April 26, 1938. S. DARLINGTON 2,115,138

WAVE TRANSMISSION NETWORK

Filed March 20, 1935 2 Sheets-Sheet 1

FIG. 1

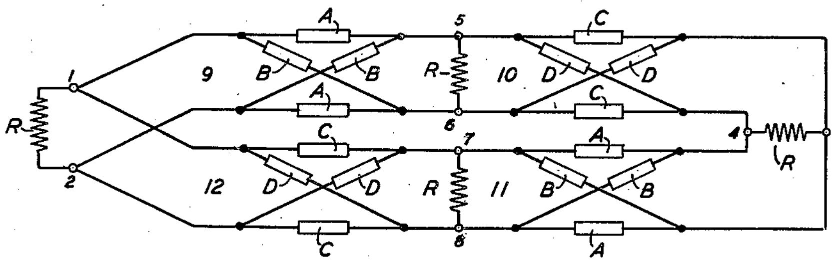


FIG. 2

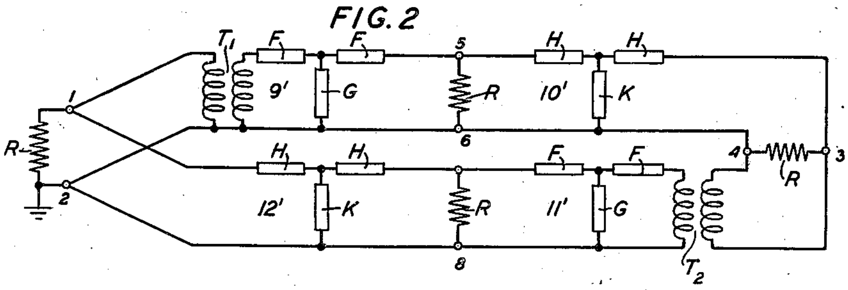


FIG. 3

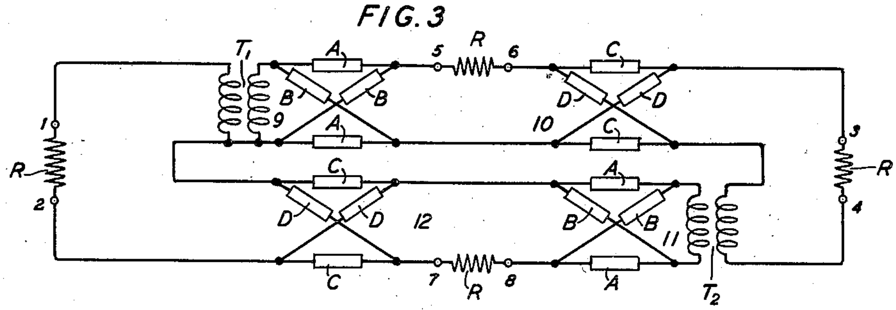
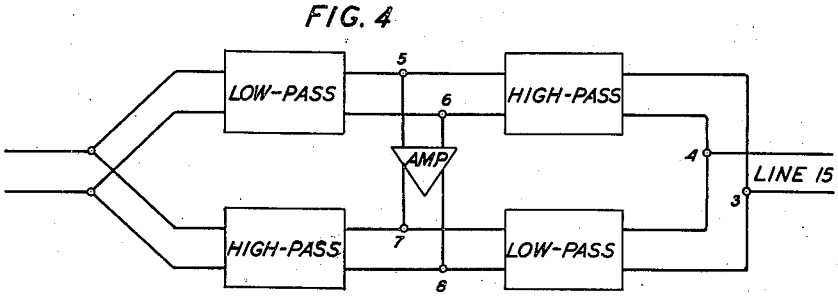


FIG. 4



INVENTOR
S. DARLINGTON
BY G H Stevenson
ATTORNEY

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FIG. 5

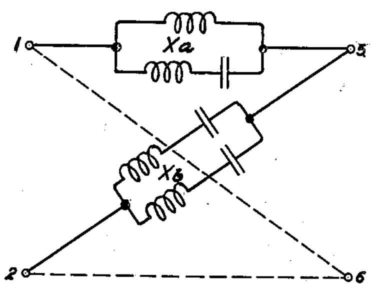


FIG. 7

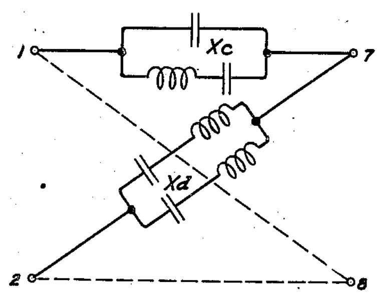


FIG. 6

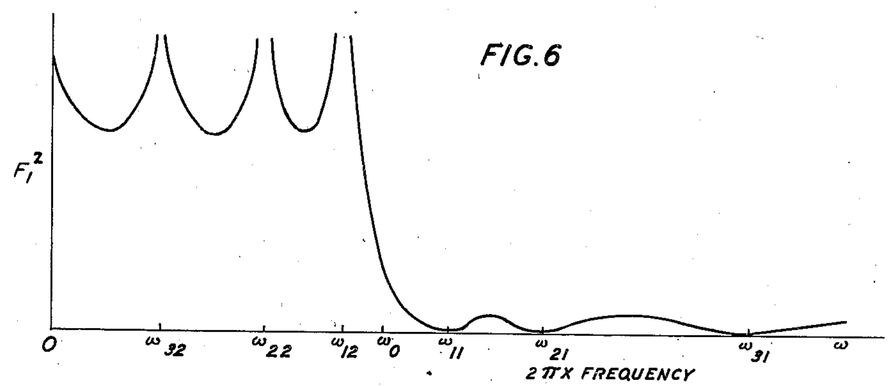
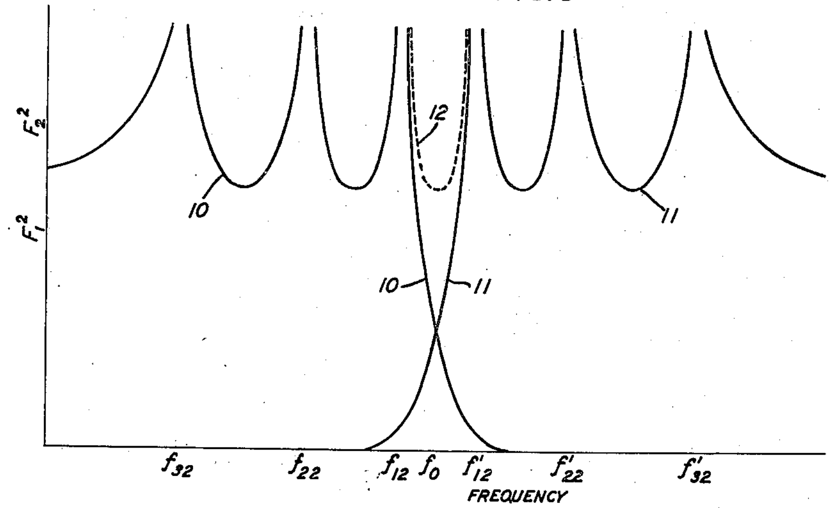


