

July 29, 1924.

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1,502,794

APPARATUS FOR AUTOMATICALLY SOLVING SPHERICAL TRIANGLES

Filed Aug. 2, 1920

6 Sheets-Sheet 1

FIG. 1

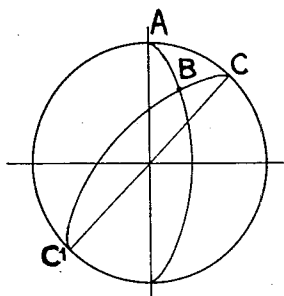


FIG. 2

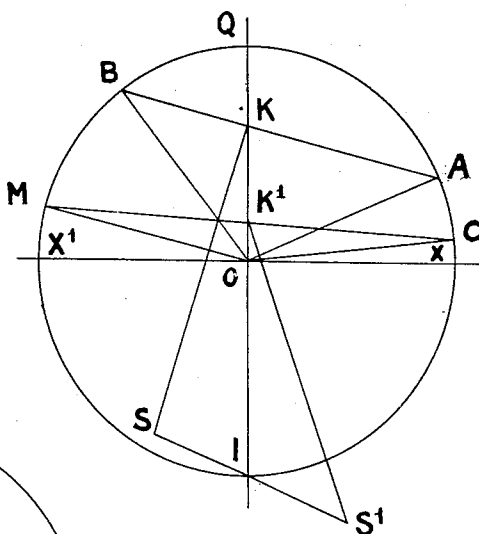


FIG. 3

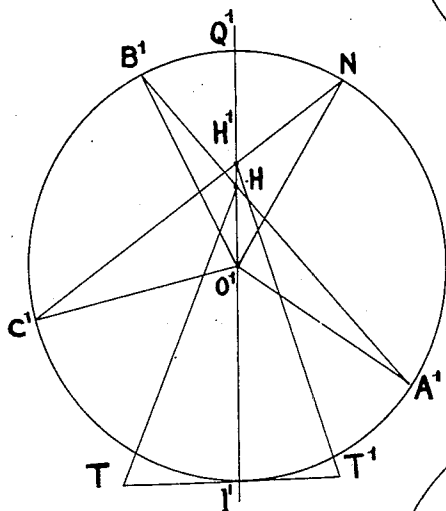
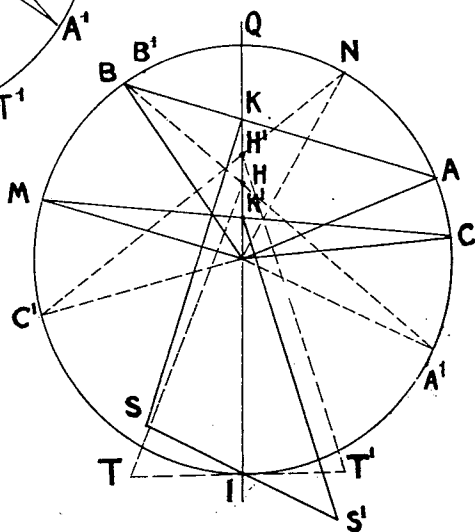


FIG. 4



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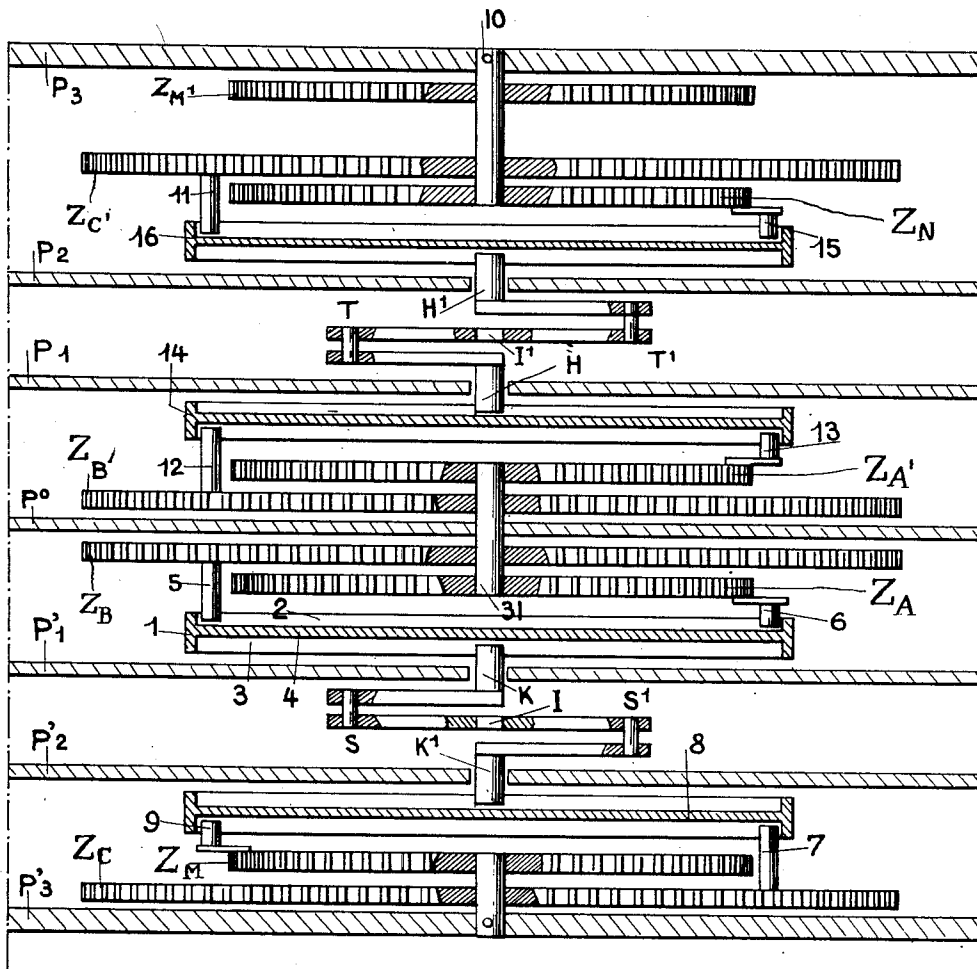
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FIG. 5



