May 22, 1934. H. NYQUIST 1,959,494

SYSTEM FOR VOLTAGE TRANSFORMATION OF CURRENTS OF WIDE FREQUENCY RANGE

Filed April 9, 1932 2 Sheets-Sheet 1

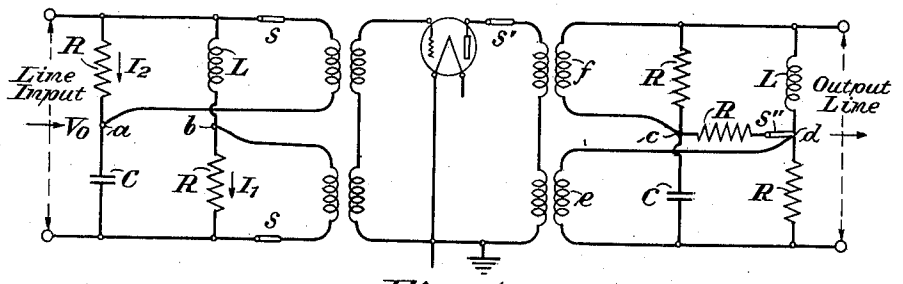


Fig. 1

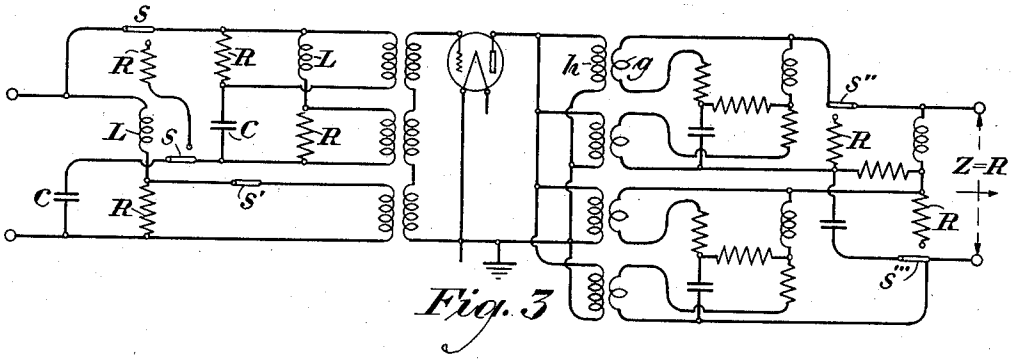


Fig. 3

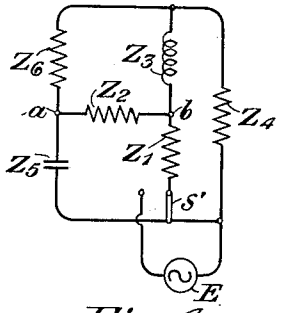


Fig. 4

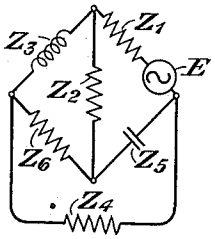


Fig. 5

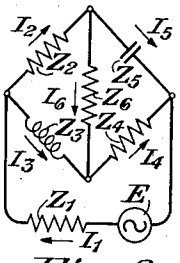


Fig. 6

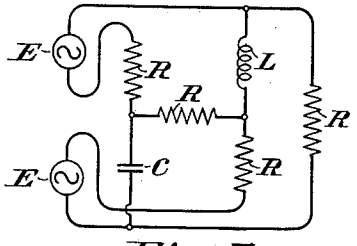


Fig. 7

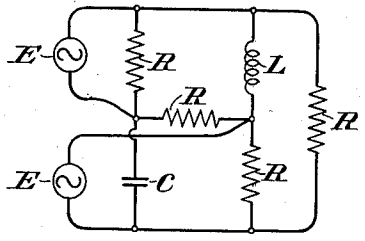


Fig. 8

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SYSTEM FOR VOLTAGE TRANSFORMATION OF CURRENTS OF WIDE FREQUENCY RANGE

