July 6, 1926. 1,591,073

O. J. ZOBEL

ELECTRICAL NETWORK

Filed Dec. 15, 1922 2 Sheets-Sheet 1

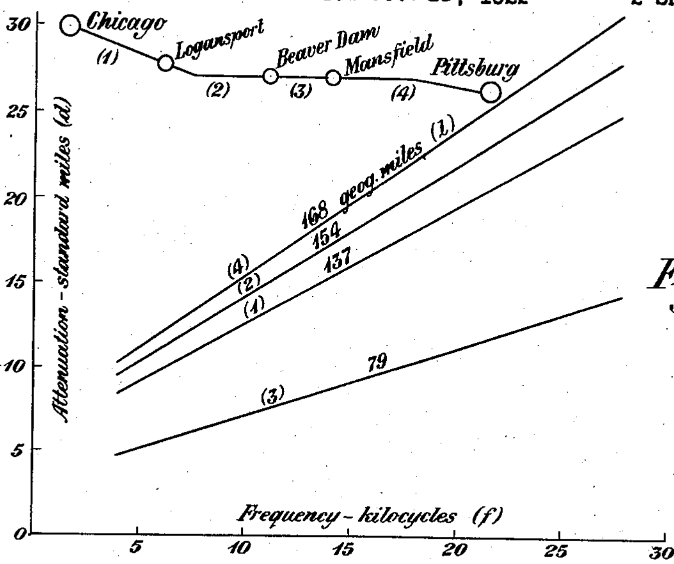

Fig. 1

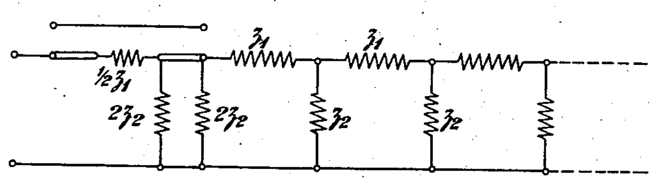

Fig. 2

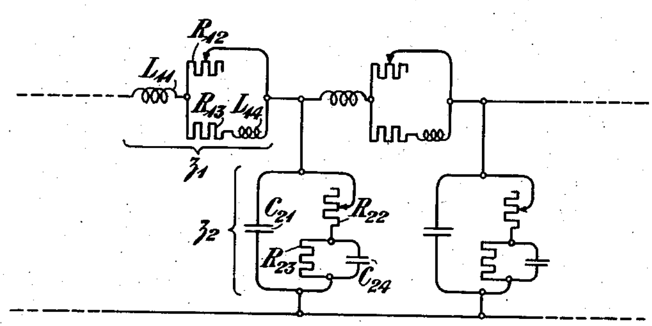

Fig. 3

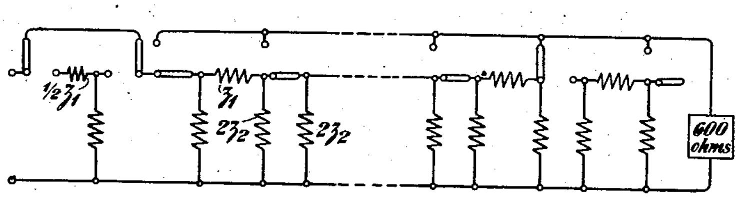

Fig. 4

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2 Sheets-Sheet 2

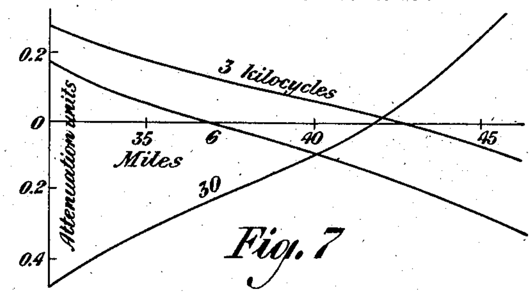

Fig. 7

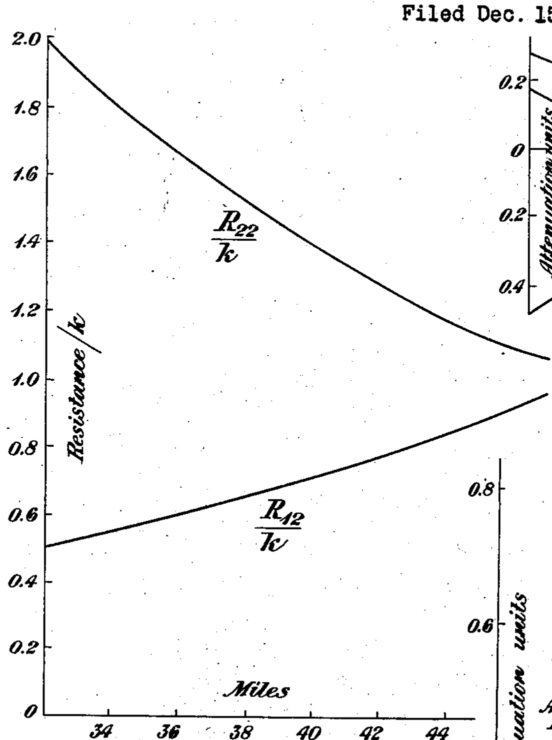

Fig. 5

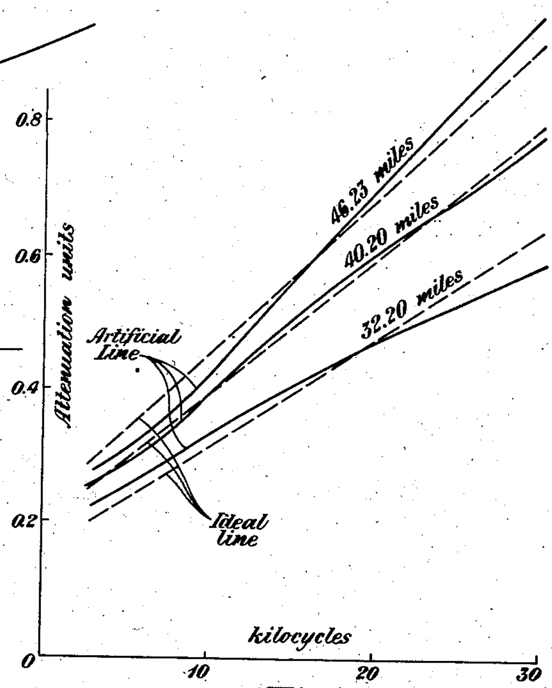

Fig. 6

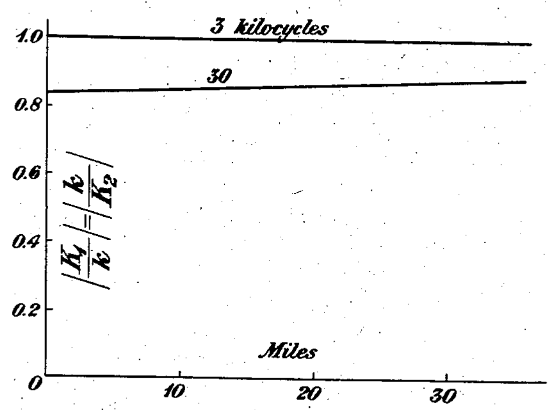

Fig. 8

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BY [signature]

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Patented July 6, 1926. **1,591,073**

UNITED STATES PATENT OFFICE.

OTTO J. ZOBEL, OF NEW YORK, N. Y., ASSIGNOR TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

ELECTRICAL NETWORK.

Application filed December 15, 1922. Serial No. 607,186.

An object of my invention is to provide a new and improved network for electric currents that shall have certain desired properties in respect to its characteristic impedance and attenuation. Another object of my invention is to provide a network that shall simulate a substantial length of non-loaded open-wire transmission line. Another object is to provide a network with substantially a constant resistance characteristic impedance and having its attenuation an increasing linear function of frequency. All these objects and others will become apparent in the following specification and claims, taken with the accompanying drawings, in which I have disclosed a single specific embodiment of the invention by way of illustration and example. With the understanding that the invention is defined in the appended claims, I shall now proceed to describe the particular example thereof shown in the drawings.

Figure 1 is a diagram exhibiting attenuation characteristics of a certain non-loaded open-wire line which it is desired closely to simulate in a network for testing purposes; Fig. 2 is a diagram of a general ladder type network; Fig. 3 is a diagram of a special case of this type of network which was developed to conform to the requirement based on Fig. 1; Fig. 4 is a diagram showing how any desired number of half-sections of this network may be employed; Fig. 5 is a diagram showing how certain elements of a section of the network of Fig. 3 may be varied to adjust the equivalent length of the section; Fig. 6 is a diagram showing attenuation as a function of frequency for several different equivalent section lengths as determined in accordance with Fig. 5; Fig. 7 is a diagram showing deviations of attenuation from an assumed ideal, exhibited as a function of equivalent length of section, for several different frequencies; and Fig. 8 is a diagram showing characteristic impedance as a function of equivalent length for two extreme frequencies of the range under consideration.