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6 Sheets-Sheet 3

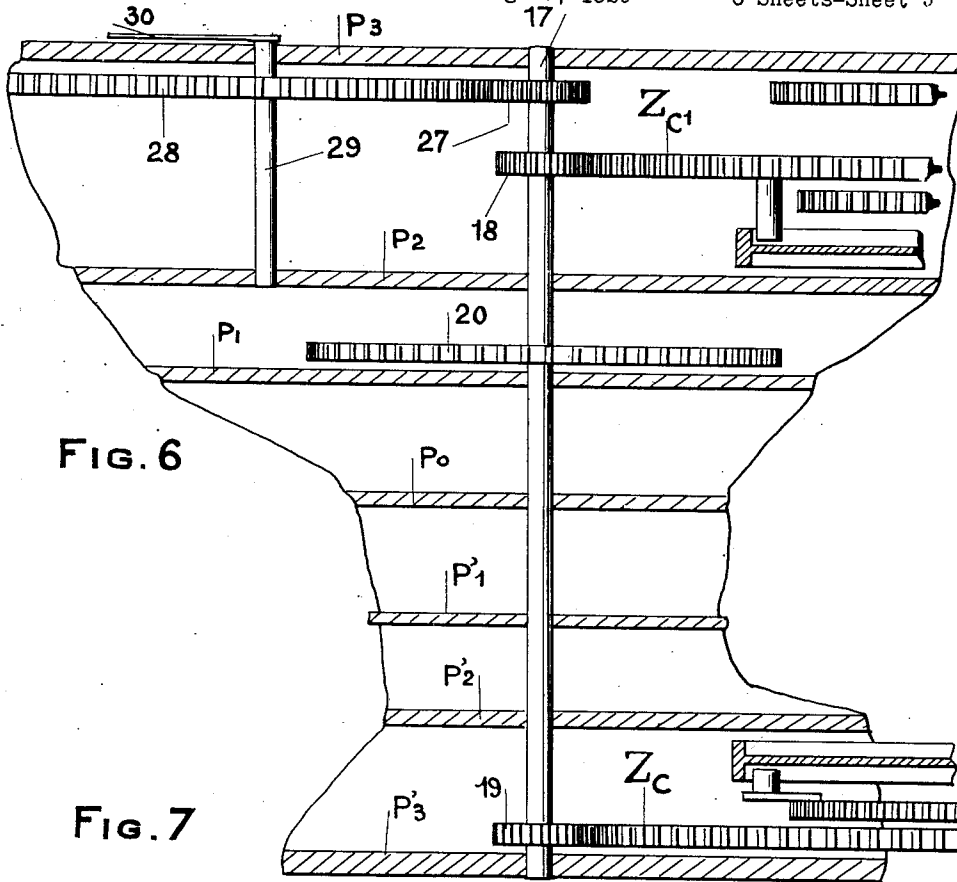
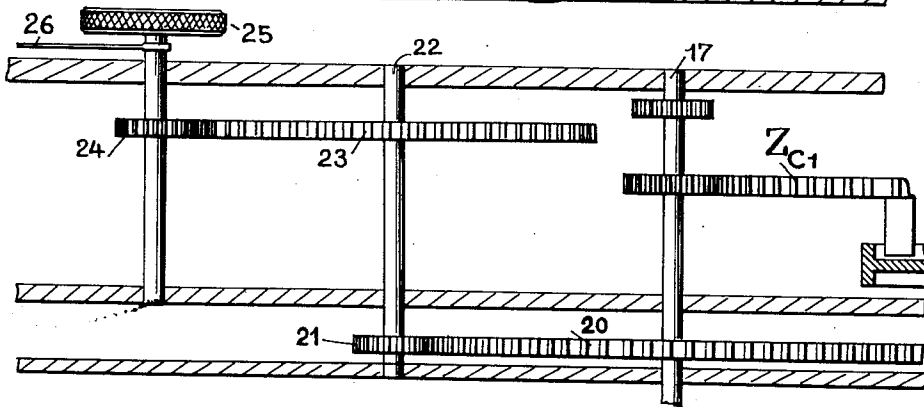