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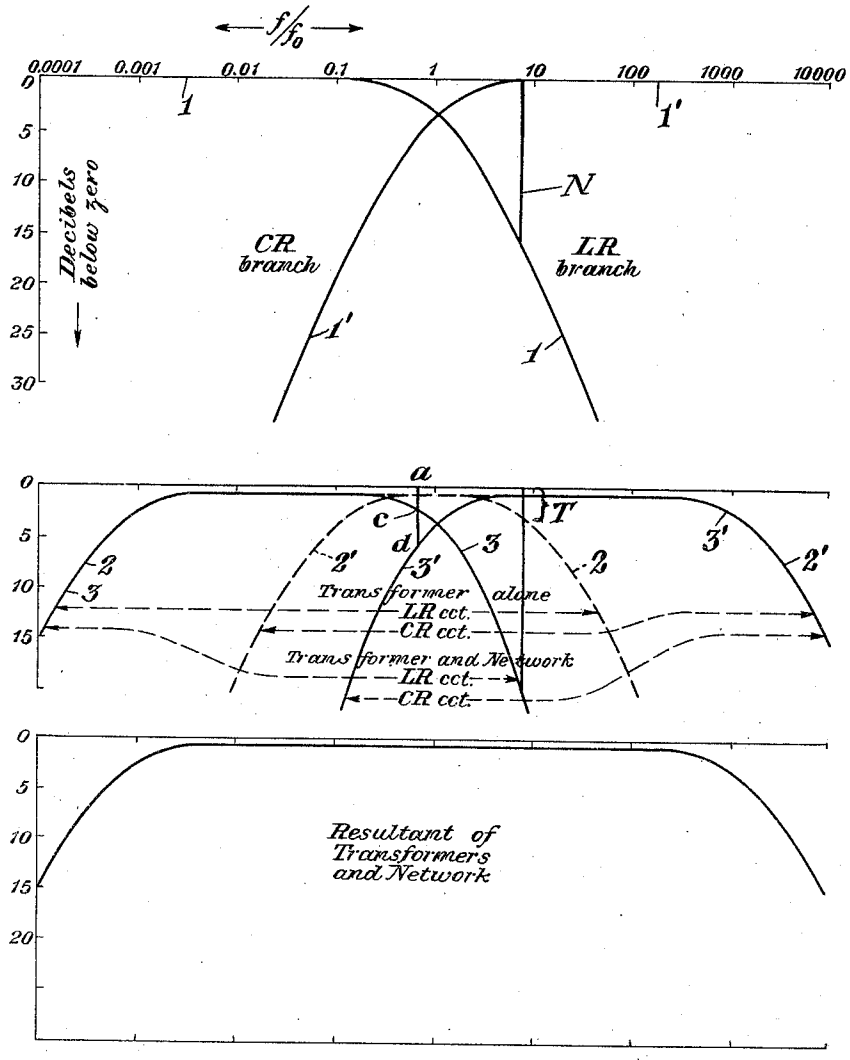


Fig. 2

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UNITED STATES PATENT OFFICE

1,959,494

SYSTEM FOR VOLTAGE TRANSFORMATION OF CURRENTS OF WIDE FREQUENCY RANGE

Harry Nyquist, Millburn, N. J., assignor to American Telephone and Telegraph Company, a corporation of New York

Application April 9, 1932, Serial No. 604,289

12 Claims. (Cl. 178—44)

It is one of the objects of the present invention to provide apparatus and a corresponding method for transforming currents of wide frequency range from one voltage to another with uniform attenuation. Another object of my invention is to provide a plurality of transformers each of limited frequency range, combined in a system so that each transformer serves for a portion of the range within its capacity and the system as a whole transforms with uniform attenuation over an extended frequency range beyond that of any one of the component transformers. Another object of my invention is to provide for transforming with uniform attenuation all the frequency components essential for television transmission. All these objects and various other objects and advantages of my invention will become apparent on consideration of the following disclosure of a limited number of examples of practice according to the invention. In this specification it will be understood that the description relates principally to these particular examples of the invention and that its scope is indicated in the appended claims.

Referring to the drawings, Figure 1 is a diagram of a system embodying my invention on the input side of a vacuum tube amplifier and likewise on its output side; Fig. 2 is a curve diagram for a network such as that of the left of Fig. 1; Fig. 3 is a diagram of a modification as compared with Fig. 1; and Figs. 4, 5, 6, 7 and 8 are diagrams that will be referred to in succession in explaining certain principles involved in my invention.