FIG. 8



INVENTOR
S. DARLINGTON
BY G H Stevenson
ATTORNEY

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2,115,138

UNITED STATES PATENT OFFICE

2,115,138

WAVE TRANSMISSION NETWORK

Sidney Darlington, New York, N. Y., assignor to Bell Telephone Laboratories, Incorporated, New York, N. Y., a corporation of New York

Application March 20, 1935, Serial No. 11,934

14 Claims. (Cl. 178—44)

This invention relates to wave transmission networks and, more particularly, to networks having frequency selective transmission characteristics.

Frequency selective transmission networks having the property of constant resistance characteristic impedance have been known and used for some time but heretofore only in the form of two-terminal impedances or of ordinary four-terminal networks. An object of the present invention is to extend this constant resistance property to more complex structures, in particular to eight terminal networks. Networks of this type comprise four individual transmission paths and are useful for the separation of currents of different frequencies as, for example, in carrier current repeaters. The problem of connecting these networks into a given transmission system is greatly simplified by the present invention since, by virtue of the constant resistance characteristic, reflection effects at the insertion points are greatly reduced and, in many cases, substantially eliminated.

The nature and the underlying principles of the invention will be more fully understood from the following detailed description and by reference to the attached drawings, of which

Fig. 1 illustrates schematically one form of network embodying the invention;

Figs. 2 and 3 illustrate alternative forms of the invention;

Fig. 4 is illustrative of the application of the invention to telephone repeaters;

Fig. 5 shows a typical arrangement of one of the component networks of the system of Fig. 1;

Fig. 6 is a diagram illustrating a characteristic of one of the networks of Fig. 5;

Fig. 7 represents another component network of Fig. 1; and

Fig. 8 illustrates a characteristic of a complete system in accordance with the invention.

The network shown in Fig. 1 comprises two parallel transmission paths extending between pairs of terminals 1, 2 and 3, 4, and each including two component networks 9, 10, in the upper path, and 11, 12, in the lower path. Resistances of value R are connected between terminals 1, 2 and 3, 4, and also between two additional pairs of terminals 5, 6 and 7, 8, located between networks 9 and 10 and networks 11 and 12, respectively.

The networks 9, 10, 11 and 12 are of the symmetrical lattice type and are similar in pairs, networks 9 and 11 being alike and also networks 10 and 12. The line and the lattice branch impedances of networks 9 and 11 have values A and B, respectively, and the corresponding branch impedances of networks 10 and 12 have values C and D, respectively. These impedances are preferably pure reactances and may be chosen to give each network a desired type of transmission characteristic. For example 9 and 11 may have low-pass characteristics while 10 and 12 may be of the high-pass type.

The order of the component networks is reversed in the one path with respect to the other, but otherwise the two paths have the same composition and therefore have equal transfer constants. There is also a reversal in the interconnection of the two paths at terminals 3 and 4 with respect to the connections at 1 and 2, the effect of which is equivalent to the addition of a phase shift of 180 degrees in one or other of the two paths.

This phase reversal, together with the similarity between the two paths gives rise to a condition of conjugacy between the resistance paths at terminals 1, 2 and 3, 4, respectively. This may be demonstrated as follows: Consider the resistance R between terminals 3 and 4 to be replaced by a short circuit and a voltage E_0 applied to terminals 1 and 2. Under this condition the total current through the short circuit will be the resultant of the separate transmissions through the individual paths, the output from the one path being unable to pass beyond the short circuit into the other path. From the principle of reciprocity it follows that the current in the short circuit from either one of the paths alone will not be changed if the path is turned end for end. Since such a reversal in one path would make the two paths exactly alike, it follows that the output currents in the short circuit will be of equal magnitude and will completely neutralize each other because of the relative reversal of phase due to the circuit connections. Since no current flows in the short circuit an impedance of any magnitude may be inserted therein without disturbing the conjugacy. Similar considerations will show that the resistance paths between terminals 5 and 6 and between terminals 7 and 8 are also conjugate. In this case the two paths include networks 10, 11 and 9, 12 respectively, with intermediate bridging resistances, and are similar in composition to the two paths considered in the previous case. A phase reversal is also present in one path with respect to the other due to the interconnections at the intermediate terminals 1, 2 and 3, 4.