A certain non-loaded open-wire line extending between Pittsburgh and Chicago is divided into sections for operating purposes, as indicated in Fig. 1. These sections have attenuation frequency characteristics as shown in the lower part of Fig. 1. It will be seen that for any one of the sections, uniform increments of frequency correspond to uniform increments of attenuation. In other words, attenuation is a linear function of frequency. In all cases, the diagrams correspond with the general equation

$$d=cl(c'+f) \quad (1)$$

where d is the attenuation in standard miles, l is the length of the line, f is the frequency, and c and c' are constants.

The characteristic impedance of these line sections is substantially a constant resistance, as is the case for any long non-loaded open-wire line. In this particular case the characteristic impedance is approximately 600 ohms of resistance.

It is desired to produce a network of lumped impedance elements adapted by adjustment to represent any section of this line, both in respect to characteristic impedance and in respect to attenuation over a range of from 3 to 30 kilocycles.

Such a network may be obtained by applying the principles of my present invention. First I assume that the desired network will be of the general ladder type shown in Fig. 2, which comprises a succession of series impedances z_1 and alternating shunt impedances z_2 connected as shown in the diagram. I impose the condition that

$$z_1 . z_2 = k^2 \quad (2)$$

where k is a real constant. As is common in the treatment of such cases of recurrent networks, it will be assumed initially that the structure extends indefinitely to the right, as suggested by the dotted lines. With the switches as shown in Fig. 2, the network has "mid-series" termination at the input end at the left,—that is, it begins with a series element of impedance value $\frac{1}{2}z_1$. It is easily shown that the mid-series characteristic impedance is

$$K_1 = k\sqrt{1+\frac{1}{4}\left(\frac{z_1}{k}\right)^2} \quad (3)$$

If the switches are changed from the positions shown in Fig. 2, it will be seen that the network begins with a shunt element of half the normal admittance value, that is the network begins with a shunt element of impedance value $2z_2$. This is a "mid-shunt" termination and it is easily shown that the mid-shunt characteristic impedance is

$$K_2 = k \frac{1}{\sqrt{1+\frac{1}{4}\left(\frac{z_1}{k}\right)^2}} \quad (4)$$

Consideration of equations (3) and (4) shows that if z_1 is kept small in absolute value relatively to k, it follows that K_1 and K_2 will each be vectors of small equal but opposite angle and of absolute value nearly equal to the real quantity k. For the present let it be assumed that $k=600$, which is the number of ohms in the characteristic resistance of the line to be simulated. It remains to find an appropriate structure for z_1, such that it will have a comparatively small absolute value of impedance over the desired frequency range of 3 to 30 kilocycles, and such that a section of the network will have an attenuation which will be an increasing linear function of frequency.

From preliminary trials it appears that a section having an attenuation about the same as 40 miles of the open-wire line will be about the limiting equivalent length that can be obtained without too great an impedance change due to the maximum value of

$$\frac{z_1}{k}$$

encountered in the contemplated frequency range. From Fig. 1 and equation (1) it is derived that the ideal attenuation for a 40-mile section increases linearly from 0.270 units at 4 kilocycles to 0.758 units at 28 kilocycles, the units being units of attenuation α.

An attenuation that increases with frequency implies inductance in the series element z_1 or capacity in the shunt element z_2 or both. Guided by this consideration and others of this nature, I arrive at the design shown in Fig. 3 where the elements of z_1 have the following values:

$$L_{11}=6.37k.10^{-6}$$
$$R_{12}=r.k$$
$$R_{13}=0.36k$$
$$L_{14}=10.37k.10^{-6}$$

In accordance with equation (2), the foregoing values determine the values of the elements of z_2, which are as follows:

$$C_{21}=\frac{6.37}{k}\cdot 10^{-6}$$
$$R_{22}=\frac{k}{r}$$
$$R_{23}=2.779\cdot k$$
$$C_{24}=\frac{10.37}{k}\cdot 10^{-6}$$

In obtaining the foregoing values for the elements of z_1, I need have no concern about the effect on K_1 or K_2 due to varying z_1, provided I keep its absolute value small as compared to k. Also, I have no concern about z_2, for its value is determined by z_1. Therefore, I have only to seek for a proper combination of elements to represent z_1, so that I shall get a straight, upward-sloping line for the attenuation frequency characteristic.

It will be seen that in the foregoing tabulated values for the elements of z_1 and z_2, R_{12} and R_{22} are expressed in terms of a constant r as yet undetermined. It will also be seen that in Fig. 3 these two resistance elements R_{12} and R_{22} are shown adjustable. It can be shown that if they are varied simultaneously in accordance with the curves shown in Fig. 5, and without varying any other elements of z_1 or z_2, the equivalent length of the section,—that is, its attenuation, will be changed without any substantial change in the characteristic impedance.