As an example of a situation in which my invention will find application, I mention the transmission of television currents. In this connection there has been considerable difficulty in employing repeaters with input and output transformers because the available transformers will not give constant attenuation over the desirable wide frequency range that is required in the television transmission currents. Specifically, the difficulty is that the ratio of the highest to the lowest frequencies which can be transmitted through a transformer with fairly uniform attenuation is not great enough for the necessary frequency range in the currents. Transformers in which this ratio is the same are about equally easy to make whether they are for a high frequency range or a low frequency range, but there is a fairly definite limit to this ratio. The greater the number of repeaters in a transmission circuit, and therefore the greater the number of transformers, the greater becomes the difficulty. By the present invention a practicable network is provided that will serve as a transformer over an extended frequency range; several transformers appropriate respectively to parts of the desired frequency range are combined in the network so that it serves with uniform attenuation for the extended frequency range.

Referring to Fig. 1, this shows a three-electrode vacuum tube amplifier with two transformers on the input side, their secondaries in series and their primaries in parallel. The input voltage from the line coming in from the left is represented by V_0, which is applied to the terminals of the network at the left as shown.

To develop the principle of operation, let it be assumed first that the switches S at the left of Fig. 1 are opened. The network comprising the two resistances R, the capacity C, and the inductance L, connected as shown, in Fig. 1, is constructed subject to the condition that,

$$R^2=L/C \qquad (1)$$

The impedance of the network looking into it across the terminals to which the voltage V_0 is applied is

$$Z=\frac{(R+i\omega L)\,(R-i/\omega C)}{R+i\omega L+R-i/\omega C} \qquad (2)$$

where

$$i=\sqrt{-1}$$

and $\omega=2\pi f$ and f is the frequency. Substituting from Equation (1) in Equation (2) it will be found that this impedance reduces to the value R.

It will now be shown that for all frequencies the potential difference V_{ab} across the points a and b is zero. Expressing the potential drop by each of the two available paths from a to b, the two following equations are obtained:

$$V_{ab}=-I_1R-iI_2/\omega C \qquad (3)$$

$$V_{ab}=i\omega LI_1-I_2R \qquad (4)$$

whence

$$I_1=I_2\frac{R-i/\omega C}{R+i\omega L} \qquad (5)$$

Substituting this value of I_1 in Equation (4), and further substituting the value of L as obtained from Equation (1), the result will be reached that the voltage across the points a and b is zero.

We now proceed to obtain the voltage drop of the current I_2 through the resistance R. The current through this resistance R is the current through the CR branch of the network

$$I_2=\frac{V_0}{\sqrt{R^2+i/\omega^2C^2}} \qquad (6)$$

Let the critical frequency of the inductance capacity combination LC be f_0 and the corresponding value of ω be ω_0.

$$\omega_0^2 = 4\pi^2 f_0^2 = 1/LC \quad (7)$$

From Equations (7) and (6)

$$\omega_0 L = \frac{1}{\omega_0 C} = R \quad (8)$$

Substituting for C from Equation (8) in Equation (6) and multiplying both sides by R to get the potential drop, we have

$$I_2 R = \frac{V_0}{\sqrt{1+f_0^2/f^2}} \quad (9)$$

Similarly it can be shown that the potential drop across the resistance R in the LR circuit is given by

$$I_1 R = \frac{V_0}{\sqrt{1+f^2/f_0^2}} \quad (10)$$

From these Equations (9) and (10) it can readily be seen that when the frequency f is equal to f_0, then the voltage drops through the resistances R in both branches are equal. As f decreases and becomes less than f_0 it can be seen from Equation (9) that the voltage drop through R in the CR circuit becomes less, but at frequencies greater than f_0 this voltage drop through R becomes greater. Similarly, it can be seen from Equation (10) that at low frequencies the voltage drop through R in the LR circuit becomes large, in fact, at zero frequency the drop through this resistance is exactly equal to the impressed voltage V_0, and at high frequencies this drop becomes small.

In the upper part of Fig. 2 the voltages across the two resistances R of Fig. 1 are plotted in ordinates as decibels below the voltage V_0 for various frequency ratios f/f_0. The two curves 1'—1' and 1—1 of the upper part of Fig. 2 are based on the foregoing Equations (9) and (10) for the CR branch and the LR branch respectively.

It will be remembered that all the foregoing discussion in connection with Equations (1) to (10) has been with the assumption that the switches S of Fig. 1 are open. Now let them be closed, thus placing a respective transformer primary across each resistance R. The secondaries of these transformers are in series in a circuit of extremely high impedance, namely, the grid circuit of the three-electrode amplifier. Accordingly, the impedance across the primaries is extremely high and, assuming that these impedances are infinite, the closing of the switches S will make no change in the operation as it has been described heretofore.