When the four pairs of terminals are bridged by equal resistances of value R it is possible, by proper choice of the impedances of the component networks, to make the input impedances of the whole network at each of the four pairs of terminals equal to the resistance R at all frequencies. This is the same thing as giving the system a constant resistance image impedance of value R at all four of its pairs of terminals, the image impedances of a transmission network being defined as the impedances measured at each of the several pairs of terminals when the other pairs are closed through impedances which produce zero reflection effects at all frequencies, that is when they are closed through their respective image impedances. In symmetrical systems the image impedances are equal and are equal to the characteristic impedance. The necessary relationships between the component network impedances to achieve this constant resistance characteristic may be found as follows:

Consider the input impedance at terminals 1 and 2. Since the resistance path between 3 and 4 is conjugate to the path between 1 and 2, the input impedance will not be affected by the value of the resistance in the branch 3, 4. The terminals 3 and 4 may therefore be assumed to be short-circuited in which case the input impedance at 1 and 2 is simply the impedance of the two transmission paths in parallel. Let Z_1 denote the impedance of the path including networks 9 and 10 with the output terminals of 10 short-circuited and let Z_2 denote the impedance of the other transmission path also short-circuited at the output end, then, if Z denote the total input impedance at 1 and 2,

$$\frac{1}{Z}=\frac{1}{Z_1}+\frac{1}{Z_2} \tag{1}$$

The values of Z_1 and Z_2 may be determined from the open circuit and short circuit impedances of the component networks. Since each of these networks is symmetrical its open and short circuit impedances will be the same at both ends. Networks 9 and 11, being similar, will have the same impedances and networks 10 and 12 likewise. Let the open circuit and short circuit impedances of 9 and 11 be denoted Z_o and Z_s respectively and let the corresponding impedances of networks 10 and 12 be denoted by Y_o and Y_s.

The impedance Z_1 is that of the network 9 terminated by an impedance made up of the parallel combination of resistance R and the short circuit impedance of network 10 and has the value

$$Z_1=Z_o\frac{Z_{r1}+Z_s}{Z_{r1}+Z_o} \tag{2}$$

where

$$Z_{r1}=\frac{RY_s}{R+Y_s}$$

Substituting the value of Z_{r1} in Equation 2 and inverting, gives

$$\frac{1}{Z_1}=\frac{1}{Z_o}\left[\frac{RY_s+Z_o(R+Y_s)}{R(Y_s+Z_s)+Y_sZ_s}\right] \tag{3}$$

In a like manner the value of Z_2 is found to be

$$\frac{1}{Z_2}=\frac{1}{Y_o}\left[\frac{RZ_s+Y_o(R+Z_s)}{R(Y_s+Z_s)+Y_sZ_s}\right] \tag{4}$$

If the input impedance at terminals 1, 2 is to be a constant resistance of value R, then Z_1 and Z_2 must be so related that

$$\frac{1}{R}=\frac{1}{Z_1}+\frac{1}{Z_2}$$

or

$$\frac{1}{R}=\frac{1}{Z_o}\left[\frac{RY_s+Z_o(R+Y_s)}{R(Y_s+Z_s)+Y_sZ_s}\right]+\frac{1}{Y_o}\left[\frac{RZ_s+Y_o(R+Z_s)}{R(Y_s+Z_s)+Y_sZ_s}\right] \tag{5}$$

from which, by simplification, the relationship is obtained

$$\frac{1}{R^2}=\frac{1}{Z_oZ_s}+\frac{2}{Y_sZ_s}+\frac{1}{Y_oY_s} \tag{6}$$

In terms of the line and lattice impedances, A, B, C and D, which make up the networks 9 to 12, Equation 6 becomes

$$\frac{1}{R^2}=\frac{1}{AB}+\frac{1}{CD}+\frac{1}{2}\left(\frac{1}{A}+\frac{1}{B}\right)\left(\frac{1}{C}+\frac{1}{D}\right) \tag{7}$$

Equations 6 and 7 set forth relationships of the network impedances, and of the branch impedances when lattice type networks are used, which must be fulfilled in order that the constant resistance requirement may be met. A procedure which may be followed in choosing the impedances A, B, C and D in accordance with these requirements will be described later. If will be sufficient at this point to note that the requirements can always be met with physical impedance elements and that these may be selected and proportioned to provide desired transmission characteristics between adjacent pairs of terminals.

In the network of Figure 1 the two transmission paths are of the balanced type and the network is therefore suited for direct connection to transmission lines having both sides balanced to ground. A modified form of the invention in which the two transmission paths are of the unbalanced type is shown in Figure 2. In this network the component networks are designated 9', 10', 11', and 12' and consist of series-shunt impedance combinations with the series impedances inserted in only one line of each path. Terminals 2, 4, 6 and 8 are connected together and to ground. To permit the grounding of one side of each path a transformer T_1 is included at one end of the upper path and a second transformer T_2 at the other end of the lower path. These transformers should be similar in their characteristics and should have unity transformation ratios. If they are similarly poled the relative phase shifts of the two paths will not be affected by their inclusion and the conjugacy of the opposite pairs of terminals will not be impaired. The component networks in Figure 2 are shown as symmetrical T networks, 9' and 11' having series impedances F and shunt impedances G and 10' and 12' having corresponding impedances H and K. Single section networks only are shown, but it will be understood that as many sections as desired may be used, the showing being intended to represent simply the unbalanced equivalents of the generalized lattices of Figure 1. It should be noted, however, that the component networks are of the series terminated type. This is necessary where the resistances R are connected in shunt to the component network terminals, as in Figures 1 and 2. Otherwise, a shunt path would be provided at each pair of terminals which would have zero impedance at some frequency and would therefore prevent the realization of the constant resistance characteristic.

Another form of the invention is shown in Figure 3 which differs from that shown in Figures 1 and 2 in that the resistances R are connected in series between the component networks instead of in shunt. The component networks 9, 10, 11 and 12, are shown as symmetrical lattices as in Figure 1, but the unbalanced equivalents of these lattices may also be used as in Figure 2. In the latter case the lower line of the upper path and the upper line of the lower path would form the common grounded conductor. Transformers T_1 and T_2 inserted as in Figure 2 permit the grounding to be effected and, with appropriate poling, maintain the conjugacy of the opposite pairs of terminals. When the unbalanced networks are used they should be of the shunt terminated type for reasons converse to those dictating the use of series terminations in the network of Figure 2.

From considerations similar to those applied to the analysis of the circuit of Figure 1 it may be shown that the relationship required in this case for securing a constant resistance impedance characteristic at each of the four pairs of terminals is expressed by

$$R^2=Z_oZ_s+2Y_oZ_o+Y_oY_s \quad (8)$$

or, in terms of the lattice impedances A, B, C and D

$$R^2=AB+\tfrac{1}{2}(A+B)(C+D)+CD \quad (9)$$

An example of the use of the invention is illustrated in Figure 4 which shows its application to a carrier telephone or telegraph repeater of the single amplifier type shown in U. S. Pat. 1,874,492 issued Aug. 30, 1932, to A. G. Ganz. The network illustrated corresponds to that of Figure 1 in which the resistances are bridged across the terminals of the component networks. The two portions **14** and **15** of a transmission line in which the repeater is inserted are connected to terminals **1**, **2** and **3**, **4**, respectively and furnish the requisite bridging resistances. An amplifier **13** is connected between terminals **5**, **6** and **7**, **8**. The bridging resistances being provided by suitable elements associated with the input and output circuits of the amplifier.

