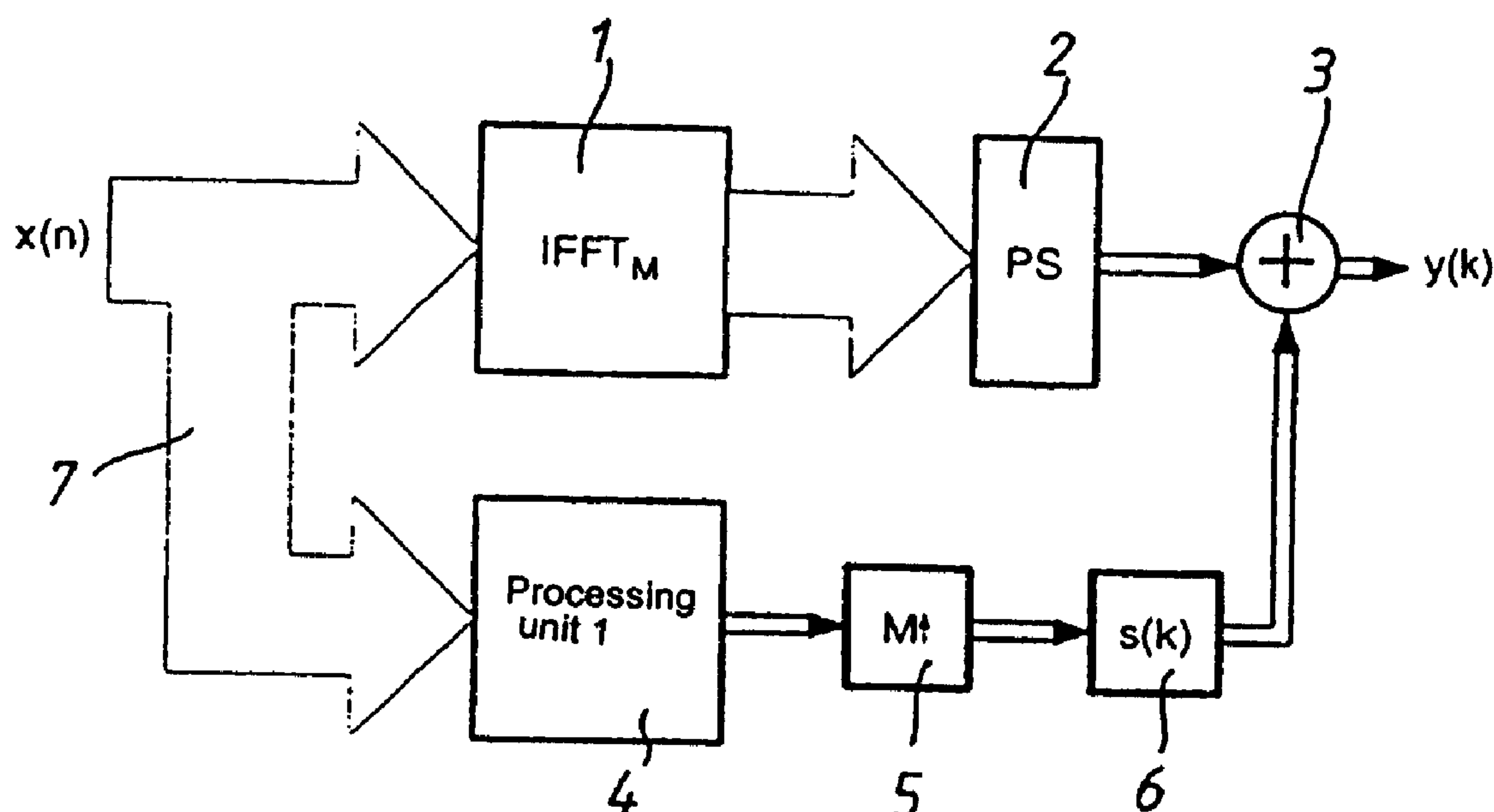




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(71) Demandeur/Applicant:
TELEFONAKTIEBOLAGET LM ERICSSON, SE
(72) Inventeur/Inventor:
BALDEMAIR, ROBERT, AT
(74) Agent: MARKS & CLERK

(54) Titre : PROCEDE POUR ANNULER DES BANDES DE FREQUENCES ETROITES
(54) Title: METHOD FOR SUPPRESSING NARROW FREQUENCY BANDS



(57) Abrégé/Abstract:

The invention relates to a method for suppressing narrow frequency bands during transmission of data by means of a multi-carrier method, e.g. DMT (discrete multitone). A predetermined broad frequency band is divided into numerous subchannels having sub-carriers assigned thereto. The data to be transmitted is modulated in the transmitter by means of inverse discrete Fourier transformation (IDFT) and is demodulated in the receiver by discrete Fourier transformation (DFT). A pulse for compensating the side lobes which appear in the phase-out area is transmitted for each frequency range having a zero charge and extending between the subcarriers which are contained in the phase-out area or are adjacent to the phase-out area. Said pulse is respectively provided with a frequency range which resembles the side lobes that appear in the intermediate area. Said pulse is controlled according to the data values of the side lobes which appear in the intermediate areas, whereby the compensation pulse/s is/are orthogonally transmitted to the information-carrying subcarriers.



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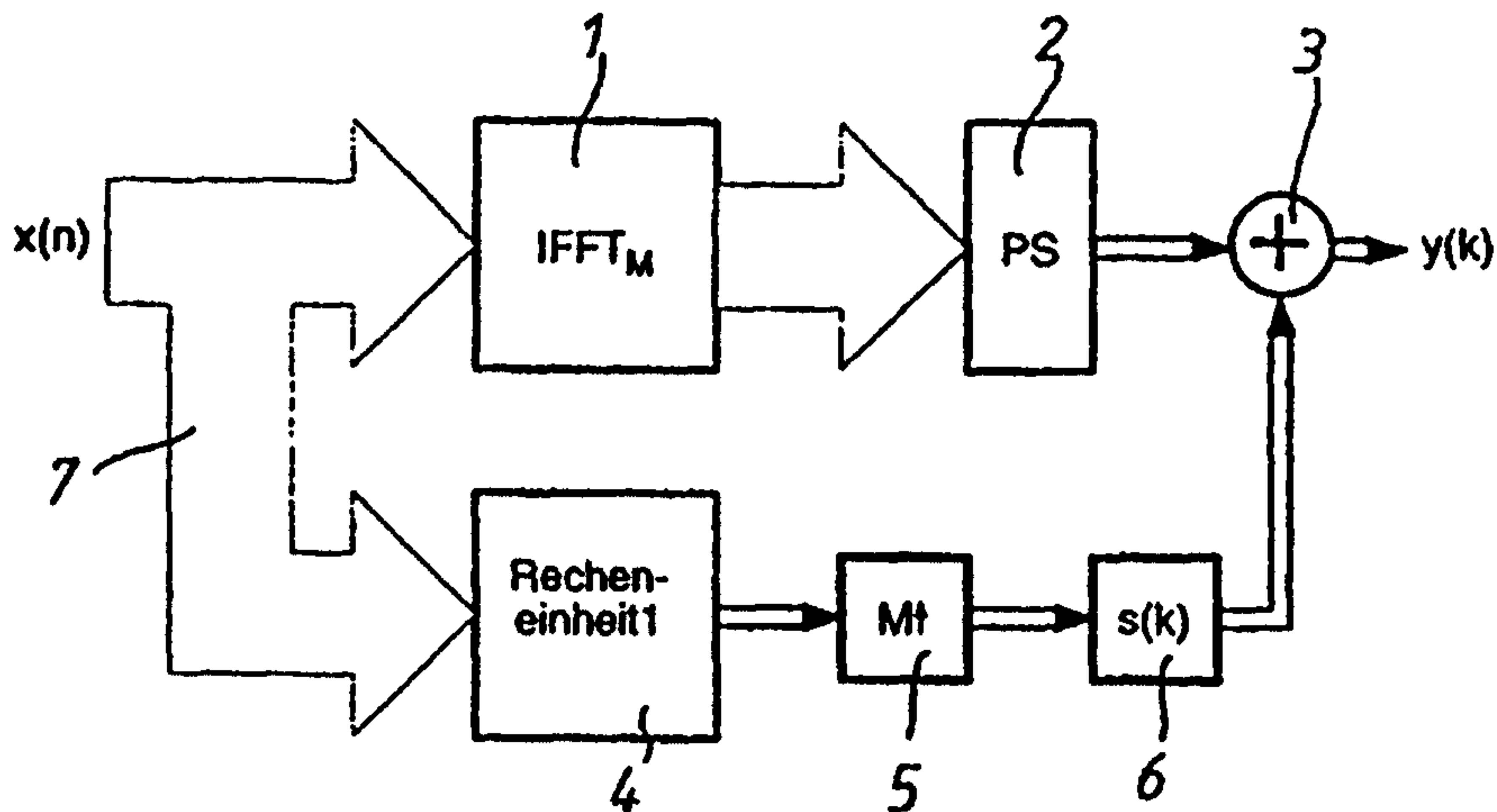
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(54) Title: METHOD FOR SUPPRESSING NARROW FREQUENCY BANDS

(54) Bezeichnung: VERFAHREN ZUR UNTERDRÜCKUNG VON SCHMALEN FREQUENZBÄNDERN

(57) Abstract

The invention relates to a method for suppressing narrow frequency bands during transmission of data by means of a multi-carrier method, e.g. DMT (discrete multitone). A predetermined broad frequency band is divided into numerous subchannels having sub-carriers assigned thereto. The data to be transmitted is modulated in the transmitter by means of inverse discrete Fourier transformation (IDFT) and is demodulated in the receiver by discrete Fourier transformation (DFT). A pulse for compensating the side lobes which appear in the phase-out area is transmitted for each frequency range having a zero charge and extending between the subcarriers which are contained in the phase-out area or are adjacent to the phase-out area. Said pulse is respectively provided with a frequency range which resembles the side lobes that appear in the intermediate area. Said pulse is controlled according to the data values of the side lobes which appear in the intermediate areas, whereby the compensation pulse/s is/are orthogonally transmitted to the information-carrying subcarriers.



4...WHITHMETIC UNIT 1

Method of suppressing narrow frequency bands

The invention relates to a method of suppressing narrow frequency bands in fade-out ranges during transmission of data by means of a multiple carrier method, e.g. DMT (Discrete Multitone) in which a predetermined broad frequency band is divided into a plurality of subchannels having subcarriers assigned thereto and in which the data to be transmitted are modulated, divided into data blocks, in the transmitter with Inverse Discrete Fourier Transform (IDFT) and are demodulated in the receiver with Discrete Fourier Transform (DFT), each subchannel being thus provided in the spectrum with a major lobe and several side lobes occurring in the range of nearby subcarriers, wherein at least part of the subcarriers contained in at least one fade-out range and of the subcarriers adjacent to this fade-out range are used as compensation sounds having a frequency spectrum which resembles the side lobes of subcarriers located outside the fade-out range that occur within the fade-out range and are modulated according to the data values of these side lobes in order to suppress said side lobes.

In a plurality of data transmission systems of the art, transmission occurs by frequency-division multiple access. The methods used thereby have become known to be the multiple carrier method, Orthogonal Frequency Division Multiplexing (OFDM) and Discrete Multitone (DMT). A predetermined, broad frequency band is thereby subdivided into a host of very narrow frequency bands or subchannels, over which data may be transmitted with various methods of modulation and bit rates. To distribute the data in the transmitter the Inverse Fast Fourier Transform (IFFT) can be used and, correspondingly, to reconstruct it in the receiver, the Fast Fourier Transform (FFT). The problem therewith is the strong overlap of the subchannels in the frequency range since the side lobes of several neighboring subchannels blanket each subchannel which consists of major and side lobes. The IFF Transform effects a filtration of the subchannels with frequency-shifted versions of one unique prototype filter. The low attenuation of the neighboring subchannels causes the side lobes to overlap as mentioned above.

A conventional transmission by frequency-division multiple access as it may be realized by means of the DMT method can sweep over a very broad frequency band, ranging, e.g. from 300 kHz to 30 MHz, which is filled with evenly spaced carrier frequencies.

According to the frequency range standards that vary from one nation to the other, there are in practically every broad transmission spectrum prohibited ranges that are reserved for specific applications. These ranges may be occupied by amateur radio, emergency call or other well-known transmitters. It is therefore absolutely necessary to keep certain frequency ranges free in order not to interfere with the transmitting operation of these assigned ranges.

As already mentioned herein above, each subchannel is provided, in addition to a central major lobe, with side lobes that symmetrically drop about the carrier frequency.

In order to achieve sufficient suppression of a certain frequency range, it is not sufficient to use the subchannels within this range without modulation, which is also called zero charge of the subchannels, since the crosstalk arising in the neighboring channels on account of the low attenuation of the side lobes is so strong that the noise being emitted by said channels is still too strong to keep the desired fade-out range free. On account of the side lobes, the power density in this fade-out range then still has a value not to be neglected.

Accordingly, with the systems of the art, many of the channels neighboring the range to be kept free had to be left unmodulated in order to thus achieve a sufficient drop of the side lobes in the fade-out range. The disadvantage thereof is the high number of channels that have to be left unused, the overall frequency range of the method of transmission employed being relatively poorly exploited as a result thereof.

WO 97/40609 disclosed a multiple carrier method for xDSL applications in which one or several compensation sounds are additionally transmitted at the border of the frequency band to be suppressed in order to suppress a narrow frequency band, the phase and amplitude of said compensation sounds being selected in such a manner that they suppress within the narrow frequency band the transmitting power occasioned by side lobes of carriers located outside said

frequency band.

The number of usable subchannels may thus be increased.

It may be shown that the amounts of the side lobes of each subchannel substantially only differ in the amplitude and in a constant phase offset. The interference caused in the fade-out range by any subchannel therefor has a spectrum which is similar to that of all the other interferences so that the resulting overall interference also resembles that of a side lobe.

The amplitude and the phase of the side lobe spectra for the fade-out range can be calculated from the data values of a number of subchannels and the compensation pulse assigned to each intermediate frequency range can be determined by adding the individual complex side lobe spectra that have been calculated for this purpose. Prior to transmission, the thus determined compensation pulse(s) are superimposed to the transmitter signal in such a manner that the fade-out range is freed from interfering side lobes.

By subtracting a compensation pulse having the same amplitude frequency response and the same phase frequency response as the interference in the fade-out range, the interference spectrum in the fade-out range may be sufficiently reduced as to achieve the desired attenuation factor.

Except for the subcarriers contained in the fade-out range, only those subcarriers are to be zero charged that are located at the border of the fade-out range and, if need be, one or a few located directly outside the border of the fade-out range.

Accordingly, only one or a few of the subcarriers neighboring the border of the fade-out range are zero charged in addition to those subcarriers that are located within the fade-out range, so that only a very reduced number of subcarriers is left unmodulated, the number of subcarriers that cannot be used for transmission being thus minimized as a result thereof.

It is the object of the present invention to indicate a method by means of which it is possible to carry out efficient suppression of narrow frequency bands in fade-out ranges and in which only

a relatively low number of subcarriers needs to be kept free for fading out the narrow frequency ranges.

According to the invention, this is achieved in that the modulation of the compensation sounds is calculated in such a way that the integral of the weighted, sent power density spectrum is minimized over the entire frequency range.

Thanks to the statistical method of calculation employed, the thus transmitted compensation sounds may be calculated very accurately and with relatively little mathematical expenditure.

According to the invention this object is also achieved in that the modulation of the compensation sounds is calculated in such a way that the integral is minimized over the entire frequency range of the weighted, squared amplitude of the Fourier transformed of the sent data signal by way of a number of data blocks that may be predetermined.

The accuracy of the deterministic method of calculation utilized for this purpose increases with the number of data blocks that are available for the calculation. Since storage capacity cannot be increased ad lib, the result depends on the capacity of the processing unit.

According to another development of the present invention, data that have already been sent may be considered in the calculation. The accuracy of the calculation can be improved by including the data already sent.

According to another feature of the invention, either a Guard Interval or a cyclical prefix may be transmitted between the data combined to blocks. The method according to the invention may be utilized for either of the two ways of forming intervals.

The invention will be explained more fully hereinafter with reference to the exemplary

embodiments illustrated in the drawing.

- Fig. 1 shows the amplitude frequency response of a prototype filter;
- Fig. 2 shows the amplitude frequency response called forth by the interference of three subchannels;
- Fig. 3 shows an equivalent circuit of an Inverse Discrete Fourier Transform;
- Fig. 4 shows the amplitude frequency response of a prototype filter;
- Fig. 5 shows the phase frequency response of a prototype filter;
- Fig. 6 shows the amplitude of the transmission functions of three subcarriers;
- Fig. 7 shows the phase response of the transmission functions of three subcarriers;
- Fig. 8 shows the overlapping and standardized side lobes of a prototype filter for $M=16$;
- Fig. 9 shows the amount frequency response with a fade-out range;
- Figs. 10a and 1b each show a schematic representation of a fade-out range;
- Fig. 11 shows a block diagram of the transmitter of an embodiment of the transmission system according to the invention;
- Fig. 12 shows a block diagram of the transmitter of another embodiment of the transmission system according to the invention;
- Fig. 13 shows the amplitude frequency response of transmission functions with a fade-out range;
- Fig. 14 shows the amplitude of the nominal transmission functions for a compensation pulse;
- Fig. 15 shows a schematic representation of the transmitter signal when using a cyclical prefix;
- Fig. 16 and Fig. 17 show amplitude frequency and phase frequency response of the transmission functions of subcarriers;
- Fig. 18 shows a schematic representation of the vectors $g(n)$;
- Fig. 19 shows the nominal transmission function and two compensation pulses of various length;
- Fig. 20 shows a schematic representation of $v(n)$;
- Fig. 21 shows a power density spectrum for transmission with $M=512$ subchannels;
- Fig. 22 to 24 show enlarged portions of the fade-out ranges of Fig. 21;

- Fig. 25 shows power density spectrum for transmission with $M=1024$ subchannels;
- Fig. 26 to 28 show enlarged portions of the fade-out ranges of Fig. 25;
- Fig. 29 shows power density spectrum for transmission with $M=2048$ subchannels;
- Fig. 30 to 32 show enlarged portions of the fade-out ranges of Fig. 21;
- Fig. 33 shows a block diagram of a transmission system for carrying out an embodiment of the method according to the invention;
- Fig. 34 shows a chart with the zero charged subcarriers used for a conventional method of transmission,
- Fig. 35 shows a diagram of the power density spectrum for a method of transmission according to Fig. 34;
- Fig. 36 shows details of the diagram according to Fig. 35;
- Fig. 37 shows a chart with the zero charged subcarriers used for an embodiment of the method according to the invention;
- Fig. 38 shows a diagram of the power density spectrum for a method of transmission according to Fig. 37;
- Fig. 39 shows details of the diagram according to Fig. 38;
- Fig. 40 shows a chart with the zero charged subcarriers used for an embodiment of the method according to the invention;
- Fig. 41 shows a diagram of the power density spectrum for a transmission method according to Fig. 40;
- Fig. 42 shows details of the diagram according to Fig. 41;
- Fig. 43 shows a chart with the zero charged subcarriers used for an embodiment of the method according to the invention;
- Fig. 44 shows a diagram of the power density spectrum for a method of transmission according to Fig. 43 and
- Fig. 45 shows details of the diagram according to Fig. 44.

In transmission systems based on frequency-division multiplexing that have become known under the designations multiple carrier method, Orthogonal Frequency Division Multiplexing (OFDM) and Discrete Multitone (DMT), a broad frequency band is subdivided into a plurality of very narrow frequency bands or subchannels which are allocated evenly spaced subcarriers.

The xDSL methods of transmission, e.g. ADSL, brought about a plurality of applications for the DMT method. The modulation of the transmittal data on the side of the transmitter is accomplished with an Inverse Discrete Fourier Transform (IDFT) while the transmitted data are demodulated on the receiver side with the assistance of the Discrete Fourier Transform (DFT).

For the sake of simplifying the considerations set forth herein after, the transmission over an entirely dispersion-free channel will be considered first so that the transmitted transmitter signals are not being distorted.

The data stream to be transmitted $A_k = 0, 1, 2, \dots$ is combined into blocks of a length M , M designating the number of subchannels. Simultaneously, M is the block length of the IDFT:

$$\begin{aligned} 0^{\text{th}} \text{ block} \quad A_0 &= [A_0 \quad A_1 \quad \dots \quad A_{M-1}]^T \\ 1^{\text{st}} \text{ block} \quad A_M &= [A_M \quad A_{M+1} \quad \dots \quad A_{2M-1}]^T \\ m. \text{ block} \quad A_{mM} &= [A_{mM} \quad A_{mM+1} \quad \dots \quad A_{mM+M-1}]^T \end{aligned}$$

For a real transmitter signal, only $M/2$ of the data may be selected freely, whereas the remaining $M/2$ data is conjugate-complex relative to the first mentioned $M/2$ data (e.g., ADSL with 256 sounds yields $M=512$)

$$\begin{aligned} a_0 &= [a_0 \quad a_1 \quad \dots \quad a_{M-1}]^T = \sqrt{M} \cdot \text{IDFT}_M\{A_0\} \\ a_M &= [a_M \quad a_{M+1} \quad \dots \quad a_{2M-1}]^T = \sqrt{M} \cdot \text{IDFT}_M\{A_M\} \\ a_{mM} &= [a_{mM} \quad a_{mM+1} \quad \dots \quad a_{mM+M-1}]^T = \sqrt{M} \cdot \text{IDFT}_M\{A_{mM}\} \end{aligned}$$

The blocks a_{kM} , $k=0, 1, 2, \dots$ are serially laid at the output and transmitted.

In Fig. 1, crosstalk of a subchannel 0 on the other subchannels is illustrated for a transmission system with $M=16$ subchannels. Accordingly, a subchannel is composed of one major lobe and of several side lobes. Superimposition of three of the overall sixteen subcarriers is shown in Fig. 2.

The Inverse Discrete Fourier Transform (IDFT) may be represented by a transmultiplexer which is illustrated in Fig. 3, the serial data being led parallel to a set of filters $h_k(n)$, $k=0, 1, 2, \dots, M-1$, after oversampling has been performed, in which $M-1$ zeros are added to each datum. The filter $h_0(n)$ is hereby a prototype filter whose time domain runs over the length M , all the other filters $h_k(n)$ represent frequency-shifted versions of this prototype filter $h_0(n)$.

$$h_0(n) = \begin{cases} \frac{1}{\sqrt{M}} & \text{für } n = 0, 1, \dots, M-1 \\ 0 & \text{sonst} \end{cases} \iff H_0(e^{j\theta}) = \frac{1}{\sqrt{M}} \frac{\sin \frac{M\theta}{2}}{\sin \frac{\theta}{2}} e^{-j\theta \frac{M-1}{2}}$$

The other filters $h_k(n)$, $k=1, 2, \dots, M-1$ are obtained by shifting the prototype filter $H_0(e^{j\theta})$ by $(2\pi/M) \cdot k$.

$$h_k(n) = h_0(n) e^{j \frac{2\pi}{M} kn} \iff H_k(e^{j\theta}) = H_0(e^{j(\theta - \frac{2\pi}{M} k)}) \quad (1)$$

In the Figs. 4 and 5, the corresponding Bode's diagram of the prototype filter for an IDFT Transform is represented with a block length of $M=16$. The side lobes differ quite clearly with regard to their amplitudes which drop symmetrically relative to the major lobe at $\theta/\pi=0$. By contrast, there is no essential difference to be observed in the frequency responses, the prototype filter being provided with a linear phase in all the side lobes. As a result thereof, the interference spectrum occasioned by any effective channel in a fade-out range resembles the interference spectra occasioned by other effective channels except for a complex scaling factor.

According to Fig. 3, the datum k triggers the filter $h_k(n)$ or $H_k(e^{j\theta})$ respectively. The greatest part of the signal power is transmitted in the band $(k-1)2\pi/M \leq \theta < (k+1)2\pi/M$.

On account of the side lobes of the transmission function $H_k(e^{j\theta})$, a not to be neglected part is also transmitted in the adjacent channels, though. If the power density in a certain frequency range is intended to remain below a certain value, it is not sufficient not to trigger the filter(s) corresponding to this range since the side lobes in the transmission functions of adjacent channels cause the power density to still have a value not to be neglected. In that the side lobes slowly

decay, they turn out to be crosstalk in the neighboring channels, the first side lobe having a maximum which is less than the major lobe by only 13 dB (Fig. 4).

For purposes of clarity, only the amplitudes and the phases of the transmission functions for the channels $k=2, 3$ and 13 are illustrated in Figs. 6 and 7 in a frequency range with $M=16$ subchannels. It is evident from the Figs. 6 and 7 that triggering the three filters shown brings about considerable power densities, not only in their own subchannels but also in the other subchannels on account of the low side lobe attenuation, each channel having an effect on all of the subchannels in the case shown so that a total of fifteen side lobes superimpose in each subchannel. Each subchannel thereby corresponds to a frequency range of $2\pi/16$. When the number of subchannels is substantially higher, the effective adjacent interaction is only limited to the nearest subchannels respectively.

Considering the side lobes of the three transmission functions shown in a subchannel, e.g. $4 \cdot 2\pi/16 \leq \theta < 5 \cdot 2\pi/16$, it is seen that although their maximum has different values, all the overlapping side lobes have a similar curve.

This is evident from the illustration according to Fig. 8 in which all the side lobes of the prototype filter $h_0(n)$ for $M=16$ are scaled to the value 1 and are superimposed in the frequency range. All the side lobes have a similar curve with regard to their amplitude spectrum.

As can already be surveyed from the equation 1, the prototype filter and all the other shifted filters have a linear phase. In Fig. 7, the phase curves for the transmission functions of three different subchannels are reproduced, they all have the same gradient and may be transferred into each other by adding a constant phase.

In the frequency band shown it may be necessary to lower the power density of certain forbidden ranges in such a manner that they cannot interfere with already existing transmitting ranges e.g., amateur radio and rescue radio. A concrete example for such a reduction may consist in reliably reducing the power density in a range of 7 to 7.1 MHz from -60 dBm to -80 dBm (VDSL).

In the following it will first be assumed that the fade-out range is localized exactly between two subcarriers k and $k+1$ so that the corresponding frequency range lies between $k \cdot 2\pi/M \leq \theta < (k+1)2\pi/M$. The two assumed carriers k and $k+1$ transmit the major portion of their transmitter performance in the selected fade-out range and must therefore be zeroed in any case. Carriers that are located farther away, e.g. $k-1$, $k-2$, $k+2$, $k+3$ do not act on the frequency range to be faded out by their major lobe but by their side lobes. Accordingly, the overall interference of the adjacent carriers may be calculated by the complex summation of all the side lobes that are still relevant in regard to intensity.

The interference of an adjacent channel with the fade-out range is the datum of the adjacent channel multiplied by the action of the side lobe in the fade-out range.

Fig. 9 represents the interferences of the adjacent channels for a system with $M=8$ subcarriers. The selected fade-out range is $2 \cdot 2\pi/8 \leq \theta < 3 \cdot 2\pi/8$. Carriers 2 and 3 are zeroed, the occupancy of the remaining carriers is discretionary. According to prior art it was up to now common practice to also charge subcarriers located farther outside the immediate fade-out range with zero in order to thus achieve that the side lobes they occasioned be not capable of interfering with the fade-out range. As a result thereof, it was compelling to relinquish a relatively high number of subchannels outside the fade-out range. The realization of the method according to the invention overcomes this disadvantage in the way described herein after.

Since, as already observed herein above, all the side lobes have a similar amplitude curve, the overall interference in the fade-out range must have an amplitude curve that resembles that of the side lobes. This property depends upon the data of the adjacent channels which only determine the maximum and the phase of the overall interference.

It is therefore possible to devise a pulse that has, within the fade-out range, a spectrum that resembles as far as possible the spectrum of the overall interference and to transmit this pulse with the transmitting spectrum. Outside this range, its spectrum should be the smallest possible. The data of the adjacent channels only determine the excitation of the filter.

If the fade-out range is not limited by two neighboring subcarriers, all the subcarriers within the fade-out range must additionally be zeroed. Such a case is represented in Fig. 10a. The subcarriers labeled with „*“ have to be zeroed.

If the fade-out range does not exactly end at one subcarrier but between two subcarriers, the outermost must also be zeroed, as can be visualized in Fig. 10b. If need be, the next adjacent subchannels must be charged with zero too.

If the fade-out range comprises several subcarriers, it is not sufficient to transmit only one compensation pulse since the interfering maximum of the side lobes arises between two adjacent subcarriers respectively. Therefor, five compensation pulses must be generated in Fig. 10a and six in Fig. 10b.

As a result thereof and according to the invention, a pulse for compensating the side lobes occurring in the fade-out range is additionally transmitted for each frequency range extending between the subcarriers contained in the fade-out range and the thereto adjacent subcarriers with zero charge respectively, said pulse being provided with a frequency spectrum that resembles the side lobes occurring in the intermediate ranges and being modulated according to the data values of the side lobes occurring in the respective intermediate ranges, the compensation pulse(s) being transmitted orthogonal to the information transmitting subcarriers.

The amplitude and phase of the side lobe spectra for the fade-out range is calculated from the data values of a number of subchannels that can be predetermined and the compensation pulse pertaining to each frequency intermediate range is obtained by summation of the discrete complex side lobe spectra that are calculated for this purpose. Prior to transmission, the thus obtained compensation pulse(s) are superimposed on the transmitter signal in such a manner that the fade-out range is freed from interfering side lobes.

