

March 12, 1940.

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2,192,991

TAPERED BAND WIDTH DEAD-END FILTER

Filed Oct. 7, 1938

2 Sheets-Sheet 1

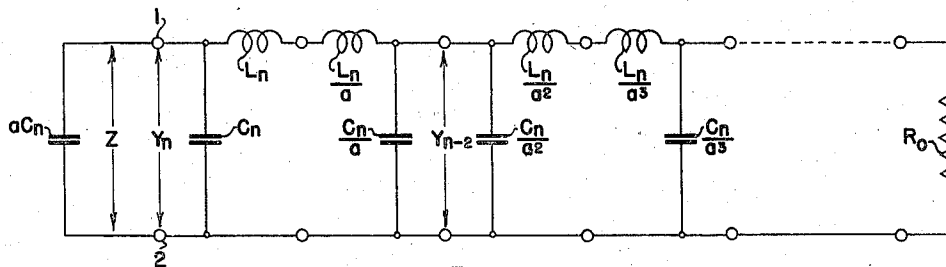


FIG. 1.

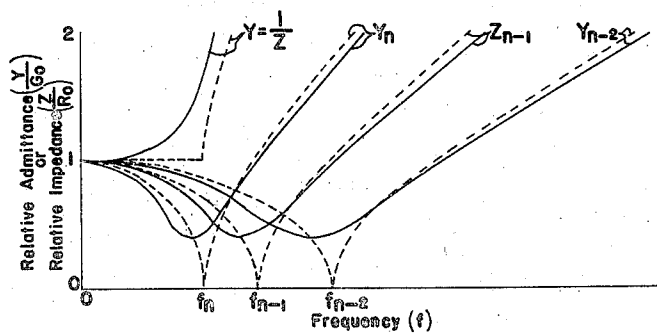


FIG. 2.

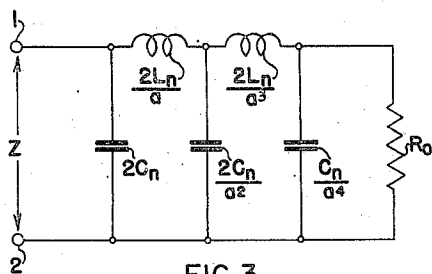


FIG. 3.

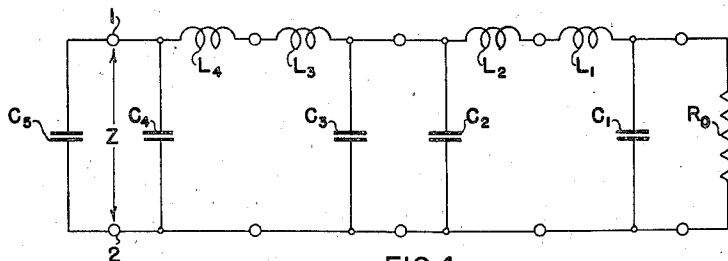


FIG. 4.

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2 Sheets-Sheet 2

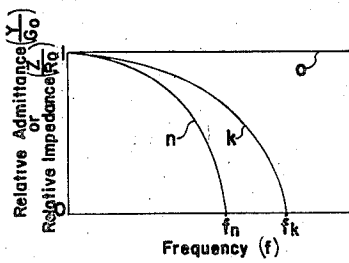


FIG. 5.

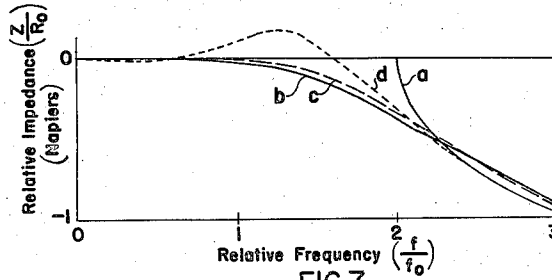


FIG. 7.

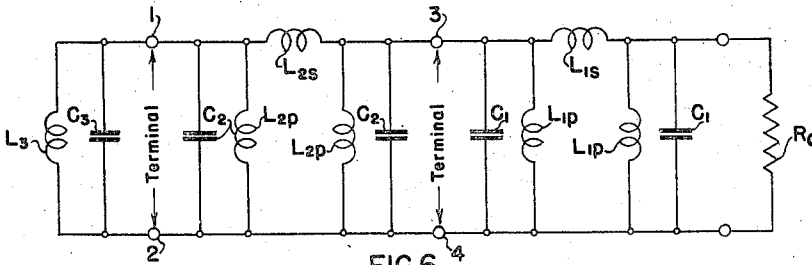


FIG. 6.

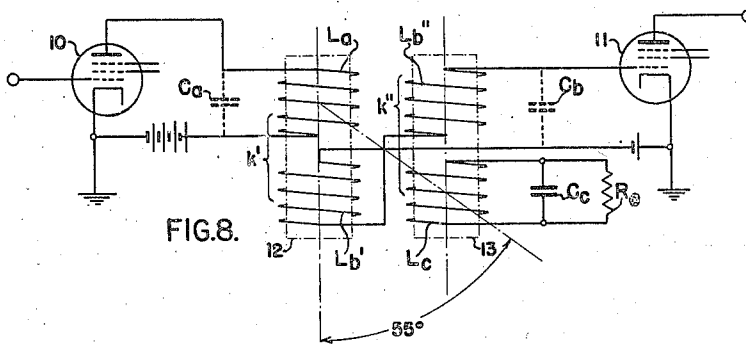


FIG. 8.

