Oct. 22, 1929. H. NYQUIST 1,732,311

METHOD AND APPARATUS FOR TESTING NETWORKS

Filed Feb. 25, 1926 2 Sheets-Sheet 1

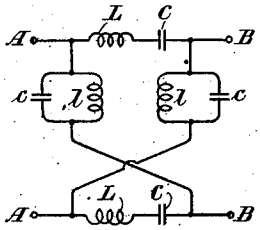


Fig. 1

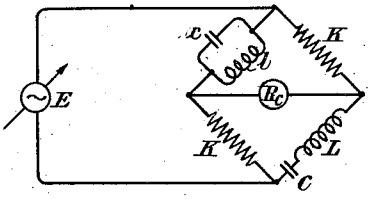


Fig. 1a

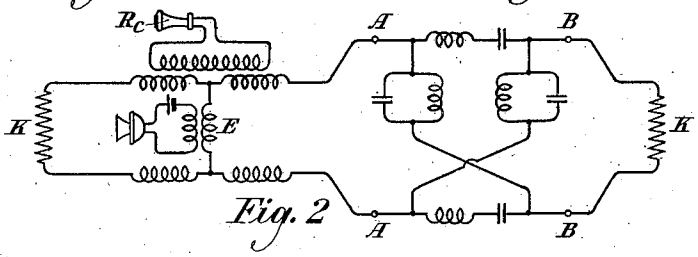


Fig. 2

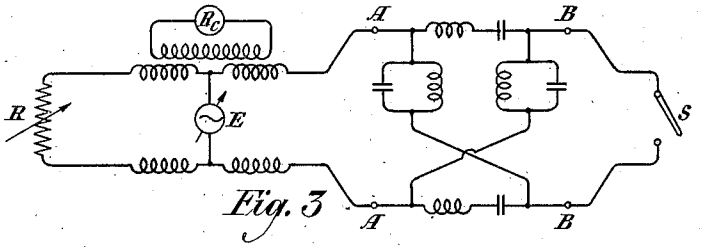


Fig. 3

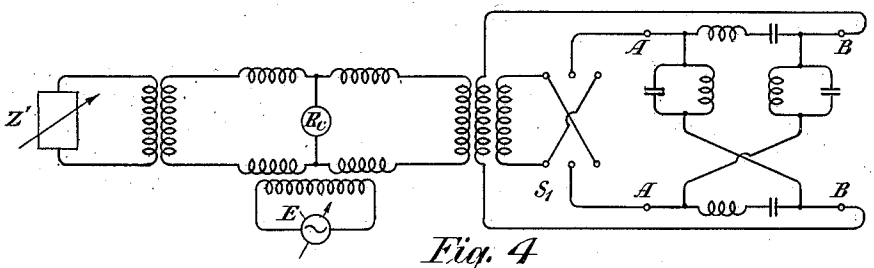


Fig. 4

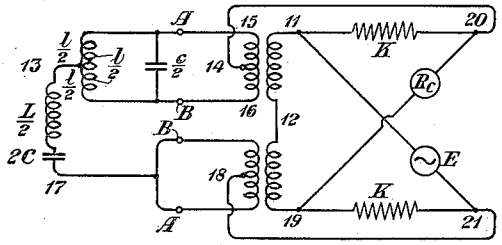


Fig. 5

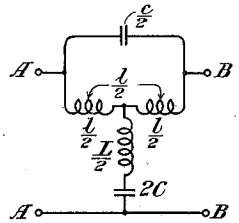


Fig. 5a

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METHOD AND APPARATUS FOR TESTING NETWORKS

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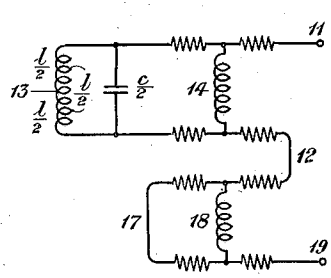