In the intermediate part of Fig. 2 the curve 2—2 labeled "Transformer alone—LR cct." is a typical attenuation characteristic of a low frequency transformer such as might be used in this connection. Likewise the curve 2'—2' labeled "Transformer alone—CR cct." is a typical high frequency transformer characteristic. These curves are in accordance with the well known fact that a well made transformer will give a substantially uniform small attenuation over a certain frequency range, and then the attenuation will increase rather rapidly at the ends of that range.

The horizontal frequency scale for all the curves in Fig. 2 is given at the top of this figure and being logarithmic, these curves indicate the practicable frequency range for these transformers as a frequency ratio.

To get the attenuation through the combined LR branch and its respective transformer, we have simply to add the corresponding ordinates of the curves 1—1 and 2—2. A representative ordinate for the LR branch curve is shown at N in the upper part of Fig. 2, and the corresponding ordinate for the transformer that goes with the LR branch is shown at T in the middle part of Fig. 2, and the sum of the two ordinates is also shown at the same place in the middle part of Fig. 2. Thus the curve 3—3 is obtained in the middle part of Fig. 2; its ordinates are respectively the sums of the corresponding ordinates of the curves 1—1 and 2—2. This curve 3—3 gives the attenuation in decibles below zero for the combined LR branch and its respective transformer.

Similarly, without following through in details, we get the curve 3'—3' for the combined CR branch and its respective transformer.

From the two transformers the power is combined in a single secondary output circuit by connecting the secondaries in series so that in that output circuit we have to add not attenuation in decibels but to combine the voltages vectorially. Now the two voltages are substantially in phase with the currents I_1 and I_2 and these considered vectorially are at right angles to each other as will be seen by substituting the value for L obtained from Equation (1) in the specification and in Equation (5) of the specification. Notice the vertical line a, c, d in the middle part of Fig. 2. We want to find the power level in decibels that corresponds to adding the power for the ordinate ac and the power for the ordinate ad, these ordinates being expressed in decibels, when the voltages combine vectorially.

First considering the LR and CR network alone, i. e., the left-hand portion of Fig. 1 of the specification with the switches S open, it has been shown in connection with Equation (3), (4) and (5) that points a and b are at the same potential. Consequently, the vector sum of the potential drops across the resistances in the two branches of the network is equal to the impressed voltage V_0, with no phase shift, at all frequencies.

Then considering the voltages on the secondary sides of the transformers when the switches S in Fig. 1 are closed, there is a region in the neighborhood of f_0 in the middle part of Fig. 2 where these two transformers give substantially equal and flat losses, i. e., in which curves 2—2 and 2'—2' follow a common horizontal trace, and also in which they give a comparatively small and negligible relative phase shift. In this region, therefore, the vector sum of the voltages of the secondaries of the transformers will be the impressed voltages V_0 (multiplied by the common voltage ratio of the transformers) reduced by the common attenuation of the transformers. The power level in decibels will be the zero or input level reduced by the common attenuation of the transformers, expressed in decibels. This has been indicated in the lower part of Fig. 2 by a horizontal line showing a constant reduction in power level equal to the common constant attenuation of the two transformers in this region.

To either side of this region about f_0 in Fig. 2 there will be another region, characterized by the facts that the voltage drop across the resistance in one branch will be appreciably greater, numerically, than that across the resistance in the other branch; that the transformer connected to the resistance having the greater drop will have in this region a small flat attenuation which

is the same as in the region close to f_0, and a comparatively small and negligible phase shift relative to that at f_0; and that the transformer connected to the resistance having the lesser drop will have a large attenuation varying rapidly with frequency, and a phase shift relative to that at f_0 which may be substantial. In this region, therefore, the resultant secondary voltage will consist of a relatively large component undistorted with respect to magnitude and phase, and a relatively small component suffering distortion as to magnitude and possibly phase. Consequently, if the curve in the lower part of Fig. 2, showing the resultant power level in decibels, be examined minutely, it should show a small amount of deviation from a horizontal straight line in this frequency region, which deviation diminishes again at frequencies still further remote from f_0, where the magnitude of the larger undistorted component sufficiently dominates that of the smaller component. This deviation in the curve in the lower part of Fig. 2 can be made small, and indeed negligible, by arranging a sufficient overlap between the non-distorting regions of the two transformers, and it is such a design which has been assumed throughout in Fig. 2.