In a multiplex carrier transmission system it is customary to include all channels transmitting in one direction in a low frequency group and those transmitting in the opposite direction in a separate high frequency group. By making networks **9** and **11** of the character of low-pass filters a path through the repeater from line **14** to line **15** is provided for the low frequency channels, this path including filter **9**, amplifier **13** and filter **11**. By making networks **10** and **12** of the character of high pass filters a corresponding path through the repeater from line **15** to line **14** is provided for the high frequency channels. These paths may be made mutually exclusive by the proper choice of the filter cut-off frequencies and by providing adequate attenuation in the filters.

The conjugacy existing between the opposite pairs of terminals because of the reversal of the interconnections at terminals **3** and **4** with respect to those at terminals **1** and **2** not only permits the constant resistance characteristic to be obtained but also minimizes the possibility of singing in the amplifier by the reduction or elimination of feed-back from the amplifier output terminals **7**, **8** to the input terminals **5**, **6**.

The general requirements for the constant resistance condition of the networks have been set forth in Equations 6, 7, 8 and 9. It remains to be shown how the individual impedances of the networks may be determined in accordance with these requirements. This will be done by developing design formulae for a network of the type of Figure 1, the general procedure outline being applicable to the other forms of network.

Equation 7 which expresses the requirement for the constant resistance condition in terms of the branch impedances is general to the extent that it does not depend on the character of the impedances, which may be resistive or reactive or may include both resistance and reactance. However, since it is desirable in practice that the system should have definite selective properties the component networks will preferably be composed of substantially pure reactances and will have individual selective characteristics corresponding to the prescribed requirements of the system. With this in mind the illustrative example will correspond to the arrangement shown in Figure 4 in which networks **9** and **11** have low-pass transmission characteristics and networks **10** and **12** have high-pass characteristics.

Assuming the lattice branches to be substantially pure reactances and writing in place of the impedances A, B, C and D the corresponding reactances X_a, X_b, X_c, and X_d, Equation 7 can be transformed to

$$\left(\frac{1}{X_a}-\frac{1}{X_b}\right)^2+\left(\frac{1}{X_c}-\frac{1}{X_d}\right)^2=\left(\frac{1}{X_a}+\frac{1}{X_b}+\frac{1}{X_c}+\frac{1}{X_d}\right)^2+\frac{4}{R^2} \quad (10)$$

or

$$F_1^2+F_2^2=\left(\frac{1}{X_a}+\frac{1}{X_b}+\frac{1}{X_c}+\frac{1}{X_d}\right)^2+\frac{4}{R^2} \quad (11)$$

where

$$F_1=\left(\frac{1}{X_a}-\frac{1}{X_b}\right)$$

and

$$F_2=\left(\frac{1}{X_c}-\frac{1}{X_d}\right)$$

It is to be noted that the right hand side of Equation 11 is always positive and ranges in value between $4/R^2$ and infinity, infinite values occurring at each of the resonances of the reactances X_a, X_b etc. To meet the constant resistance requirements the reactances must be so proportioned that the sum $F_1^2+F_2^2$ varies in the same manner as the right hand side of the equation.

It may be shown by ordinary network analysis that the insertion loss between terminals **1**, **2** and **5**, **6** is given by

$$e^{2\alpha_1}=1+\left(\frac{F_2}{F_1}\right)^2 \quad (12)$$

and that between terminals **1**, **2** and **7**, **8** by

$$e^{2\alpha_2}=1+\left(\frac{F_1}{F_2}\right)^2 \quad (13)$$

where α_1 and α_2 are the respective insertion losses.

Since it is desired that the path through network **9** have a low-pass characteristic and that through network **12** a high-pass characteristic, it is apparent from Equations 12 and 13 that the ratio F_2/F_1 must be small in the low-pass range and large in the high-pass range. That is, F_2 must be small in the range where F_1 is large and vice versa.

The character of the frequency variations of F_1^2 and F_2^2 may readily be determined in any particular case. A suitable form for the low-pass networks **9** and **11** is illustrated in Figure 5 in which X_a is characterized by a single finite resonance and X_b by two finite resonances. From considerations of ordinary filter theory the resonance of X_a will occur at a frequency lying between the two resonance frequencies of X_e.

In terms of the resonance frequencies F_1^2 may be expressed by

$$F_1^2=\left[\frac{A_0}{\omega}-\frac{A_2\omega}{\omega^2-\omega_{12}^2}+\frac{A_4\omega}{\omega^2-\omega_{22}^2}-\frac{A_6\omega}{\omega^2-\omega_{32}^2}\right]^2 \quad (14)$$

where ω denotes 2π times frequency, ω_{12}, ω_{22}, and ω_{32} correspond to the three resonance frequencies, and A_0, A_2, A_4 and A_6 are constants. The resonance frequencies all occur in the low-pass transmission range and at these frequencies F_1^2 become infinite.

Equation 14 may be transformed by ordinary algebraic processes to the form

$$F_1^2=\left[\frac{C_1}{\omega}\left(\frac{\omega^2-\omega_{11}^2}{\omega^2-\omega_{12}^2}\right)\left(\frac{\omega^2-\omega_{21}^2}{\omega^2-\omega_{22}^2}\right)\left(\frac{\omega^2-\omega_{31}^2}{\omega^2-\omega_{32}^2}\right)\right]^2 \quad (15)$$

where

$$C_1\left(\frac{\omega_{11}\omega_{21}\omega_{31}}{\omega_{12}\omega_{22}\omega_{32}}\right)^2=A_0$$

and ω_{11}, ω_{21}, ω_{31}, correspond to frequencies at which F_1^2 has zero values.