Fig. 6

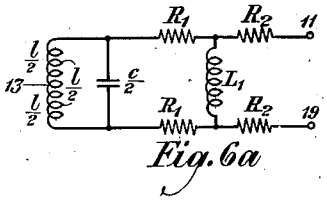


Fig. 6a

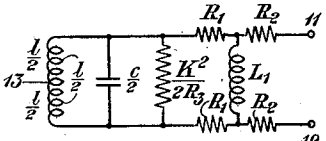


Fig. 8

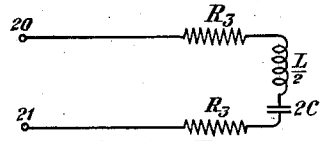


Fig. 7

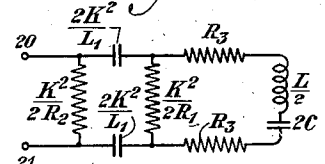


Fig. 9

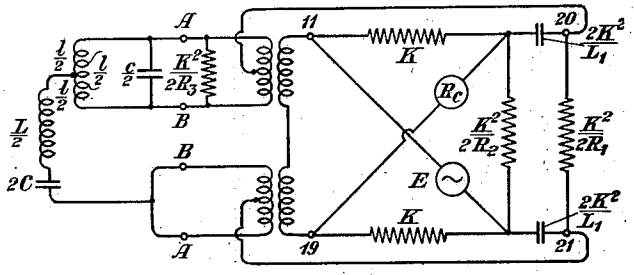


Fig. 10

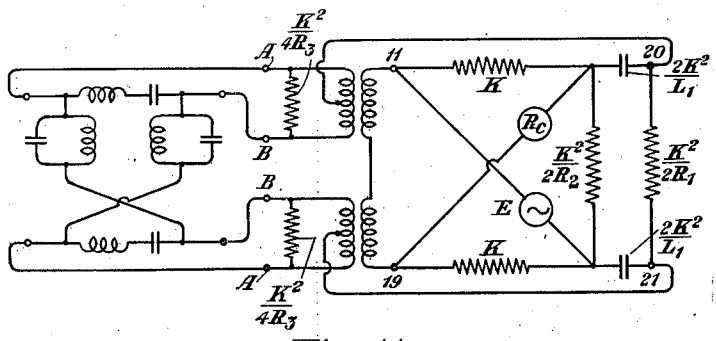


Fig. 11

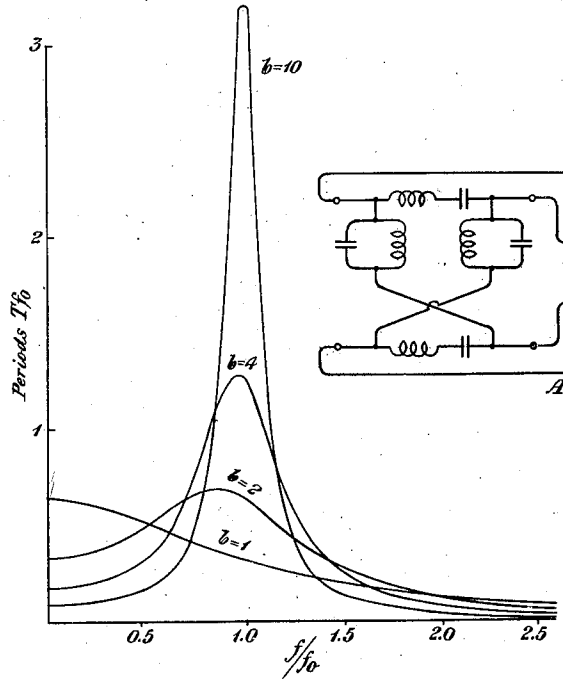


Fig. 12

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1,732,311

UNITED STATES PATENT OFFICE

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METHOD AND APPARATUS FOR TESTING NETWORKS

Application filed February 25, 1926. Serial No. 90,654.

An object of my invention is to provide a new and improved method and suitable apparatus for testing networks for the constant K relation. As will be explained herein, a constant K network has the property that if terminated on the output side by a resistance K, then its input impedance at all frequencies will be the constant resistance K. Constant K networks are useful for such applications as attenuation equalizers, phase compensators, delay networks, etc. It may be easier to design a network having a constant resistance for all frequencies as compared with a network having its resistance a variable function of frequency; also it will often be preferable to have the resistance constant so that there will be no discrimination among frequencies in this respect. A constant K network can be interposed in a line whose characteristic impedance is the resistance K without producing reflection effects. Another object of my invention relates to testing such a network without disassembling its elements. These and other objects of my invention will become apparent on consideration of a limited number of specific examples of practice according to the invention which I have chosen to disclose in this specification. It will be understood that the invention is defined in the appended claims and that the following description relates to these examples of the invention.