FIG. 6

FIG. 7



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6 Sheets-Sheet 4

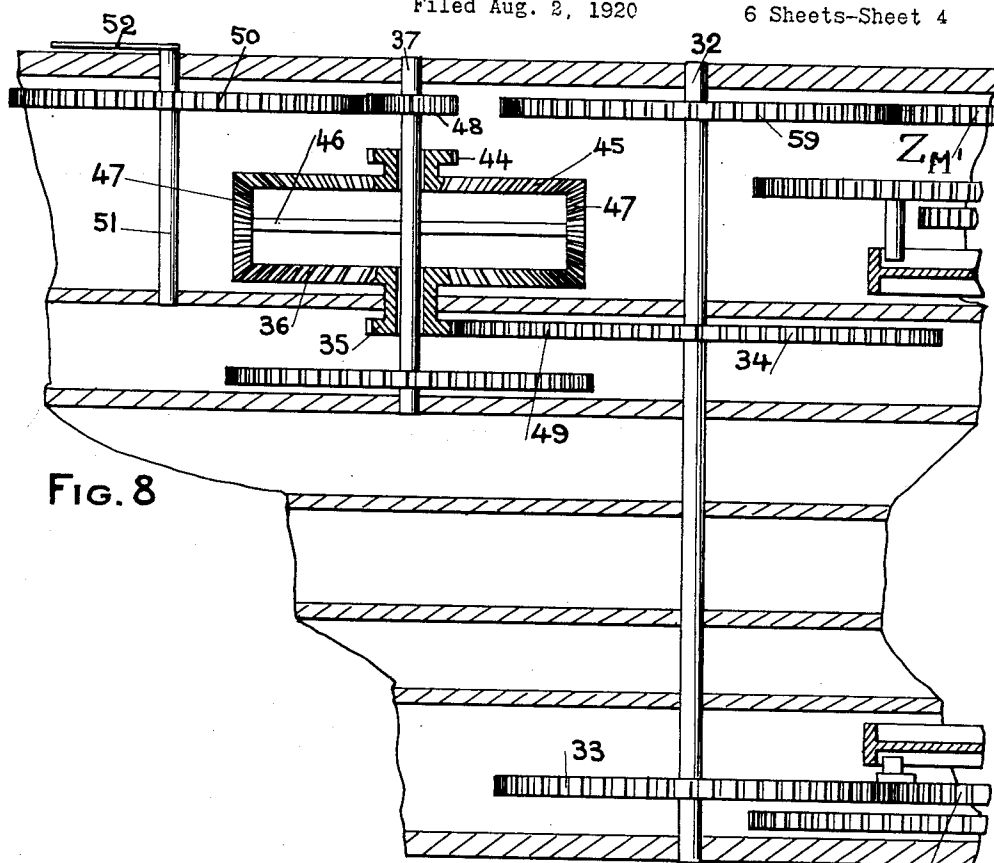


FIG. 8

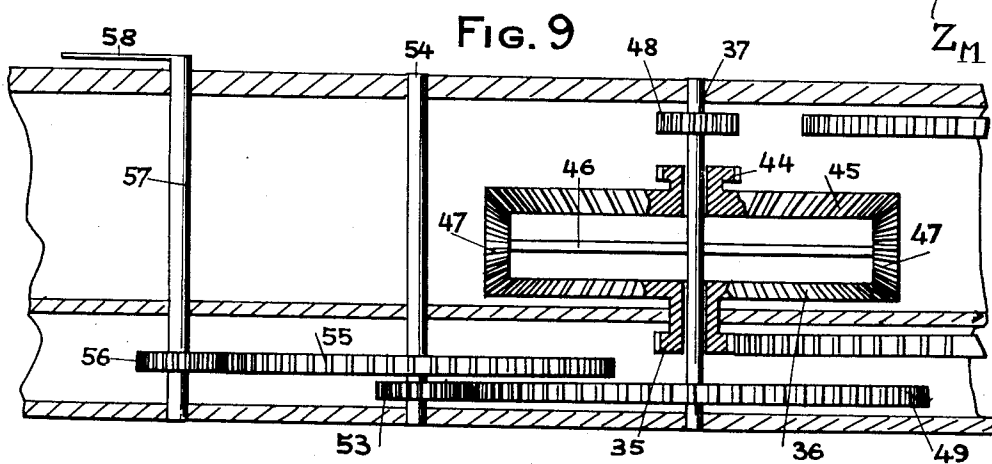


FIG. 9