In the frequency regions of Fig. 2 furthest remote from f_0, the large component noted above again becomes distorted. Its magnitude dominates that of the smaller component so much, however, that the latter can be neglected entirely. The edges of the curve in the lower part of Fig. 2 reproduce with substantial exactness, therefore, the portions, in the same regions, of the curves 3—3 and 3′—3′ in the middle part of Fig. 2.

Thus we have shown how the curves in the middle part of Fig. 2 are combined to give that in the lower part, and how the manner of this combination proceeds over the entire frequency range.

By the principles that have been developed heretofore, more than two transformers may be employed up to any number. Referring to Fig. 3, the use of three transformers is shown on the input side at the left. Suppose the two switches S are shifted from their positions shown in the drawings and the switch S′ is opened. This network at the left of Fig. 3 will then be the same as at the left of Fig. 1. But the network that extends to the right from the switches S is the same as at the left to Fig. 1 and likewise has the impedance R looking into it across these switches. Hence the switches can be shifted back to the positons shown in the drawings without altering the impedance looking from the line terminals at the left. Hence on the same principles as explained for Fig. 1 we have here in Fig. 3, three consecutive frequency ranges with respective transformers, thus extending the frequency range of uniform attenuation farther than in Fig. 1. Here again the voltages across the three resistances R of Fig. 3 have to be added vectorially. It will be seen that this process can be continued indefinitely, replacing any resistance R with a typical network containing an L, a C, and two R's.

In all that has gone before we have assumed a practically infinite impedance on one side of the network as in the grid circuits of Fig. 1 and Fig. 3. My improved network system can readily be adapted or modified for connection with circuits of finite impedance, as will be shown presently. It will be convenient to explain the principles involved in connection with Figs. 4, 5, 6, 7 and 8.

As discussed heretofore, the operation of the network would be unchanged with any impedance connected across the points a and b of Fig. 1, for the reason that the voltage across ab is zero. If a voltage is interposed in series with either R of Fig. 1, then a current would, in general, flow across ab if they were connected by a finite impedance. But in that case if the connection across ab has the value R, then we shall show that no component would flow in the remaining R due to an electromotive force through the first R.

In Fig. 4, with the switch S′ in the position shown, we have the same network as at the left of Fig. 1 when the switches S in Fig. 1 are open, except that a resistance marked Z_2 is connected across the points a and b. The impedance Z_4 in Fig 4 represents the impedance of the line connected at the left in Fig. 1. It will now be shown that if an electromotive force E is introduced in series with impedance Z_1 (the upper left resistance R) and if the impedance Z_2 is embodied in a resistance R, then there will be no current across Z_6 due to the electromotive force E. To show this, let the switch S′ be shifted from its position in Fig. 4 and let the figure be redrawn in conventional bridge fashion as in Fig. 5. As is well known, a bridge diagram like Fig. 5 can be redrawn so that two non-adjacent arms in the first case become the bridging members in the second case; thus Fig. 6 is derived from Fig. 5.

In Fig. 6, according to Kirchhoff's first law, we have the folowing equations:

$$I_1=I_2+I_3;\quad I_4=I_3+I_6;\quad I_5=I_2-I_6 \qquad (11)$$

and, according to Kirchhoff's second law, we have the following equations:

$$\begin{aligned} I_1Z_1+I_2Z_2+I_5Z_5&=E\\ I_2Z_2+I_6Z_6-I_3Z_3&=0\\ I_5Z_5-I_4Z_4-I_6Z_6&=0 \end{aligned} \qquad (12)$$

substituting from Equations (11) for I_1, I_4 and I_5 in Equations (12) and then solving by determinants for I_6, the determinant solution will be

$$I_6=N/D \qquad (13)$$

where N stands for the numerator determinant and D for the denominator determinant. This numerator determinant is

$$N=\begin{vmatrix} Z_1+Z_2+Z_5 & Z_1 & E\\ Z_2 & -Z_3 & 0\\ Z_5 & -Z_4 & 0 \end{vmatrix}=E(Z_3Z_5-Z_2Z_4) \qquad (14)$$

Let Z_4 of Figs. 4, 5 and 6 have the value R. As already mentioned Z_2 has the value R. Comparing Fig. 4 with Fig. 1 on which it is based, we see that Z_3 has the value ωL and Z_5 has the value $1/\omega C$. Substituting these values in the right hand member of Equation (14) and relying on Equation (1), we see that $N=0$ and accordingly $I_6=0$. Thus we have proved that with a resistance R introduced across the points a and b with a resistance R at Z_4 across the output side of the network, the introduction of an electromotive force E at any frequency in series with the resistance R in the RL branch, gives rise to no current in the other R of the CR branch.