Referring to the drawings, Figure 1 is a diagram of a certain constant K network; Fig. 1[a] shows certain elements of this network arranged in a bridge for testing the constant K relation; Fig. 2 shows how one kind of test may be made without disassembling the network; Fig. 3 shows a method for testing a constant K network for certain other constants; Fig. 4 shows a method for measuring impedances of parts of the network without disassembly thereof; Fig. 5 shows a bridge method in which uncompensated transformers are employed; Fig. 5[a] shows a network equivalent to that in Fig. 1 and of the same type as incorporated in Fig. 5; Fig. 6 shows equivalent T networks substituted for the transformers of Fig. 5; Fig. 6[a] shows these equivalent T networks consolidated to one equivalent T network; Fig. 7 shows the substitution of resistances for the transformers of Fig. 5 in their aspect from another arm of the bridge of Fig. 5; Fig. 8 shows the proper compensating resistance introduced as compared with Fig. 6[a]; Fig. 9 shows the proper compensating condensers and resistances introduced as compared with Fig. 7; Fig. 10 shows these compensating elements incorporated in Fig. 5; Fig. 11 shows the change in Fig. 10 made by substituting the form of network of Fig. 1 for that of Fig. 5[a]; and Fig. 12 is a delay-frequency diagram to which reference will be made in explaining the principles involved in my invention.

Referring to the network of Fig. 1 with the two input terminals A, A and the two output terminals B, B, it can easily be proved that if Z_1 is the impedance of the combination L, C and if Z_2 is the impedance of the combination l, c, and if $Z_1Z_2=K^2$, where K is a constant, then if the output terminals are connected with a resistance of value K the input impedance will be the resistance K for all frequencies. Accordingly this network is an example of a constant K network.

It can also be shown that if the signal transmission delay through such a network is plotted against frequency, the plots will have the character shown in Fig. 12 where b is a parameter of design that determines the shape of the characteristic. The nature and significance of this parameter b and certain essential relations in which it is involved will be discussed in connection with the equations given hereinafter.

To test the network of Fig. 1 for the constant K relation, the bridge of Fig. 1[a] is arranged with the equal resistances K in one pair of opposite arms and with one combination L, C and one combination l, c, respectively, in the arms of the other pair. If an alternating current of varying frequency is applied across the bridge terminals, as indicated by E, and if the effect is null in the receiver Rc at all the frequencies of the desired

range, and if the same is true for the remaining combinations L, C and l, c, then it proves that the network has the constant K relation for that range.

If a balance is not obtained with the arrangement shown in Fig. 1[a], it is possible to adjust either resistance and any one of the reactance elements until a balance is obtained. The characteristic impedance of the network section will then be equal to the square root of the product of the two resistance arms. Another method of adjustment to obtain a balance is to adjust two of the reactance elements, and this will be preferable when it is desired not to alter the characteristic impedance of the network. In this case the two reactance elements chosen for adjustment should not consist of an inductance in one arm of the bridge and a capacity in the other arm, because in that case it will not, in general, be possible to obtain a balance. The choice of any other pair of elements will make it possible to get a balance.

In the case of Fig. 2, the network of Fig. 1 is incorporated as a whole, and its parts are not separated as is the case in Fig. 1[a]. The source E is of complex frequency, a voice-varied electromotive force as indicated in the diagram, and the receiver Rc is a telephone receiver. It will be seen that the currents from the source divide, going partly to each of the two resistances K, and if the network between the input terminals A, A and the output terminals B, B is, indeed, a constant K network, it will present a constant input impedance which is the resistance K at all frequencies, so that there will be a balance to the receiver Rc, and no sound will be heard in it.

If the balance is not perfect, two reactance elements in the network may be altered (subject to the same restriction against an inductance in one member and a capacity in the other, as mentioned before) until a balance is obtained, or one reactance element can be adjusted, and at the same time the two resistance elements K must be adjusted alike.