A particularly high number of usable subcarriers may be achieved in that, except for the subcarriers contained in the fade-out range, only the subcarriers that are located at the border of the fade-out range or outside the border in proximity thereto be charged with zero. By charge,

the modulation of a subcarrier is meant.

In Figs. 11 and 12 variants of transmitting parts of a transmission system according to the invention are indicated in the form of a basic block diagram by means of which the method according to the invention may be carried out.

In Fig. 11, the transmission unit comprises an Inverse Discrete Fourier Transform unit (IDFT) 1, by means of which a plurality of subchannels that subdivide the transmitting frequency range with allocated subcarriers may be modulated. The receiver unit not here presented contains a corresponding Discrete Fourier Transform unit (DFT) by means of which the transmitted data may be demodulated.

Through the IDFT-unit 1, all the subcarriers contained in the fade-out range and the subcarriers adjacent the fade-out range may be charged with zero, so that no major lobes from subcarriers can occur in the desired fade-out range.

The data to be transmitted are redirected as a vector $x(n)$ through the input unit 7 to the IDFT-unit 1 and to a processing unit 4. Said processing unit serves to compute side lobes occasioned by subchannels located outside the fade-out range. From these side lobes, the amplitude and phase of the overall interference in the fade-out range can be calculated by summation of the individual interferences. If the fade-out range comprises several subcarriers, a processing unit 4 is provided for each frequency range occurring either completely or in parts within the fade-out range between two subcarriers, said processing unit being connected at its output with the input of an allocated compensation filter 6 via a unit for overscanning 5, said filter having a transmission function that equals or resembles the spectrum of the side lobes of the corresponding frequency intermediate range. Fig. 11 shows a block diagram for only one frequency intermediate range.

The output of the compensation filter $s(k)$ 6 is connected to a first input of a subtraction member 3 and the output of the IDFT-unit 1 is connected to a second input of the subtraction member 3 so that an interference compensated transmitter signal may be detected at the output of the

subtraction member 3.

If the filter 6 is excited by a pulse having the amplitude and phase of the overall interference computed in the processing unit 4, a compensation signal is obtained in the fade-out range, the spectrum of which is very similar to the spectrum of the interference.

The output of the IDFT-unit 1 calculates the inverse discrete Fourier transform of the adjacent data vector $x(n)$ and a parallel-to-serial converting unit 2 converts the parallel data stream emerging the IDFT-unit 1 into a serial flow of symbols. The entire signal in this frequency range is merely composed of crosstalk portions because the subcarriers adjacent the fade-out range have been charged with zero in the data vector $x(n)$. The output signal of the filter 6 has inside the fade-out range a spectrum which resembles the spectrum of the crosstalk signal. By subtracting these two signals, the transmitter spectrum in the fade-out range is strongly reduced, e.g. by more than 20 dB.

If the fade-out range is not located exactly between two adjacent subcarriers, but rather extends over several subchannels or if the spectrum of power density is to be suppressed in several separate bands, the branch with the processing unit 4 and the filter 6 must be performed additionally for each frequency range between two subcarriers. The filters $s(k)$ 6 of each of the discrete branches must then copy the interference spectrum in each intermediate range. The variant illustrated in Fig. 12 uses the subcarriers contained in the fade-out range to carry out the compensation of the interfering side lobes. The transmitting functions of the filters $s_i(k)$ form a vector space. In order to be capable of recovering the data in the receiver through the application of a Discrete Fourier Transform, it is necessary to select the transmitting functions of the filters $s_i(k)$ to be orthogonal to the transmitting functions of the sounds used with the Inverse Fourier Transform unit. In this case, the set of the transmitting functions of the unused IFFT-channels can be used as a base for the vector space formed by the $s_i(k)$. When these functions are used as a base it is possible to have the filterings drawn into the Inverse Fourier Transform by means of the $s_i(k)$. In this case, the subchannels which overlap the fade-out ranges are not charged with zero but with values computed in the processing unit 4' so that IDFT and filtering yield the same results. The processing unit 4' calculates the new values with which the subchannels overlapping

the fade-out ranges must be charged. The data in the other subchannels are not altered in the process. For this purpose, the data to be transmitted may be fed at the input of the processing unit 4' and, at the output of the processing unit 4', the subcarriers contained in the fade-out range and the thereto adjacent subcarriers with a charge compensating the side lobes may be detected, whereas these subcarriers may be read by the IDFT-unit 1 together with the unaltered charges of the remaining subcarriers located outside the fade-out range. In unit 2, the parallel data are converted into a serial transmitter signal.

As a result thereof, the side lobes occurring in the frequency intermediate ranges is calculated for each frequency range extending between the subcarriers contained in the fade-out range and the thereto adjacent subcarriers which yields the required charge of the subcarriers contained in the fade-out range and the thereto adjacent subcarriers so that a compensation of the side lobes occurring in the fade-out range is obtained, the subcarriers contained in the fade-out range and the thereto adjacent subcarriers being transmitted with the computed charge and the remaining subcarriers remaining unaltered. To demonstrate the method according to the invention, three examples were calculated for a VDSL transmission route. 10.5 MHz was selected as a Nyquist frequency. The analogous transmission filter circuit has a passband of 0.3 MHz to 10.1 MHz. Three amateur radio bands are located within this range, namely 1.81 MHz - 2.00 MHz, 3.50 MHz - 3.80 MHz and 7.00 MHz - 7.10 MHz.

The examples according to the Figs. 21 - 24, Figs. 25 - 28 and Figs. 29 - 32, in which the achieved power density spectra are illustrated, show a possibility of suppressing the amateur radio bands with the method according to the invention, the power density spectrum must be lowered to below 0.3 MHz or to above 10 MHz respectively by means of the analogous transmission filter circuit and is not further taken into consideration. The number of the channels M amounts to 512 (Figs. 21-24), 1024 (Figs. 25-28) and 2048 (Figs. 29-32). The areas with a grey background represent the various amateur radio bands, Figs. 22-24, Figs. 26-28 and Figs. 30-32 being enlargements of the fade-out ranges.

The parameters for the various ranges are indicated in the following charts. The first column indicates in which subchannels a compensation pulse is transmitted. The second column states

which subcarriers are used for generating the compensation pulses and are not charged with information symbols. If the method according to the invention is not made use of, the channels indicated in the last column must be charged with zero.

	compensation pulses	needed subchannels	set to zero
1 st band	$k = 43, 44, \dots, 49$	$k = 42, 43, \dots, 51$	$k = 37, 38, \dots, 56$
2 nd band	$k = 85, 86, \dots, 92$	$k = 84, 85, \dots, 94$	$k = 79, 80, \dots, 99$
3 rd band	$k = 170, 171, 172, 173$	$k = 170, 171, \dots, 174$	$k = 163, 164, \dots, 180$

$M = 512$

	compensation pulses	needed subchannels	set to zero
1 st band	$k = 87, 88, \dots, 97$	$k = 86, 87, \dots, 99$	$k = 82, 83, \dots, 103$
2 nd band	$k = 170, 171, \dots, 185$	$k = 168, 169, \dots, 188$	$k = 165, 166, \dots, 191$
3 rd band	$k = 340, 341, \dots, 346$	$k = 339, 340, \dots, 347$	$k = 335, 336, \dots, 353$

$M = 1024$

	compensation pulses	needed subchannels	set to zero
1 st band	$k = 176, 177, \dots, 195$	$k = 175, 176, \dots, 197$	$k = 171, 172, \dots, 200$
2 nd band	$k = 341, 342, \dots, 349$ $363, 364, \dots, 371$	$k = 339, 340, \dots, 373$	$k = 336, 337, \dots, 376$
3 rd band	$k = 682, 683, \dots, 692$	$k = 681, 682, \dots, 694$	$k = 677, 678, \dots, 698$

$M = 2048$

In all of the three examples, the lower most and the central subcarriers are not charged. This zero setting is not needed for fading out the amateur radio bands but is intended to reduce the power

density spectrum of low and high frequencies.

For the symbols of the subcarriers $M/2+1$ to $M-1$, $A_l = A_{M-l}^*$, $l = M/2+1, M/2+2, \dots, M-1$. This charging regulation is necessary for a real transmitter signal.

In the third example (Figs. 29 - 32), the second amateur radio band is allocated the channels 341 to 370. Compensation pulses are only devised for the subbands 341, 342, ..., 349, 363, 364, ..., 371, though. In the central subchannels the interferences are already attenuated enough, compensation pulses do not have to be employed.

The calculation of the compensation pulse will be indicated herein after.

It may be shown that the energy of a pulse response of a filter in a frequency range of θ_1 to θ_2 may be represented as a square form. The pulse response $s(n)$ be M taps long so that the energy E_s is defined as

$$E_s = \frac{1}{2\pi} \int_{\theta_1}^{\theta_2} s^t \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) s \, d\theta = \frac{1}{2\pi} s^t \int_{\theta_1}^{\theta_2} \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) \, d\theta s = \frac{1}{2\pi} s^t \Theta(\theta_1, \theta_2) s \quad (7)$$

$s = [s(0)s(1) \dots s(M-1)]^T$ is the pulse response, $\psi(e^{j\theta})$ is

$$\psi(e^{j\theta}) = [1 \, e^{-j\theta} \, \dots \, e^{-j\theta(M-1)}]^T. \quad (6)$$

whereas

$$\Theta(\theta_1, \theta_2) = \int_{\theta_1}^{\theta_2} \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) \, d\theta.$$

s^t signifies s transposed and conjugated.

A compensation pulse has to be devised for every frequency intermediate range (or subband) that is limited by two adjacent subcarriers and is located within the fade-out range.

Within this frequency intermediate range, the compensation pulse must reproduce the spectrum of the interference in the best possible way. For this purpose, the compensation pulse has to have, within this intermediate range, a transmission function that has still to be determined.

Outside this frequency intermediate range there has to be made a distinction between the intermediate ranges that are also located within the fade-out range and those intermediate ranges that are outside the fade-out range. Since the compensation pulses themselves act as interferences in other frequency bands, they have to be provided with the least possible transmission function for the adjacent channels located within the fade-out range in order not to occasion additional interferences there. In frequency ranges located outside the fade-out range, the demand for a highly attenuated transmission function is not as strict but it still has to be taken into consideration. The reason is that, on account of the deterministic interrelationship between the excitation of the compensation pulse and the data of the subcarriers used, constructive interferences leading to an excess of power may arise outside the fade-out range.

The data are to be demodulated in the receiver by means of a Discrete Fourier Transform (DFT). The DFT as well as the IDFT can be transferred to a transmultiplexer whose filters are arranged orthogonal to one another. In order to obtain that all the data be independent from each other after demodulation through DFT, all the transmission filters must be orthogonal. The prototype filter and the filters derived therefrom through shifting already meet this requirement. In addition, the compensation pulse must be orthogonal to the filters of those subcarriers that transmit useful data. The discrete compensation pulses need not be orthogonal to each other.

As already mentioned herein above, the compensation pulse has to best approximate a

transmission function that will be more fully explained herein after within the frequency intermediate range for which it was devised. The spectrum within the intermediate range needs to be as similar as possible to the spectrum of the interference.

The interference is composed of the superimposition of several side lobes, the side lobes with the greatest amplitude maximum having the strongest interfering influence. On this account, the nominal transmission function of the compensation pulse is composed of the transmission functions of the two large side lobes. Outside its subband, the nominal transmission function equals zero.

If the compensation pulse for the subband $k \cdot 2\pi/M \leq \theta < (k+1) \cdot 2\pi/M$ is to be devised, the two neighboring pulses $H_{k-1}(e^{j\theta})$ and $H_{k+2}(e^{j\theta})$ are mainly responsible for the interferences in the frequency intermediate range considered, as is represented in Fig. 13 for $M=16$ and $k=2$ and in the amplitude of the transmission functions $H_1(e^{j\theta})$ and $H_4(e^{j\theta})$. The transmission functions for the two subcarriers $k-1$ and $k+2$ read

$$H_{k-1}(e^{j\theta}) = \frac{1}{\sqrt{M}} \frac{\sin \frac{M}{2} \left(\theta - \frac{2\pi}{M}(k-1) \right)}{\sin \frac{1}{2} \left(\theta - \frac{2\pi}{M}(k-1) \right)} e^{-j \left(\theta - \frac{2\pi}{M}(k-1) \right) \frac{M-1}{2}} \quad (10)$$

$$H_{k+2}(e^{j\theta}) = \frac{1}{\sqrt{M}} \frac{\sin \frac{M}{2} \left(\theta - \frac{2\pi}{M}(k+2) \right)}{\sin \frac{1}{2} \left(\theta - \frac{2\pi}{M}(k+2) \right)} e^{-j \left(\theta - \frac{2\pi}{M}(k+2) \right) \frac{M-1}{2}} . \quad (11)$$

In the frequency range of $0.25 \leq \theta/\pi < 0.375$ of concern, the side lobes of these two transmission functions were assumed as the main source of interference. The transmission functions located farther away participate accordingly less in the interference, their part in it being neglected in calculating the nominal transmission function.

The maximum of the right and left main side lobe of $H_{k-1}(e^{j\theta})$ and $H_{k+2}(e^{j\theta})$ occurs at $\theta = (2\pi/M)$

(k+0.5). Substitute at this place yields

$$H_{k-1}(e^{j\theta}) \Big|_{\theta=(k+\frac{1}{2})\frac{2\pi}{M}} = \frac{1}{\sqrt{M}} \frac{-1}{\sin \frac{3\pi}{2M}} e^{-j\frac{3\pi}{2}(1-\frac{1}{M})} \quad (12)$$

$$H_{k+2}(e^{j\theta}) \Big|_{\theta=(k+\frac{1}{2})\frac{2\pi}{M}} = \frac{1}{\sqrt{M}} \frac{-1}{\sin \frac{3\pi}{2M}} e^{j\frac{3\pi}{2}(1-\frac{1}{M})} . \quad (13)$$

As can be seen in Fig. 13, the two side lobes have the same amplitude maximum but a different phase. The phase difference of the two side lobes at $\theta = (2\pi/M)(k+0.5)$ is

$$\Delta\phi = \arg \{ H_{k+2}(e^{j\theta}) \} - \arg \{ H_{k-1}(e^{j\theta}) \} \Big|_{\theta=(k+\frac{1}{2})\frac{2\pi}{M}} = 3\pi - \frac{3\pi}{M} . \quad (14)$$

For this reason, the nominal transmission function

$$S(e^{j\theta}) = \begin{cases} -\frac{1}{2} \left(e^{j\frac{\Delta\phi}{2}} H_{k-1}(e^{j\theta}) + e^{-j\frac{\Delta\phi}{2}} H_{k+2}(e^{j\theta}) \right) & \text{for } k\frac{2\pi}{M} \leq \theta < (k+1)\frac{2\pi}{M} \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

is selected, as can be visualized in the Figs. 13 and 14. This choice of the nominal transmission function is more or less dictated by heuristic principles and can possibly be improved by an optimized criterion. Since the nominal transmission function only has a value other than zero within the fade-out range, it does not interfere with other frequency ranges.

Another restriction of the class of potential compensation pulses that has to be considered though is the criterion of orthogonality in order to keep the reception of the various filters free from any interference. In order for the data in the receiver to be capable of being demodulated and separated with the help of a DFT-Transform, it is necessary that the compensation pulse is orthogonal to the transmission functions of all the subchannels used.

The indices of all the charged subcarriers may be combined in the quantity M . The K functions

$$h_k(n) = \frac{1}{\sqrt{M}} e^{j \frac{2\pi}{M} kn} \quad \text{für } k \in \mathcal{K}, n = 0, 1, \dots, M-1 \quad (16)$$

form an orthonormal base for the K dimensional subspace K . The indices of all the subcarriers be combined in the quantity M , $M = 0, 1, \dots, M-1$. The functions

$$h_k(n) = \frac{1}{\sqrt{M}} e^{j \frac{2\pi}{M} kn} \quad \text{für } k, n = 0, 1, \dots, M-1 \quad (17)$$

form the M dimensional space M , K being a subspace of M . The transmission functions of the subcarriers used are located in the subspace K . The compensation pulse must be orthogonal to these functions, i.e., it must lie in a subspace $L = K^\perp$ which is normal to K . The space that

presents itself here is the difference $L = M/K$. For this $L = M - K$ dimensional subspace, the functions

$$h_k(n) = \frac{1}{\sqrt{M}} e^{j \frac{2\pi}{M} kn} \quad \text{für } k \in \mathcal{L}, n = 0, 1, \dots, M - 1 \quad (18)$$

constitute an orthonormal base. The quantity L is defined as $L = M/K$. The compensation pulse can now be represented by the linear combination of the base vectors (18)

$$g(n) = \sum_{l \in \mathcal{L}} c_l h_l(n) \quad \text{bzw.} \quad g = Hc \quad (19)$$

in the manner of writing vectors, whereas $g = [g(0)g(1)\dots g(M - 1)]^T$. In the column vector c , the coefficients c_l of the linear combination are combined. The columns of the matrix H are the base vectors (18).

$$H = [h_{l_0} h_{l_1} \dots h_{l_{L-1}}] \quad \text{mit} \quad \{l_0 l_1 \dots l_{L-1}\} = \mathcal{L} . \quad (20)$$

To calculate the compensation pulse, the following optimization calculation can be written down:

$$g(n) = \arg \min_{g(n) \in \mathcal{L}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} |G(e^{j\theta}) - S(e^{j\theta})|^2 d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l1}}^{\theta_{l2}} |G(e^{j\theta})|^2 d\theta \quad (21)$$

$G(e^{j\theta})$ is the Fourier Transform of $g(n)$ which is the compensation pulse looked for. Accordingly, the minimization occurs through all the functions of the space $\mathcal{L}$ which is normal to the transmission functions used. The first integral constitutes the deviation of $G(e^{j\theta})$ from the nominal transmission function $S(e^{j\theta})$. This deviation is calculated within the subband

As already mentioned, the nominal transmission function $S(e^{j\theta})$ equals zero outside this band. The second integral calculates the energy of $G(e^{j\theta})$ within the bands $\theta_{l1} \leq \theta < \theta_{l2}$. As already mentioned, the compensation pulse is to possess a highly attenuated transmission function outside its band. Summation occurs over ranges in which suppression is desired to be of different strength. Attenuation will need to be higher within the fade-out range than outside the same. This behavior can be adjusted by means of the weighting coefficients W_l .

Multiplying out (21) yields

$$g(n) = \arg \min_{g(n) \in \mathcal{L}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} \left(G(e^{j\theta}) - S(e^{j\theta}) \right)^* \left(G(e^{j\theta}) - S(e^{j\theta}) \right)^T d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l1}}^{\theta_{l2}} G(e^{j\theta})^* G(e^{j\theta})^T d\theta \quad (22)$$

$$= \arg \min_{g(n) \in \mathcal{L}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} \left(g^T \psi^*(e^{j\theta}) - s^T \psi^*(e^{j\theta}) \right) \left(\psi^T(e^{j\theta}) g - \psi^T(e^{j\theta}) s \right) d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l1}}^{\theta_{l2}} g^T \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) g d\theta \quad (23)$$

$$= \arg \min_{g(n) \in \mathcal{L}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} \left(g^T \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) g - s^T \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) g - g^T \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) s + s^T \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) s \right) d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l1}}^{\theta_{l2}} g^T \psi^*(e^{j\theta}) \psi^T(e^{j\theta}) g d\theta \quad (24)$$

$$\begin{aligned}
&= \arg \min_{g(n) \in \mathcal{L}} W_1 \left(g^t \Theta(\theta_{1,1}, \theta_{1,2}) g - s^t \Theta(\theta_{1,1}, \theta_{1,2}) g - g^t \Theta(\theta_{1,1}, \theta_{1,2}) s + s^t \Theta(\theta_{1,1}, \theta_{1,2}) s \right) + \\
&\quad \sum_{l=2}^Q W_l g^t \Theta(\theta_{l,1}, \theta_{l,2}) g
\end{aligned} \tag{25}$$

$$\begin{aligned}
&= \arg \min_{g(n) \in \mathcal{L}} \sum_{l=1}^Q W_l g^t \Theta(\theta_{l,1}, \theta_{l,2}) g - \\
&\quad W_1 \left(s^t \Theta(\theta_{1,1}, \theta_{1,2}) g + g^t \Theta(\theta_{1,1}, \theta_{1,2}) s - s^t \Theta(\theta_{1,1}, \theta_{1,2}) s \right)
\end{aligned} \tag{26}$$

In the penultimate line, equation (8) was substituted. To shorten writing, the two quantities $\theta_{1,1} = k \frac{2\pi}{M}$ and $\theta_{1,2} = (k+1) \frac{2\pi}{M}$ were introduced. The pulse response of the nominal transmission function is combined in the column vector s ,

$$[s]_n = -\frac{1}{2} \left(e^{j\frac{\Delta\epsilon}{2}} h_{k-1}(n) + e^{-j\frac{\Delta\epsilon}{2}} h_{k+2}(n) \right) \quad \text{for } n = 0, 1, \dots, M-1. \tag{27}$$

The direct minimization via $g(n)$, as it is executed in (26) does not make sense for two reasons: first, minimization has to be done over a large number of parameters (M coefficients) and second, optimization has to take place under the boundary condition $g(n) \in L$. It makes more sense to minimize, starting from the coefficient vector c (19): first, the number of the parameters to be optimized is reduced and second, minimization may occur without secondary conditions as the statement (19) already takes the secondary condition $g(n) \in L$ into consideration. Substitution of (19) in (26) yields the following:

$$c_{opt} = \arg \min_c \sum_{l=1}^Q W_l c^t H^t \Theta(\theta_{l_1}, \theta_{l_2}) H c - \quad (28)$$

$$W_1 \left(s^t \Theta(\theta_{l_1}, \theta_{l_2}) H c + c^t H^t \Theta(\theta_{l_1}, \theta_{l_2}) s - s^t \Theta(\theta_{l_1}, \theta_{l_2}) s \right) \quad (29)$$

The solution to this optimization problem reads

$$c_{opt} = H^{-1} \left(H^t \sum_{l=1}^Q W_l \Theta(\theta_{l_1}, \theta_{l_2}) \right)^{-1} W_1 H^t \Theta(\theta_{l_1}, \theta_{l_2}) s . \quad (30)$$

Substitution in (19) yields the pulse response of the compensation pulse looked for.

$$g = H c_{opt} = \left(H^t \sum_{l=1}^Q W_l \Theta(\theta_{l_1}, \theta_{l_2}) \right)^{-1} W_1 H^t \Theta(\theta_{l_1}, \theta_{l_2}) s \quad (31)$$

Simulations showed that this pulse only insufficiently meets the required properties. For this reason, the compensation pulse must be allowed to have a length superior to M . At this point it also makes sense to relinquish the restriction of the distortion-free channel.

It is assumed that the channel possesses a memory length of maximum P , the pulse response of the channel accordingly having a length of maximum $P + 1$ taps. For DMT systems, a cyclical

prefix is used which substantially simplifies distortion in the receiver.

With the cyclical prefix, the last P symbols of a data block are sent first at the beginning of a block, compare *FIG. 15*. Considering the transmission of one single pulse only, it may be written for the transmission sequence $y(n)$

$$y(n) = \begin{cases} a_{M-P+n} & \text{für } n = 0, 1, \dots, P-1 \\ a_{n-P} & \text{für } n = P, P+1, \dots, N+P-1 \end{cases} \quad (32)$$

Substitute the IDFT for a_n yields

$$y(n) = \begin{cases} \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j \frac{2\pi}{M} k(M-P+n)} = \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j \frac{2\pi}{M} k(n-P)} & \text{für } n = 0, 1, \dots, P-1 \\ \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j \frac{2\pi}{M} k(n-P)} & \text{für } n = P, P+1, \dots, N+P-1. \end{cases} \quad (33)$$

As can be seen, the case discrimination is abolished on account of the M periodicity of the IDFT, the transmission sequence reads

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} A_k e^{j \frac{2\pi}{M} k(n-P)} \quad \text{for } n = 0, 1, \dots, N+P-1. \quad (34)$$

This sequence too can again be generated by a transmultiplexer, whereas the following is valid:

$$h_k(n) = \frac{1}{\sqrt{M}} e^{j\frac{2\pi}{M}(n-P)}, \quad n = 0, 1, \dots, N + P - 1, \quad (35)$$

Fig. 16, 17 shows some transmission functions for a system with $M = 16$ and $P = 5$. It is evident that here, overlap of the side lobes occurs which is not very nice. For this reason, the compensation method cannot be used here.

For this reason, no cyclical prefix is employed in transmission but a Guard Interval of the length P .

The compensation pulse computed in the previous section complies with this condition. If however the compensation pulse is to be longer than M taps, M coefficients must always be followed by P zeros. This construction effects that the transmission sequence contains a Guard Interval of the length P after M values.

$$g = [g_0^T 0_P g_1^T 0_P \dots g_{R-1}^T]^T \quad (36)$$

The vectors g_k each contain M coefficients. The following P zeros are included in the line vector 0_P . In order for the demodulation and separation of the data in the receiver to be capable of being carried out through a DFT, all the vectors g_k must lie in the subspace L . The following optimization problem may be written down for the compensation pulse:

$$g(n) = \arg \min_{\substack{g_0(n) \in \mathcal{L} \\ g_1(n) \in \mathcal{L} \\ \vdots \\ g_{R-1}(n) \in \mathcal{L}}} W_1 \int_{k \frac{2\pi}{M}}^{(k+1) \frac{2\pi}{M}} |G(e^{j\theta}) - S(e^{j\theta})|^2 d\theta + \sum_{l=2}^Q W_l \int_{\theta_{l,1}}^{\theta_{l,2}} |G(e^{j\theta})|^2 d\theta \quad (37)$$

On account of the secondary factor $g_k \in L$

$$g_k = H c_k \quad (38)$$

may be stated again. The matrix H is defined in (20). Taking (36) and (38) into consideration, the frequency response of the compensation pulse becomes

$$G(e^{j\theta}) = \sum_{n=0}^{(R-2)(M+L)+M} g(n) e^{-j\theta n} = \sum_{k=0}^{R-1} g_k^T \psi_k(e^{j\theta}) = \sum_{k=0}^{R-1} c_k^T H^T \psi_k(e^{j\theta}) . \quad (39)$$

Fig. 18: diagrammatic illustration of $g(n)$ when the length is greater than M .

The newly introduced value $\psi_k(e^{j\theta})$ reads

$$\psi_k(e^{j\theta}) = [e^{-jk(M+L)\theta} e^{-j(k(M+L)+1)\theta} \dots e^{-j(k(M+L)+M-1)\theta}]^T . \quad (40)$$

If in (37) one substitutes and multiplies out, the following minimization problem occurs:

$$\begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{R-1} \end{pmatrix} = \arg \min_{\substack{c_0 \\ c_1 \\ \vdots \\ c_{R-1}}} \sum_{l=1}^Q W_l \sum_{k=0}^{R-1} \sum_{\kappa=0}^{R-1} c_k^t H^t \Theta_{k,\kappa}(\theta_{l_1}, \theta_{l_2}) H c_\kappa - \quad (41)$$

$$W_1 \left(\sum_{m=0}^{R-1} \left(s^t \Theta_{0,m}(\theta_{l_1}, \theta_{l_2}) H c_m + c_m^t H^t \Theta_{m,0}(\theta_{l_1}, \theta_{l_2}) s \right) - s^t \Theta_{0,0}(\theta_{l_1}, \theta_{l_2}) s \right) \quad (42)$$

The matrix $\Theta_{k,\kappa}(\theta_{l_1}, \theta_{l_2})$ is

$$\Theta_{k,\kappa}(\theta_{l_1}, \theta_{l_2}) = \int_{\theta_{l_1}}^{\theta_{l_2}} \psi_k^*(e^{j\theta}) \psi_\kappa^T(e^{j\theta}) d\theta . \quad (43)$$

The solution to the above indicated minimization problem reads

$$\begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{R-1} \end{pmatrix} = \begin{pmatrix} A_{0,0} & A_{0,1} & \dots & A_{0,R-1} \\ A_{1,0} & A_{1,1} & \dots & A_{1,R-1} \\ \vdots & \vdots & \ddots & \vdots \\ A_{R-1,0} & A_{R-1,1} & \dots & A_{R-1,R-1} \end{pmatrix}^{-1} \begin{pmatrix} B_0 \\ B_1 \\ \vdots \\ B_{R-1} \end{pmatrix} \quad (44)$$

with

$$A_{m,n} = H^t \sum_{l=1}^Q W_l \Theta_{m,n}(\theta_{l_1}, \theta_{l_2}) H \quad \text{und} \quad B_m = H^t \Theta_{m,0}(\theta_{l_1}, \theta_{l_2}) \quad (45)$$

The compensation pulse can be calculated with (36) and (38).

Fig. 19 illustrates the nominal transmission function as well as compensation pulses of various length for a system with $M = 16$ and $P = 2$. In this example, the fade-out range was selected to range from $0.25 \leq \theta/\pi < 0.625$. The compensation pulses are devised for the band $0.375 \leq \theta/\pi < 0.5$. The quality factors outside the fade-out range are selected to be very low, this being the reason for the excesses. The compensation pulse of the length 34 has considerably improved properties over the compensation pulse of the length 16.