These frequencies, if real, all lie above the low-pass range, but in the general case certain of them may be imaginary or complex.

The general expression for F_1 in the factorial form of Equation 15 may be written as

$$F_1=\frac{C_1}{\omega}\prod_{s=1}^{s=m}\frac{(\omega^2-\omega_{s1}^2)}{(\omega^2-\omega_{s2}^2)} \quad (16)$$

where Π indicates a product of terms of the type following it and m is the number of factors in F_1^2, that is, the number of poles or the number of zeros.

By a proper choice of critical frequencies defining the zeros and poles in Equations 15 and 16 the frequency variation of F_1^2 may be made to take the form illustrated by the curve in Fig. 6 which is characterized by a series of equal minima between the poles in the low frequency range and a corresponding series of equal maxima between the zeros in the high frequency range. Moreover the values of F_1^2 in the high frequency range may be made so small as to be completely negligible.

The design proceeds by determining the critical frequencies and the element values of the low-pass network so that the minima of F_1^2 in the low frequency range are equal to $4/R^2$ and so that the descending portion of the F_1^2 characteristic in the range between the high and the low values passes through the value $2/R^2$ at a predetermined frequency marking the division of the ranges.

A complementary characteristic is next determined for F_2^2 having minima in the high frequency range likewise equal to $4/R^2$ and having its descending portion intersect that of F_1^2 at the predetermined dividing frequency, its slope at this point being equal that of F_1^2 but of opposite sign. This ensures a minimum at the dividing frequency the value of which will be $4/R^2$.

Since the values of F_1^2 in the high frequency range are negligible and those of F_2^2 in the low frequency range are likewise negligible the sum of the two quantities gives a series of minima each of value $4/R^2$ and alternating in the frequency scale with a series of infinite values as required by Equation 11. It develops also that the form of $F_1^2+F_2^2$ agrees with that of the right hand side of Equation 11 and hence that the constant resistance requirement is met. I have found that the neglect of the low values of F_1^2 in the high frequency range and of the corresponding values of F_2^2 affects the resistance of the system to an extent of less than one part in 5000 in practical cases.

The choice of the critical frequencies of F_1^2 to give the required frequency variation is based on the following expansion theorems for elliptic functions which follow from relationships given in Elliptic Functions, by A. Cayley, published by G. Bell and Sons, London, England, second edition, 1895, pages 265 and 267.

$$C_4\sqrt{k_1}\,sn(nuK_1,k_1)=C_3\sqrt{k}\,sn(uK,k)\left[1+\sum_{s=1}^{s=m}\frac{(-1)^s 2U_sV_s}{1-k^2sn^2\left(\frac{2s}{n}K,k\right)sn^2(uK,k)}\right] \quad (17)$$

$$=C_2\sqrt{k}\,sn(uK,k)\prod_{s=1}^{s=m}\left[\frac{k\,sn^2\left(\frac{2s}{n}K,k\right)-k\,sn^2(uK,k)}{1-k^2sn^2\left(\frac{2s}{n}K,k\right)sn^2(uK,k)}\right] \quad (18)$$

Wherein the following notations are used:

Expressions of the form $sn\ (y, a)$ denote elliptic sines of modulus a and argument y, the modulus being a positive numeric of value less than unity;

K is the complete elliptic integral of the first kind, of modulus k;

K_1 is the complete elliptic integral of the first kind, of modulus k_1;

$$U_s=\sqrt{1-sn^2\left(\frac{2s}{n}K,k\right)}$$
$$V_s=\sqrt{1-k^2sn^2\left(\frac{2s}{n}K,k\right)} \quad (19)$$

$$n=2m+1$$

C_2, C_3 and C_4 are numerical constants and $sn(uK,k)$ is the variable parameter.

The quantities K, K_1, k, k_1, are related in the following manner.

Let K′ and K_1' denote the complete elliptic integrals of the first kind, of moduli

$$\sqrt{1-k^2}$$

and

$$\sqrt{1-k_1^2}$$

respectively, and let

$$q=e^{-\pi\frac{K'}{K}}$$

and

$$q_1=e^{-\pi\frac{K_1'}{K}} \quad (20)$$

then K_1 and K_1' are such that

$$\frac{K_1'}{K_1}=n\frac{K'}{K} \quad (21)$$

and

$$q_1=q^n \quad (22)$$

Knowing q and n the value of q_1 is determined and the corresponding value of k_1 can be found from standard tables of elliptic functions. For the cases of interest in connection with the present invention k_1 is small and may be determined with great accuracy from the approximate relationship

$$k_1=4\sqrt{q_1} \quad (23)$$

The value of K_1 follows from its definition.

From Equation 18 it will be seen that the quantity

$$C_4\sqrt{k_1}\,sn(nuK_1,\ k_1)$$

expressed as a function of $sn\ (u, K, k)$ has m poles, or infinite values, corresponding to the zero values of the denominator factors and has an equal number of related zeros which occur in a different range of values of the argument $sn\ (u, K, k)$. In accordance with the known principles of elliptic functions the square of the above indicated quantity goes through a series of equal minima between the poles and a series of equal maxima between the zeros.

The values of the minima are given by

$$\frac{C_4^2}{k_1} \tag{24}$$

and the maxima by

$$C_4^2 k_1 \tag{25}$$

the ratio of the maxima to the minima being equal to k_1^2.

The constants C_3 and C_4 are found by comparing the expressions in which they appear as $sn\ (u\ K,\ k)$ approaches infinity. This gives rise to the relationship

$$\frac{C_3}{K\sqrt{k}} = \frac{C_4}{nK_1\sqrt{k_1}} \tag{26}$$

Since the variation of the function

$$C_4\sqrt{k_1}\,sn(nuK_1,\ k_1)$$

corresponds to the desired variation of the quantity F_1, the design of the low-pass networks is readily accomplished by identifying F_1 with the summation expression of Equation 17 or with the product expression of Equation 18.