Similarly the introduction of an electromotive force E in series with the resistance R of the CR branch can be shown to give rise to no current in the resistance R of the LR branch. These

conclusions will be readily appreciated in connection with Fig. 7.

Moreover, if the electromotive forces E are applied in shunt to the R's as in Fig. 8, the current component in each R due thereto will correspond to a certain electromotive force in series with that R, and the foregoing reasoning will apply.

This shows that when the network at the left of Fig. 1 is used in connection with apparatus where it must work into a circuit having finite impedance (for example, the plate circuit of a vacuum tube) a resistance R having the same value as the other R's shown must be connected between the points *a* and *b*, since without it any current flow will produce a voltage across one resistance arm, say R in the LR circuit which would cause current to flow through the other resistance R in the CR circuit. Similarly, a corresponding potential will be produced across R in the CR circuit at some other frequency which would cause additional current to flow through R of the LR circuit. The voltages produced in this manner across the resistances R in each circuit would consequently be different than those which will be produced when R is used to connect the points *a* and *b*. This would give some undesirable resultant characteristic for the transformers. The new network with R across *a* and *b* is not restricted to the input circuit of an amplifier, since it can work into a finite impedance.

Referring to the right side of Fig. 1, if the switches S' and S'' were opened and if the input were where the output is shown and vice versa, the arrangement would be the same as at the left of Fig. 1. Considering that the plate circuit is of infinite impedance, it follows that closure of switch S' leads to the development of superposed electromotive forces in the windings *e* and *f* and hence through the two resistances R. But if the switch S'' is closed, neither such electromotive force through one of these R's develops any current through the other R. Thus far we have dealt with the network at the right of Fig. 1 as if the input were at the right and its output at the left. But in a network without internal sources of energy the transmission therethrough is the same if the input and output terminals are interchanged, hence the output line at the right will draw its voltage from the two transformers at *e* and *f* according to the apportionment that has been considered in connection with Fig. 2.

Now in the arrangement which has been described so far it will be noted that, since the plate circuit impedance is not infinite, when the transformer *e* in Figure 1 for example is connected across the terminals of the resistance R, this will tend, if there is no other modification, to reduce the value of the net effective resistance in the LR branch. In order that the network may function properly it is necessary, therefore, to modify the fixed resistance, and so to proportion the contributions of resistance from it and from the vacuum tube plate circuit that the net effective resistance in the LR branch has the value previously assigned to R.

In a practical case it is desirable to dissipate as little energy as possible in the fixed resistance shown. This can be achieved by making this resistance infinite or in other words simply leaving it out, and making the contribution of resistance from the vacuum tube plate circuit constitute the only contribution of resistance to the LR branch. Exactly the same line of reasoning will apply in the case of the resistance R in the CR branch.

Referring to the right hand side of Fig. 3, suppose first the switches S'' and S''' were thrown from the positions shown in the drawings. Then we should have at the right of these switches a single network such as that at the right of Fig. 1. But with the switches S'' and S''' as shown, each of the two resistances R is replaced by a similar network of resistance R. If each of the four windings *g* has low impedance across its terminals and has electromotive force induced therein, the operation will be on the same principle as for Fig. 7. Thus on the right of Fig. 3, thinking of transmission from left to right, the output from the plate circuit goes into the four transformer primaries *h* in parallel, and corresponding electromotive forces are induced in the secondaries *g*. Each of the associated resistances R carries the current due to a respective electromotive force without affecting the current in any other one of these resistances R, and the ultimate output from the network (at the right) takes off currents over a wide range of frequencies, each of the four transformers *hg* independently contributing the part for which it is adapted and giving a flat overall characteristic.

Here as in the case of the network shown at the right hand side of Fig. 1, the contributions of resistance to the LR and CR branches from the plate circuit of the tube, operating through the respective transformers, and of the respective fixed resistances shown, must be so proportioned as to give a net effective resistance of the value R in each branch. In the particular case shown in Fig. 3, if it is desired to reduce the dissipation of energy in the network to a minimum the fixed resistance should be made zero, or a short circuit.