The fade-out range is assumed to be the range $k \frac{2\pi}{M} \leq \theta < (k+1) \frac{2\pi}{M}$, the compensation filter calculated therefor, $S(e^{j\theta})$. If the information symbol A_l is transmitted in the channel l , said symbol acts on the channel (the prefactor $\frac{1}{\sqrt{M}}$ comes in the IDFT because of $\frac{1}{M}$) with the transmission function $N_l(e^{j\theta}) = \frac{1}{\sqrt{M}} H_l(e^{j\theta})$. The spectrum at the output of the compensation filter $S(e^{j\theta})$ is to coincide as far as possible with $N_l(e^{j\theta})$ within the fade-out range.

$$K_l S(e^{j\theta}) \approx A_l N_l(e^{j\theta}) \quad \text{for } k \frac{2\pi}{M} \leq \theta < (k+1) \frac{2\pi}{M} \quad (46)$$

The factor K_l is the excitation of the filter $G(e^{j\theta})$ with which $S(e^{j\theta})$ is approximated. Total coincidence is not possible within the overall fade-out range. This is the reason why the above equation is to be exactly satisfied with an equal sign for the frequency $(k + \frac{1}{2}) \frac{2\pi}{M}$.

$$K_l S(e^{j\theta}) \Big|_{\theta=(k+\frac{1}{2})\frac{2\pi}{M}} = A_l N_l(e^{j\theta}) \Big|_{\theta=(k+\frac{1}{2})\frac{2\pi}{M}} \quad (47)$$

Evaluated at $\theta = (k + \frac{1}{2}) \frac{2\pi}{M}$, the transmission functions yield

$$N_l(e^{j\theta}) \Big|_{\theta=(k+\frac{1}{2})\frac{2\pi}{M}} = \frac{(-1)^{k-l}}{M \sin\left(\frac{\pi}{M}(k-l+\frac{1}{2})\right)} e^{j\pi(k-l+\frac{1}{2})(-1+\frac{1}{M})} \quad \text{or} \quad (48)$$

$$S(e^{j\theta}) \Big|_{\theta=(k+\frac{1}{2})\frac{2\pi}{M}} = \frac{1}{\sqrt{M} \sin \frac{3\pi}{2M}}, \quad \text{resp.} \quad (49)$$

accordingly, the excitation of the filter $G(e^{j\theta})$ is

$$K_l = A_l \frac{N_l(e^{j\theta})}{S(e^{j\theta})} \Big|_{\theta=(k+\frac{1}{2})\frac{2\pi}{M}} = A_l \frac{\sin \frac{3\pi}{2M}}{\sqrt{M} \sin\left(\frac{\pi}{M}(k-l+\frac{1}{2})\right)} e^{j\pi(k-l+\frac{1}{2})(-1+\frac{1}{M})} (-1)^{k-l}. \quad (50)$$

Crosstalk that is occasioned by the information symbol A_l of the channel l in the fade-out range is compensated with this excitation. Each charged channel generates an interference in the fade-out range through the side lobes of its transmission function. The indices of all the charged subcarriers be combined in the quantity K . In order to compensate crosstalk of all the charged subcarriers in the range $k\frac{2\pi}{M} \leq \theta < (k+1)\frac{2\pi}{M}$, the filter $G(e^{j\theta})$ must be excited with

$$K = \sum_{l \in \mathcal{K}} K_l = \sum_{l \in \mathcal{K}} A_l \frac{\sin \frac{3\pi}{2M}}{\sqrt{M} \sin \left(\frac{\pi}{M} \left(k - l + \frac{1}{2} \right) \right)} e^{j\pi(k-l+\frac{1}{2})(-1+\frac{1}{M})(-1)^{k-l}} \quad (51)$$

It turns out that not all the charged channels have to be taken into consideration. It is generally sufficient merely to take into consideration the channels within a certain range about the fade-out range.

Generally speaking, the fade-out range will extend over several subbands. In this event, each subband $k \frac{2\pi}{M} \leq \theta < (k+1) \frac{2\pi}{M}$, $k \in \mathcal{U}$, must be provided with a compensation filter $G_k(e^{j\theta})$ of its own with appropriate excitation. The indices of all the subchannels in which a compensation pulse is to be transmitted are summed up in the quantity U .

If the compensation pulse $g(n)$ is directly implemented as a FIR filter, each coefficient of the filter must be multiplied by the excitation. With long filter lengths the calculation they imply becomes unacceptable. A more efficient implementation is possible, when it is taken into consideration that the compensation pulse can be represented as a linear combination of the base functions h_l , $l \in L$ (compare equation (19)).

$$Kg = K[g_0^T 0_P g_1^T 0_P \dots g_{R-1}^T]^T = K[H^T c_0^T 0_P H^T c_1^T 0_P \dots H^T c_{R-1}^T]^T \quad (52)$$

The matrix H is defined in the equation (20). The columns of the matrix H are the base functions

of the linear combination. K is the necessitated excitation of the compensation pulse as it has been calculated in the previous chapter. Substituting (20) yields

$$Kg = K \left[\begin{pmatrix} h_{l_0}^T \\ h_{l_1}^T \\ \vdots \\ h_{l_{L-1}}^T \end{pmatrix} \cdot c_0^T \quad 0_P \quad \begin{pmatrix} h_{l_0}^T \\ h_{l_1}^T \\ \vdots \\ h_{l_{L-1}}^T \end{pmatrix} \cdot c_1^T \quad 0_P \quad \dots \quad \begin{pmatrix} h_{l_0}^T \\ h_{l_1}^T \\ \vdots \\ h_{l_{L-1}}^T \end{pmatrix} \cdot c_1^T \right]^T \quad (53)$$

The above equation signifies that the base functions $h, l \in L$ must be excited with $K c_o^T$ at the time of transmission of the actual data block. At the time of transmission of the next data block, these base functions must be modulated with $K c_1^T$, and so on. This is only true when zero blocks only are sent after the first block. In normal transmission operation, the excitation vectors $v(n)$ are calculated by convolution at the instant of time n , compare also *Fig. 20* in this connection.

$$v(n) = \sum_{l=0}^{R-1} K(n-l)c_l \quad (54)$$

Overlapping of the discrete excitation sequences is occasioned by the length of the compensation pulse. If the length of the compensation pulse $g(n)$ exactly equals one symbol period (M taps), the excitation vector v_n becomes $v(n) = K(n)c_o^T$.

The base function h_l , $l \in L$ is nothing else than the transmission functions of the IDFT channel l , scaled with $\sqrt{M}$. At the instant of time n , the channels l , $l \in L$, of IDFT must be occupied with $\sqrt{M} v(n)$.

If the fade-out range extends over several subbands, the above factorization is necessary for each compensation filter. If one base function is contained in several compensation filters, the excitations for this base function have to be summarized.

Fig. 33 shows another embodiment of the method according to the invention.

In this method, at least part of the subcarriers contained in at least one fade-out range and of the subcarriers adjacent to said fade-out range respectively are used as compensation sounds, their charge being calculated in such a way that the integral of the weighted, transmitted power density spectrum is minimized over the entire frequency range.

The subcarriers that are not used for transmitting information thus form compensation sounds which allow the power density spectrum within the fade-out range to be reduced. The number of fade-out ranges within the frequency band intended to be used for transmission is submitted to no restriction whatever. Not all of the subcarriers contained in a fade-out range need to be actually used as compensation sounds, if it is not necessary. When fade-out ranges are very closely adjacent, it is also conceivable to use the subcarriers of one fade-out range as compensation sounds within the neighboring fade-out range.

The computation of the charge of the compensation sounds is based on the considerations set forth below.

In a conventional DMT-transmission system, the modulation of the subcarriers is carried out through an Inverse Discrete Fourier Transform (IDFT). On application of the sequence $A_k[n]$ on the sound u , the output of IDFT is given by

$$s_u[n] = \sum_{l=-\infty}^{\infty} A_u[l] h_u[n - lN]. \quad (101)$$

$N = M + P$ is the length of the time domain symbol plus the Guard Interval. $h_u[n]$ is the pulse response of the sound u and may be written as

$$h_u[n] = \begin{cases} e^{j\frac{2\pi}{M}un} & \text{for } n = 0, 1, \dots, M-1 \\ 0 & \text{else.} \end{cases} \quad (102)$$

In order to guarantee a real value time signal, $A_{u^*}[n]$ is applied to the sound $M-u$, M being the block length of the IDFT processing.

$$s_u[n] = \sum_{l=-\infty}^{\infty} \left(A_u[l] h_u[n - lN] + A_{u^*}[l] h_{M-u}[n - lN] \right) \quad (103)$$

$$= \sum_{l=-\infty}^{\infty} \left(A_u[l] h_u[n - lN] + A_u^*[l] h_u^*[n - lN] \right) \quad (104)$$

In order to reduce the power density spectrum (PSD) of $s_u[n]$ within the fade-out range, the sounds $i, i \in K_1$ are used to transmit the compensation signals. The set K_1 contains the index of those sounds that should be used for compensation.

$$\begin{aligned}
 s_u[n] = & \sum_{l=-\infty}^{\infty} \left(A_u[l] h_u[n-lN] \right. \\
 & + A_u[l] \left([c_u[0]]_1 h_{i_1}[n-lN] + \cdots + [c_u[0]]_I h_{i_I}[n-lN] \right) \\
 & \vdots \\
 & + A_u[l-R+1] \left([c_u[R-1]]_1 h_{i_1}[n-lN] + \cdots + [c_u[R-1]]_I h_{i_I}[n-lN] \right) \Big) \\
 & + \text{CC}
 \end{aligned} \tag{105}$$

The first row corresponds to a conventional DMT signal when $A_u[l]$ is applied to the sound u . I is the number of elements that are contained in the set K_1 . In the second line, the actual data $A_u[l]$ are not transmitted by the sound u , but weighted versions of $A_u[l]$ are transmitted by the sounds $i, i \in K_1$. The weighting of $A_u[l]$ is effected by the weighting vector $c_u[0]$. $[c_u[0]]_i$ is the i^{th} coordinate of the vector $c_u[0]$. The transmission of the weighted versions of $A_u[l]$ by the sounds $i, i \in K_1$ should minimize the effect of $A_u[l]$ within the fade-out range. The following lines correspond to the transmission of the weighted and delayed $A_u[l-n]$, $r = 1, 2, \dots, R-1$. This should minimize the effect of the past values $A_u[l-n]$ within the fade-out range. The number R determines the memory. The optimal selection of the weighting factors $c_u[r]$, $r = 0, 1, \dots, R-1$ will be explained herein after.

A more compact notation can be achieved by using vectors.

$$s_u[n] = \sum_{l=-\infty}^{\infty} \left(A_u[l] h_u[n - lN] + \sum_{r=0}^{R-1} A_u[l - r] c_u^T[r] h_I[n - lN] \right) + \text{CC} \quad (106)$$

The column vector $h_I[n]$ contains the pulse responses at the instant of time n of the sounds used for compensation and may be written as

$$h_I^T = [h_{i_1}[n] \ h_{i_2}[n] \ \dots \ h_{i_I}[n]] \quad \text{with} \quad \{i_1, i_2, \dots, i_I\} = \mathcal{K}_I. \quad (107)$$

In the case considered up to now, only one sound u is being transmitted. Now we will discuss the case in which not only one sound, but all the sounds u , $u \in \mathcal{K}_u$ are transmitted.

$$s[n] = \sum_{k \in \mathcal{K}_u} s_u[n] \quad (108)$$

$$= \sum_{k \in \mathcal{K}_u} \left(\sum_{l=-\infty}^{\infty} \left(A_u[l] h_u[n - lN] + \sum_{r=0}^{R-1} A_u[l - r] c_u^T[r] h_I[n - lN] \right) + \text{CC} \right)$$

$$= \sum_{l=-\infty}^{\infty} \left(A^T[l] h_u[n - lN] + \sum_{r=0}^{R-1} A^T[l - r] C[r] h_I[n - lN] \right) + \text{CC} \quad (109)$$

The set K_u consists of the index of the sounds used for the transmission of information. The column vector $h_u[n]$ is defined by

$$h_u^T = [h_{u_1}[n] \ h_{u_2}[n] \ \dots \ h_{u_U}[n]] \quad \text{with} \quad \{u_1, u_2, \dots, u_U\} = K_u. \quad (110)$$

$h_u[n]$ contains the pulse responses of the sounds used at the instant of time n . U is the number of the elements in the set K_u . The column vector $A_u[n]$ contains values that are applied to the sounds used.

$$A^T[n] = [A_{u_1}[n] \ A_{u_2}[n] \ \dots \ A_{u_U}[n]] \quad \{u_1, u_2, \dots, u_U\} = K_u. \quad (111)$$

The x^{th} line of the matrix C_m is set up by the line vector $c_{u_x}^T[r]$,

$$C[r] = \begin{pmatrix} c_{u_1}^T[r] \\ c_{u_2}^T[r] \\ \vdots \\ c_{u_U}^T[r] \end{pmatrix} \quad \text{for } r = 0, 1, \dots, R-1. \quad (112)$$

As already mentioned herein above, the signals to be obtained should be real value time domain signals. This includes that only the sounds $1, 2, \dots, M/2-1$ can carry any desired complex values, the sounds $M-1, M-2, \dots, M/2+1$ must therefore transmit the conjugate complex values. The sounds 0 and $M/2$ are charged with zero. The sets K_u and K_l only contain the sounds below $M/2$ so that $K_u \leq \{1, 2, \dots, M/2-1\}$ and $K_l \leq \{1, 2, \dots, M/2-1\}$ has to be true. Charging of the sounds above $M/2$ with conjugate complex values is considered by the term CC, CC standing for conjugate complex. One sound can either be used for transmitting information or for compensation, it cannot be used for both though. The intersection of the sets K_u and K_l must be empty.

The equation (109) allows to specify the block diagram of the embodiment of the method according to the invention which is shown in Fig. 33. The upper portion of the block diagram consists of an IDFT 10 and is a customary DMT transmitter. The already mentioned influence arises from the weak spectral limitation of the base functions of the IDFT. The data vector A_m is applied to the sounds $u, u \in K_u$. In order to ensure a real value time domain signal, the conjugate complex data are applied to the sounds $v, v \in K_l^c = \{v \mid v = M - u, u \in K_u\}$.

To ensure simple computation on the side of the receiver, a Guard Interval or a cyclical prefix has to be introduced.

First, the case of a Guard Interval will be considered, but later on, the results obtained will be generalized to a system with a cyclical prefix. If the Guard Interval used has a length P , P zeros are introduced between two consecutive time domain blocks.

Each block 11, 12, 13 in Fig. 33 designated with $C_m, m = 0, 1, 2, \dots, R-1$ effectuates a multiplication of its input vectors by the matrix C_m in using the method according to the

invention. The collected results of these multiplications are applied to the compensation sounds. The input vectors applied to the blocks C_n are the actual data vectors A_m and delayed versions thereof, A_{n-r} , $r = 1, 2, \dots, R-1$. At each time stage, the combined result is applied to the sounds i , $i \in K_1$. In order to ensure a real value time domain signal, the sounds i , $i \in K_1^c = \{i \mid i = M - i, i \in K_1\}$ must again be charged with the complex conjugate values.

Calculation of the weighting factors is carried out as follows. The matrices C_n , $r = 0, 1, \dots, R-1$ have to be calculated in order to minimize the integral of the weighted power density spectrum. In order to be able to do this, an analytic expression of the power density spectrum has to be indicated. This is achieved by calculating the autocorrelation function $R_s[n, m]$ of the transmitted sequence s_n and by applying the Fourier transformed thereto, this yielding the power density spectrum $S_s(e^{j\theta})$.

As s_n is a cyclostatic process, the time-dependent function $R_s[n, m]$ must first be calculated at the time n and the delay m

$$R_s[n, m] = E\{s^*[n]s[n + m]\} \quad (113)$$

Upon substituting the equation (108) in (113) and some algebraic conversions, the equation (113) becomes

$$\begin{aligned}
R_s[n, m] = & \sum_{l=-\infty}^{\infty} \left(h_u^t[n - lN] P h_u[n + m - lN] \right. \\
& + \sum_{r=0}^{R-1} h_I^t[n - lN] C^t[r] P h_u[n + m - (l - r)N] \\
& + \sum_{r=0}^{R-1} h_u^t[n - lN] P C[r] h_I[n + m - (l + r)N] \\
& \left. + \sum_{r=0}^{R-1} \sum_{\rho=0}^{R-1} h_I^t[n - lN] C^t[r] P C[\rho] h_I[n + m - (l - r + \rho)N] \right) + CC.
\end{aligned} \tag{114}$$

In order to obtain equation (114), statistically independent data with the same power in the real and the imaginary portion were assumed. P is a diagonal matrix whose elements match the power of the channels used

$$P = \text{diag}\{\sigma_{A_{u_1}}^2, \sigma_{A_{u_2}}^2, \dots, \sigma_{A_{u_U}}^2\} \quad \text{with} \quad \sigma_{A_{u_i}}^2 = E\{|A_{u_i}|^2\}, \quad \{u_1, u_2, \dots, u_U\} = \mathcal{K}_U. \tag{115}$$

(114) discloses the periodic nature of R_{sn}, m , each $n + pN$ yields the same result as n . To eliminate this periodicity, R_{sn}, m is taken the mean of over one period.

$$R_s[m] = \frac{1}{N} \sum_{n=0}^{N-1} R_s[n, m] \quad (116)$$

$$\begin{aligned}
&= \frac{1}{N} \sum_{n=0}^{N-1} \left(\sum_{l=-\infty}^{\infty} \left(h_u^t[n - lN] P h_u[n + m - lN] \right. \right. \\
&\quad + \sum_{r=0}^{R-1} h_I^t[n - lN] C^t[r] P h_u[n + m - (l - r)N] \\
&\quad + \sum_{r=0}^{R-1} h_u^t[n - lN] P C[r] h_I[n + m - (l + r)N] \\
&\quad \left. \left. + \sum_{r=0}^{R-1} \sum_{\rho=0}^{R-1} h_I^t[n - lN] C^t[r] P C[\rho] h_I[n + m - (l - r + \rho)N] \right) \right) + CC \quad (117)
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{N} \sum_{n=0}^{N-1} \left(h_u^t[n] P h_u[n + m] \right. \\
&\quad + \sum_{r=0}^{R-1} h_I^t[n] C^t[r] P h_u[n + m + rN] + \sum_{r=0}^{R-1} h_u^t[n] P C[r] h_I[n + m - rN] \\
&\quad \left. + \sum_{r=0}^{R-1} \sum_{\rho=0}^{R-1} h_I^t[n - lN] C^t[r] P C[\rho] h_I[n + m + (r - \rho)N] \right) + CC \quad (118)
\end{aligned}$$

Since $\mathbf{h}_u[n]$ and $\mathbf{h}_I[n]$ only have a support in the range $0 \leq n < N$, summation over l in equation (117) is reduced to the term for $l = 0$, whereas all the other terms are equal to zero.

The use of the discrete Fourier transformed on $R_s[m]$ yields the power density spectrum

$$S_s(e^{j\theta}) = \sum_{m=-\infty}^{\infty} R_s[m] e^{-j\theta m} \quad (119)$$

$$\begin{aligned}
&= \frac{1}{N} \sum_{m=-RN+1}^{RN-1} \left(\sum_{n=0}^{N-1} \left(h_u^t[n] P h_u[n+m] \right. \right. \\
&\quad + \sum_{r=0}^{R-1} h_I^t[n] C^t[r] P h_u[n+m+rN] + \sum_{r=0}^{R-1} h_u^t[n] P C[r] h_I[n+m-rN] \\
&\quad \left. \left. + \sum_{r=0}^{R-1} \sum_{\rho=0}^{R-1} h_I^t[n-lN] C^t[r] P C[\rho] h_I[n+m+(r-\rho)N] \right) + \text{CC} \right) e^{-j\theta m}
\end{aligned} \quad (120)$$

Since $R_s[m]$ only differs from zero in the range $-RN < m < RN$, summation in the equation (120) only has to be carried out in a finite interval.

As we now have an expression for the power density spectrum $S_s(e^{j\theta})$, a criterion may now be formulated which is to be optimized with regard to the coefficients $C[r]$, $r = 0, 1, \dots, R-1$. It is the object of the present invention to suppress the transmitted signal within the fade-out range. The criterion used for optimization according to the invention is the weighted integral of $S_s(e^{j\theta})$.

$$\psi_1(C[0], C[1], \dots, C[R-1]) = \int_0^{2\pi} W(e^{j\theta}) S_s(e^{j\theta}) d\theta \quad (121)$$

$W(e^{j\theta})$ is the weighting function. When $W(e^{j\theta})$ is set to 1 or outside zero within the fade-out range, the equation (121) is the transmitted power within the fade-out ranges. This choice

corresponds to the minimal power transmitted in the fade-out range. The minimal power transmitted is not exactly the value of interest, though. The exact goal consists in suppressing the maximum value of the power density spectrum $S_s(e^{j\theta})$ within the fade-out range below a certain value. Of course, a minimum/maximum criterion may be formulated therefor, but this criterion is more difficult to manipulate mathematically.

This is the reason why the function used is not a rectangular window function $W(e^{j\theta})$, but rather a relatively smooth weighting function. The transitions from 1 to zero and reverse are thereby carried out with linear gradients. This choice does not correspond to the minimum/maximum solution, neither does it to the minimization of the power transmitted within the fade-out range, but simulations proved that this choice yields better results than the minimization of the transmitting power within certain frequency bands.

The coefficients C_n , $n = 0, 1, \dots, R-1$ may now be written down as the solution to the optimization problem

$$\begin{pmatrix} C[0] \\ C[1] \\ \vdots \\ C[R-1] \end{pmatrix} = \arg \min_{\substack{C[0] \\ C[1] \\ \vdots \\ C[R-1]}} \psi_1(C[0], C[1], \dots, C[R-1]) . \quad (122)$$

Because (121) is a square function in the coefficients C_n , the equation (121) has one single minimum. This minimum is described by the equations

$$\frac{\partial}{\partial [C[r]]_{ui}} \psi_1(C[0], C[1], \dots, C[R-1]) = 0 \quad \text{for} \quad \begin{array}{l} r = 0, 1, \dots, R-1 \\ u = u_1, u_2, \dots, u_U \\ r = i_1, i_2, \dots, i_I \end{array} \quad (123)$$

that may be solved by linear systems of equations that are independent of U . Each value of u corresponds to a linear system of equations with RI unknowns. The system of equations that pertains to a fixed value of $u_x \in U$, describes the coefficients that are contained in the x^{th} line of all the R matrices C_{r} , $r = 0, 1, 2, \dots, R - 1$. If the x^{th} line of the matrix C_{r} is designated as $\mathbf{c}_{\text{ux}}^T$ (compare equation (112)), the vector of the unknowns can be written down as $\mathbf{c}_{\text{ux}}^T = [\mathbf{c}_{\text{ux}}^T(0) \mathbf{c}_{\text{ux}}^T(1) \dots \mathbf{c}_{\text{ux}}^T(R-1)]$. At first sight it seems strange that the system of equations described by equation (123) splits into U -independent systems of equation. The coefficients contained in the x^{th} line are the weighting factors that have to be applied to the I compensation sounds $i_1, i_2, \dots, i_I$ in order to minimize the effect of the sound u_x . The statistically independent data that were assumed in connection with the derivation of $S_s(e^{j\theta})$ imply that the data transmitted by one sound do not contain information of the data transmitted by the other sounds. Accordingly, minimization of the effect of the data transmitted by one single sound is only carried out by considering this single sound and none else so that the equations described by equation (123) split into as many systems as sounds are used for transmitting information.

The coefficient matrix of all the U systems of equations are the same and can be represented as a block matrix.

$$A = \begin{pmatrix} A_{00} & A_{01} & \dots & A_{0R-1} \\ A_{10} & A_{11} & \dots & A_{1R-1} \\ \vdots & \vdots & \ddots & \vdots \\ A_{R-10} & A_{R-11} & \dots & A_{R-1R-1} \end{pmatrix} \quad (124)$$

With the matrix A_{kl} , $k, l = 0, 1, \dots, R-1$ defined as

$$A_{kl} = \int_0^{2\pi} W(e^{j\theta}) H_I(e^{j\theta}) H_I^l(e^{j\theta}) e^{-j\theta(l-k)N} d\theta . \quad (125)$$

The newly introduced value $H_I(e^{j\theta})$ contains the Fourier transformed $H_i(e^{j\theta})$ of the compensation sounds $h_{i[n]}$, $i \in K_I$

$$H_I^T(e^{j\theta}) = [H_{i_1}(e^{j\theta}) \ H_{i_2}(e^{j\theta}) \ \dots \ H_{i_I}(e^{j\theta})] \quad \{i_1, i_2, \dots, i_I\} = K_I \quad (126)$$

The right side of the system of equations, which corresponds to $u = u_x$, may be written down as a block vector

$$d_{u_x}^T = [d_{u_x}^T[0] \ d_{u_x}^T[1] \ \dots \ d_{u_x}^T[R-1]] \quad \text{with}$$

$$d_{u_x}[r] = - \int_0^{2\pi} W(e^{j\theta}) H_{u_x}(e^{j\theta}) H_I^*(e^{j\theta}) e^{j\theta r N} d\theta \quad \text{for} \quad r = 0, 1, \dots, R-1 . \quad (127)$$

$H_{u_x}(e^{j\theta})$ is the Fourier transformed of the base function $h_{u_x[m]}$ of the sound u_x . With the equations (124) and (127), a system of equations is obtained for each $u_x \in U$

$$Ac_{u_x} = d_{u_x} , \quad (128)$$

that can be solved for c_{u_x} . All the possible values for u_x yield the complete set of coefficients C_m (compare equation (112)) required for the method according to the invention.

Another possible calculation is the deterministic calculation. Statistic is not used as in the previous method of calculation to calculate the optimum selection of the coefficients C_m , $r = 0, 1, 2, \dots, R - 1$. Since the actual data and all the previous data are known, it is possible to calculate the Fourier transformed of the transmitted sequence. The minimized criterion is the energy of the Fourier transformed contained in the fade-out range.

At least part of the subcarriers contained in at least one fade-out range and of the subcarriers adjacent to said fade-out range will be used as compensation sounds, their charge being thus calculated that the integral is minimized over the entire frequency range of the weighted, squared amplitude of the Fourier transformed of the data signal sent over a predetermined number of data blocks.

The system used for this embodiment of the method according to the invention is very similar to the system in the block diagram according to Fig. 33. The unique difference consists in the calculation unit for the compensation vector. As opposed to the previous statistical calculation, the actual and past data are not limited to a linear transformation.

The transmitting signal may be written as

$$s[n] = \sum_{l=-\infty}^{\infty} (A^T[l]h_u[n-lN] + c^T[l]h_x[n-lN]) + CC . \quad (129)$$

The vector c_l contains the amplitudes of the compensation sounds. In the ideal case, the Fourier transformed of the complete sequence is calculated to the present and the coefficients are tried to be found that correspond to the smallest share of energy within the fade-out range. This creates two problems. First, for calculating the Fourier transformed, the complete sequence of all the past data is needed, which requires an infinitely large memory. Second, when the sequence starts in the infinite past, the sequence cannot be absolutely summed up and accordingly, the Fourier transformed cannot exist.

In order to overcome these problems, the sequence considered is not the complete one but only those sequences that are occasioned by the actual and the last $R-1$ data vectors. As a result thereof, the memory required becomes $R-1$ and the Fourier transformed exists on account of the finite sequence. The time domain considered may be written as

$$s[n] = \sum_{l=m-R+1}^m (A^T[l]h_u[n-lN] + c^T[l]h_r[n-lN]) + CC. \quad (130)$$

The number m is the time index of the actual data vector and $m = n/N$. A_l and c_l with $l = m$ corresponds to the past data vectors and to the compensation vectors. A_m is the actual data vector, c_m is the actual compensation vector that has to be calculated.

The Fourier transformed of the equation (129) equals

$$S(e^{j\theta}) = \sum_{l=m-R+1}^m \left(A^T[l]H_u(e^{j\theta}) + c^T[l]H_I(e^{j\theta}) + A^l[l]H_u^*(e^{-j\theta}) + c^l[l]H_I^*(e^{-j\theta}) \right) e^{-j\theta l N}. \quad (131)$$

The newly introduced value $H_u(e^{j\theta})$ contains the Fourier transformed $H_u(e^{j\theta})$ of the information transmitting sounds $h_u[m]$, $u \in K_u$.

$$H_u^T(e^{j\theta}) = [H_{u_1}(e^{j\theta}) \ H_{u_2}(e^{j\theta}) \ \dots \ H_{u_U}(e^{j\theta})] \quad \{u_1, u_2, \dots, u_U\} = K_u. \quad (132)$$

The function that is to be optimized by the optimal choice of the $c[m]$ now is the integral of the squared value $|S(e^{j\theta})|^2$ weighted by $W(e^{j\theta})$.

$$\psi_2(c[m]) = \int_0^{2\pi} W(e^{j\theta}) |S(e^{j\theta})|^2 d\theta \quad (133)$$

When the weighting function $W(e^{j\theta})$ is selected as a rectangular window function, which corresponds to 1 within the fade-out range and to zero everywhere outside this range, the integral in the equation (133) corresponds to the energy inside the fade-out range.

