially equal attenuation.

12. In a transmission system, a transmission line, and an attenuation equalizer serially connected with said line said attenuation equalizer comprising resistance and capacity elements so proportioned and related to each other and the line that all frequencies within a desired range will be transmitted over the system with substantially equal attenuation.

13. In a transmission system, a transmission line, and an attenuation equalizer, said attenuation equalizer comprising localized inductance and capacity elements associated with said line at a point along its length and so proportioned and related to each other and the line that all frequencies within a desired range will be transmitted over the system with substantially equal attenuation.

14. In a transmission system, a transmission line the attenuation of which varies with frequency in accordance with a known law, and a localized network associated with said line at a point along its length, said network being so connected and proportioned with respect to said line as to cause the attenuation of the system comprising the line and network to vary with frequency in a different predetermined manner.

15. In a transmission system, a transmission line the attenuation of which varies with frequency in accordance with a known law, and an auxiliary network the attenuation of which is predeterminable at different frequencies, said auxiliary network being localized with respect to the line and so designed and so associated with the transmission line at a point along the length thereof that the resultant attenuation varies with the frequency in accordance with a predetermined law.

In testimony whereof, I have signed my name to this specification this 25th day of June 1918.

RAY S. HOYT.