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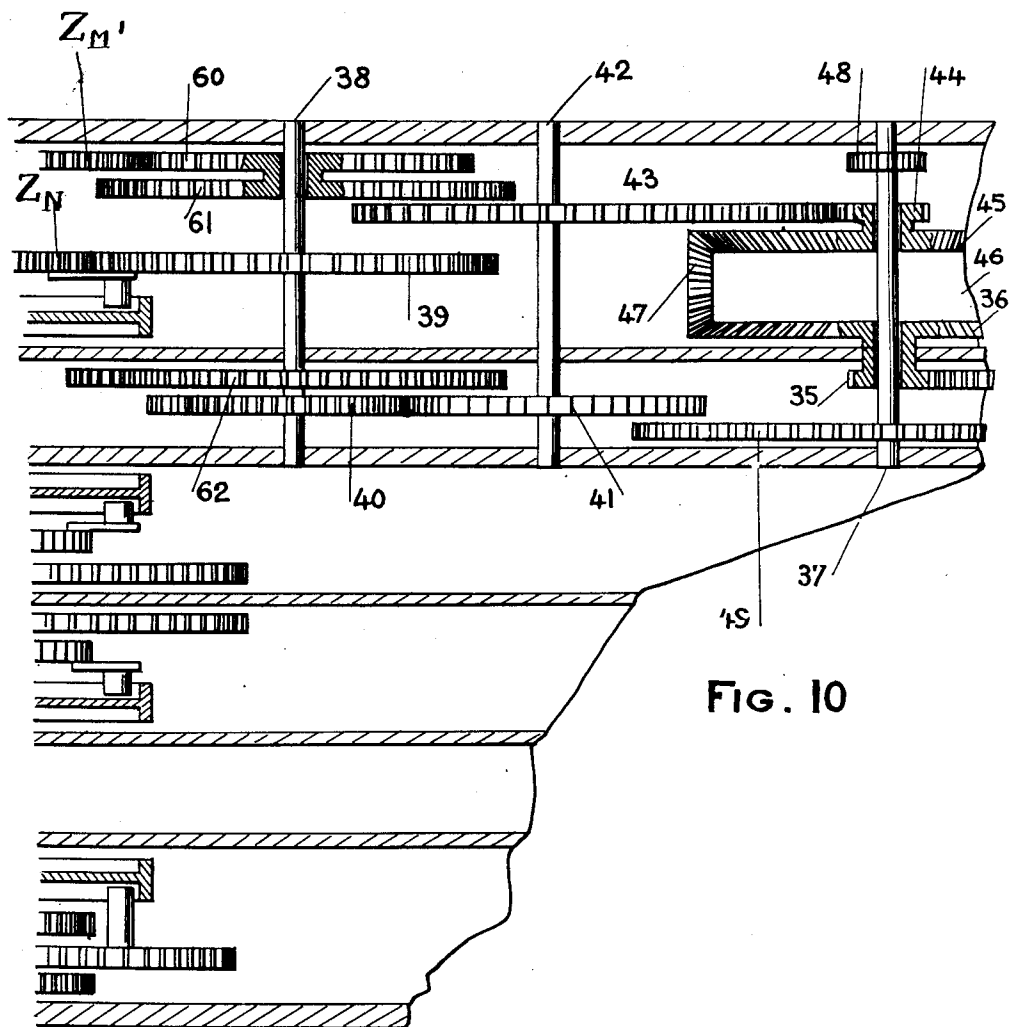


FIG. 10

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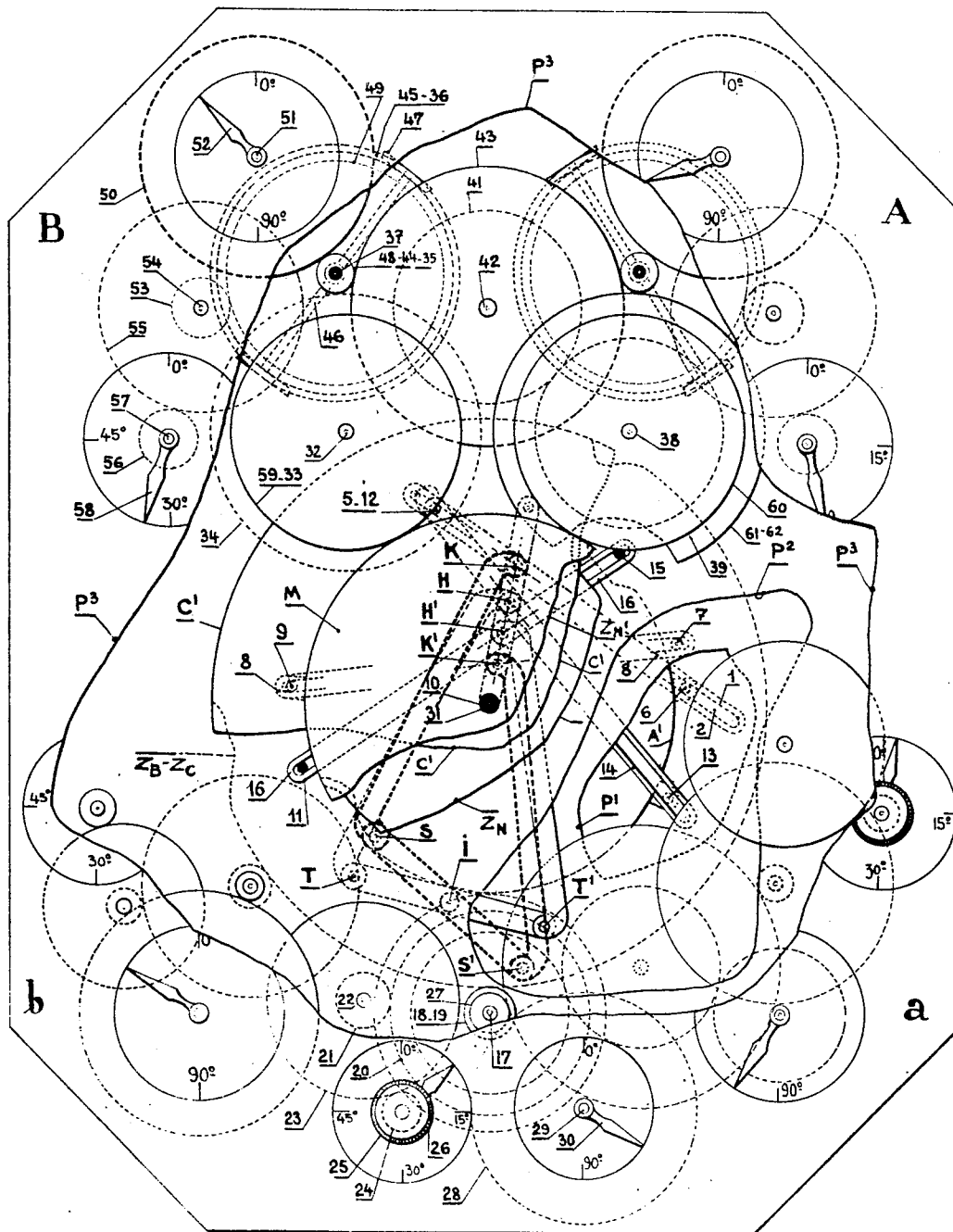