Whereas in Fig. 1[a] the adjustment for various frequencies was made by varying the frequency of the source E, in Fig. 2 the various frequencies are obtained by employing a composite electromotive source in which a variety of frequencies are superposed. Fig. 2 not only relieves the operator from disassembling the network, but it also tests whether the section is properly put together, for example, whether it is wired correctly.

In Fig. 3 the network is balanced against an adjustable resistance R, and a switch S is provided by which the output terminals of the network can be directly connected or left open. When the switch S is closed, this puts L, C and l, c in parallel, and such combination in series with a like combination. The balancing resistance R and the frequency are adjusted until balance is obtained with a low resistance value. This frequency will be the resonance frequency f_0 of each inductance-capacity combination. Care should be taken that the resistance R is at the lowest value for which such a balance can be obtained, for it is possible to get a balance at two other points at large resistance values.

Next, the switch S is opened, which puts a combination L, C in series with a combination l, c, and the remaining combinations are likewise put in series, and the two series in parallel. Again adjustment is made of frequency and resistance R until balance is obtained for a minimum value of resistance. This will occur at two different frequencies which correspond to a quarter wave length in the section, that is, at the two frequencies f_1 and f_2 for which the phase shift in traversing the section is 90°.

Equations will now be deduced which are useful in computing the parameters of the network, when f_1 and f_2 have been found. Let the network be of the lattice form such as illustrated in Figure 1. It has been established (see, for example, Equations 23 on page 19 of "Theory and design of uniform and composite electric wave-filters", by Otto J. Zobel, in The Bell System Technical Journal, January, 1923) that

$$\cosh \Gamma = 1 + \frac{2z_1}{4z_2 - z_1}$$

and

$$K = \sqrt{z_1 z_2}$$

Where Γ is the propagation constant per section
K is the characteristic impedance
z_1 is twice the impedance of the combination L, C
z_2 is one-half the impedance of the combination l, c

It is desirable that the characteristic impedance should be a real constant. This is insured if we make

$$z_1 = Kz \text{ and } z_2 = K/z.$$

This gives

$$\cosh \Gamma = 1 + \frac{2z}{4/z - z}$$

Now

$$\tanh \frac{\Gamma}{2} = \sqrt{\frac{\cosh \Gamma - 1}{\cosh \Gamma + 1}}$$

$$\tanh \frac{\Gamma}{2} = \frac{z}{2}$$

Let z be made up of a series resonant circuit.

$$z = \frac{i4\pi fL}{K} + \frac{1}{i\pi fCK}$$

Let two constants b and f_0 be defined such that

$$\frac{4\pi L}{K}=\frac{b}{f_0} \text{ and } \frac{1}{\pi CK}=f_0 b$$

Then

$$\frac{z}{2}=i\frac{fb}{2f_0}+\frac{f_0 b}{i2f}$$

The constants b and f_0 give the network a particular delay characteristic, several characteristics being illustrated by the curves of Figure 12, wherein the symbol T denotes the delay, $d\beta/d\omega$. For a non-dissipative network

$$\tanh\frac{\Gamma}{2}=\tanh\frac{i\beta}{2}=\frac{ib}{2}\left(\frac{f}{f_0}-\frac{f_0}{f}\right)$$

$$\therefore \frac{i\beta}{2}=\tanh -l\frac{ib}{2}\left(\frac{f}{f_0}-\frac{f_0}{f}\right)$$

$$\beta=2\tan -l\frac{b}{2}\left(\frac{f}{f_0}-\frac{f_0}{f}\right)$$

When there is dissipation in the coils and condensers of the network this expression is usually found to be a satisfactory approximation. When β is $-90°$ at the frequency f_1,

$$-90°=2\tan -l\frac{b}{2}\left(\frac{f_1}{f_0}-\frac{f_0}{f_1}\right)$$

$$\therefore \frac{b}{2}\left(\frac{f_1}{f_0}-\frac{f_0}{f_1}\right)=\tan(-45°)=-l$$

Whence,

$$2/b=f_0/f_1-f_1/f_0.$$

When β is 90° at the frequency f_2, a similar line of reasoning gives

$$2/b=f_2/f_0-f_0/f_2$$

Equating the right-hand members of the last two equations and solving for f_0,

$$f_0=\sqrt{f_1 f_2}$$

Substituting from this equation in the previous equation,

$$\frac{2}{b}=\sqrt{f_2/f_1}-\sqrt{f_1/f_2}.$$

Whereas the procedure discussed in connection with Fig. 1[a] and Fig. 2 gave a test for the constant K relation, the procedure in connection with Fig. 3 ascertains the values for the important parameters f_0 and b.