For this purpose let a value of ω, denoted by ω_0, be chosen in the neighborhood of the desired cut-off of the low-pass network. For the present this may be chosen arbitrarily. The critical values of ω, namely ω_{s1} and ω_{s2}, in Equation 16 are now chosen such that

$$\frac{\omega_{s2}}{\omega_0} = \frac{\omega_0}{\omega_{s1}} = p_s$$

where

$$p_s = \sqrt{k}\,sn\left(\frac{2s}{n}K,\ k\right)\Bigg]_{s=1}^{s=m} \tag{27}$$

and the variable frequency ratio ω_0/ω is identified with

$$\sqrt{k}\,sn(uK,\ k)$$

The value of the modulus k is chosen with reference to the degree of discrimination required of the network. As the value of the modulus approaches unity the poles and zeros of F_1^2 move close to ω_0 and the characteristic is marked by low minima and high maxima. Reducing the value of the modulus separates the critical frequencies and at the same time increases the ratio of the minima to the maxima. In the case of a network of small degree of complexity, for example one having two poles and two zeros, a modulus value of 0.9 will ensure a satisfactorily sharp discrimination by the network and will result in a ratio of the minima to the maxima greater than 5000. For more complex networks higher values of the modulus may be taken and because of the larger number of poles and zeros both the sharpness of discrimination and the ratio of the minima to the maxima are greatly increased.

In the case of the network illustrated in Fig. 5, for which the function F_1^2 has three poles and three zeros, Equations 26 and 27 give the following values for the critical frequencies

$$\omega_{11} = \omega_0 \div \sqrt{k}\,sn\left(\frac{2}{7}K,\ k\right) = \omega_0/p_1$$

$$\omega_{21} = \omega_0 \div \sqrt{k}\,sn\left(\frac{4}{7}K,\ k\right) = \omega_0/p_2$$

$$\omega_{31} = \omega_0 \div \sqrt{k}\,sn\left(\frac{6}{7}K,\ k\right) = \omega_0/p_3$$

$$\omega_{12} = \omega_0^2/\omega_{11}$$

$$\omega_{22} = \omega_0^2/\omega_{21}$$

and

$$\omega_{32} = \omega_0^2/\omega_{31}$$

Substituting these values in the right hand side of Equation 18 and identifying this with F_1 gives

$$F_1 = \left[\frac{C_2\omega_0 p_1^2 p_2^2 p_3^2}{\omega}\frac{(\omega^2-\omega_{11}^2)(\omega^2-\omega_{21}^2)(\omega^2-\omega_{31}^2)}{(\omega^2-\omega_{12}^2)(\omega^2-\omega_{22}^2)(\omega^2-\omega_{32}^2)}\right] \tag{28}$$

which agrees with Equation 15 when

$$C_2 = \frac{C_1}{\omega_0}\frac{1}{p_1^2 p_2^2 p_3^2}$$

Substitution of the same values in the right hand side of Equation 17 gives

$$F_1 = C_3\left[\frac{\omega_0}{\omega} - \frac{2\omega\omega_0\sqrt{(1-kp_1^2)(1-p_1^2/k)}}{\omega^2-\omega_{12}^2} + \frac{2\omega\omega_0\sqrt{(1-kp_2^2)(1-p_2^2/k)}}{\omega^2-\omega_{22}^2} - \frac{2\omega\omega_0\sqrt{(1-kp_3^2)(1-p_3^2/k)}}{\omega^2-\omega_{32}^2}\right] \tag{29}$$

This equation corresponds to Equation 14 and enables the constants A_0, A_2, etc. in that equation to be determined in terms of the chosen elliptic function parameters so that the desired variation of F_1^2 will be secured. Since each term in Equation 14 represents the susceptance of a branch path containing only a simple inductance or the combination of an inductance and capacity in series the determinations of the A's leads directly to the element values.

The required value of C_3 follows from Equations 25 and 26. Equation 25 gives the value of the minima of F_1^2 between the poles, which, in accordance with the design requirements must be equal to $4 \div R^2$. Accordingly

$$C_4 = \frac{2\sqrt{k_1}}{R} \tag{30}$$

and hence, from Equation 26

$$C_3 = \frac{2\sqrt{k}K}{R\,nK_1} \tag{31}$$

For the cases of interest, that is where the networks have relatively sharp discrimination and high attenuation, the value of K_1 will not differ sensibly from $\pi/2$ and the approximation for C_3

$$C_3 = \frac{4\sqrt{k}K}{R\ n\pi} \tag{32}$$

may be used.

The high pass network, for which the function F_2^2 has to be complementary to F_1^2, has to meet the requirement that its function F_2^2 must have negligibly small values in the low pass range, have minima equal to $4 \div R^2$ in the high pass range, and must have the rising part of its characteristic intersect the descending part of F_1^2 at the point $2 \div R^2$ with a slope equal to that of F_1^2 and opposite in sign.

These requirements are most readily met by making the schematic form of the network such that F_2 has the same number of poles and zeros as F_1 and locating the poles and zeros symmetrically with the corresponding poles and zeros of F_1 with respect to the cross over frequency, which will be denoted by $\omega_c \div 2\pi$. The poles of F_1 and F_2 will thus occur in pairs having ω_c as their geometric mean and the zeros will likewise occur in similarly related pairs.

The cross over point ω_c will not be the same as ω_0 which appears in the formulae for F_1 but will be slightly lower. The mathematical determination of this point is rather lengthy and only the explicit formula for the ratio of ω_c to ω_0 will be given here.

$$\frac{\omega_c}{\omega_0}=\frac{\sqrt{k}}{\sqrt{1-(1-k^2)\left[sn\frac{2\log(1+\sqrt{2})}{-n\log q}K',\sqrt{1-k^2}\right]^2}} \tag{33}$$

In design practice the value ω_c will generally be assigned. Equation 33 then permits the value ω_0 for the function F_1 to be determined and also, from the required symmetry of the two systems, a corresponding value which will be denoted by ω_0' for the function F_2. The values of ω_0 and ω_0' are related to ω_c by the equation

$$\omega_0\omega_0'=\omega_c^2 \tag{34}$$

Using the same principles as discussed above formulae for F_2 corresponding to Equations 28 and 29 can be developed, but it is simpler to determine F_2 directly from Equation 29 taking advantage of the symmetry of the F_1 and F_2 characteristics. For each value of ω above ω_c the function F_2^2 will have the same value as F_1^2 at the proportionality lower frequency $\omega_c^2\div\omega$. F_2 may therefore be derived from Equation 29 by the simple expedient of replacing ω therein by $-\omega_c^2\div\omega$. The negative sign appears in this transformation to take account of the difference in sign of the reactances of coils and of condensers which represent complementary impedances.