Fig. 11 *Inventor*
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Patented July 29, 1924.

1,502,794

UNITED STATES PATENT OFFICE.

LUCIEN JOSEPH LOUIS MOUREN, OF ASNIERES, FRANCE.

APPARATUS FOR AUTOMATICALLY SOLVING SPHERICAL TRIANGLES.

Application filed August 2, 1920. Serial No. 400,825.

To all whom it may concern:

Be it known that I, LUCIEN JOSEPH LOUIS MOUREN, a citizen of the Republic of France, residing at No. 51 Rue Chanzy, Asnieres, France, have invented certain new and useful Apparatus for Automatically Solving Spherical Triangles (for which I have filed applications in France February 26, 1919, No. 508,982, and in England July 10, 1920, No. 20,592), of which the following is a specification.

This invention relates to an apparatus for automatically solving a spherical triangle, three elements of which are known (angles or sides): the apparatus may be used more especially for solving most of the problems of navigation.

The principle of the present apparatus is supplied by certain geometrical constructions proposed by applicant to solve a spherical triangle, three elements of which are known; the said three constructions are explained hereinafter.

In the accompanying drawings:—

Figs. 1, 2, 3 show the said geometrical constructions.

Fig. 4 represents diagrammatically the principle of an apparatus based on these constructions.

Figs. 5 to 11 relate to the construction of an apparatus of this type.

Figs. 5 to 10 are detailed views.

Fig. 11 is a plan of the assemblage with parts broken away to show the elements located at different levels.

The geometrical constructions upon which the apparatus is based are:

Let us suppose a spherical triangle ABC (Fig. 1) the sides of which are referred to as usual by a, b, c , and the angles of which by A, B, C . If two sides a, b and the included angle C are known, it is possible to obtain the sum and the difference of the two other angles A and B , and therefore to determine immediately the two angles A and B by the following construction (Fig. 2):

A diameter QI is drawn in a circle of centre O and radius R ; a radius OA is drawn such that arc $QA=a$; a radius OB such that the arc $QB=b$; a radius OC such that the angle $COI=c$; a straight line AB is drawn cutting the diameter QI at a

certain point. A line SS' such that $IS=IS'=\frac{R}{2}$ can rotate around the point I ; two arms $SK, S'K'$ having the same length $=\frac{3}{2} \times R$ are linked to the points S, S' . If the point K is brought to the point of intersection of the line AB with the diameter QI in the position illustrated in Fig. 2, the position of the line SS' is well determined and the end K' of the second arm $S'K'$ will be brought to one point of the diameter QI only, that is to the point shown in the figure. It can easily be proved that the line CK' cuts the circle at a point M , such that the arc $IM=a+b$.

If the same construction is effected on a circle of the same radius R and centre O' (Fig. 3) by taking $I'A'=a, Q'B'=QB=b$, angle $Q'O'C'=C$ and by placing the linked triangle $HT-TI'T'-T'H'$, the sides of which have the same lengths as in Fig. 2 in such a manner that H shall be at the intersection of $A'B'$ with the diameter $Q'I'$ and that H' shall be on the same diameter, then the line $C'H'$ will cut the circle at a point N such that arc $Q'N=A-B$.

The above two constructions give immediately the required angles A and B which are

$$A = \frac{\text{arc } IM + \text{arc } Q'N}{2} = \frac{m+n}{2}$$

$$B = \frac{\text{arc } IM - \text{arc } Q'N}{2} = \frac{m-n}{2}$$

if we designate the arcs IM and $Q'N$ by m and n .

The above two constructions may be superposed on the same circle as shown in Fig. 4; we will point out that arc $QA=\text{arc } IA'$, and that C' is symmetrical with C relatively to the centre O .