In Fig. 1 a ground is shown in the usual location, that is, connected directly to the filament-cathode, so that it is on that side of the grid circuit and the plate circuit which have this filament cathode as a common element. In the two transformers associated with the grid circuit, low frequencies will flow principally in the primary of the lower transformer and high frequencies principally in the primary of the upper transformer. This is a desirable arrangement, because if it were the other way, when the capacity to ground of the low frequency transformer would be added to the capacity of the vacuum tube for electromotive forces applied in the grid circuit through the high frequency transformer. This would have a considerable effect to narrow the transmission band of the high frequency transformer by reducing the frequency at which it would tend to cut off. By the arrangement of Fig. 1 this objectionable effect is obviated. The same principle has been applied in the connection of the transformer associated with the plate circuit.

Referring to Fig. 3, the same principle has been embodied as discussed above for Fig. 1. Here in Fig. 3 the highest frequency components go principally through the primary of the uppermost transformer in the grid circuit and the low frequency components go principally through the primary of the lowermost transformer in the grid circuit, and components of intermediate frequency go through the primary of the middle transformer in the grid circuit. The reason for this arrangement is apparent from the explanation given above in connection with Fig. 1.

I claim:

1. The method of transforming currents of wide frequency range from one voltage to another with substantially uniform attenuation at all frequencies within that range, which consists in separating the currents in respective channels according to different parts of said frequency range and in each channel transforming the voltage of the current at uniform attenuation within its part of the frequency range and combining the outputs in all the channels to get a resultant that is transformed uniformly over said wide frequency range.

2. The method of transforming currents of wide frequency range from one voltage to another with substantially uniform attenuation at all frequencies within that range, which consists in separating the currents into respective channels corresponding to parts of said frequency range, transforming from one voltage to the other within each channel with uniformity only over the corresponding part of the frequency range and combining the respective output current parts to get a resultant that is transformed with uniform attenuation over said wide frequency range.

3. The method of transforming currents of wide frequency range from one voltage to another with a desired degree of substantially uniform attenuation at each and all frequencies in that range, which consists in separating the currents in respective channels corresponding to parts of said frequency range, in each channel transforming from the one voltage to the other at the desired attenuation for the corresponding frequencies, and combining the outputs from these channels to get the desired resultant transformed current.

4. Means to transform a current of a wide range of frequencies from one voltage to another, consisting of a transducer with a set of input terminals, and a set of output terminals and a plurality of transformers, means within said transducer to separate input currents received through the input terminals according to consecutive parts of said frequency range and to apply them to respective transformers, and means to combine the outputs from said transformers to said output terminals.

5. Means to employ the available frequency ranges of two transformers over respective consecutive parts of an extended frequency range comprising a network associated with said transformers and adapted to give low attenuation at low frequencies through one transformer and low attenuation at high frequencies through the other transformer, whereby the resultant output of both transformers is of uniform attenuation over the extended combined ranges of both transformers.

6. Means to transform a current of a wide range of frequencies from one voltage to another consisting of a transducer with a pair of input terminals and a pair of output terminals, said transducer comprising two parallel current paths across one pair of terminals, one such path comprising a resistance R and an inductance L in series, the other path comprising a resistance of like value R and a capacity C in series, and two transformers having one winding of each associated with a respective resistance.

7. A combination according to claim 6, subject to the condition that $R^2=L/C$.

8. A combination according to claim 6, with a third resistance R connected across the point between R and L in one said path to the point between R and C in the other said path.

9. A combination according to claim 6, with a third resistance R connected across from the point between R and L in one said path to the point between R and C in the other said path and subject to the condition that $R^2=L/C$.

10. The combination of claim 6 with a line of impedance R connected across one pair of terminals, and subject to the condition that $R^2=L/C$.

11. In combination with two transformers, means to extend a frequency range of constant attenuation beyond that obtainable with a single transformer consisting of respective networks adapted to pass different frequency ranges with low attenuation, said transformers being respectively adapted for those frequency ranges, and connections to separate a composite current into two paths, each path consisting of a network and its associated transformer, and to combine the outputs therefrom.

12. The method of transforming currents of wide frequency range from one voltage to another with substantially uniform attenuation at all frequencies within that range which consists in separating the currents in respective networks selectively adapted for different parts of said frequency range and transforming the voltage of these current parts, each part at uniform attenuation within its frequency range, and combining the outputs from all the networks to get a resultant that is transformed with uniform attenuation over said wide frequency range.

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