As already explained in connection with the statistical calculation, this choice needs not be

optimal. Again, equation (133) is a square function of the coefficient vector $\mathbf{c}[m]$ with one single minimum with respect to $\mathbf{c}[m]$. The minimum is achieved by setting the derivation of equation (133) zero, and by resolving for $\mathbf{c}[m]$. As a result thereof, the compensation vector $\mathbf{c}[m]$ can be denoted by

$$\begin{aligned} \mathbf{c}[m] = & \sum_{l=m-R+1}^m \left(\mathbf{F}[m-l]\mathbf{A}[l] + \mathbf{G}[m-l]\mathbf{A}^*[l] \right) \\ & + \sum_{l=m-R+1}^{m-1} \left(\mathbf{Q}[m-l]\mathbf{c}[l] + \mathbf{R}[m-l]\mathbf{c}^*[l] \right) . \end{aligned} \quad (134)$$

The matrices $\mathbf{F}(1)$, $\mathbf{G}(1)$, $\mathbf{Q}(1)$, $\mathbf{R}(1)$, $\mathbf{A}(1)$, $\mathbf{B}(1)$, $\mathbf{C}(1)$, $\mathbf{D}(1)$, , $l = 0, 1, \dots, R-1$ are defined as

(135)

$$\begin{aligned} \mathbf{F}[l] &= (\mathbf{B}^{-1}[0]\mathbf{D}[0] - \mathbf{D}^{\star^{-1}}[0]\mathbf{B}^{\star}[0])^{-1} (-\mathbf{B}^{-1}[0]\mathbf{C}[l] + \mathbf{D}^{\star^{-1}}[0]\mathbf{A}^*[l]) \\ \mathbf{G}[l] &= (\mathbf{B}^{-1}[0]\mathbf{D}[0] - \mathbf{D}^{\star^{-1}}[0]\mathbf{B}^{\star}[0])^{-1} (-\mathbf{B}^{-1}[0]\mathbf{A}[l] + \mathbf{D}^{\star^{-1}}[0]\mathbf{C}^*[l]) , \end{aligned}$$

(136)

(137)

$$\begin{aligned} \mathbf{Q}[l] &= (\mathbf{B}^{-1}[0]\mathbf{D}[0] - \mathbf{D}^{\star^{-1}}[0]\mathbf{B}^{\star}[0])^{-1} (-\mathbf{B}^{-1}[0]\mathbf{D}[l] + \mathbf{D}^{\star^{-1}}[0]\mathbf{B}^{\star}[l]) \\ \mathbf{R}[l] &= (\mathbf{B}^{-1}[0]\mathbf{D}[0] - \mathbf{D}^{\star^{-1}}[0]\mathbf{B}^{\star}[0])^{-1} (-\mathbf{B}^{-1}[0]\mathbf{B}[l] + \mathbf{D}^{\star^{-1}}[0]\mathbf{D}^{\star}[l]) . \end{aligned}$$

(138)

(139)

$$A[l] = \int_0^{2\pi} W(e^{j\theta}) H_I^*(e^{-j\theta}) H_U^l(e^{j\theta}) e^{-j\theta l N} d\theta$$

$$B[l] = \int_0^{2\pi} W(e^{j\theta}) H_I^*(e^{-j\theta}) H_I^l(e^{j\theta}) e^{-j\theta l N} d\theta$$

(140)

(141)

$$C[l] = \int_0^{2\pi} W(e^{j\theta}) H_I^*(e^{-j\theta}) H_U^T(e^{-j\theta}) e^{-j\theta l N} d\theta$$

$$D[l] = \int_0^{2\pi} W(e^{j\theta}) H_I^*(e^{-j\theta}) H_I^T(e^{-j\theta}) e^{-j\theta l N} d\theta .$$

(142)

The matrices $F(1)$, $G(1)$, $Q(1)$, $R(1)$, , $1 = 0, 1, \dots, R-1$ do not alter with time. They can be calculated once and then memorized. The only calculations that need to be carried out for each time stage is the evaluation of equation (134). As compared to the statistical calculation, the deterministic calculation requires more calculation because the matrix multiplications $2R(I \times U)$ and $2R - 2(I \times I)$ have to be carried out as compared to the matrix multiplications $R(I \times U)$ in the statistical calculation. The additional complexity is due to the fact that c_m also depends on the complex conjugate values of the data.

As will be still explained herein after, the statistical and the deterministic calculation almost yield the same results so that the statistical calculation is preferable on account of the fact that less

calculation is required.

If the data blocks are transmitted with a cyclical prefix, the last P values of each time domain block are set in front of said block. The cyclical prefix may be described by using slightly modified base functions for the information transmitting sounds and for the compensation sounds.

$$h_u[n] = e \begin{cases} j^{\frac{2\pi}{M}} u(n-P) \\ 0 \end{cases} \quad u \in \mathcal{K}_U, n=0,1,\dots,N+P \quad (143)$$

$$h_i[n] = \begin{cases} e^{j^{\frac{2\pi}{M}} i(n-P)} \\ 0 \end{cases} \quad i \in \mathcal{K}_I, n=0,1,\dots,N+P \quad (144)$$

The vectors $\mathbf{h}_u[n]$ and $\mathbf{h}_i[n]$ are still composed in the same way as in equation (110) and (107) but with the new prefixed values of $h_u[n]$ and $h_i[n]$.

In the complete derivations of the statistical and the deterministic methods of calculation, the particular form of the base functions (110) and (107) are not taken into consideration in the calculations. When the base functions are replaced by equation (144) and (143), the other equations still remain valid.

In conclusion, a simulation of the statistical method is now indicated when said method is applied to a system with a cyclical prefix.

For this purpose, a VDSL transmission system is taken. A major problem with VDSL transmissions is the existence of very narrow bands, so-called HAM bands, that are occupied by amateur radio. The HAM bands are located in the ranges of 1.81 - 2.00 MHz, 3.50 - 3.80 MHz and 7.00 - 7.10 MHz. Within these bands, the transmitted power density spectrum has to be reduced by 20 dB. In order to reduce the power density spectrum within these bands, it is not sufficient not to transmit any information in the subchannels overlapping the HAM bands. A series of neighboring channels must be charged with zero in order to achieve the desired reduction of the power density spectrum. The following examples show the applicability of the method according to the invention of reducing the number of the subchannels charged with zero.

All the systems use a channel number of $M = 512$. The length of the Guard Intervals or of the cyclical prefix is 32.

The first simulation relates to a system that does not use any specific method of suppressing the power density spectrum within the HAM bands. The required suppression is achieved through some sounds charged with zero. A cyclical prefix is used. The power density spectrum and the required zero charged sounds of the system A are indicated in Figs. 35 and 36 and in the chart according to Fig. 34.

In the second simulation, the required suppression of the power density spectrum within the HAM band is achieved by the statistical method according to the invention. System B uses a Guard Interval. The power density spectrum and the zero charged sounds are indicated in the Figs. 38 and 39 and in the chart according to Fig. 37.

System C corresponds to a system that uses the deterministic method according to the invention. Figs. 41, 42 and the chart according to Fig. 40 show the power density spectrum and the zero charged sounds.

The system D employs the statistical method according to the invention for the case in which a cyclical prefix is used. The results of the simulation are summed up in the chart according to Fig. 43, the power density spectrum is shown in the Figs. 44, 45.

On comparing the charts according to the Figs. 34, 37, 40 and 43 it can be visualized that the systems that use the method according to the invention require the same number of zero charged sounds in order to achieve the desired suppression of the power density spectrum within the HAM bands. System A, which does not use a method according to the invention, needs 35 more channels to achieve the same goal than the systems B and C. A comparison of the power density spectra of the systems B and C shows that they are almost completely equivalent. As already mentioned above, the system B is less complex than system C so that the statistical system will be preferred. The behavior of system D, which uses a cyclical prefix instead of a Guard Interval, is just as good as the one of system B.

The method according to the invention may also be used for a system with a cyclical prefix. On comparing the suppression of the power density spectrum of the systems A and D, which are both used for the cyclical prefix, it may be seen that system A very poorly meets the requirements, whereas system D yields very good results.

International Patent Application PCT/AT99/00310

Applicant: Telefonieaktiebolaget LM Ericsson et al.

NEW CLAIMS

1. Method of suppressing narrow frequency bands in fade-out ranges during transmission of data by means of a multiple carrier method, e.g. DMT (Discrete Multitone) in which a predetermined broad frequency band is divided into a plurality of subchannels having subcarriers assigned thereto and in which the data to be transmitted are modulated, divided into data blocks, in the transmitter with Inverse Discrete Fourier Transform (IDFT) and are demodulated in the receiver with Discrete Fourier Transform (DFT), each subchannel being thus provided in the spectrum with a major lobe and several side lobes occurring in the range of nearby subcarriers, wherein at least part of the subcarriers contained in at least one fade-out range and of the subcarriers adjacent to this fade-out range are used as compensation sounds having a frequency spectrum which resembles the side lobes of subcarriers located outside the fade-out range that occur within the fade-out range and are modulated according to the data values of these side lobes in order to suppress said side lobes, **wherein** the modulation of the compensation sounds is calculated in such a way that the integral of the weighted, sent power density spectrum is minimized over the entire frequency range.

2. Method of suppressing narrow frequency bands in fade-out ranges during transmission of data by means of a multiple carrier method, e.g. DMT (Discrete Multitone) in which a predetermined broad frequency band is divided into a plurality of subchannels having subcarriers assigned thereto and in which the data to be transmitted are modulated, divided into data blocks, in the transmitter with Inverse Discrete Fourier Transform (IDFT) and are demodulated in the receiver with Discrete Fourier Transform (DFT), each subchannel being thus provided in the spectrum with a major lobe and several side lobes occurring in the range of nearby subcarriers, wherein at least part of the subcarriers contained in at least one fade-out range and of the subcarriers adjacent to this fade-out range are used as compensation sounds having a frequency spectrum which resembles the side lobes of subcarriers located outside the fade-out range that occur within the fade-out range and are modulated according to the data values of these side lobes in order to

suppress said side lobes, **wherein** the modulation of the compensation sounds is calculated in such a way that the integral is minimized over the entire frequency range of the weighted, squared amplitude of the Fourier transformed of the sent data signal by way of a number of data blocks that may be predetermined.

3. Method according to claim 1 or 2, **wherein** computation takes already sent data into consideration.

4. Method according to one of the claims 1, 2 or 3, **wherein** either a Guard Interval or a cyclical prefix is transmitted between the data combined to blocks.

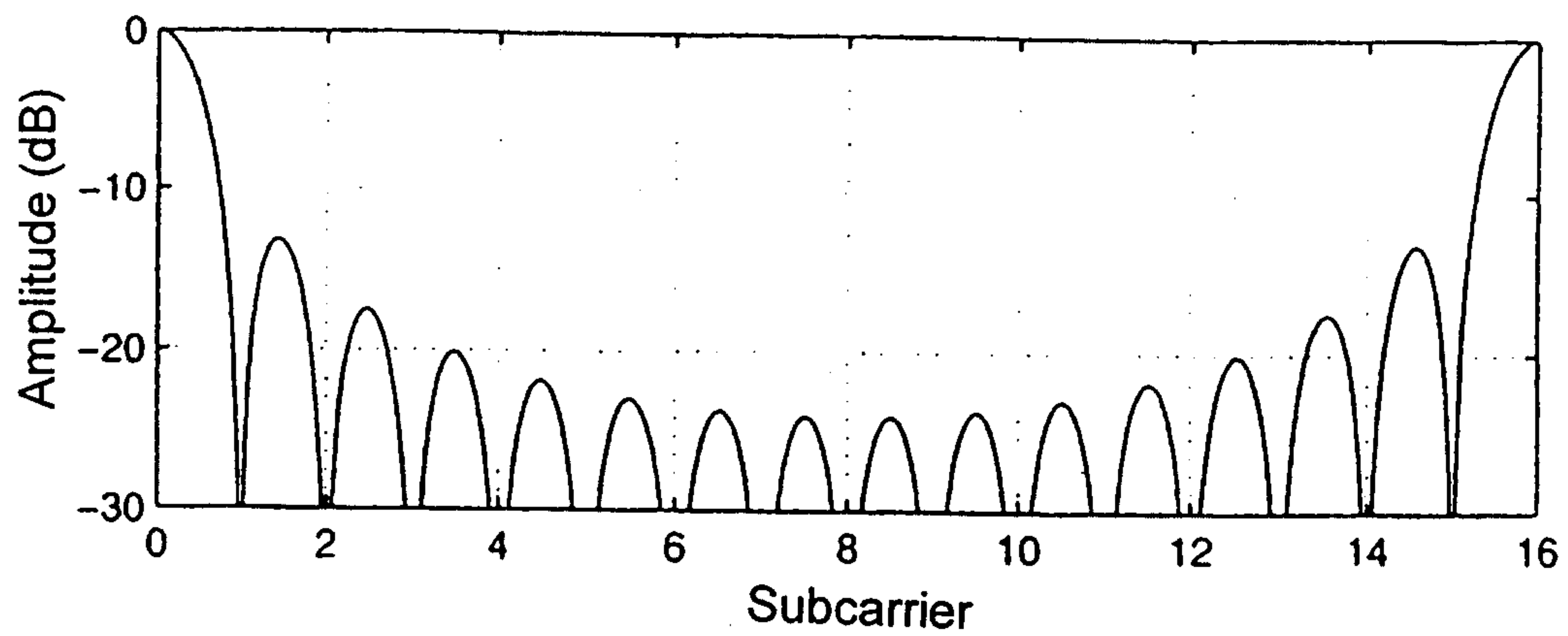


FIG. 1

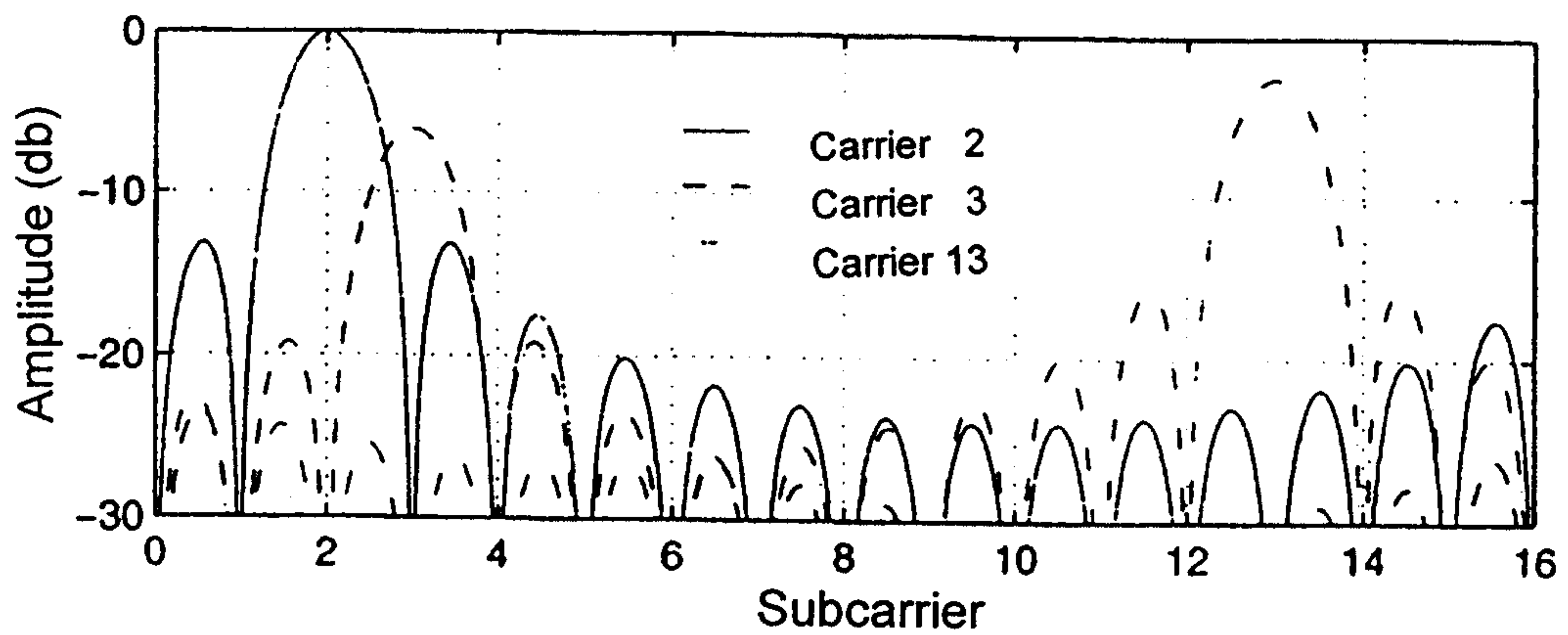


FIG. 2

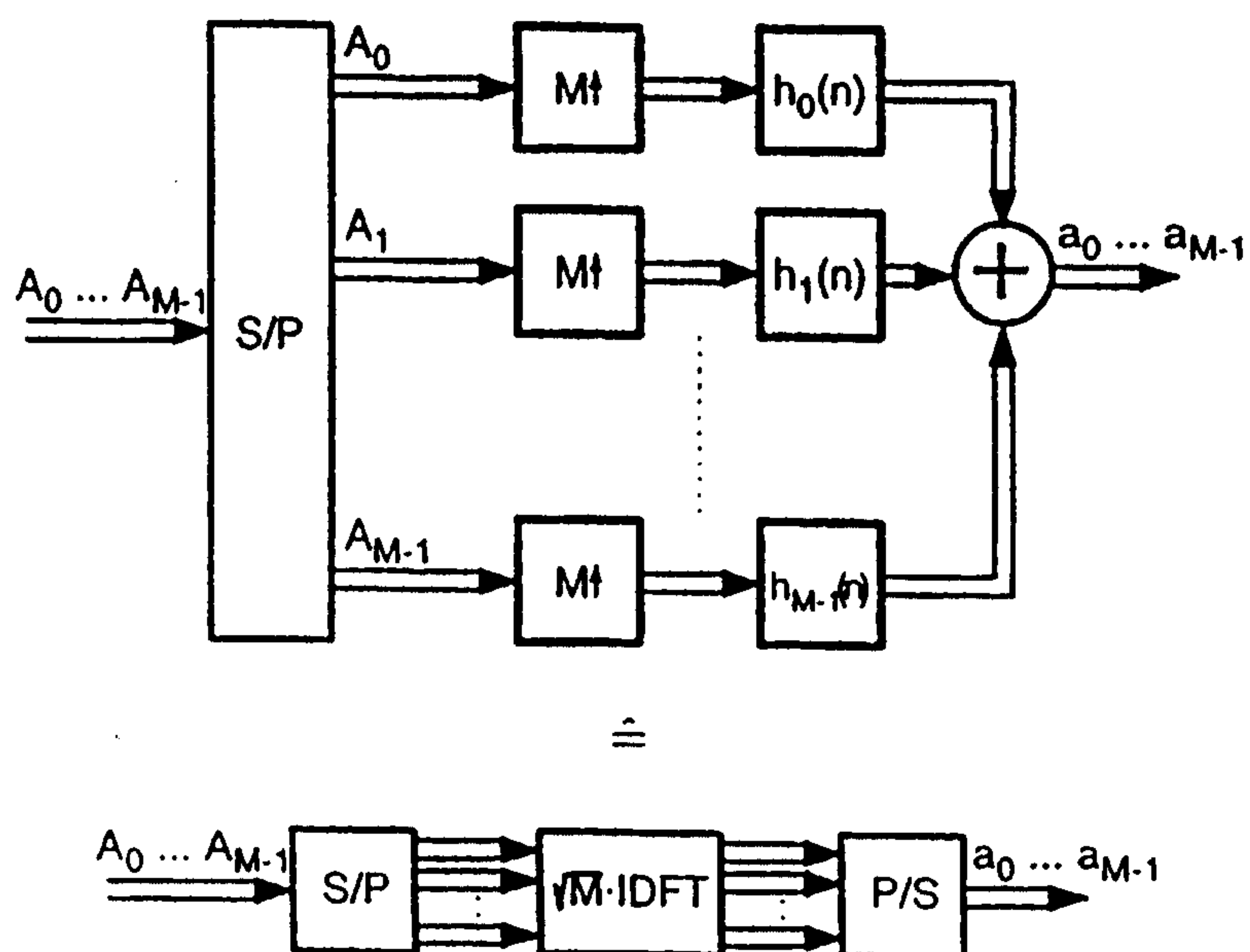


FIG. 3

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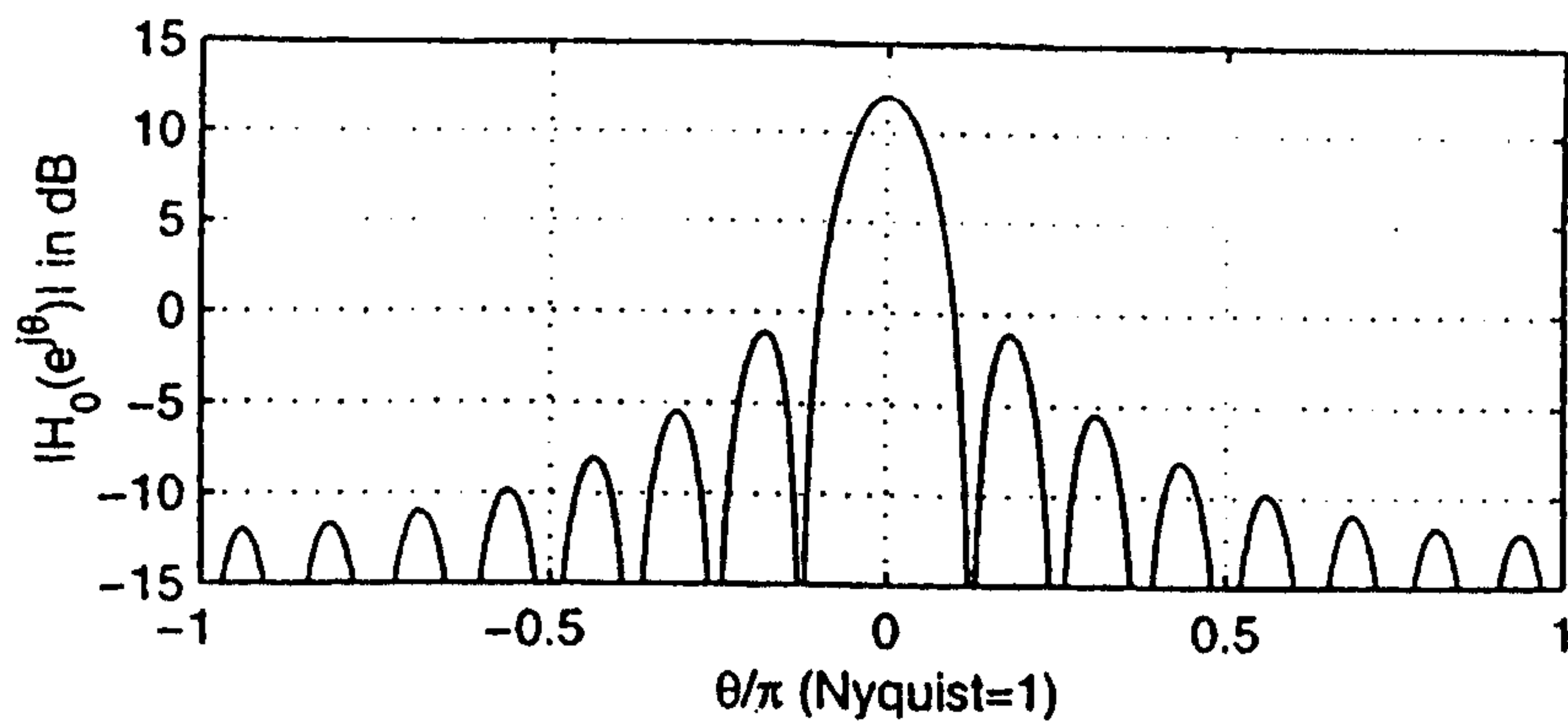


FIG. 4

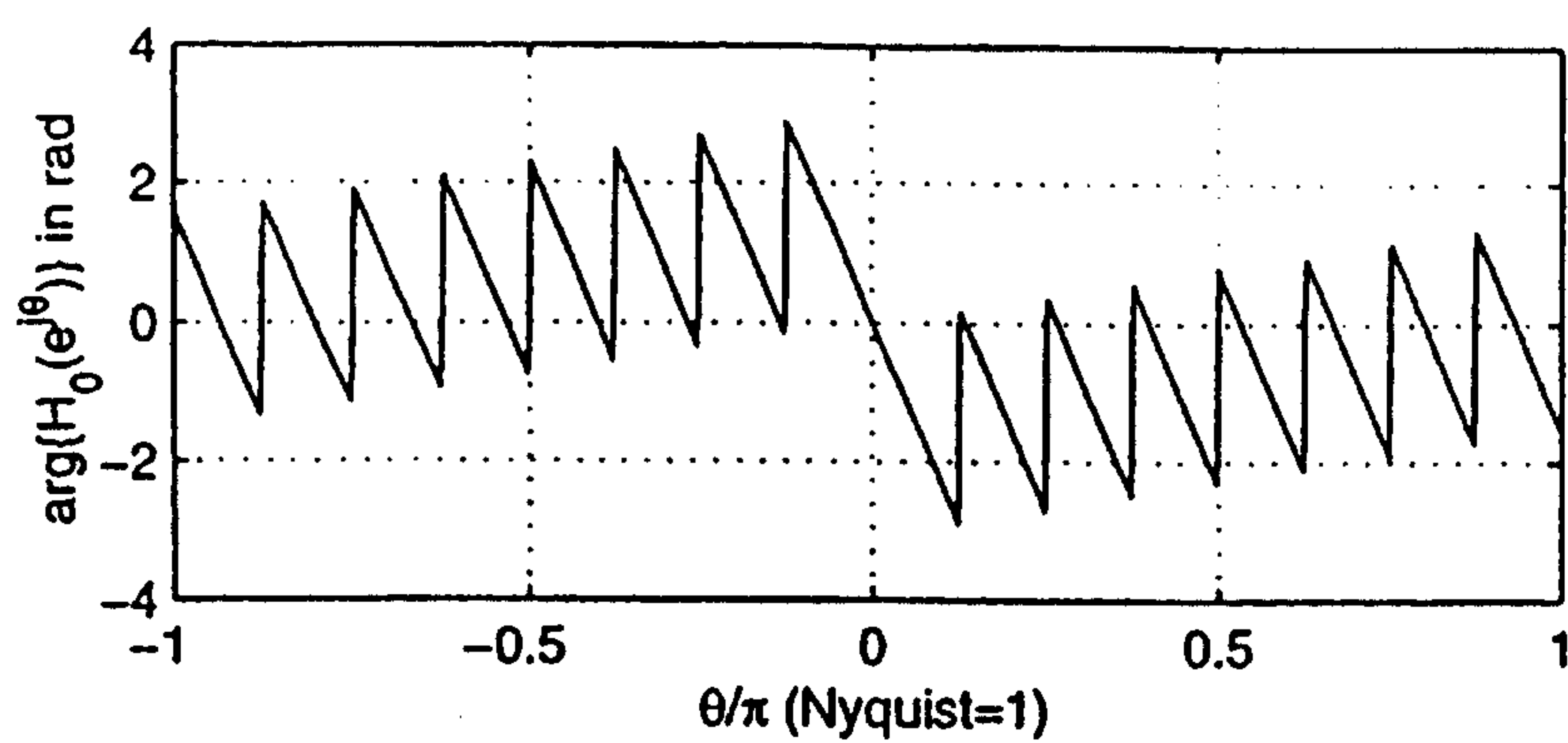


FIG. 5

prototype filter $H_0(e^{j\theta})$ for $M = 16$.