It can be easily provided that the condition necessary and sufficient in order that the two constructions shall be possible is:

$$a > b$$

$$a+b < \pi$$

It is always possible to apply the problem to a particular case when the above two conditions are fulfilled. Indeed,

(1) If we have $a < b$ it is sufficient to

interchange a and b , that is to mark a by means of the radius OB and b by means of the radius OA . The angles A and B are then given by

$$A = \frac{m-n}{2}$$

and

$$B = \frac{m+n}{2}$$

(2) If $a+b > \pi$, we will point out that in the triangle ABC' (Fig. 1) the apex C' of which is diametrically opposed to the apex C , the following relations exist between the sides $a'b'c'$ and the angles $A'B'C'$:

$$\begin{aligned} a' &= \pi - a & b' &= \pi - b \\ A &= \pi - A' & B &= \pi - B' \end{aligned}$$

Therefore, if $a+b$ is $> \pi$, then we have $a'+b' < \pi$. The triangle ABC' fulfills the condition for being solved by the above mentioned method; the values of A and B are found immediately from those of A' and B' .

In order to solve a triangle, two angles A and B and the included side c of which are known, the same construction may be used by substituting A for a , B for b , $\pi - c$ for C . In these conditions it will be obtained

$$a = \frac{m+n}{2} \quad b = \frac{m-n}{2}$$

It is therefore possible by the above constructions to solve a spherical triangle, three consecutive elements of which are known.

An apparatus can be easily constructed on the principle of Fig. 4, which apparatus would comprise one or several fixed graduated circles of centre O , and which may have different diameters, movable radii OA , OA' , OB , and a substantial diameter COC' which are displaced on the fixed graduated circle or circles, a link AB constrained to pass through the points A and B , a linked open triangle $KSIS'K'$ having the dimensions indicated above, the apex K of which is constrained to remain on the link AB , and to move along the diameter QI , the middle point of the side SS' of the said triangle being linked in I and the other apex K' of the triangle being constrained to move along QI and to remain on a link CM which is constrained to pass through the point C and which carries the radius OM with it. The apparatus comprises also a second open linked triangle $HTIT'H'$ of the same size, the point H of which is moved along by the link $A'B$ and the point H' of which carries with it a link which is constrained to pass through the point C' and which carries with it the radius ON , the points H and H' being, on the other side, constrained to remain on the diameter QI .

It will be seen that by means of this apparatus after having placed the four radii

OA , OA' , OB , OC , according to the three given elements of the spherical triangle the arcs $m=IM$, $n=QN$ can be read immediately for instance on graduations originating at I and Q .

The use of the apparatus may be rendered easier by the following arrangement of details:

The two radii OA , OA' which are symmetrical relatively to the diameter perpendicular to QI may be interconnected in such a manner that it suffices to place one in position in order that the other one also shall be placed in position; there are several known mechanical devices enabling to obtain the said result; one operation is thus sufficient.

The divisions of the graduations which correspond to the radii OM and ON may be marked with numbers which will be only half the numbers of normal divisions. It

will be thus possible to read directly $\frac{m}{2}$ and

$\frac{n}{2}$, instead of m and n respectively, which will render the calculations of the required elements easier.

Practically, in the apparatus constructed, such as described in Figs. 5 to 11, each of the radii OA , OA' , OB , OC , OC' , OM , ON by a circle on which the radius is represented by means of a projection placed near the periphery. The periphery of the circles is provided with teeth. The placement at a suitable angle of those of the circles which correspond to the given elements of the problem is effected by means of gears which operate them and at the same time operate one or several pointers for the angle; the circles which represent the radii OM and ON are carried along by the first circles and are placed at the desired angle by the play of the links and of the open linked triangles, the mechanism of which has been explained above; the said circles operate gears which in their turn operate one or several pointers which indicates directly on the dials the arcs m and n and the unknown elements of the problem. By combining the movements of the two circles by means of differential mechanisms, it is possible to have the required values of the angles A and B which are the $\frac{1}{2}$ sum and $\frac{1}{2}$ difference of the arcs m and n marked directly by means of pointers. In the mode of construction of the apparatus illustrated in Figs. 5 to 10 all the circles are mounted in a box comprising a bottom P_3 and a cover P_3 . The said box is divided towards its middle by means of a horizontal partition P_0 above which are two partitions P_1 , P_2 and below which are also two partitions P'_1 , P'_2 . It can be seen by what we said above, that the apparatus which is diagrammatically illustrated in Fig. 4 is for solving spherical triangles

three consecutive elements of which are known; it is therefore a general solution of the problem. The construction shown in Fig. 4 is the combination of the two constructions illustrated diagrammatically in Figs. 2 and 3 and which give the arcs m and n separately. The two constructions of Figs. 2 and 3 are effected by means of the apparatus illustrated in Figs. 5 to 10, one of them being above, the other one below the middle plane P_0 which divides the apparatus into two almost symmetrical parts.