The terms of the expression thus found for F_2 will include two kinds; one set will directly represent physically realizable susceptances and the remainder will represent physically realizable susceptances when the sign is changed. Those representing directly realizable susceptances are identified with X_c and the others with X_d. The schematic form of the high-pass network thus obtained is illustrated in Fig. 7. The reactance X_c comprises a capacity and a resonant circuit connected in parallel and the reactance X_d comprises two simple resonant circuits connected in parallel.

Fig. 8 shows the curves of the two functions F_1^2 and F_2^2 and their sum plotted against the logarithm of frequency. Curve 10 represents F_1^2, curve 11 represents F_2^2 and the looped portions of the two curves together with dotted curve 12 represents their sum. The cross over frequency is designated by f_c, the poles of F_1^2 by f_{12}, f_{22}, and f_{32}, and the poles of F_2^2 by the inversely related frequencies f'_{12}, f'_{22}, and f'_{32}. The symmetry of the two characteristics about f_c is clearly shown by this figure.

While the foregoing procedures give the desired networks as symmetrical lattices, these may be reduced by known procedures to unbalanced types of networks such as bridged—T or ladder networks suitable for use in the unbalanced type of network shown in Fig. 2.

What is claimed is:

1. A wave transmission network comprising two paths connected in parallel between a first pair of terminals and a second pair of terminals, a pair of symmetrical frequency selective networks having complementary transmission bands connected in tandem in one of said paths, a pair of respectively similar networks connected in tandem in the other of said paths, but in reverse order with respect to the networks in said first path, and means for reversing the phase of the currents in one of said paths with respect to the currents in the other whereby said pairs of terminals are made to be conjugate to each other.

2. A wave transmission network comprising two paths connected in parallel between a first pair of terminals and a second pair of terminals, a pair of symmetrical frequency selective networks having complementary transmisison bands connected in tandem in one of said paths, a pair of respectively similar networks connected in tandem in the other of said paths, but in reverse order with respect to the networks in the said first paths, said networks having open circuit and short circuit impedances related in accordance with the equation

$$\frac{1}{R^2}=\frac{1}{Z_oZ_s}+\frac{2}{Y_sZ_s}+\frac{1}{Y_oY_s}$$

where Z_o and Z_s are respectively the open circuit and the short circuit impedances of the one pair of similar networks, Y_o and Y_s are the corresponding values for the other pair of similar networks, and R is an arbitrarily assigned resistance.

3. A wave transmission network comprising two paths connected in parallel between a first pair of terminals and a second pair of terminals, a pair of symmetrical frequency selective networks having complementary transmission bands connected in tandem in one of said paths, a pair of respectively similar networks connected in tandem in the other of said paths, but in reverse order with respect to the networks in the said first paths, said networks having open circuit and short circuit impedances related in accordance with the equation

$$R^2=Z_oZ_s+2Y_oY_s+Y_oY_s$$

where Z_o and Z_s are respectively the open circuit and the short circuit impedances of the one pair of similar networks, Y_o and Y_s are the corresponding values for the other pair of similar networks, and R is an arbitrarily assigned resistance.

4. A transmission network comprising two paths extending in parallel between a pair of input terminals and a pair of output terminals, two frequency selective networks having substantially complementary transmission bands disposed in tandem in one of said paths, and two respectively similar networks disposed in tandem in reverse order in the other of said paths, the networks of one of the similar pairs being defined by two frequency characteristics $f_1(\omega)$ and $f_2(\omega)$ which jointly determine their transmission properties, and the networks of the other similar pair being likewise defined by frequency characteristics $f_3(\omega)$ and $f_4(\omega)$, and the said networks being so proportioned that the quantity

$$[f_1(\omega)-f_2(\omega)]^{-2}+[f_3(\omega)-f_4(\omega)]^2$$

has all of its minimum values equal.

5. A transmission network comprising two paths connected in parallel between a pair of input terminals and a pair of output terminals, two frequency selective networks having substantially complementary transmission bands disposed in tandem in one of said paths, and two respectively similar networks disposed in tandem in reverse order in the other of said paths, said networks being composed of substantially pure reactance elements, the networks of one of the similar pairs being each characterized by two reactances X_a and X_b each having a plurality of critical frequencies, and the networks of the other similar pair being likewise characterized by reactances X_c and X_d, the magnitude of said

reactances and the values of their critical frequencies being so proportioned that the quantity

$$\left(\frac{1}{X_a}-\frac{1}{X_b}\right)^2+\left(\frac{1}{X_c}-\frac{1}{X_d}\right)^2$$

has all of its minimum values equal.

6. A transmission network comprising two paths connected in parallel between a pair of input terminals and a pair of output terminals, two frequency selective networks having substantially complementary transmission bands disposed in tandem in one of said paths, and two respectively similar networks disposed in tandem in reverse order in the other of said paths, said networks being composed of substantially pure reactance elements, the networks of one of the similar pairs being each characterized by two reactances X_a and X_b each having a plurality of critical frequencies, and the networks of the other similar pair being likewise characterized by reactances X_c and X_d, the magnitude of said reactances and the values of their critical frequencies being so proportioned that the quantity

$$(X_a-X_b)^2+(X_c-X_d)^2$$

has all of its minimum values equal.

7. A transmission network comprising two paths connected in parallel between a pair of input terminals and a pair of output terminals, two frequency selective networks having substantially complementary transmission bands disposed in tandem in one of said paths, and two respectively similar networks disposed in tandem in reverse order in the other of said paths, said networks being composed of substantially pure reactance elements, the networks of one of the similar pairs being each characterized by two reactances X_a and X_b each having a plurality of critical frequencies, and the networks of the other similar pair being likewise characterized by reactances X_c and X_d, the magnitude of said reactances and the values of their critical frequencies being so proportioned that the quantity

$$\left(\frac{1}{X_a}-\frac{1}{X_b}\right)^2$$

has a plurality of infinite values alternating with uniform minimum values in a prescribed frequency range, and the quantity

$$\left(\frac{1}{X_c}-\frac{1}{X_d}\right)^2$$

has a like plurality of infinite values at frequencies in an adjacent range and has minimum values equal to those of the first mentioned quantity.