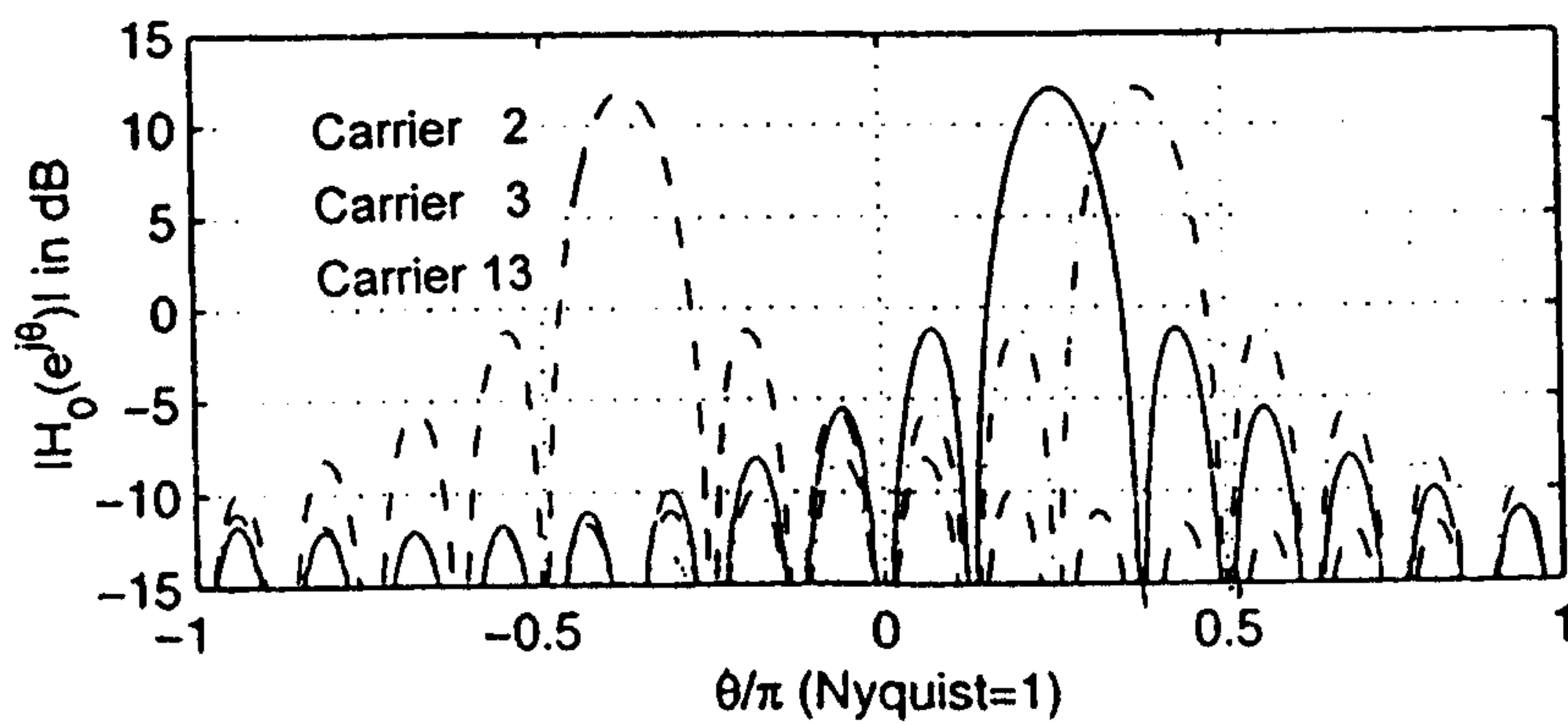


FIG. 6

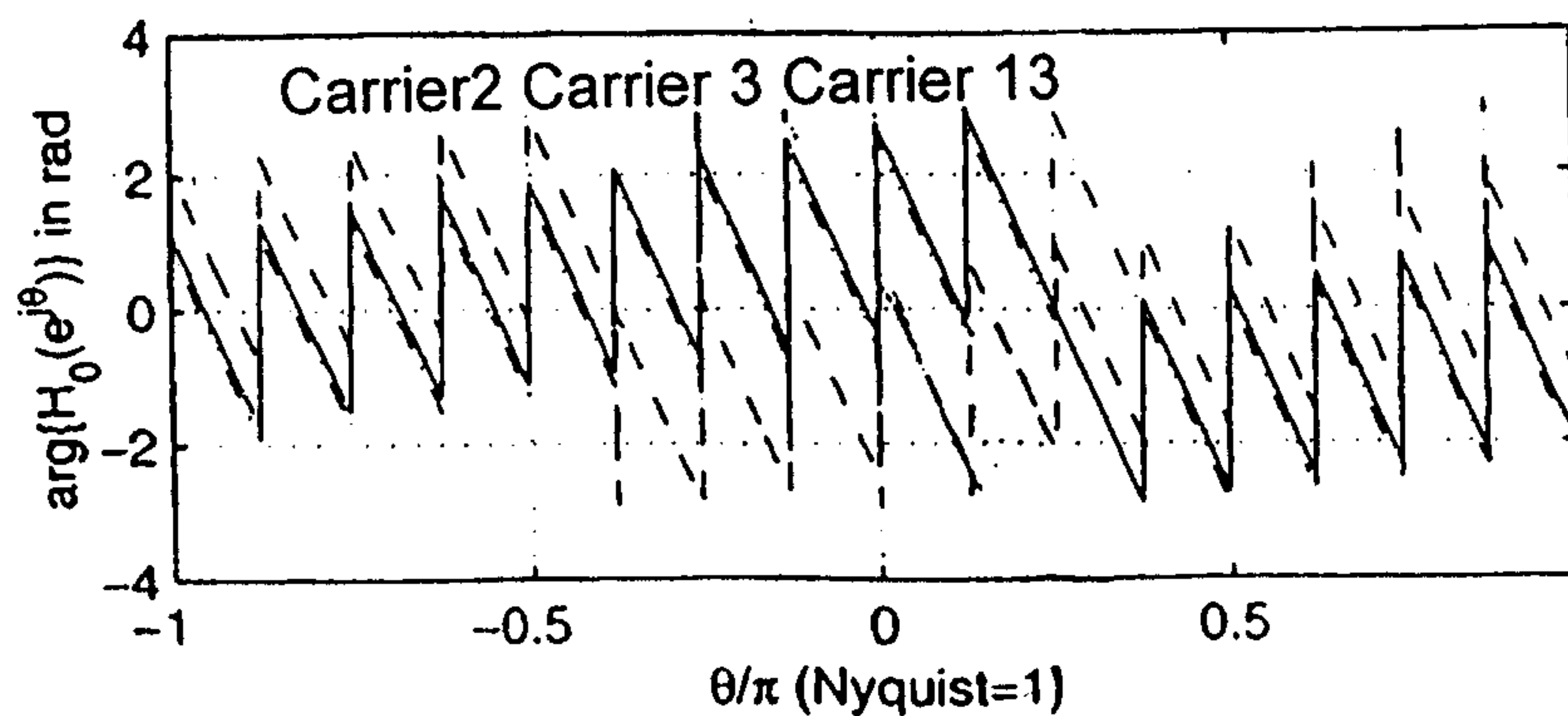


FIG. 7

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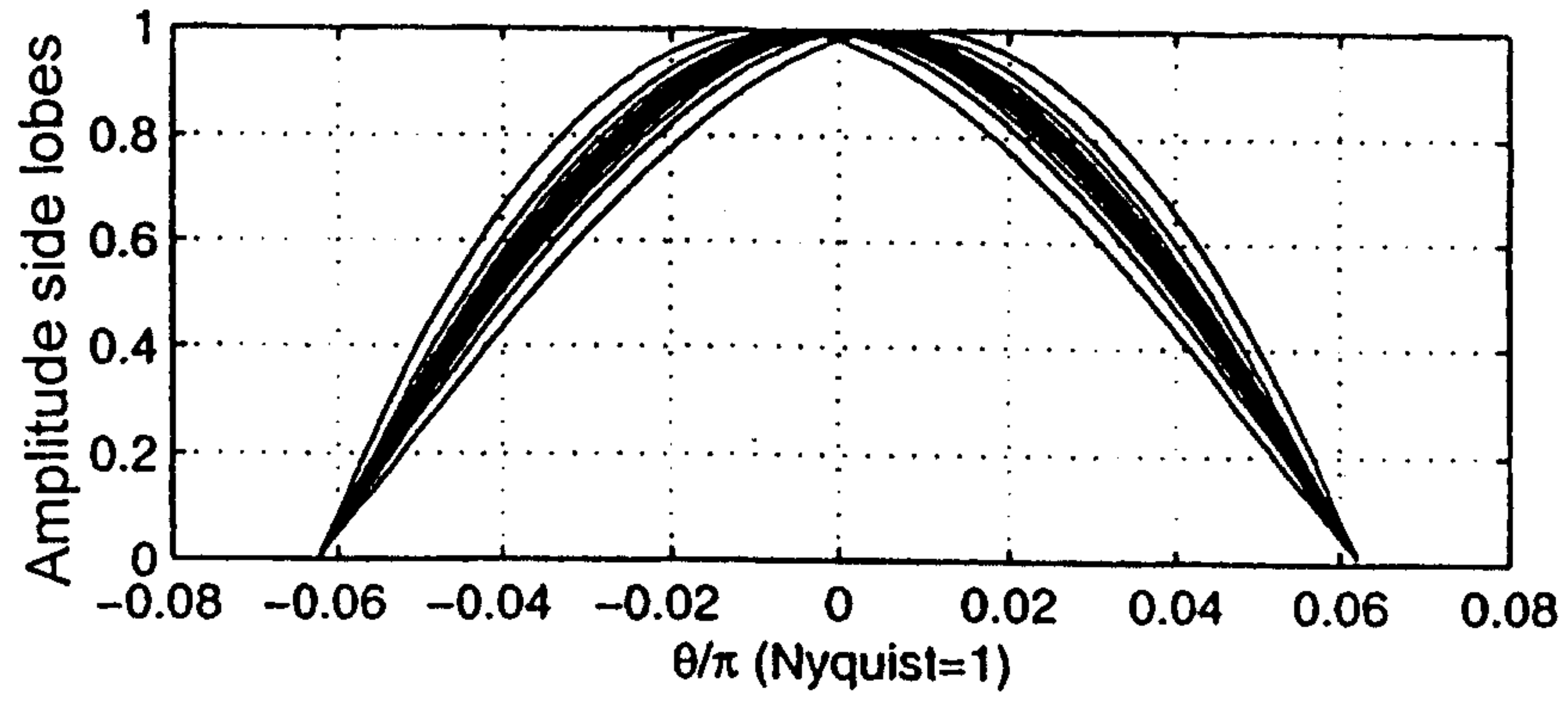


FIG. 8

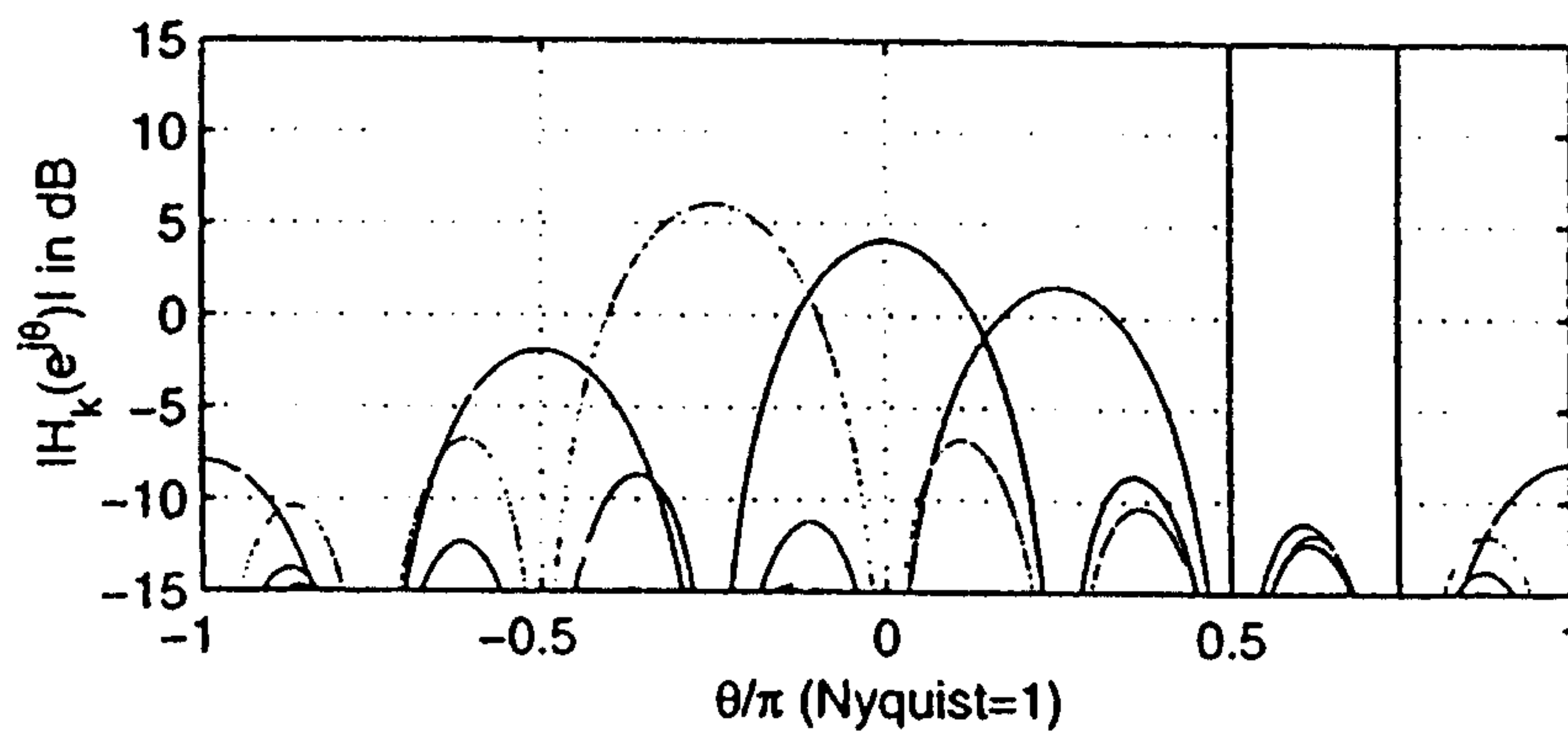


FIG. 9

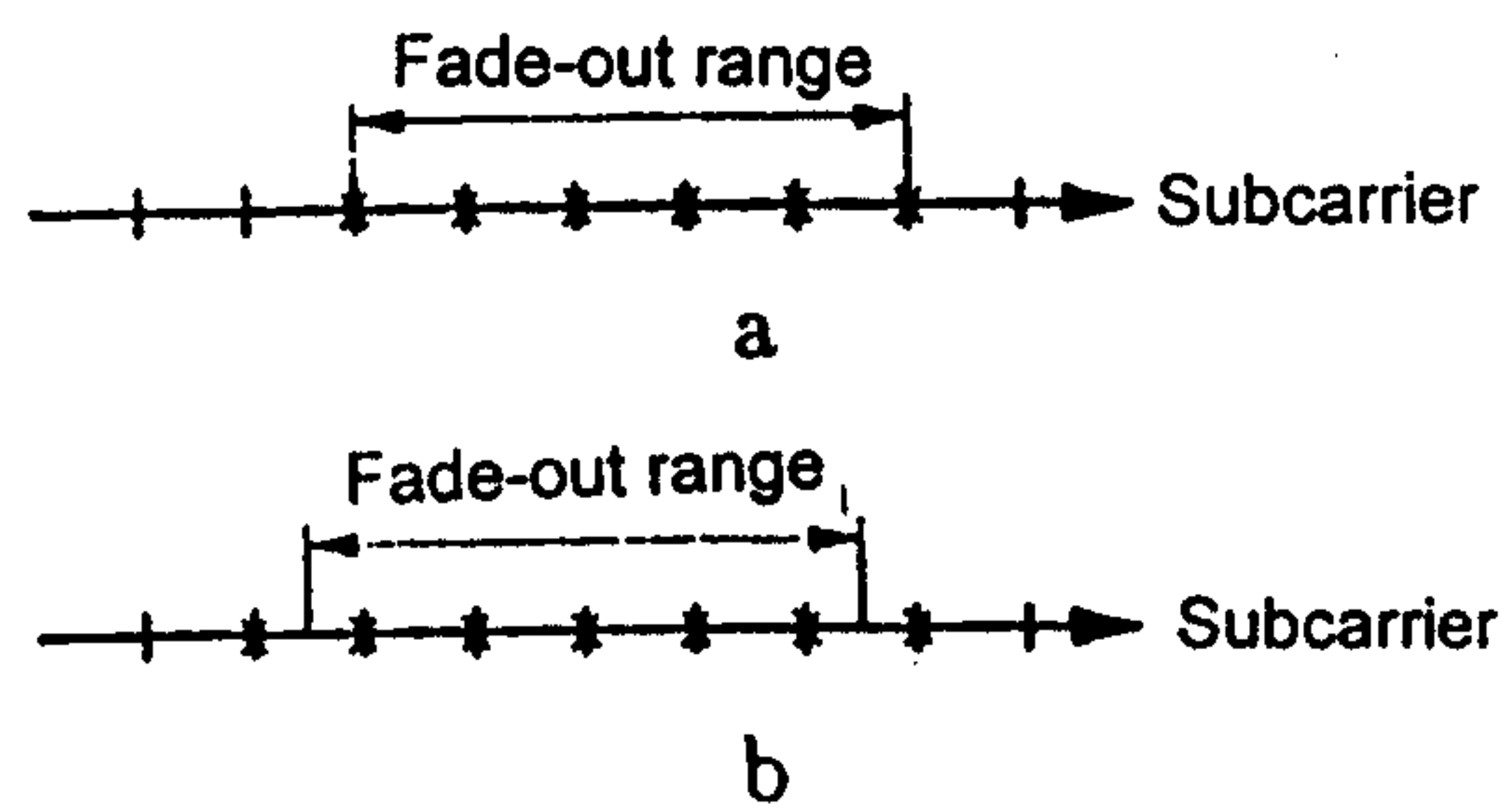


FIG. 10

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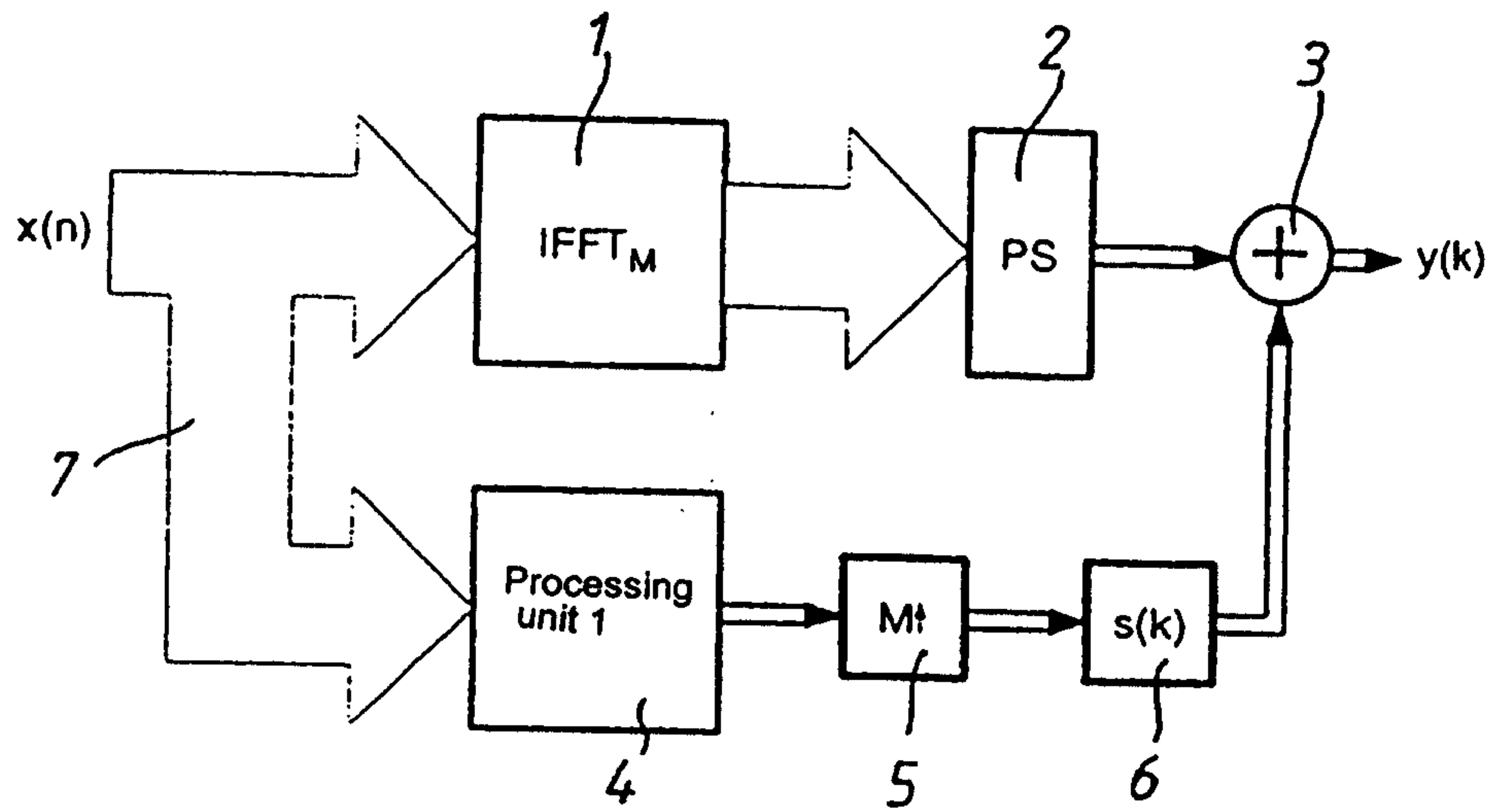


FIG. 11

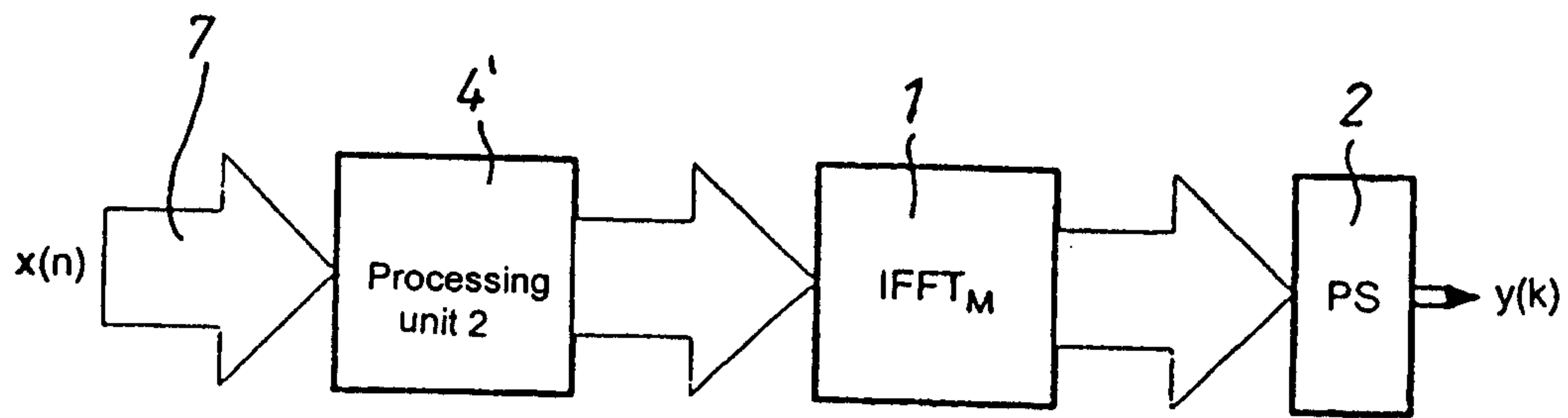


FIG. 12

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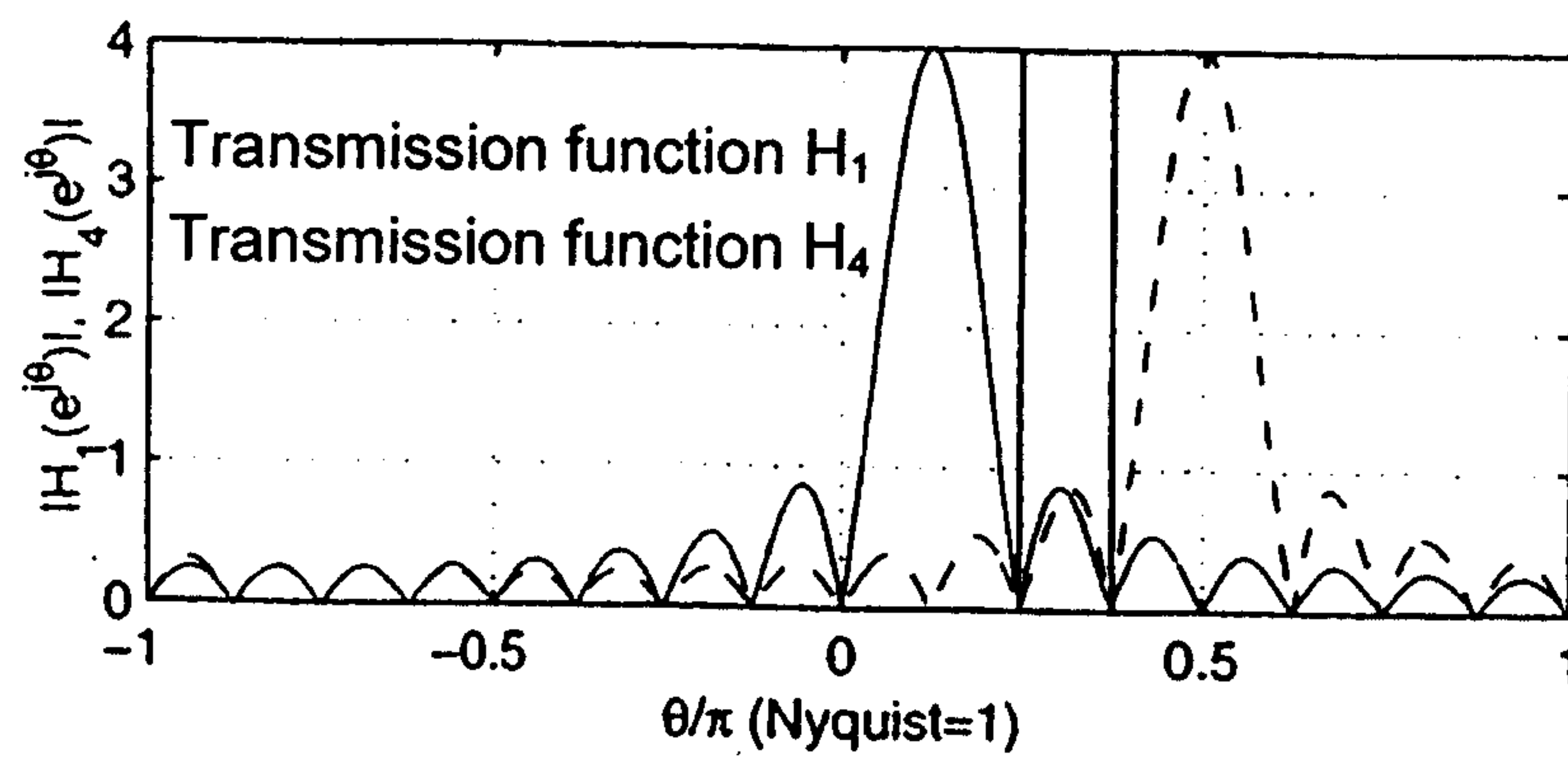