Fig. 4 shows an arrangement which not only tests indirectly the network for the constant K relation, but permits an investigation of the individual members without taking the network apart. When the reversing switch S_1 is closed in either direction, voltages are applied to the two ends of the section. These voltages are equal in magnitude and may be either in phase or in phase opposition, according to the position of the switch S_1. If these voltages are in phase, there will be no current in the combinations L, C, and the impedance measurement obtained by adjusting Z′ to get a balance is a measurement of the two combinations l, c in parallel. Next, the reversing switch is thrown to the other position, so that the two voltages are in opposite phase and there is no current in the two combinations l, c, and the adjusted value of Z′ to give a balance measures the impedance of the two combinations L, C in parallel.

In Fig. 11 a network section like that of Fig. 1 is shown in a bridge arrangement that has the advantages of Fig. 1[a] without its disadvantage of requiring the network to be disassembled. Figs. 5[a] to 10 are arrangements that are auxiliary or alternative and which may be considered to lead in order to Fig. 11.

It can be shown that the network represented in Fig. 5[a] is equivalent to that of Fig. 1 both with respect to impedance and propagation constant over a range comprising all frequencies, with the assumption that the capacities and inductances of Fig. 5[a] have the values indicated by the attached legends based on the values in Fig. 1.

In Fig. 5 the equivalent network of Fig. 5[a] is connected through transformers, as shown, to two opposite arms of a Wheatstone bridge. It is apparent that if the transformers were "perfect," the arm 14—18 would virtually comprise only the elements between the points 13 and 17, and the arm 11—19 would virtually comprise only the condenser marked $c/2$ and in parallel therewith the inductance consisting of two coils on the same core each marked $l/2$ and having that inductance value alone and the two also having the mutual inductance value $l/2$. In order to arrive at a modification that will be equivalent to a system with "perfect" transformers, I first substitute the equivalent T networks as in Fig. 6 for the transformers whose primaries are 11, 12 and 12, 19. Next, in Fig. 6[a], these two equivalent T networks are consolidated to one equivalent T network.

In the bridge arm 20—21 of Fig. 5, the transformers merely introduce apparent resistances in the path through the elements L/2 and 2C. In Fig. 7 this path is shown with the equivalent resistances introduced as R_3 on each side.

Fig. 8 shows the proper correcting resistance $K^2/2R_3$ introduced as compared with Fig. 6[a], and Fig. 9 shows the proper correcting capacities and resistances introduced as compared with Fig. 7. Having determined the magnitude of these correcting elements and introduced them by the aid of the equivalent T network and equivalent resistances of Figs. 6[a] and 7, Fig. 10 is drawn, restoring the transformers of Fig. 5, but introducing the correcting elements as shown. Fig. 10 now operates with compensated transformers. One arm of the bridge now virtually comprises only the series combination L/2 and 2C, and the other arm of the bridge now virtually comprises only the parallel combina-

tion $c/2$ and the coil of two parts each $l/2$ having mutual inductance $l/2$. Hence Fig. 10 accomplishes the balance of Fig. 1 without requiring actual disassembly of the parts as in Fig. 1.

In Fig. 11 the lattice-type network of Fig. 1 is substituted for the equivalent network section forming part of Fig. 10.

With a testing system arranged as shown at the right of the points A, A, B, B in either Fig. 10 or 11, any supposed constant K network can be connected to the left of those points and tested for the constant K relation by the bridge method. In the procedure of Fig. 11, like Fig. 10, it is not necessary to disassemble the network as was the case for Fig. 1.