8. A transmission network comprising two paths connected in parallel between a pair of input terminals and a pair of output terminals, two frequency selective networks having substantially complementary transmission bands disposed in tandem in one of said paths, and two respectively similar networks disposed in tandem in reverse order in the other of said paths, said networks being composed of substantially pure reactance elements, the networks of one of the similar pairs being each characterized by two reactances X_a and X_b each having a plurality of critical frequencies, and the networks of the other similar pair being likewise characterized by reactances X_c and X_d, the magnitude of said reactances and the values of their critical frequencies being so proportioned that the quantity

$$(X_a-X_b)^2$$

has a plurality of infinite values alternating with uniform minimum values in a prescribed frequency range, and the quantity

$$(X_c-X_d)^2$$

has a like plurality of infinite values at frequencies in an adjacent range and has minimum values equal to those of the first mentioned quantity.

9. A transmission network comprising two paths connected in parallel between a pair of input terminals and a pair of output terminals, two frequency selective networks having substantially complementary transmission bands disposed in tandem in one of said paths, and two respectively similar networks disposed in tandem in reverse order in the other of said paths, said networks being composed of substantially pure reactance elements, the networks of one of the similar pairs being each characterized by two reactances X_a and X_b each having a plurality of critical frequencies, and the networks of the other similar pair being likewise characterized by reactances X_c and X_d, the said reactances X_a and X_b having poles alternately at frequencies in a prescribed range having the values

$$f_o\sqrt{k}\,sn\left(\frac{2s}{2m+1}K,\ k\right)$$

wherein f_o is a frequency marking one end of the prescribed range, m is the total number of poles, and s is an integer taking the successive values 1 to m, and the said reactances X_c and X_d have a like number of poles alternately at frequencies in an adjacent range having values inversely related by a common constant to those of reactances X_a and X_b.

10. A transmission network comprising two paths connected in parallel between a pair of input terminals and a pair of output terminals, two frequency selective networks having substantially complementary transmission bands disposed in tandem in one of said paths, and two respectively similar networks disposed in tandem in reverse order in the other of said paths, said networks being composed of substantially pure reactance elements, the networks of one of the similar pairs being each characterized by two reactances X_a and X_b each having a plurality of critical frequencies, and the networks of the other similar pair being likewise characterized by reactances X_c and X_d, the susceptances

$$\frac{1}{X_a}$$

and

$$\frac{1}{X_b}$$

having poles alternately at frequencies in a prescribed range having the values

$$f_0\sqrt{k}\,sn\left(\frac{2s}{2m+1}K,\ k\right)$$

wherein f_0 is a frequency marking one end of the prescribed range, m is the total number of poles, and s is an integer taking the successive values 1 to m, and the susceptances

$$\frac{1}{X_c}$$

and

$$\frac{1}{X_d}$$

have a like number of poles alternately at frequencies in an adjacent range having values inversely related by a common constant to those susceptances

$$\frac{1}{X_a}$$

and

$$\frac{1}{X_b}$$

11. A transmission network comprising two paths extending in parallel between a pair of input terminals and a pair of output terminals, two frequency selective networks having substantially complementary transmission bands disposed in tandem in one of said paths, and two respectively similar networks disposed in tandem in reverse order in the other of said paths, the networks of one of the similar pairs being defined by two frequency characteristics $f_1(\omega)$ and $f_2(\omega)$ which jointly determine their transmission properties, and the networks of the other similar pair being likewise defined by frequency characteristics $f_3(\omega)$ and $f_4(\omega)$, and the said networks being so proportioned that the poles of the quantity

$$[f_1(\omega)-f_2(\omega)]^2$$

lie within a prescribed frequency range and occur at frequencies having substantially the values

$$f_0\sqrt{k}sn\left(\frac{2s}{2m+1}K, k\right)$$

wherein f_0 is a frequency marking one end of the prescribed range, m is the total number of poles, and s is an integer taking the successive values 1 to m, and the quantity

$$[f_3(\omega)-f_4(\omega)]^2$$

has a like number of poles in an adjacent range occurring at frequencies inversely related by a common constant to those of the first mentioned quantity.

12. A constant resistance network combination comprising a pair of symmetrical frequency selective networks having substantially complementary transmission bands, one pair of terminals of each of said networks being connected to a common pair of input terminals, equal resistances connected to the other pairs of terminals of said networks respectively, and additional terminating impedances connected in parallel with said resistances, the additional impedance connected to each network being equal to the short-circuit impedance of the other network.

13. A constant resistance system in accordance with claim 12 in which the said frequency selective networks are proportioned in accordance with the relation

$$\frac{1}{R^2}=\frac{1}{Z_oZ_s}+\frac{2}{Y_sZ_s}+\frac{1}{Y_oY_s}$$

where R is the value of the resistances connected to the network terminals, Z_o and Z_s are respectively the open circuit and short-circuit impedances of one network and Y_o and Y_s are respectively the open circuit and short-circuit impedances of the other network.

14. A wave transmission network having six terminals arranged in three pairs, comprising three transmission paths arranged to interconnect each pair of terminals with each other pair, frequency selective networks having substantially complementary transmission bands included respectively in two of said paths and a network comprising two tandem connected portions corresponding to said complementary networks included in the third of said paths, said complementary networks being of symmetrical structure and being proportioned so that

$$\frac{1}{R^2}=\frac{1}{Z_oZ_s}+\frac{2}{Y_sZ_s}+\frac{1}{Y_oY_s}$$

wherein R is an arbitrarily assigned resistance, Z_o and Z_s are respectively the open circuit and short-circuit impedances of one complementary network and Y_o and Y_s are the corresponding impedances of the other complementary network.

SIDNEY DARLINGTON.