FIG. 13

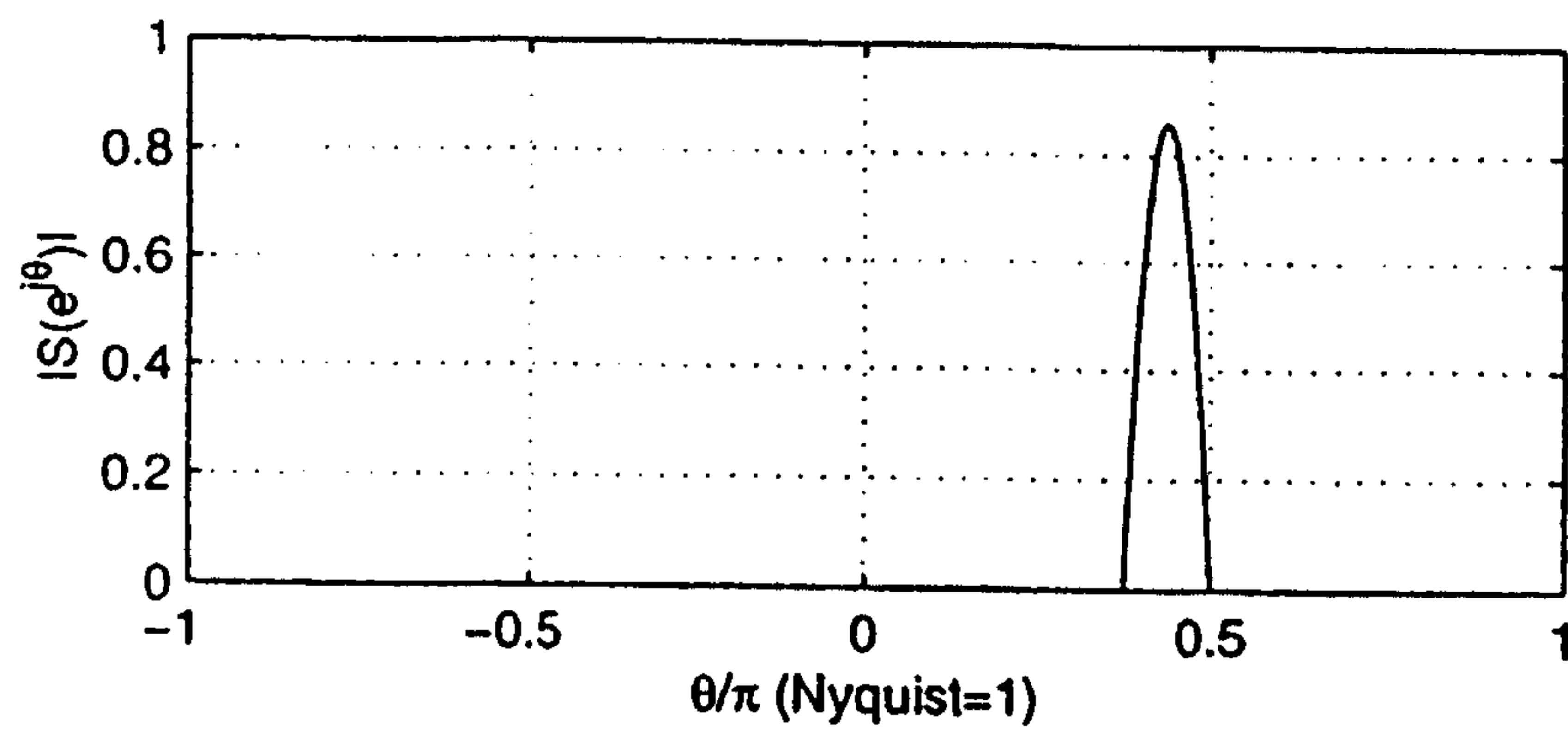


FIG. 14

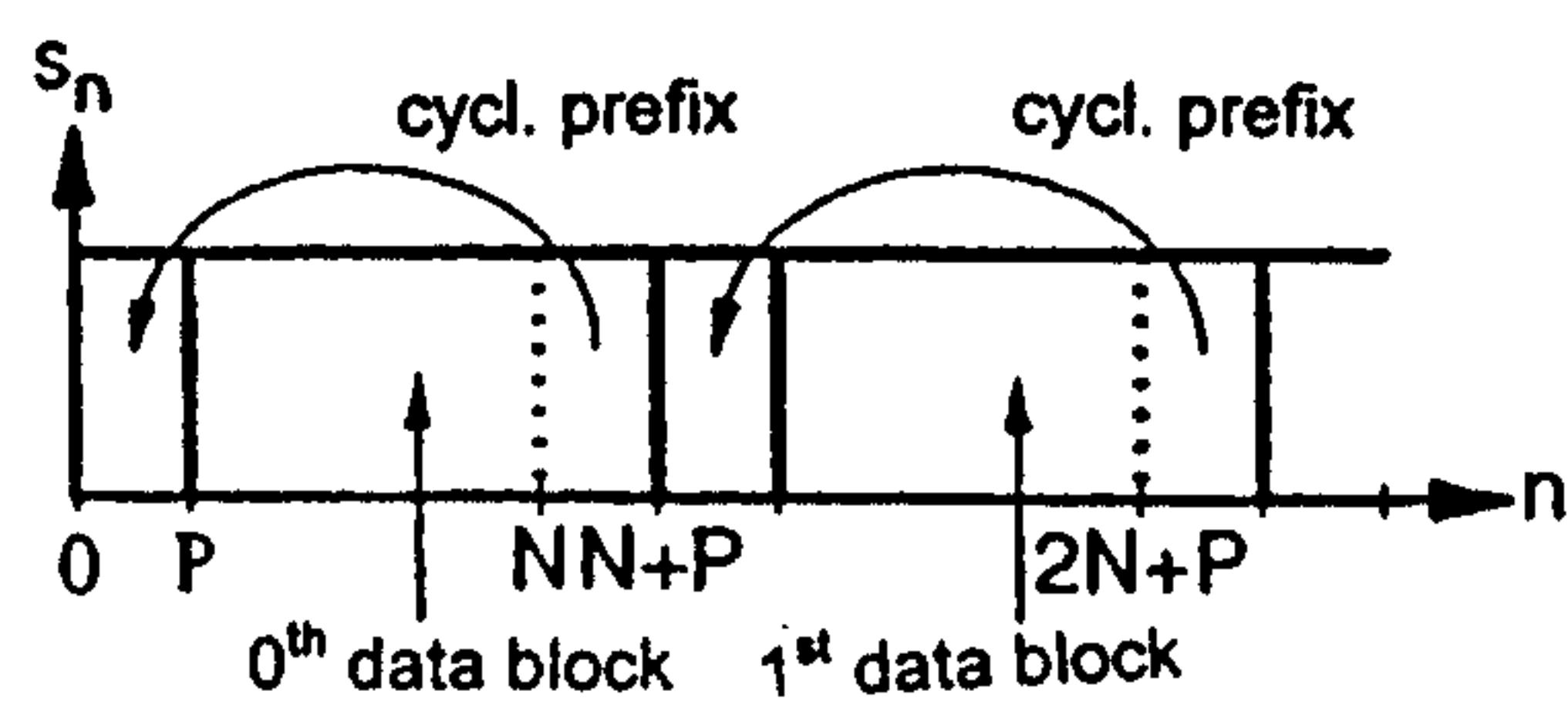


FIG. 15

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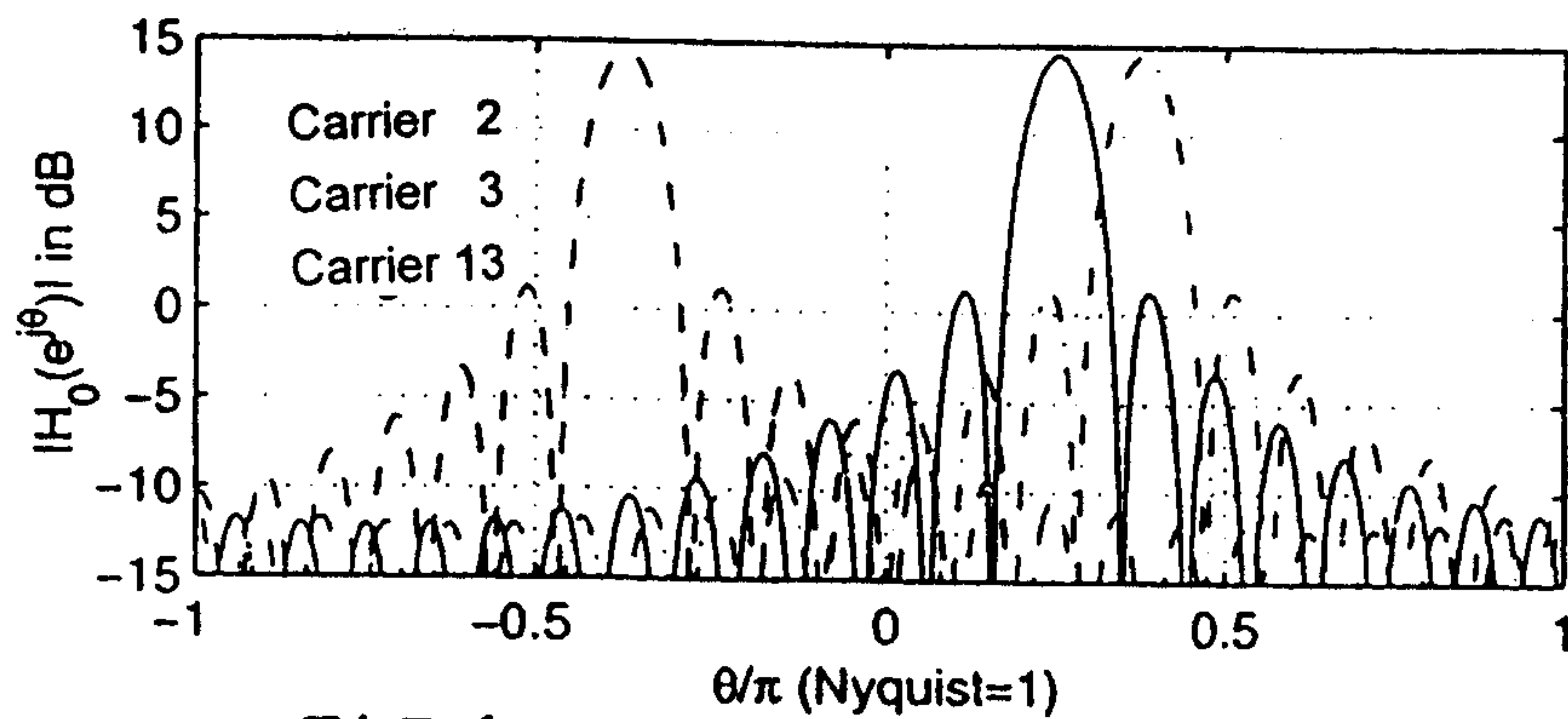


FIG. 16

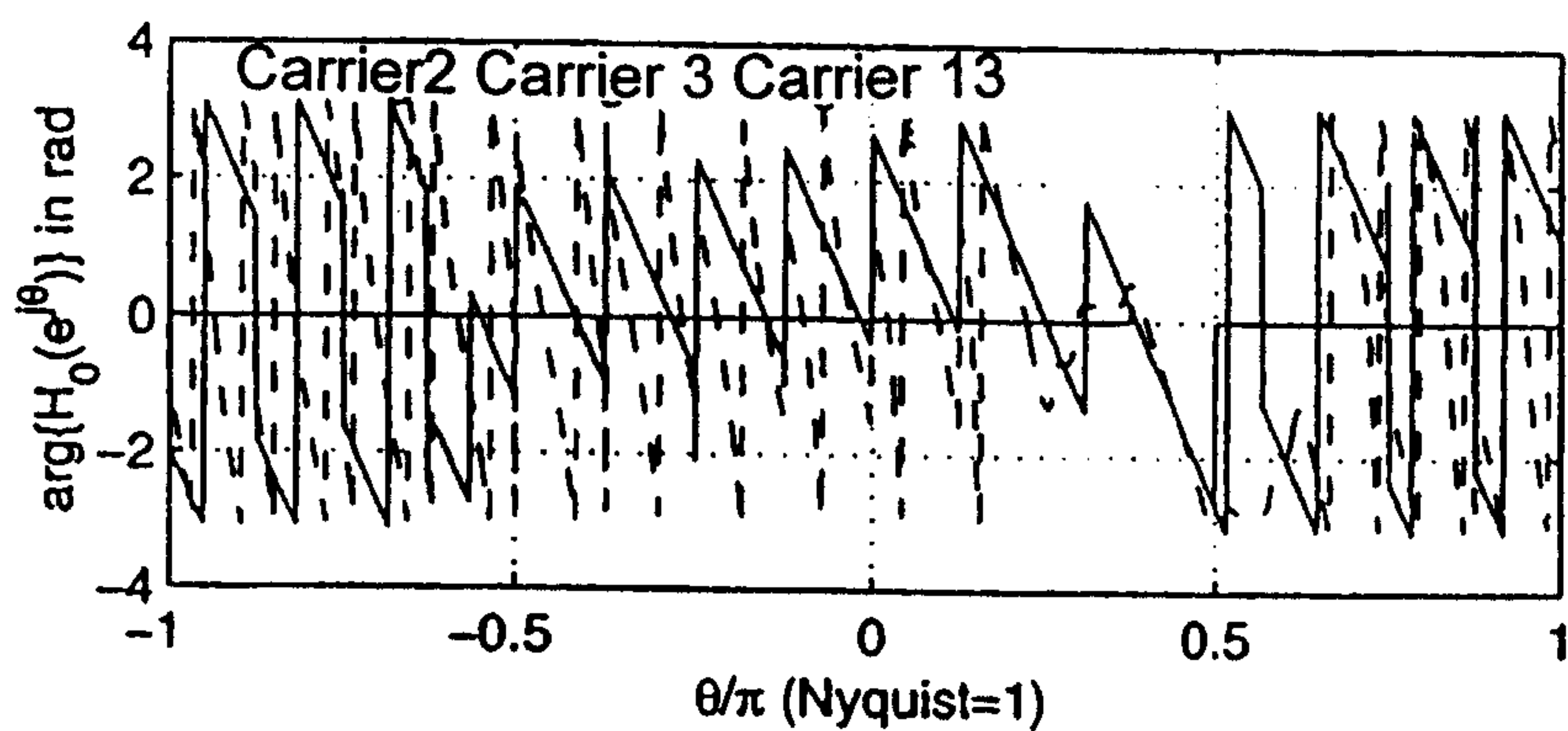


FIG. 17

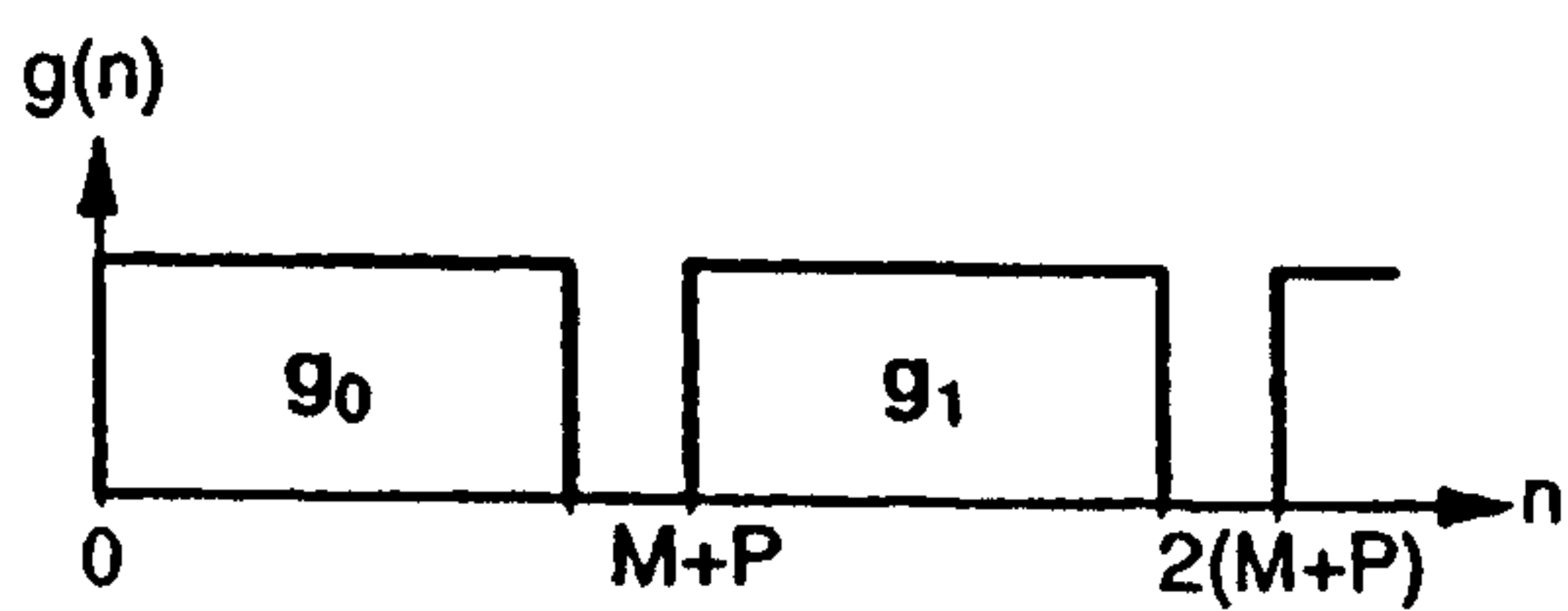


FIG. 18

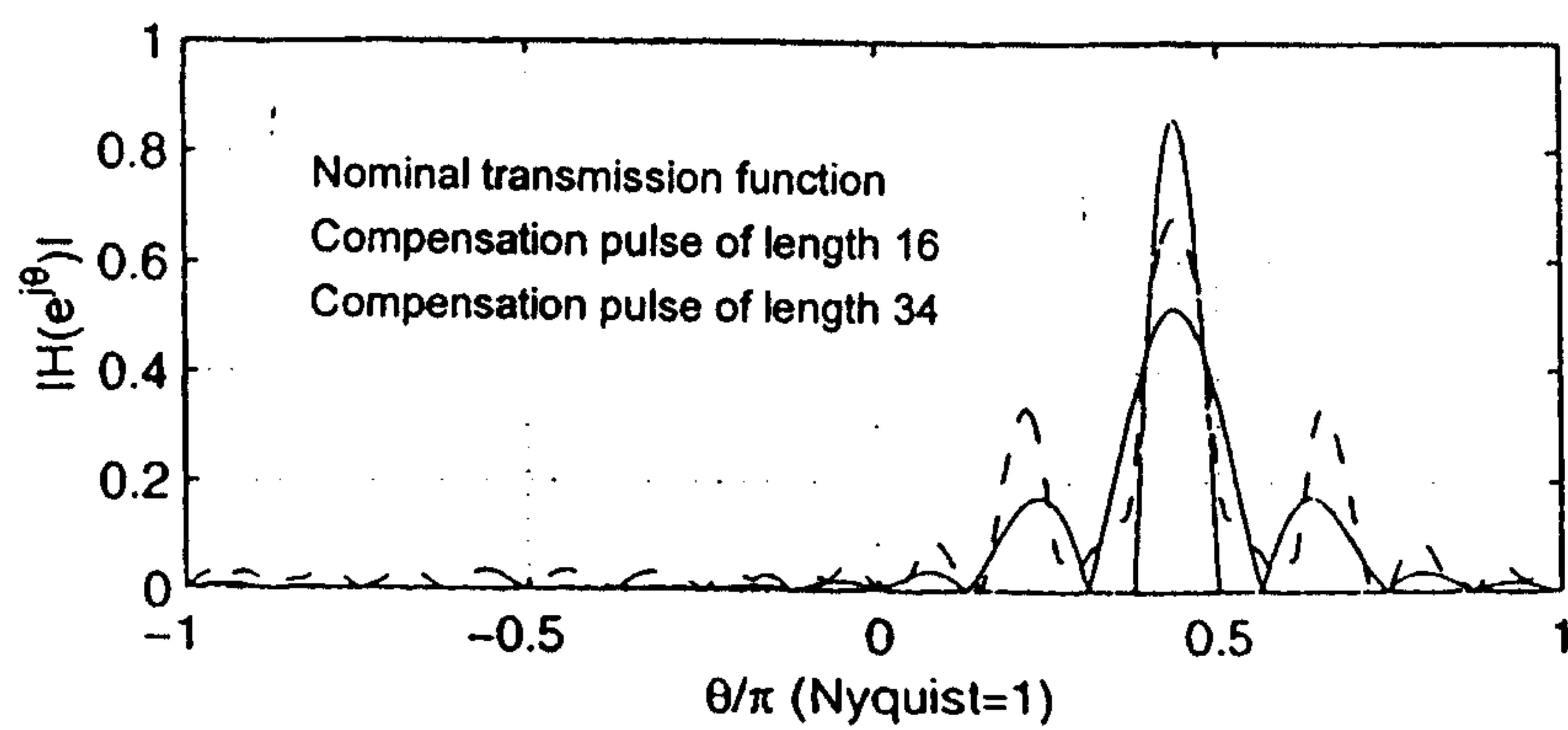


FIG. 19

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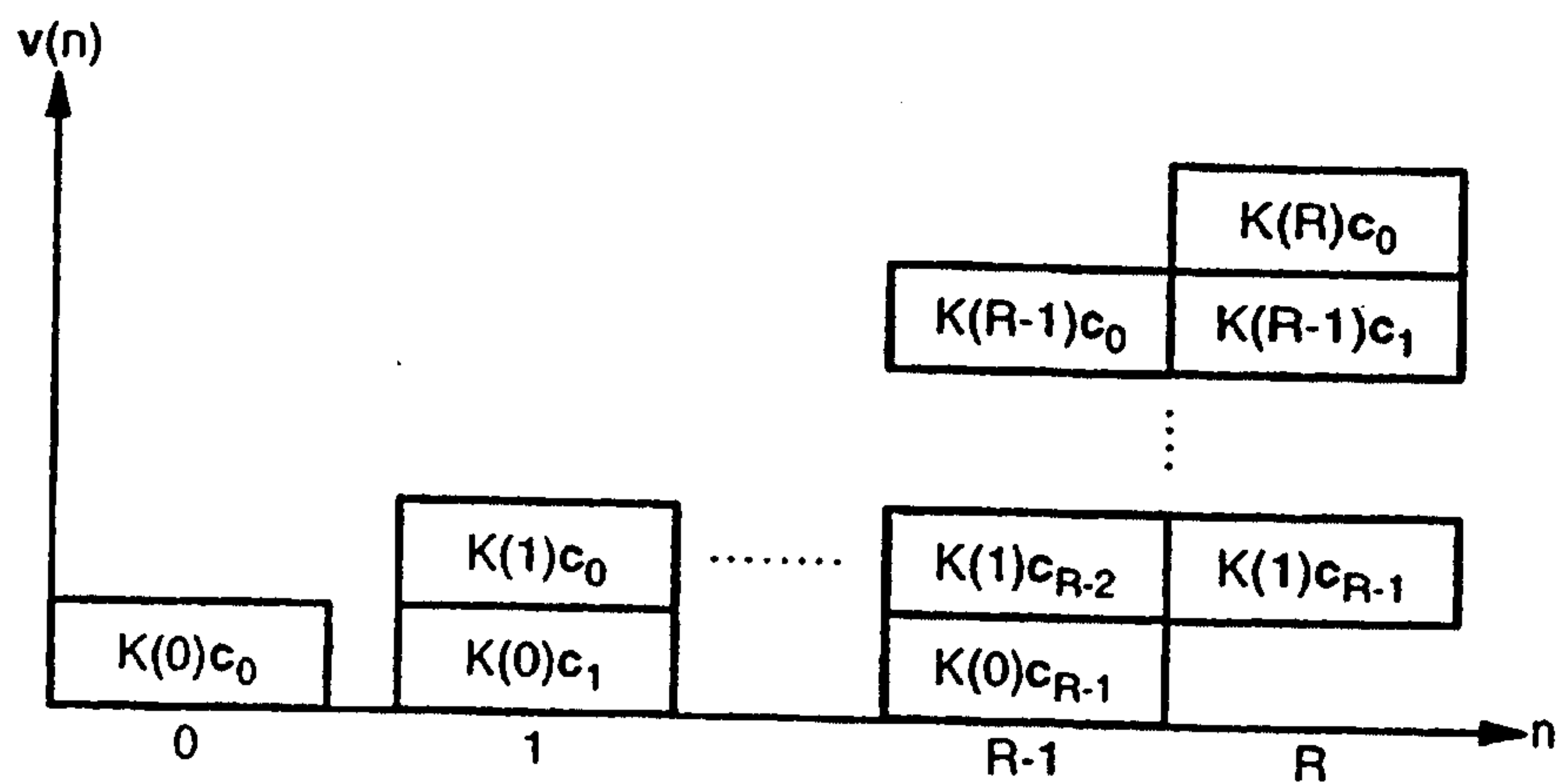


FIG. 20

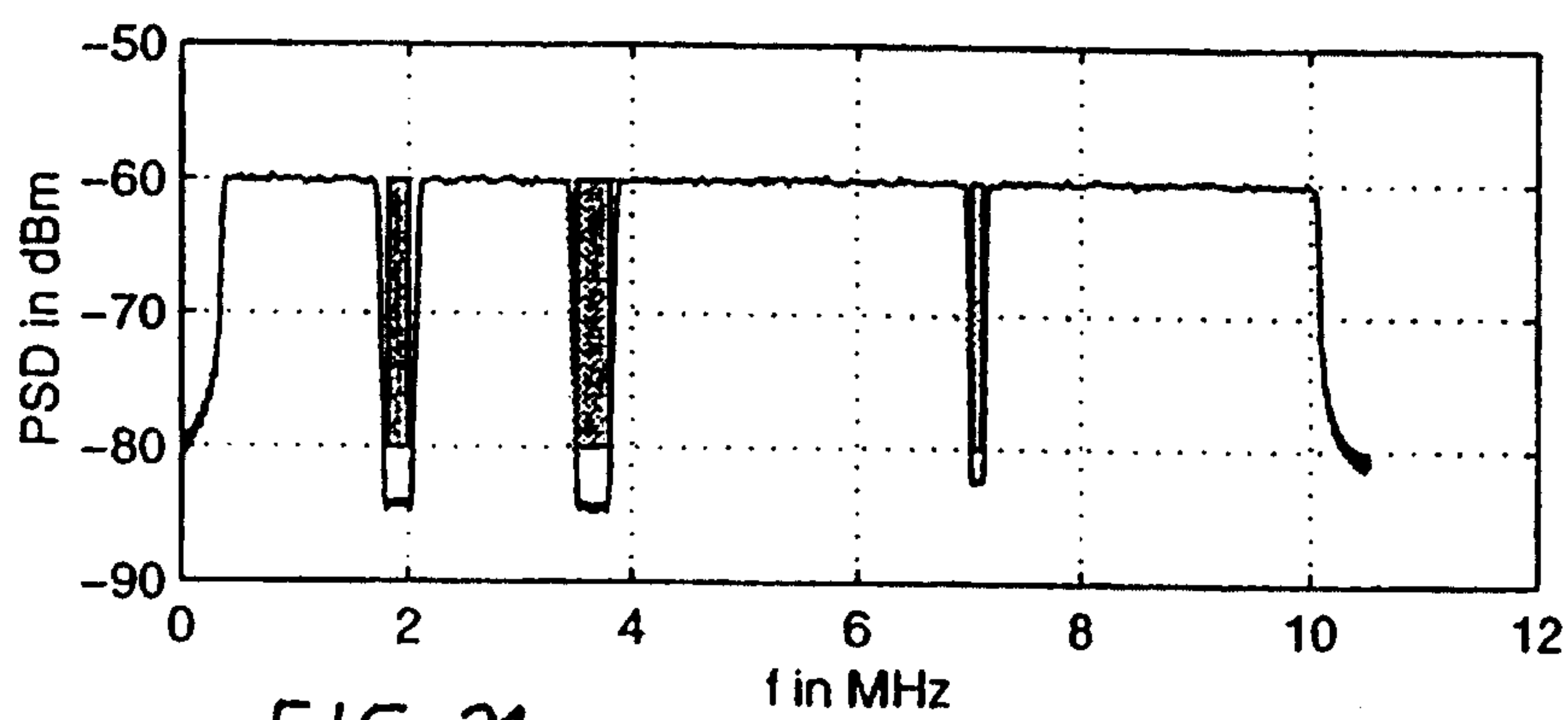


FIG. 21

a

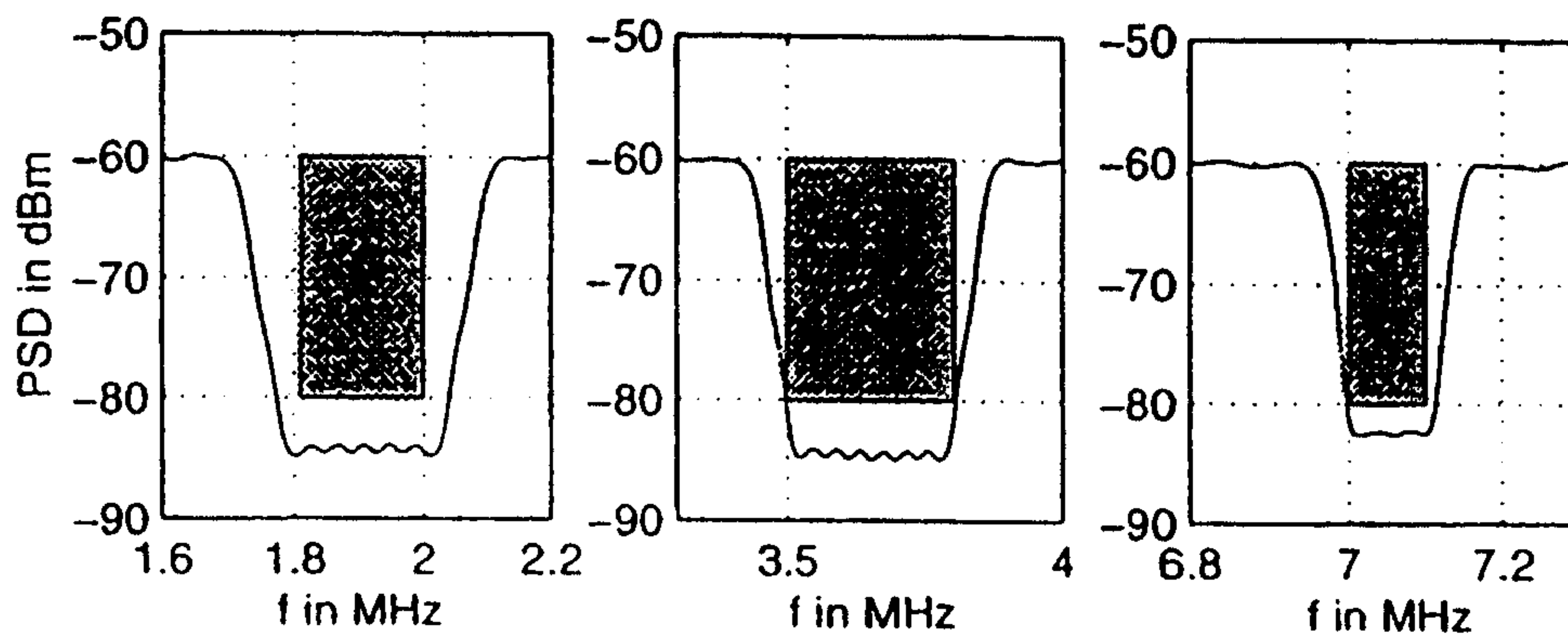
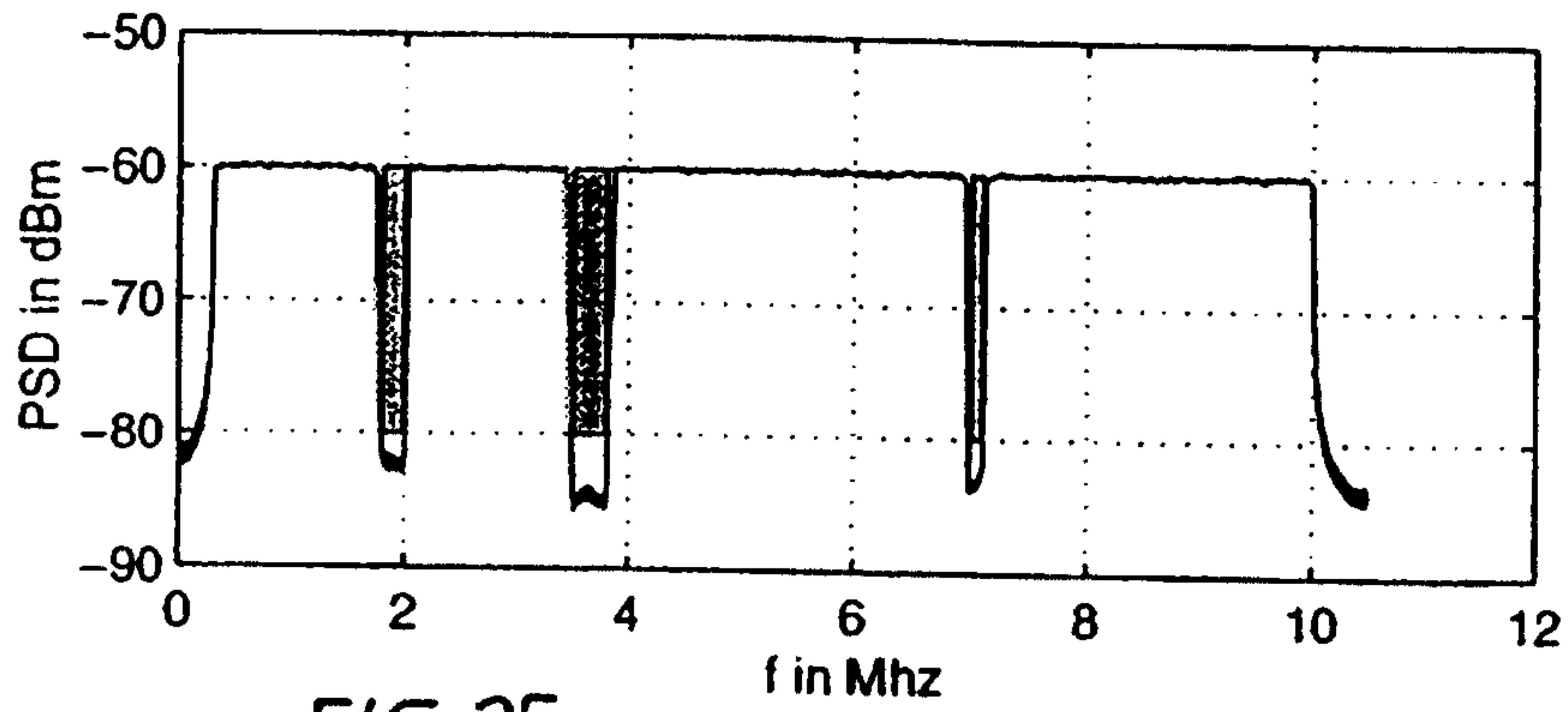