The members corresponding to the elements of Fig. 2 are found below the plane P_0 , namely: a circle Z_A corresponding to the radius OA ; a circle Z_B corresponding to the radius OB ; a circle Z_C corresponding to the radius OC ; a link 1 comprising an upper groove 2 and a lower groove 3 separated by a horizontal partition 4. A pin 5 of the circle Z_B , and a button 6 of the circle Z_A engage with the upper groove 2 in such a manner that the link 1 passes always through the two points of the said circles, corresponding to the radii OB and OA . A pin K engages with the lower groove 3 of the link 1, constrained to move in a groove of the plane P'_1 perpendicular to the plane of the figure and corresponding to the diameter QI of Fig. 2. It is fixed on an arm having for its length $\frac{3R}{2}$, linked at S on to an

arm of a length SS' equal to R , which is itself pivoted at its middle point I to the end of the diameter QI perpendicular to the plane of Figure 5. Another arm is pivoted in S' provided at its end with a pin K' constrained to move in a groove of the plane P'_2 perpendicular to the plane of the figure. A circle Z_C is placed between the planes P'_2 and P'_3 which circle corresponds to the radius OC and is provided with a pin 7 engaging in a link 8 similar to the link 1; a pin 9 of the circle Z_M corresponding to the radius OM also engages in the lower groove of the link 1. The pin K' already mentioned above, engages in the upper groove of the link 8. If the three circles Z_A , Z_B , Z_C , are placed according to the given elements of the problem, the play of the link 1 of the linked triangle $KSIS'K'$ of the link 8 will determine the position of the circle Z_M . All the pins 5, 6, 7, 9 of the various circles and the point I on which the arm SS' is pivoted are situated on one and the same circle having its centre on the axis 10 of the apparatus, the said circle being the one given by the geometrical construction of Fig. 2. But the circles themselves, Z_A , Z_B , Z_C , Z_M may have any radii. The axis 10 is interrupted in that part where the links and the linked triangles are placed in order to allow the displacement of the links and of the sides of the triangles.

Above the plane P_0 are the members corresponding to the diagrammatical construction of Fig. 3, that is: a circle Z_C' , provided with a pin 11, a circle Z_B provided with a pin 12, a circle Z_A' provided with a pin 13, a link 14 receiving in its lower groove the two pins 12 and 13 of the circles Z_B and Z_A' and in its upper groove the pin H of the open linked triangle $HTI'T'H'$, the side TT' of which is pivoted at the point I' situated at the end of the diameter perpendicular to the figure at the same distance from the centre as the pins 12 and 13. The pins H and H' are constrained to move in grooves of the planes P_1 , P_2 perpendicular to the figure and corresponding to the diameter $Q'I'$. A circle Z_N provided with a pin 15 is placed below the circle Z_C' . The two pins 11, 15 of the circles Z_C' and Z_N engage into the upper groove of a link 16, the lower groove of which receives the pin H' . It follows from the connections thus established that when the circles Z_B , Z_C' , Z_A' have been placed according to the given elements of the problem, the circle Z_N occupies the position corresponding to the radius ON shown in Fig. 3.

In order to operate easily all the above circles as described, they are made of toothed wheels actuated by gears. With a view to an easier construction of the apparatus, one and the same diameter has been given to the four circles Z_C , Z_B , Z_B' , Z_C' and one and the same but smaller diameter to the four other circles Z_M , Z_A , Z_A' , Z_N .

Figs. 6 and 7 illustrate by way of example of the device described above, the operation of the two circles Z_C , Z_C' which are placed at the top and at the bottom. It has been already seen that the two radii OC , OC' , belonging to the same diameter should have displacements which shall be equal and of the same direction. To this effect, a vertical axle 17 (Fig. 6) carries two equal pinions 18 and 19 gearing with the circles Z_C , Z_C' . It also carries a toothed wheel 20 actuated by a pinion 21 (Fig. 7) the axle 22 of which carries another toothed wheel 23 actuated by the pinion 24 which in its turn is operated by means of a disc 25. A pointer 26 moving on a dial gives a measure of the angle of which the circles Z_C and Z_C' have rotated. A pinion 27 (Fig. 6) also mounted on the axle 17 carries along a toothed wheel 28, the axle 29 of which carries a pointer 30 which serves also to measure the angle of which the circles Z_C , Z_C' have rotated. The size of the radii, pinions and wheels may be chosen in such a manner that the pointer 26 shall make one revolution whilst the circle C rotates through 1° and the pointer 30 one revolution for 180° . The pointer 30 indicates then the number of degrees of the angle, the pointer 27 the supple-

mentary number of minutes of which the circle Z_C has rotated.

The actuation of the other circles is effected in a similar manner.

5 The two circles Z_B, Z_B' situated above and below the plane P_0 corresponding to the two radii OB of Figs. 2 and 3 which should have the same position in the two combinations may be fixed by keying on to the axle 31 which carries them, and on which axle 31 the circles Z_A, Z_A' can rotate freely. They can also be operated separately by means of two equal pinions integral with the same axis. In the latter case the axis 31 may be rendered integral with the plane P_0 .

15 It is very easy by means of differential gear to combine the movements of two circles Z_M and Z_N in such a manner that the pointers shall indicate directly the value of the unknown quantities A and B .