Inasmuch as the characteristic impedance is approximately 600 ohms of resistance, the infinite network of Fig. 2, with its details shown in Fig. 3, will behave the same at its input end, if it is terminated at the end of any section by a resistance of 600 ohms. Accordingly, such a termination is shown in Fig. 4 with a system of switches by which any desired number of half-sections may be employed between the input end at the left and the terminal network at the right. Since the characteristic impedance is approximately the same at mid-series and mid-shunt as shown by equations (3) and (4), the termination of the network may be made at any mid-series or mid-shunt point and this enables us to employ any desired number of half-sections, the attenuation in each half-section being the same as for 20 miles of the open-wire line if the whole section corresponds to 40 miles. With the design of Fig. 3 and the values for the elements thereof that have heretofore been written out in this specification, and with $k=600$ ohms and with R_{12} and R_{22} given corresponding values within the range of Fig. 5, a single section has attenuation corresponding to from 32 to 47 miles of the open-wire line of Fig. 1. In Fig. 6 the attenuation is plotted against frequency for each of three equivalent lengths at the extremes and at an intermediate of the range of lengths considered. It will be seen that the agreement with the ideal is closer for a length of 40.20 miles than for the lesser length of 32.20 miles or the greater length of 46.23 miles. The same data are exhibited differently in Fig. 7, where the deviation of the attenuation from the ideal is plotted against equivalent length for each of three representative frequencies, and here again the 40 mile length shows up the most favorably. From Figs. 6 and 7 it appears that the best attenuation frequency characteristic is obtained with a 40 mile section, and, accordingly, that is assumed as the optimum, but lengths a little greater or a little less may be had by adjustment of the resistances R_{12} and R_{22} in accordance with Fig. 5. Also, a 20 mile length may be had by taking a half-section, for the reasons that have already been explained, and the same percentage variation from 20 miles may be had by adjustment of R_{12} and R_{22} in accordance with Fig. 5. Applying these principles, it is readily shown that the successive sections of Fig. 1 from Chicago to Pittsburgh may be represented respectively by:

(1) 3½ sections each 39.1 miles long;
(2) 4 sections each 38.5 miles long;
(3) 2 sections each 39.5 miles long;
(4) 4 sections each 42.0 miles long.

It will be seen from Fig. 8 that for a 40-mile equivalent section, when k=600, K_1 ranges from about 600 at 3 kilocycles to about 520 at 30 kilocycles, whereas K_2 ranges from about 600 to about 695, the extreme departures from the 600 value being about 15%. If only one kind of sections is employed, either full mid-series or full mid-shunt, a value of k different from 600 may be chosen advantageously. For full mid-series sections only, let k=640. Then K_1 will range from 640 to about 550 and the extreme deviation from 600 will be only about 8%. For full mid-shunt sections only, let k=560, and K_2 will range from 560 to about 650, giving likewise about 8% deviation.

While I have explained my network in relation to a special case, it will readily be seen that it offers a solution of the problem of getting a network of approximately constant resistance characteristic impedance and with a desired relation between attenuation and frequency.

I claim:

1. A network of the type having successive series impedances z_1 and in alternation therewith successive shunt impedances z_2, in which each of said impedances comprehends resistance elements, and subject to the condition that the product of z_1 and z_2 is substantially a real constant for various frequencies.

2. A network of ladder type with the product constant for its series impedance and its shunt impedance and with resistance elements in those impedances.

3. A ladder type network of substantially constant resistance characteristic impedance and having resistances in its impedance elements and having a predetermined attenuation frequency characteristic.

4. A network to simulate a non-loaded upon wire line with successive series impedances and alternately disposed shunt impedances, each series impedance consisting of a series inductance and two branches in parallel with respect to said inductance, one such branch comprising a resistance and another inductance and the other branch having an adjustable resistance, and each shunt impedance consisting of two parallel branches in one of which there is a condenser and in the other of which there is an adjustable resistance, and in parallel with respect thereto another resistance and another condenser.

5. A network to simulate a non-loaded open wire line with successive series impedances and alternately disposed shunt impedances, each series impedance comprising reactance and resistance elements and each shunt impedance comprising reactance and resistance elements, and the product of a series impedance value and a shunt impedance value being equal to a real constant.

6. A network of constant resistance characteristic impedance and with attenuation a linear function of frequency, said network comprising an adjustable series resistance and an adjustable shunt resistance to vary the degree of attenuation.

7. A sectional network of constant resistance characteristic impedance and with attenuation per section the same function of frequency as for a certain length of non-loaded open-wire line, and means to adjust the equivalent length of a section within a narrow range and means to combine sections in sequence.

In testimony whereof, I have signed my name to this specification this 14th day of December, 1922.

OTTO J. ZOBEL.