FIG. 22

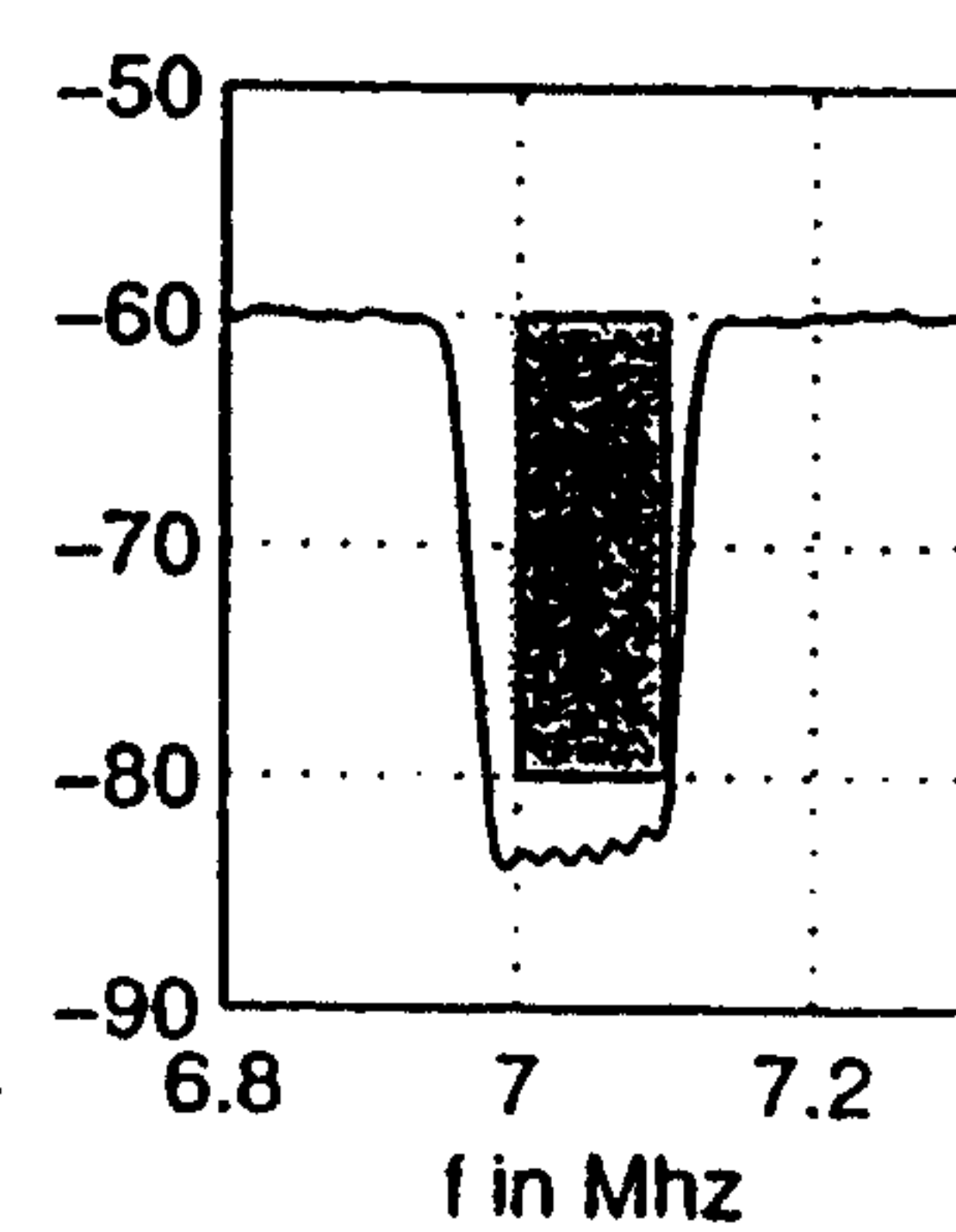
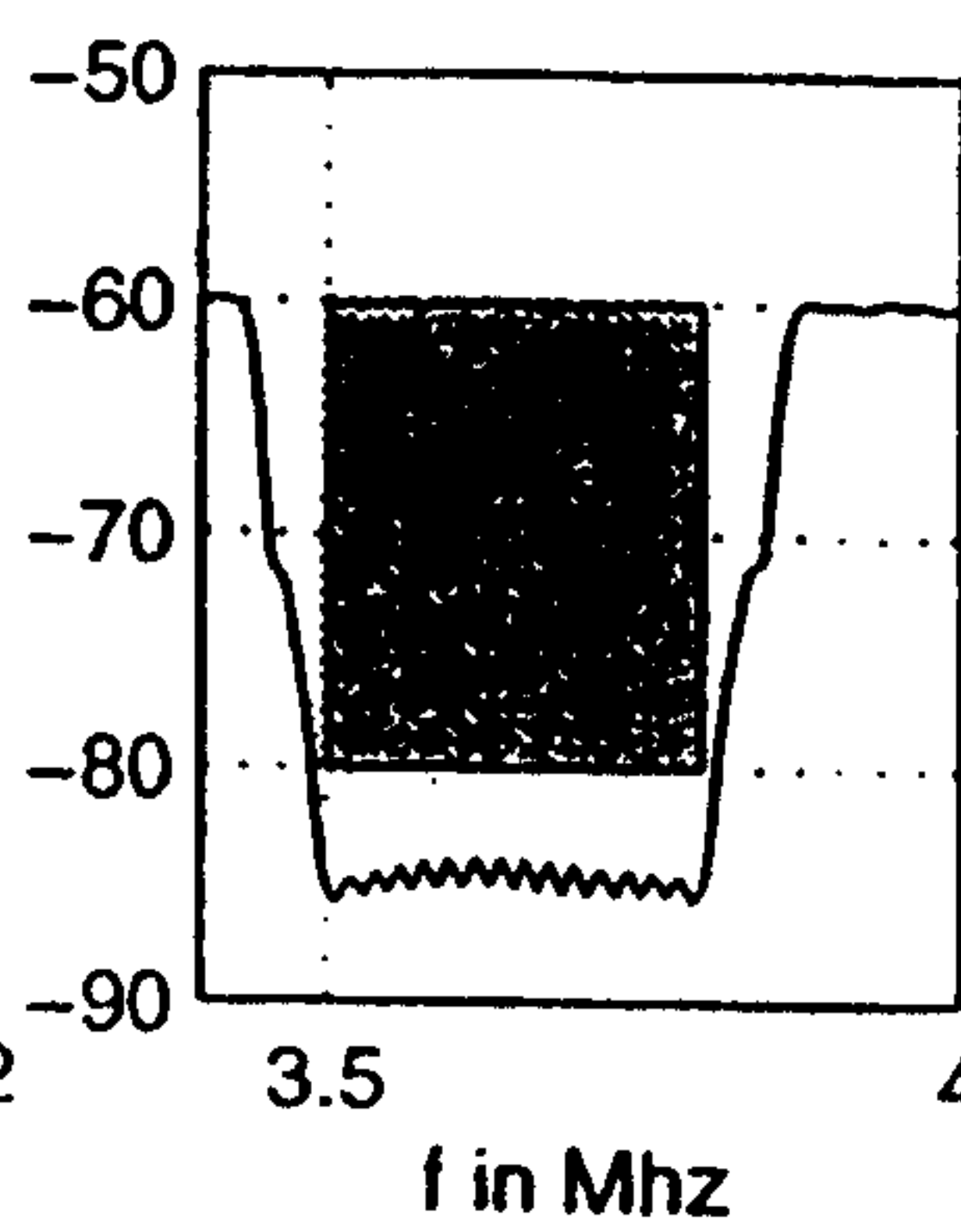
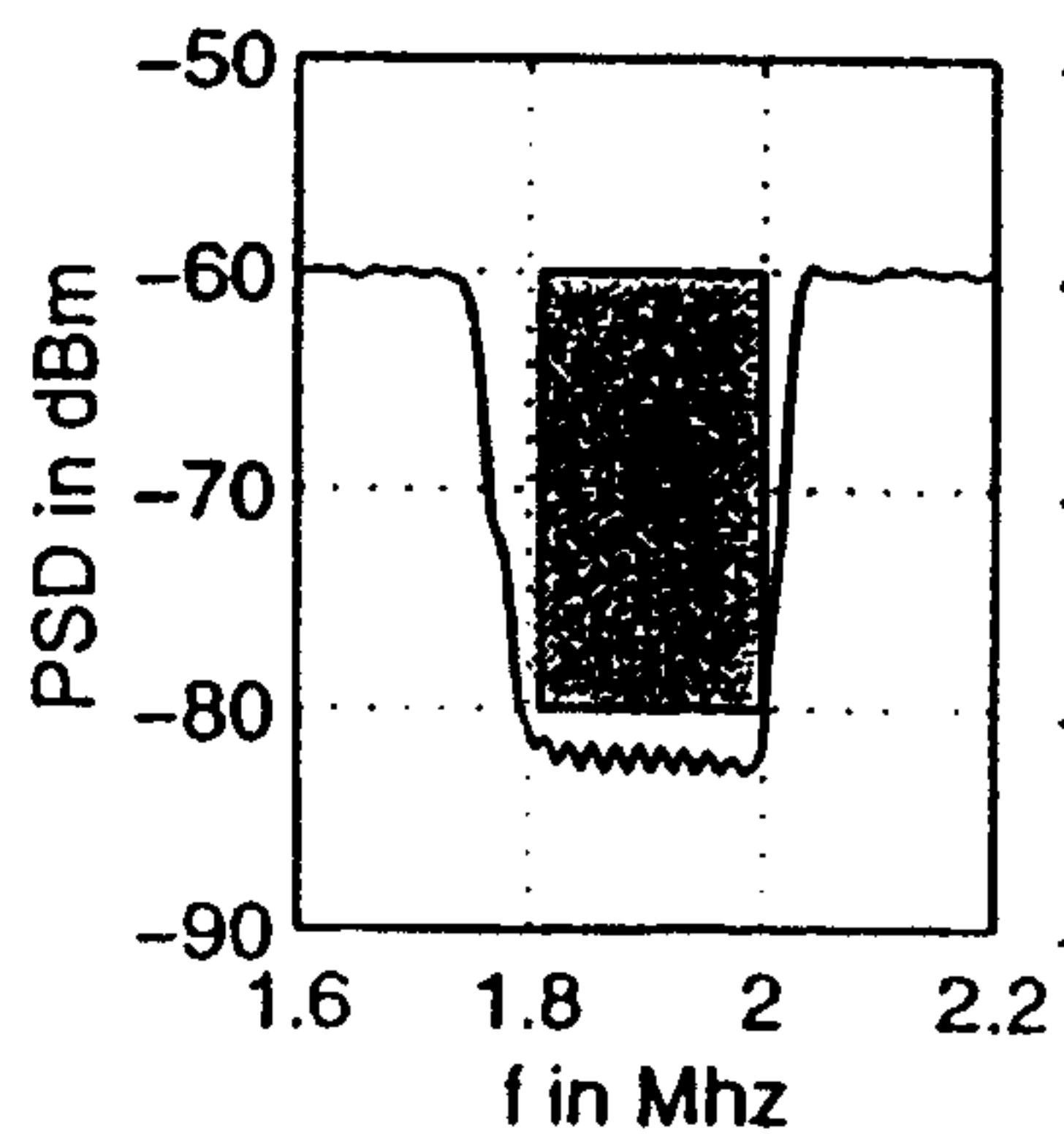
FIG. 23

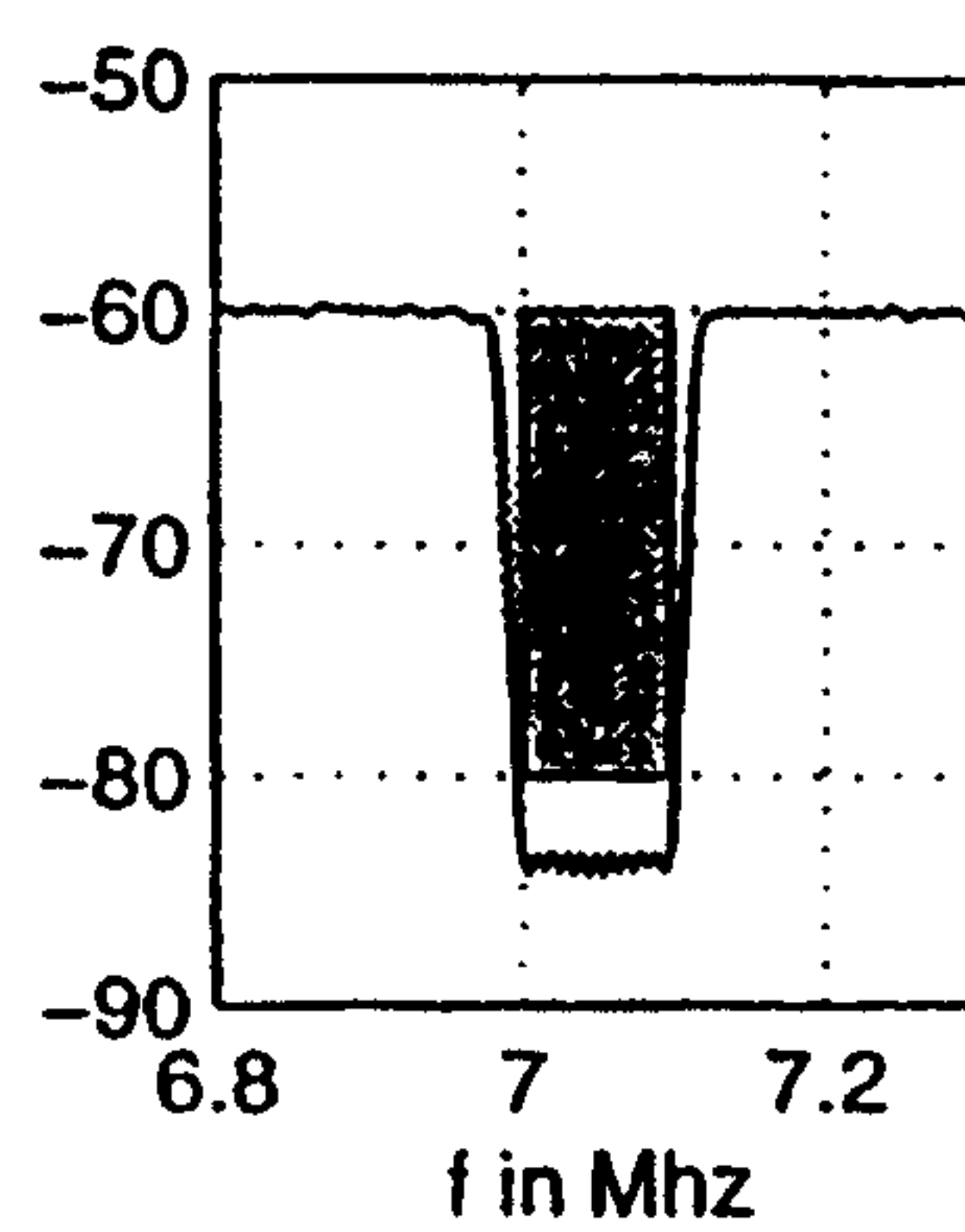
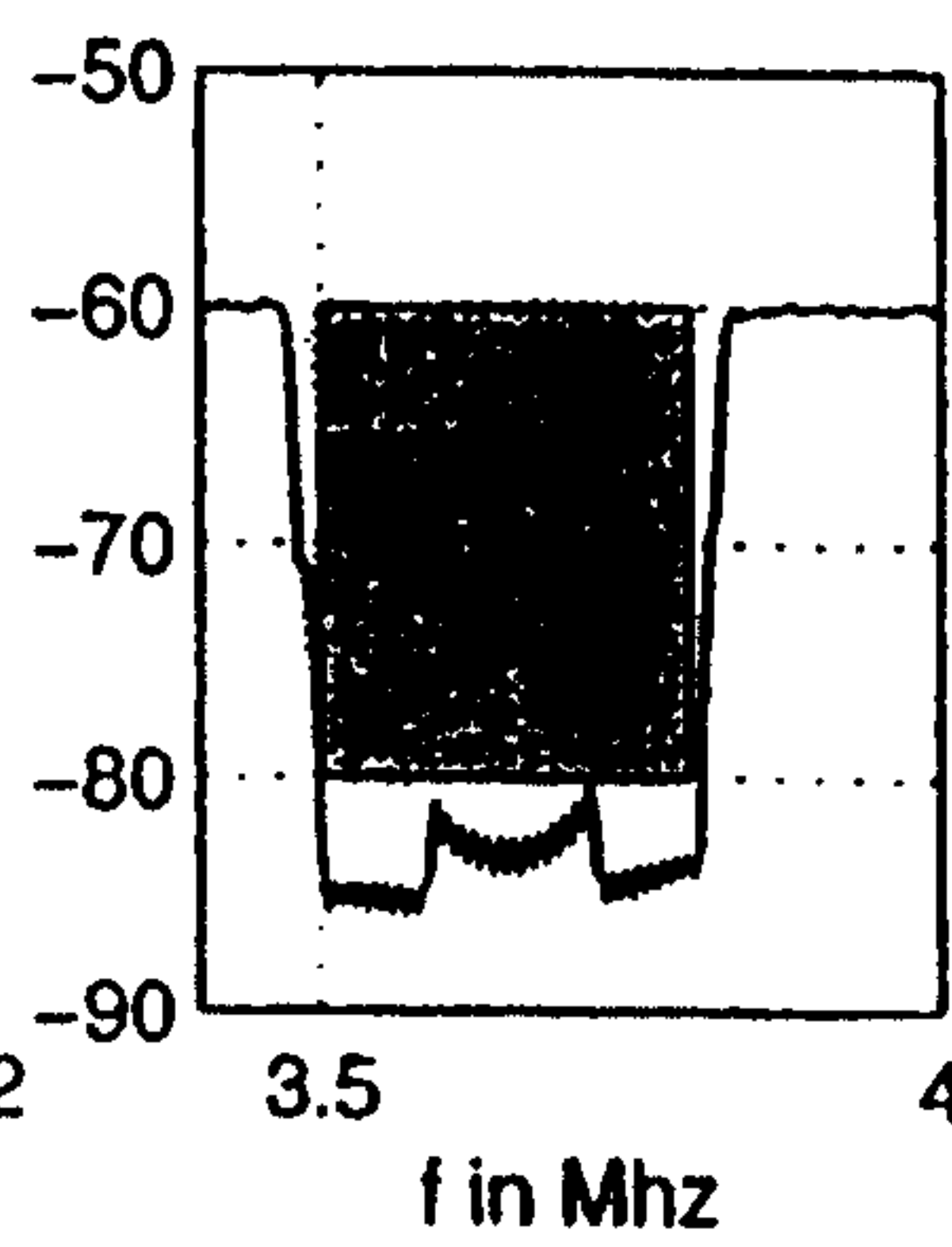
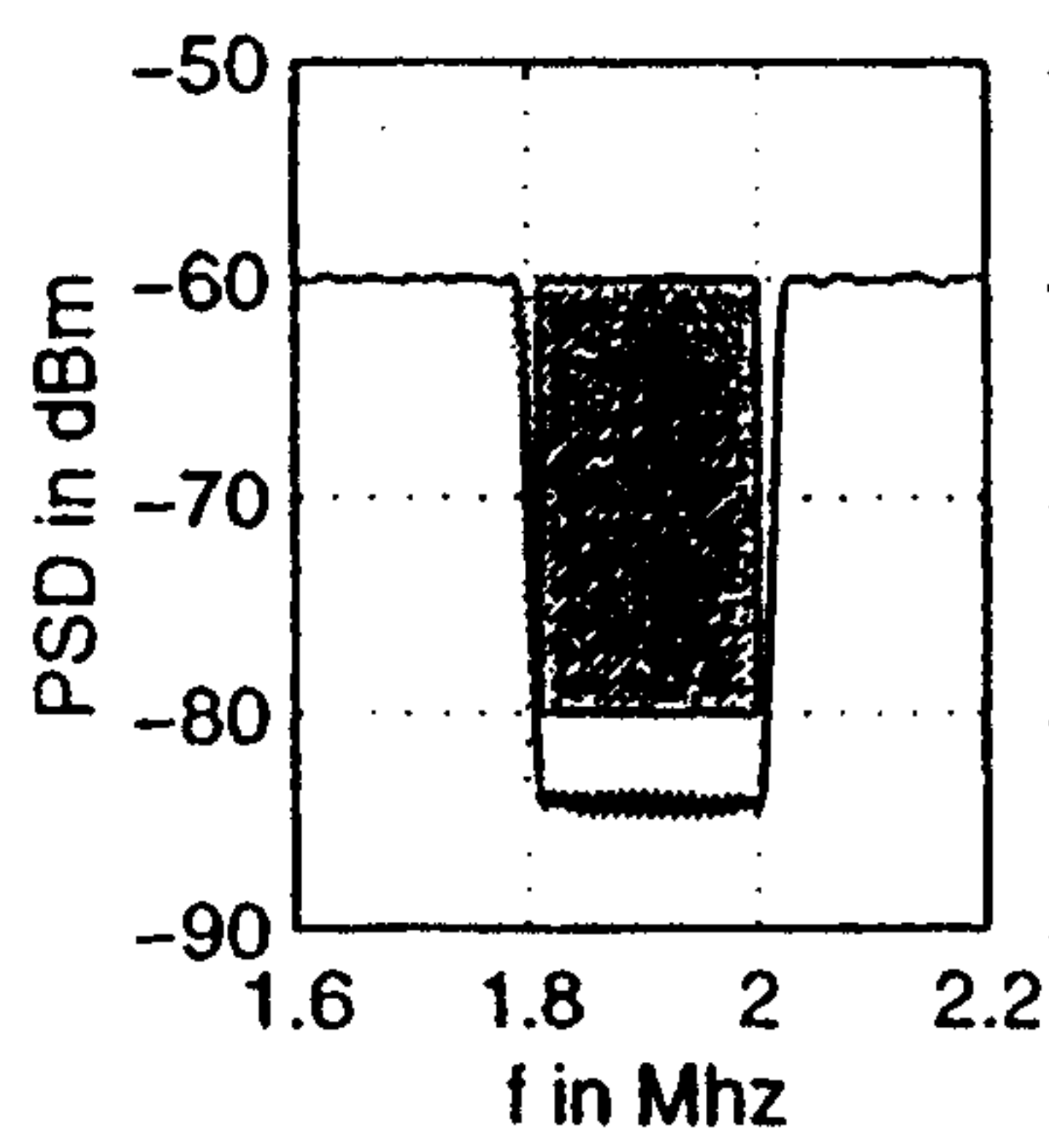
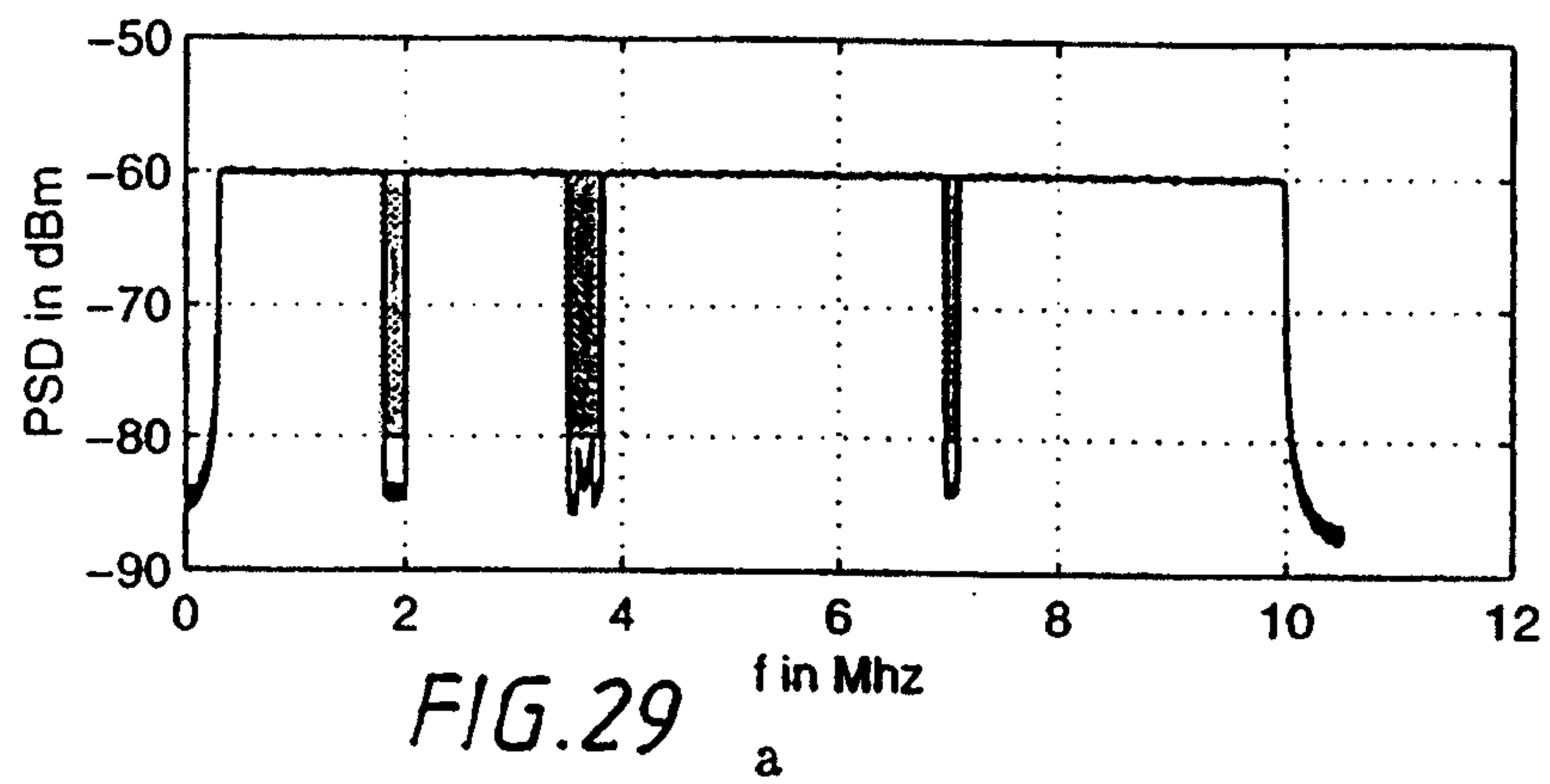
FIG. 24

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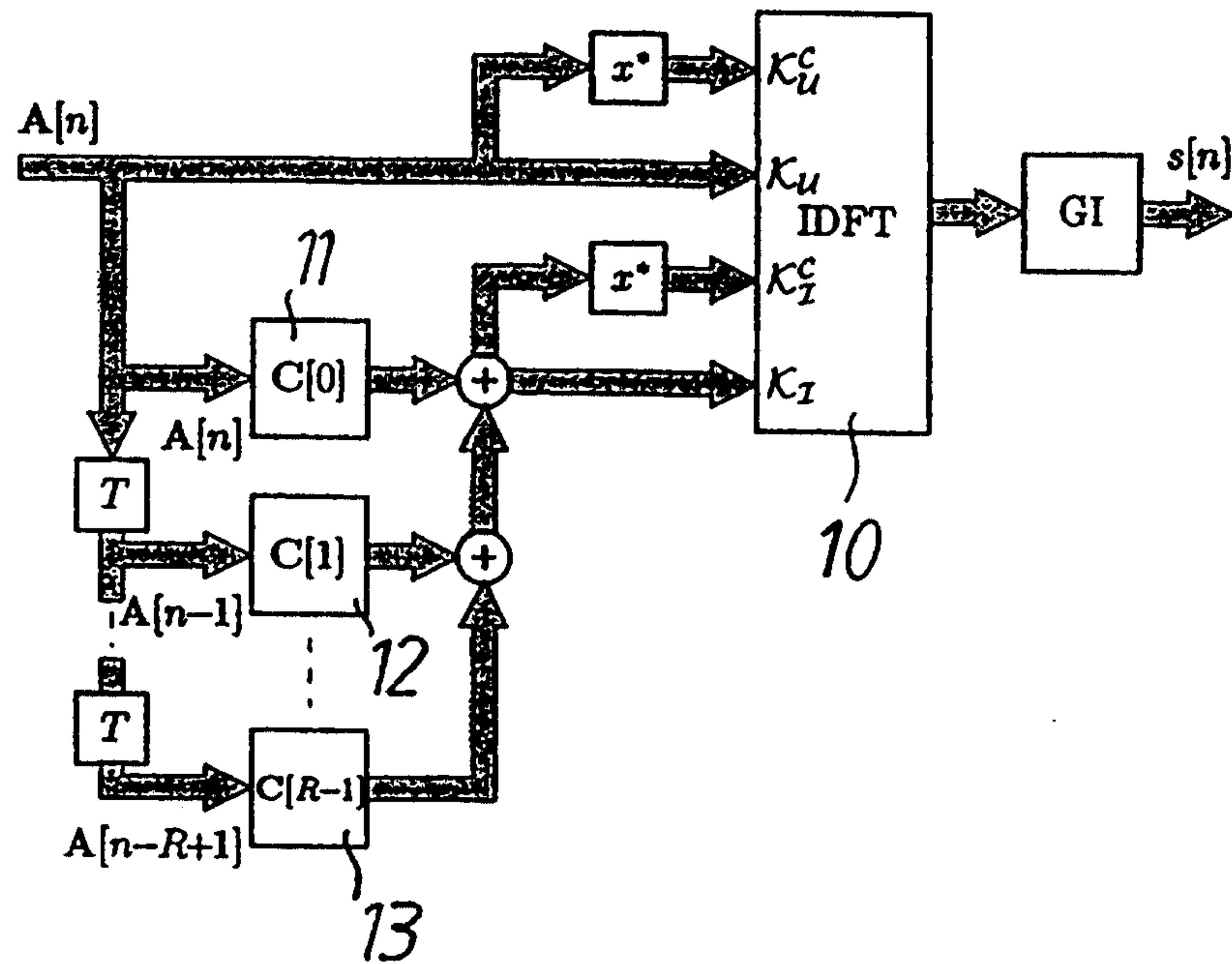