If the reactances of a network section such as that of Fig. 1 are not of the exact values required, the discrepancy will be manifested as a reflection effect, and adjustment of some or all of these reactances will be needful. I have ascertained that a slight departure from the proper impedance value for any one of the four different reactances will give approximately the same resulting reflection coefficient in each case. It follows that the reflection produced by any one of these four kinds of irregularities can be compensated substantially by a suitable change in any one of the other three elements. For example, suppose a lattice-type network is under construction with four elements; particular accuracy of reactance value is not necessary for all four elements, but three of them may be approximate and then the fourth element may be put into the combination and all the necessary corrections made by adjustment thereof to get the desired degree of accuracy in the combination.

The principle here involved is of value for manual adjustment and may also be of value for automatic adjustment. An increase in the inductance for one pair of coils as L, L of Fig. 1 requires a proportional increase for the other pair, as l, l of Fig. 1. If the coils are subject to variation in inductance with saturation, and are made of the same kind of material and the same amount of iron and the same amount of copper, then the change of inductance with saturation will be in the same ratio in the two kinds of coils, thus affording automatic compensation.

I claim:

1. In combination, a bridge comprising the usual four arms and two bridge members a section of lattice-type network forming a part of said bridge, said network having two pairs of terminals connected one way to make the network form one arm of the bridge and connected a different way to make the network form the opposite arm of the bridge, two equal resistances forming the remaining two opposite arms of the bridge, a receiver connected to form one of the bridging members and a source of electromotive force of a variety of frequencies connected to form the other bridging member.

2. In combination in a bridge, a constant K network with two pairs of terminals connected in one way to form one arm of the bridge and in another way to form the other arm, two equal resistances forming the remaining two arms, a receiver in one of the bridging members, and a source of electromotive force of a variety of frequencies in the other bridging member.

3. In combination in a bridge, a network with two pairs of terminals connected in one way to form one arm of the bridge and in another way to form the other arm, two equal resistances forming the remaining two arms, a receiver in one of the bridging members and a source of electromotive force of a variety of frequencies in the other bridging member, the said network having the property that when one pair of terminals are connected through a certain resistance then the input impedance across the other pair is equal to said resistance value.

4. In combination in a bridge, two transformers with their primaries in series in one arm of the bridge and with the midpoints of the secondaries connected to the terminals for the opposite arm, a network having two pairs of terminals, one pair connected to respective ends of the two secondaries and the other pair connected to the other ends thereof, and transformer compensating elements comprised in these connections.

5. In combination in a bridge, two transformers with their primaries in series in one arm of the bridge and with the midpoints of the secondaries connected to the terminals for the opposite arm, a section of constant K network having two pairs of terminals, one pair connected to respective ends of the two secondaries and the other pair connected to the other ends thereof, and transformer compensating elements comprised in these connections.

6. In combination in a bridge, a constant K network with two pairs of terminals connected through three winding transformers to form two opposite arms of the bridge, and equal resistances for the other two arms.

7. In combination in a bridge, two transformers with their primaries in series in one arm of the bridge and with the midpoints of their secondaries connected to the terminals for the opposite arm, a network having two pairs of terminals, one pair connected to respective ends of the two secondaries and the other pair connected to the other ends thereof, transformer compensating elements comprised in these connections, and equal resistances forming the remaining pair of arms of the bridge.

8. The method of testing a section of network with two pairs of terminals for the con-

stant K relation which consists in bridge balancing two resistances of value K and the network through transformers connected with the network terminals.

9. The method of testing a section of network having direct and across reactance combinations for the constant K relation, which consists in bridge balancing two equal resistances of value K and the impedances of the two different reactance combinations.

10. The method of testing a constant K network which consists in connecting it in a bridge in such a way that in effect one impedance combination of the network will constitute one arm of the bridge and the other impedance combination will constitute the opposite arm.

11. In combination in a bridge, a source of electromotive force in one bridging member and a receiver in the other, resistances each of value K forming two opposite arms of the bridge, and a constant K network with its two pairs of terminals connected in the remaining two arms of the bridge so that one impedance combination of the network in effect forms one of these arms and the other impedance combination of the network in effect forms the other such arm.

In testimony whereof, I have signed my name to this specification this 23rd day of February, 1926.

HARRY NYQUIST.