20 As an example of the differential gear, Figs. 8, 9 and 10 illustrate the operation of the two pointers giving the required value of the unknown element B . It has been

25 seen that $B = \frac{m}{2} - \frac{n}{2}$. The pointers will be

made to indicate the above result in the following manner: A vertical axis 32 (Fig. 8) carries a toothed wheel 33 engaging with the circle Z_M . The said axis carries also a second toothed wheel 34, gearing with a pinion 35. The pinion 35 is rendered integral with a wheel 36 which is the lower wheel of a differential gear. The whole, pinion 35 and wheel 36 are loosely mounted on the axle 37 of the differential gear. On the other hand a vertical axle 38 (Fig. 10) carries a toothed wheel 39 gearing with a circle Z_N and carries also a toothed wheel 40 gearing with a wheel 41 of the same diameter. The latter wheel is mounted on the vertical axis 42 and transmits its motion through the intermediary of the said axle to the wheel 43 gearing with the pinion 44. The pinion 44 similar to the pinion 35 is integral with the wheel 45 which is the upper wheel of the differential gear. The whole, pinion 44 and wheel 45, is loosely mounted on the axle 37 of the differential gear. It will be seen that the movements of the circle Z_M and Z_N are transmitted to the wheels 36 and 45 of the differential gear. The result of the algebraical addition of the rotations of the wheels 36 and 45 is transmitted to the axle 37 through the intermediary of an arm 46 and of the satellite pinion 47. A pinion 48 and a wheel 49 are also mounted on the axle 37. The pinion 48 (Fig. 8) gears with a wheel 50, the axle 51 of which carries a pointer 52 which gives a measure of the angle B by its displacement on a dial. The wheel 49 (Fig. 9) gears with the pinion 53 the axle 54 which carries another toothed wheel 55 actuating a pinion 56. The axle

57 of the pinion 56 moves a pointer 58 which gives also a measure of the element B .

The sizes of the pinion and of the wheels which have just been mentioned may be chosen in such a manner that for instance the pointer 52 shall make one revolution for 180° of B , and the pointer 58 one revolution per degree of B . The first one would then serve to indicate the number of degrees of B and the second one the number of additional minutes.

The wheel Z_M' illustrated in Figs. 5, 8 and 10 is a toothed wheel loosely mounted on an axle 31. The wheel Z_M' is operated by a wheel 59 (Fig. 8) identical to the wheel 33 and mounted on the same axle. The wheel Z_M' has therefore the same motion as the wheel Z_M . The motion which carries over one of the wheels of the differential gear of the element A is taken from the wheel Z_M' . The above arrangement has been adopted because in view of the small space available it would not have been possible to obtain the motion of the corresponding wheel of the differential gear of the element A from the circle Z_M .

The apparatus thus completed allows the solving not only of spherical triangles, three consecutive elements of which are known, as it could be seen from the study of the diagrammatical Figures 2, 3 and 4, but also of spherical triangles, three non-consecutive elements of which are known. Indeed, the apparatus is constructed in such a manner that by operating the members corresponding to the three known elements a, b, C of the spherical triangle, the unknown elements A and B are obtained; but if one of the three first elements a, b, C is unknown and one of the two others A and B is known it is enough to mark the two first known elements by the corresponding pointer of the apparatus and to operate the third leading element in order to obtain on the dial of the element which is known, the given value. It is then possible to read on the dial of the third element the quantity which is marked by the pointer.

What I claim is:—

1. In an apparatus for automatically resolving a spherical triangle, three elements of which are known, the system comprising in combination: four movable pivots tending to be maintained at the same distance R from the general axis of the apparatus; a slide connecting two of these pivots, another slide connected the two others, a pivot fixed at the same distance R from the general axis, a rod rotating about said fixed pivot and having at each side of said pivot a

length equal to $\frac{R}{2}$, a guide placed in the plane perpendicular to the general axis which contains the fixed pivot and along the

line which joins the fixed pivot to the general axis, and two rods having a length equal to $\frac{3}{2}R$ and one of their ends connected
 5 to the ends of the above mentioned rod, the other end of one of these rods being adapted to move at the same time in the first mentioned slide and in the guide, and the other end of the other of these rods being adapted
 10 to move at the same time in the second mentioned slide and in the guide.

2. Apparatus for automatically resolving a spherical triangle three elements of which are known, consisting of two systems as defined in claim 1.

3. Apparatus for automatically resolving a spherical triangle, three elements of which are known, consisting of two systems, each comprising in combination, four pivots each
 20 mounted on a toothed wheel at an equal distance R from the general axis of the apparatus, a slide connecting two of these pivots, another slide connecting the two others, four gear trains in mesh with the four toothed
 25 wheels, a pivot fixed at the same distance R from the axis, a rod turning around this

fixed pivot and having at each side of said pivot a length equal to $\frac{R}{2}$, a guide place in a plane perpendicular to the general axis
 30 which contains the fixed pivot and along the line which joins the fixed pivot to the general axis, and two rods having a length equal to $\frac{3}{2}R$ and connected by one of their
 35 ends to the ends of the above mentioned rod, the other end of one of these rods being adapted to move at the same time in the first mentioned slide and in the guide and the other end of the other of these rods being
 40 adapted to be moved at the same time in the second mentioned slide and in the guide.

4. Apparatus for automatically resolving a spherical triangle three elements of which are known, comprising two systems as defined in claim 3, and a differential mechanism adapted to combine the resultant movements of each of the systems.

In testimony whereof I have signed my name to this specification.

L. MOUREN.