FIG. 33

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	needed subchannels
1 st band	$k = 37, 38, \dots, 56$
2 nd band	$k = 79, 80, \dots, 99$
3 rd band	$k = 163, 164, \dots, 180$

FIG. 34

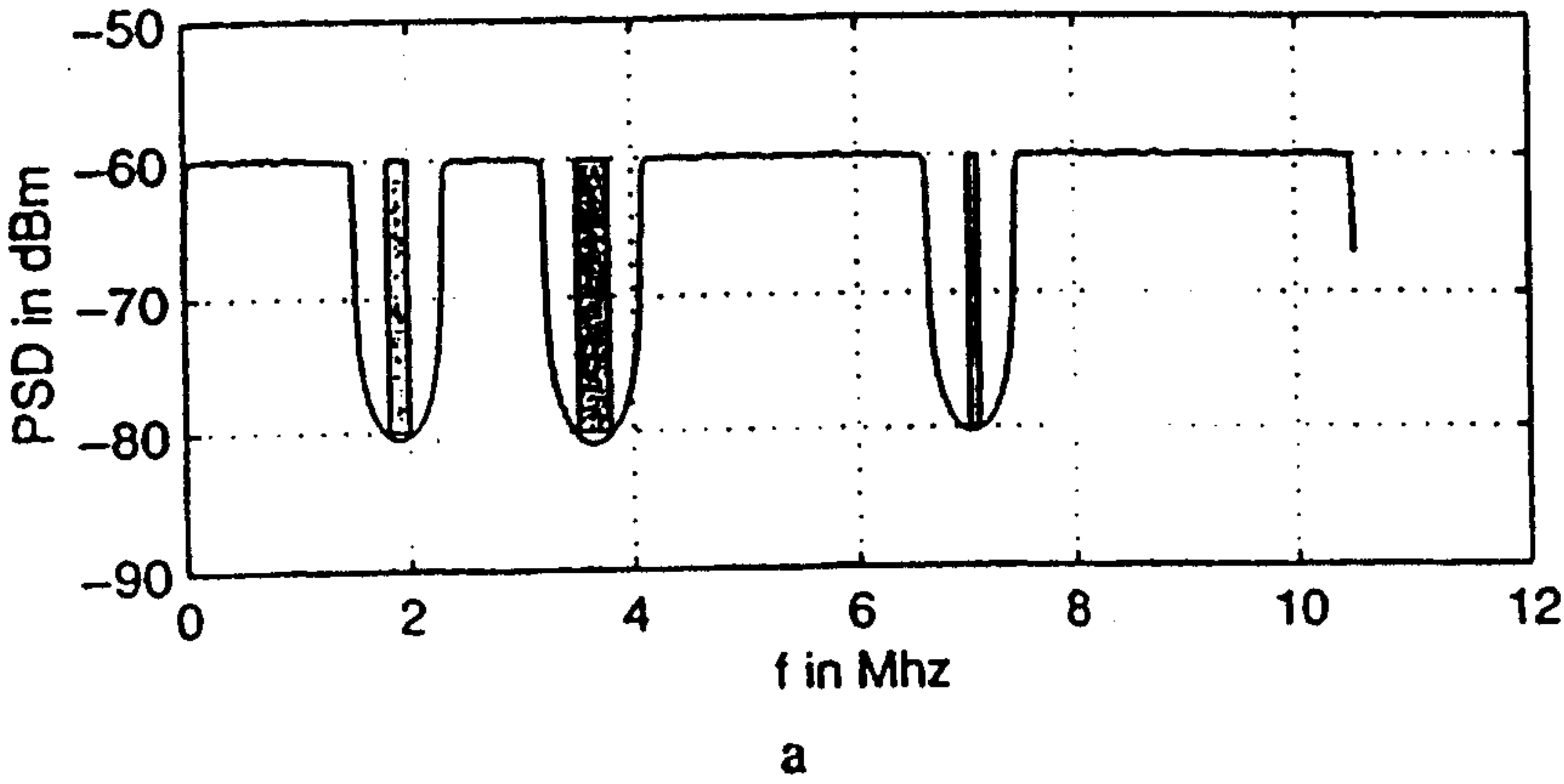


FIG. 35

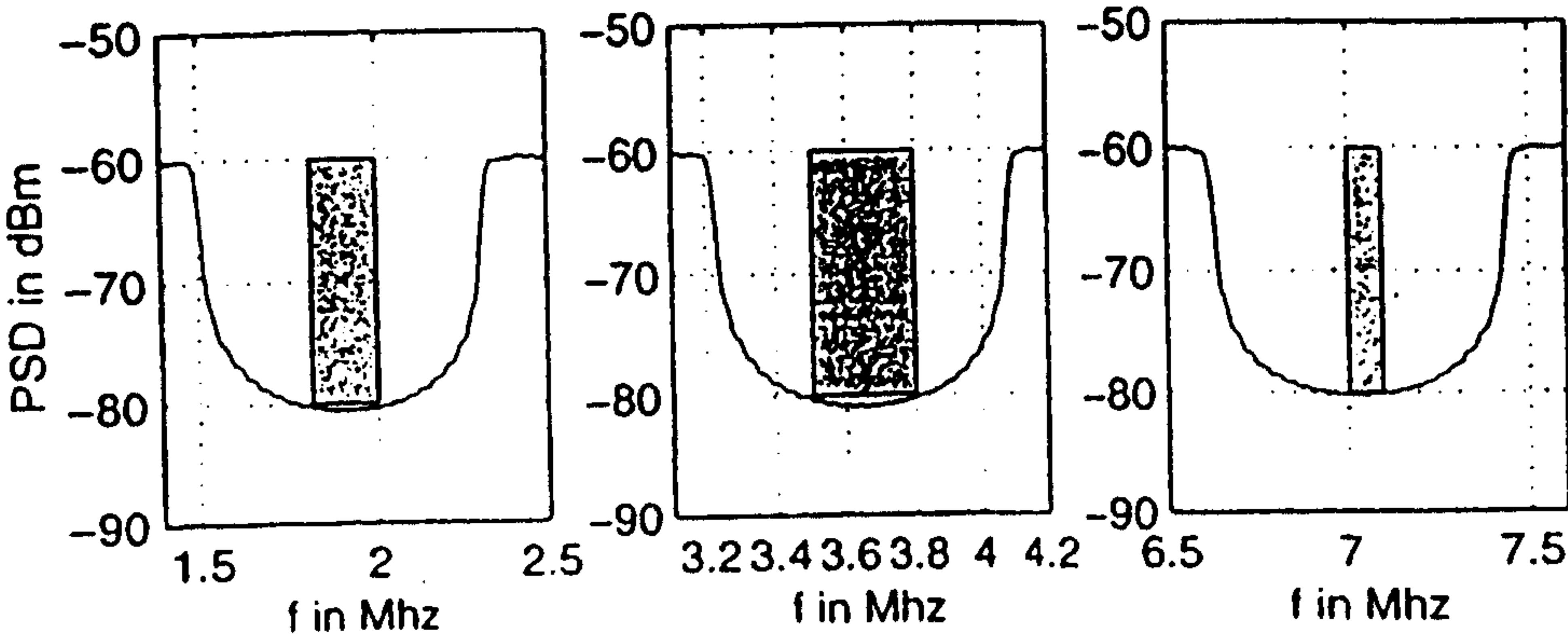


FIG. 36

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	needed subchannels
1 st band	$k = 43, 44, \dots, 50$
2 nd band	$k = 84, 85, \dots, 94$
3 rd band	$k = 170, 171, \dots, 174$

FIG.37

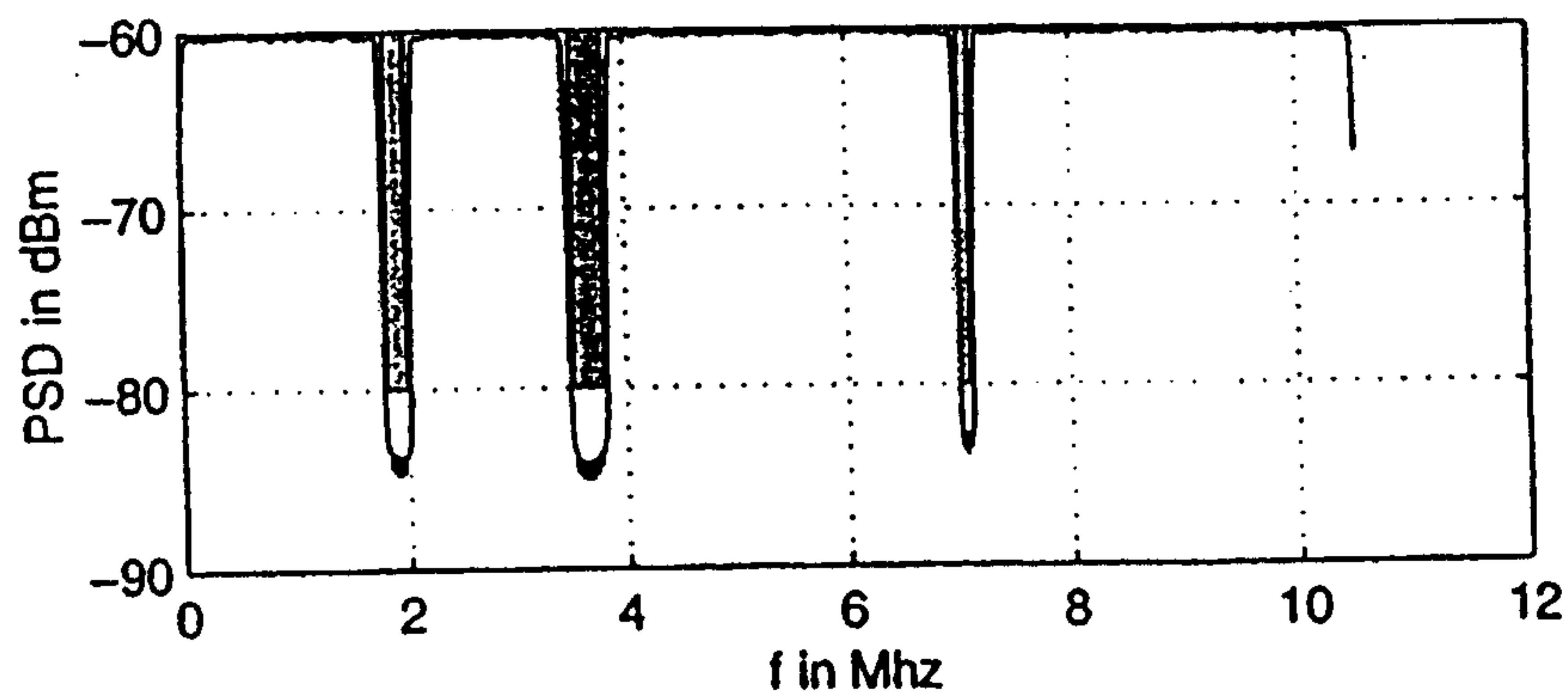


FIG.38

a

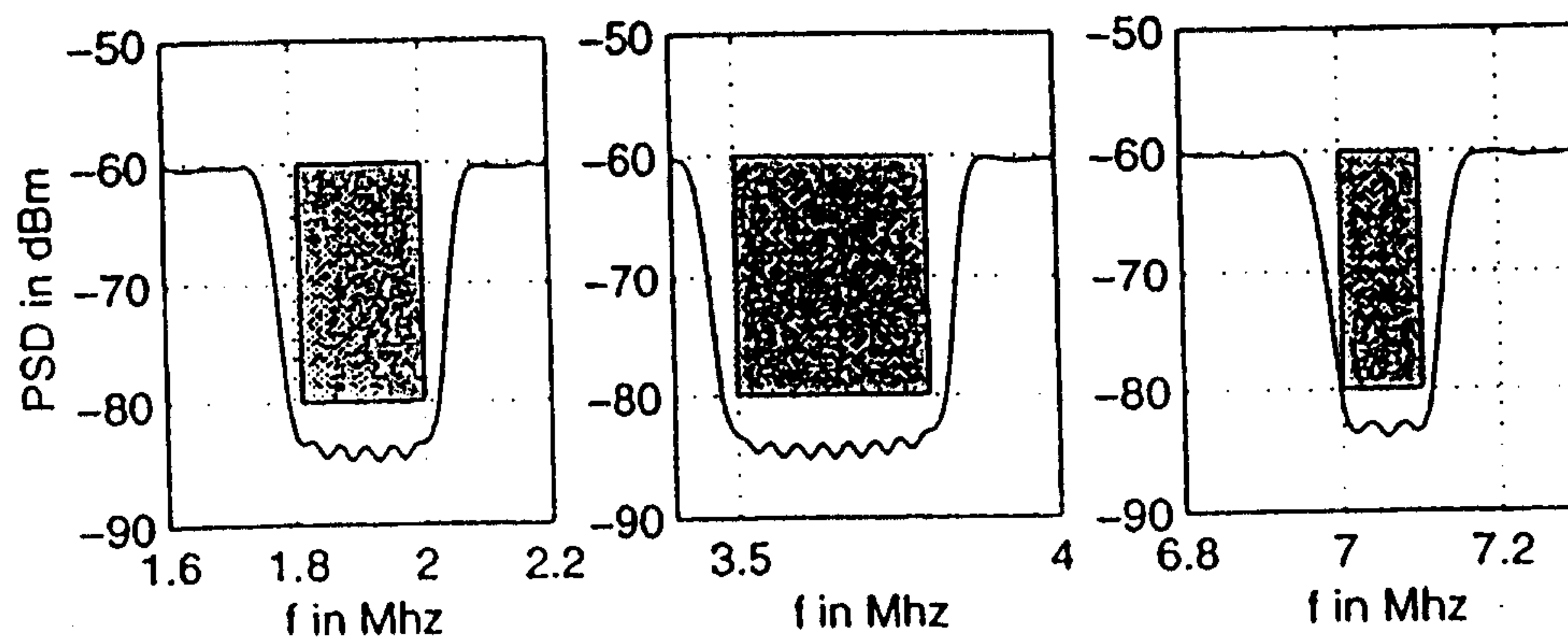
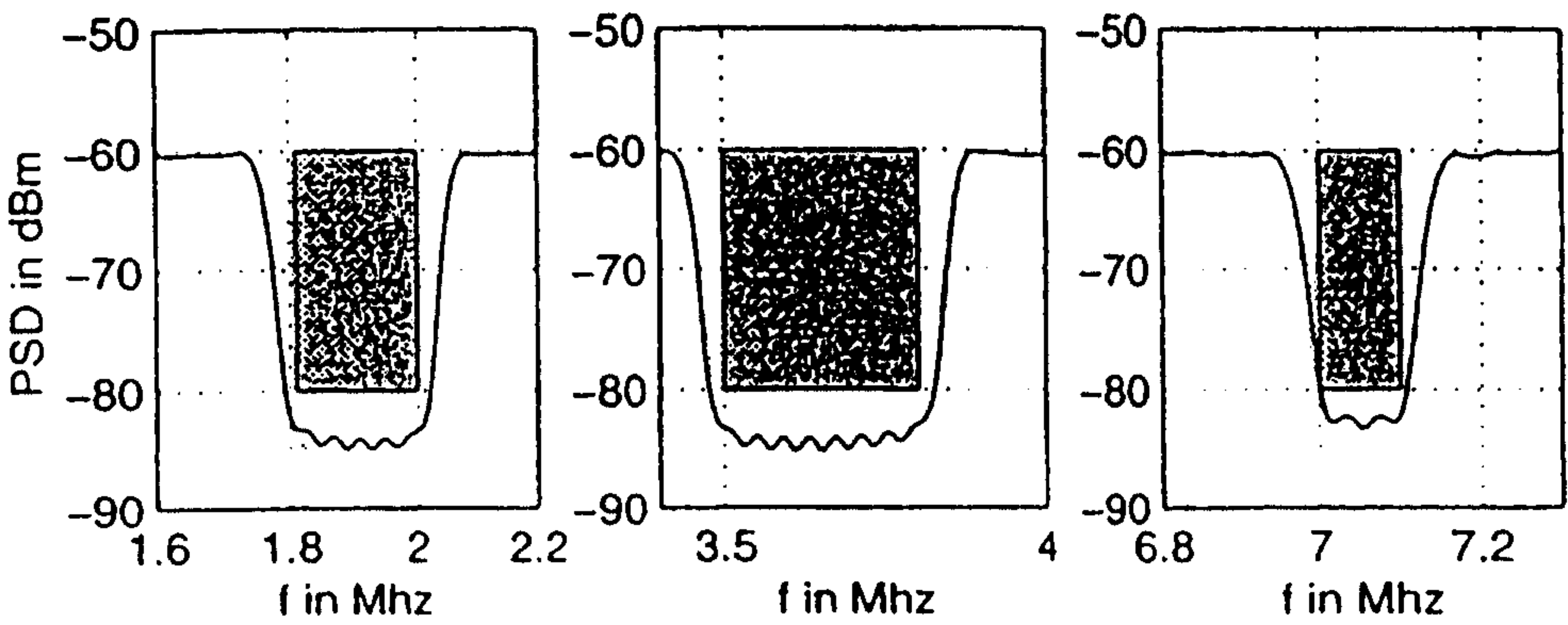
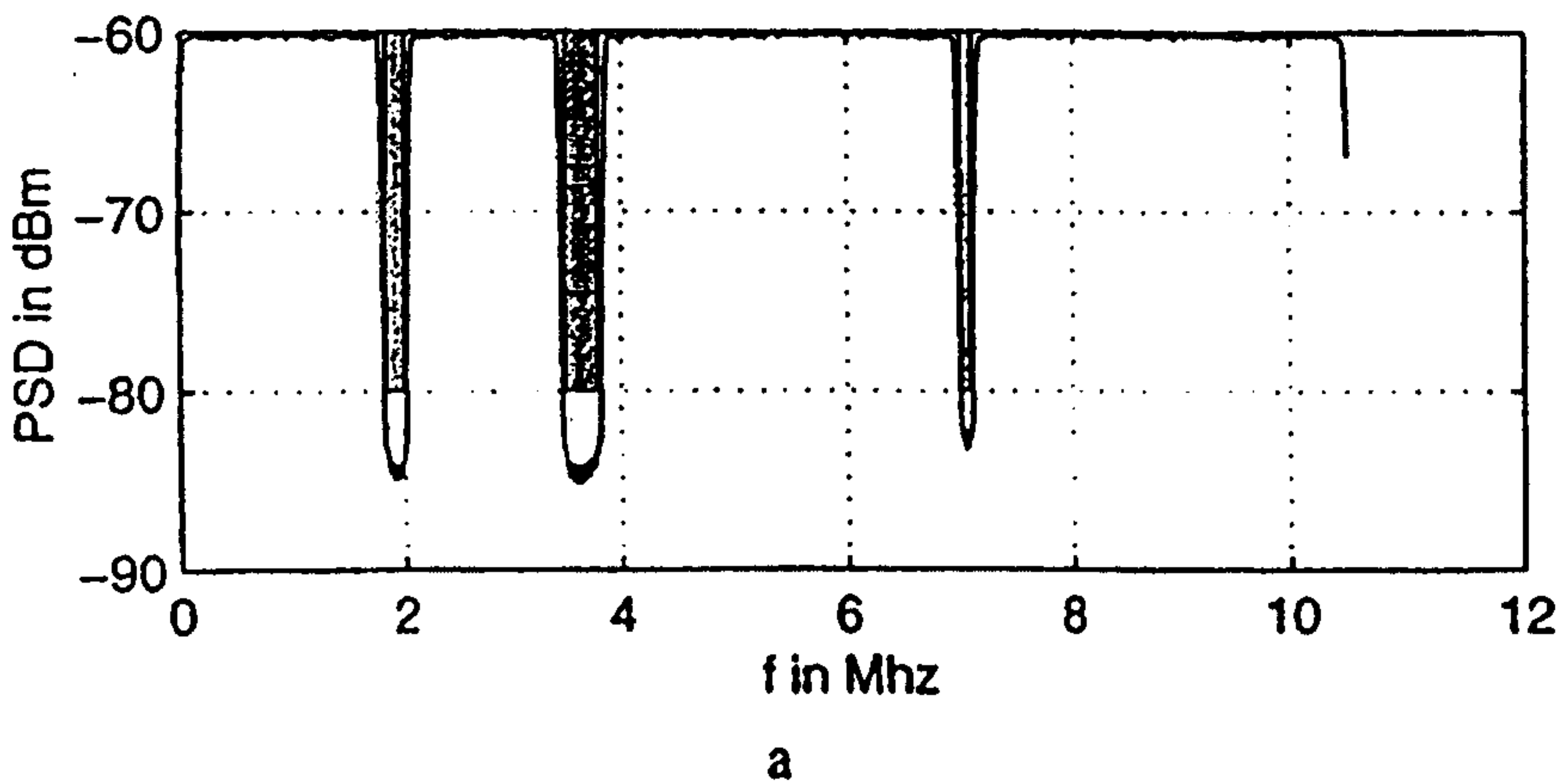


FIG.39

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	needed subchannels
1 st band	$k = 43, 44, \dots, 50$
2 nd band	$k = 84, 85, \dots, 94$
3 rd band	$k = 170, 171, \dots, 174$

FIG.40



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	needed subchannels
1 st band	$k = 43, 44, \dots, 50$
2 nd band	$k = 84, 85, \dots, 94$
3 rd band	$k = 170, 171, \dots, 174$

FIG. 43

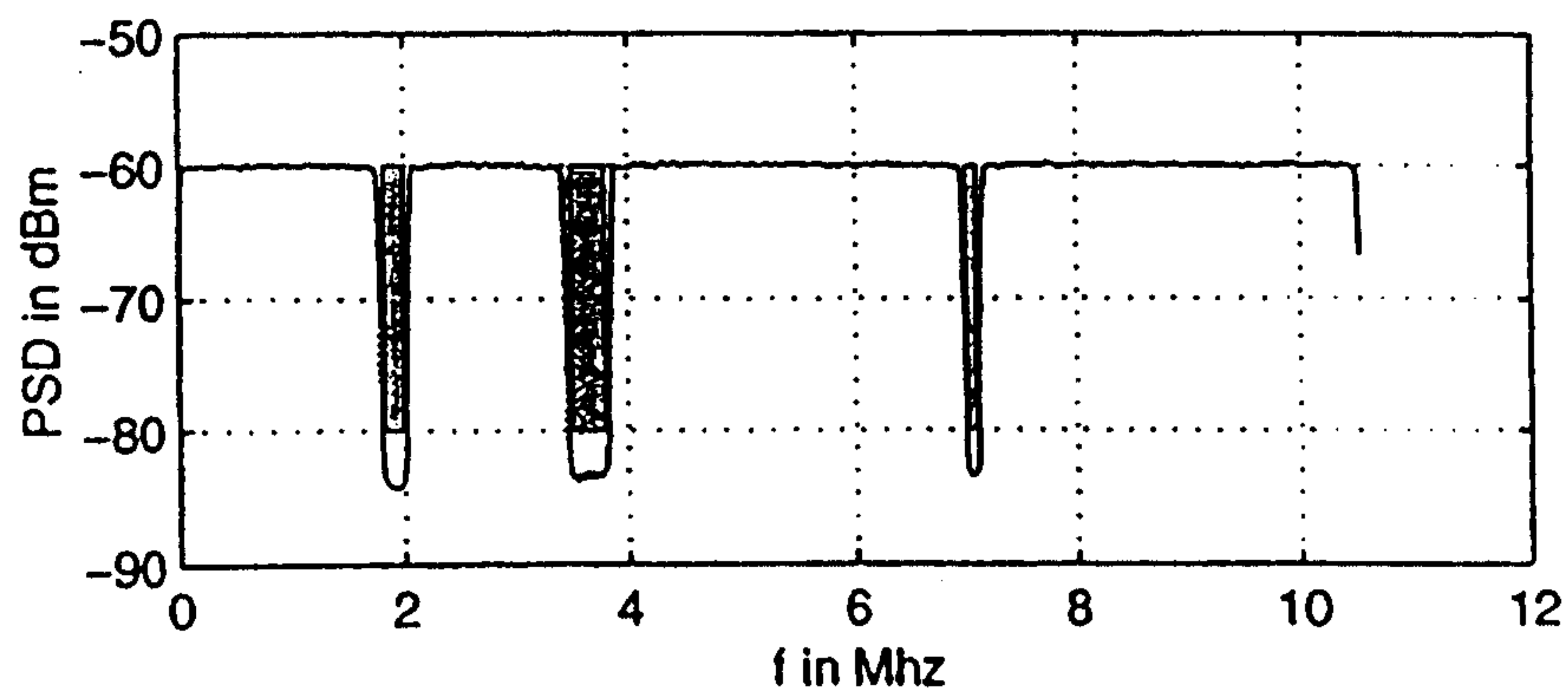


FIG. 44

a

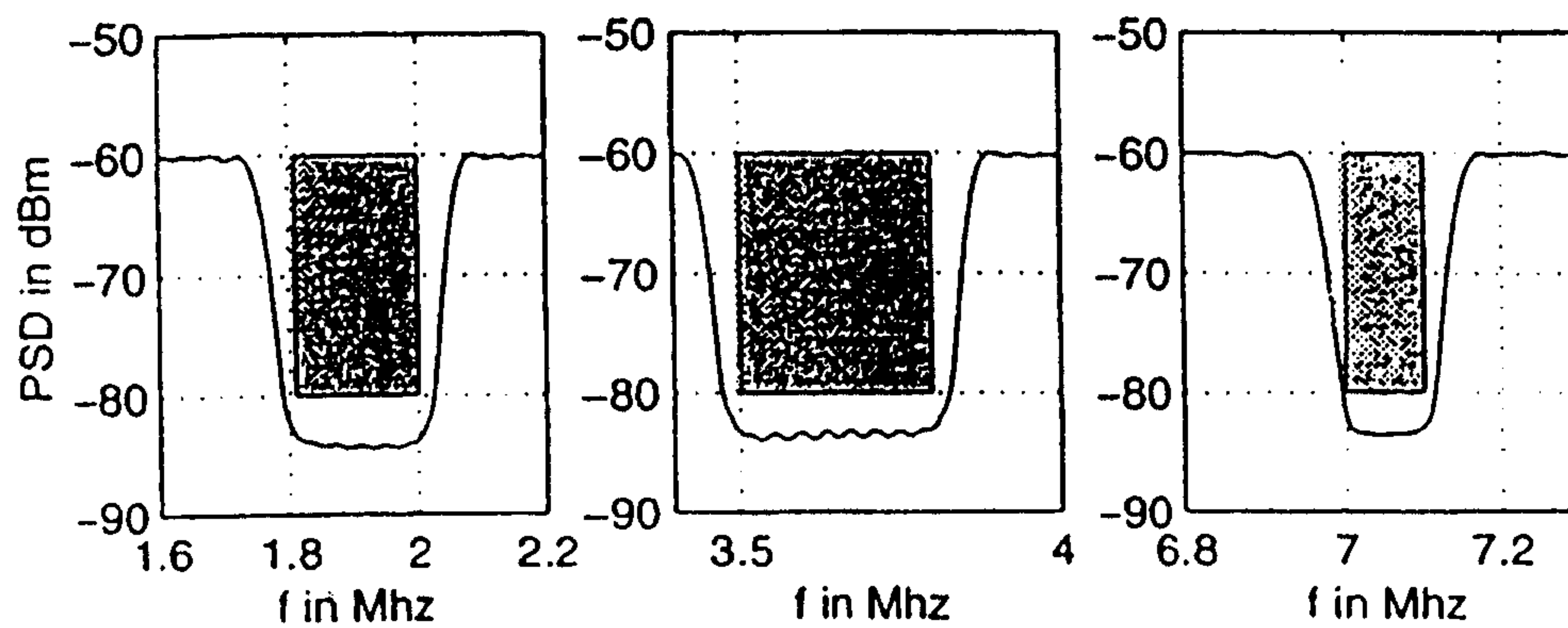


FIG. 45

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