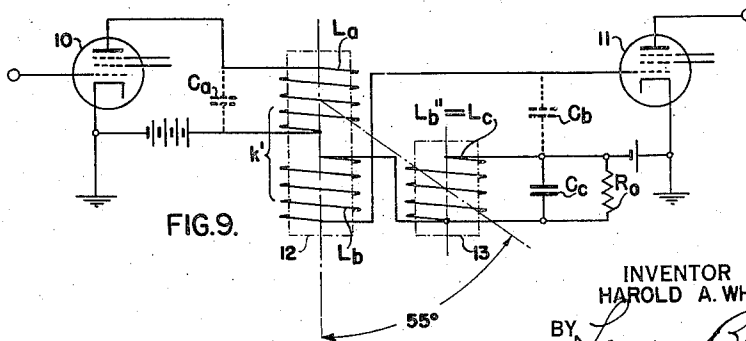


FIG. 9.

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2,192,991

TAPERED BAND WIDTH DEAD-END FILTER

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Application October 7, 1938, Serial No. 233,727

9 Claims. (Cl. 178-44)

This invention relates generally to coupling systems and particularly to coupling systems including a pair of terminals between which there is substantial capacitance and across which it is desired to build up a high impedance which is substantially uniform over a wide range of frequencies.

In many coupling arrangements, it is desirable to build up a substantially uniform impedance having a limiting and uniform value over a wide range of frequencies across terminals having susceptance thereacross, that is, it may be desirable to build up a substantially uniform impedance having the highest possible mean value over a wide range of frequencies across terminals having capacitance in parallel therewith. In other words, it may be desired to maintain uniform at approximately its maximum, the mean value of impedance across the terminals over the wide range of frequencies, or, in other words, to maintain the response of the system substantially uniform and substantially the maximum that can be maintained across the terminals over the wide frequency range. For instance, in the design of vacuum-tube amplifiers to pass a wide range of frequencies, it is desirable to build up across the inherent capacitance of the tubes and tube circuits to be coupled the maximum impedance that can be maintained substantially uniform over the operating frequency range of the amplifier. The value of the impedance which can be maintained uniform over a given frequency range is limited by the inherent capacitance and inductance of the tube circuits to be coupled. In applicant's Patents Nos. 2,167,134; 2,167,135; 2,167,136; and 2,167,137; there are disclosed coupling circuits designed for the above purpose including a dead-end filter coupled to the terminals in question and terminated in its image impedance. This means of terminating well-known types of filters is based on the concept of designing all the sections of a given filter with the same cutoff frequencies. It is not practicable with this type of filter termination, to obtain an absolutely uniform response out to the cutoff frequencies of the system, because attenuation and image impedance cannot be maintained exactly uniform as far out as the cutoff frequencies.

It is an object of the present invention, therefore, to provide a coupling system comprising terminals across which there is susceptance wherein a limiting mean value of impedance is maintained across said terminals over a wide frequency range.

It is another object of the invention to provide

a coupling system comprising terminals across which there is shunt capacitance wherein a maximum and substantially uniform impedance is maintained across said terminals over a wide frequency range and in which the response of the system is substantially constant out to the cutoff frequencies of the pass band.

It is another object of the invention to provide a coupling system of the type described having a maximum impedance over a wide frequency range and suitable for coupling two successive tubes of a vacuum-tube amplifier.

The present invention relates to filters and in accordance with one embodiment relates to dead-end filters in which a uniform image impedance over a wide band is obtained. The method utilizes a filter of tapered band width and involves a departure from confluent filter design in that the cutoff frequencies of the filter as a whole are not critically determined. The cutoff becomes more gradual and the phase distortion becomes much less near the cutoff frequencies in a filter of tapered band width. The pass bands of the sections of the filter of the invention increase gradually from section to section near the dead-end termination in order to minimize mismatching of the image impedances at the junctions of the sections. The filter section at the dead-end termination has a pass band much wider than the useful pass band of the entire filter, so that its image impedance is more nearly uniform over the useful frequency band. The variation of the image impedance of this end section near its own cutoff frequencies is of no consequence because that is outside the useful band of the system.

In dealing with filters of tapered band width, a new concept of iterative impedance is useful and is most readily understood by comparison of conventional filters with filters having tapered impedance instead of tapered band width. In the case of filters having a tapered impedance, the iterative impedance varies from section to section, for instance, exponentially, without any change of the frequency scale. On the other hand, a filter of exponentially tapered band width involves a change of the frequency scale from section to section, without any change of the scale of impedance as affecting either the magnitude or the shape of the curve of the iterative impedance. A filter of tapered band width operates as a frequency multiplier in the same manner that a filter of tapered impedance operates as an impedance multiplier (transformer). The iterative impedance characteristics of a filter of ta-

pered band width are those corresponding to the image impedance of nontapered filters.

An analysis of filters of tapered band width yields a new form of impedance characteristic which is similar to the image impedance characteristic of nontapered filters. It differs in that it has a gradual transition at the cutoff frequency instead of a critical point, as in the case of nontapered filters. The iterative impedance of filters of tapered band width is complex at all frequencies, although mainly resistive in the pass band and mainly reactive in the attenuation band. The gradualness of the transition at the cutoff frequency is proportional to the band width ratio from section to section.

In accordance with a preferred embodiment of the invention, the pass bands of the portions of a multisection dead-end filter are made to increase in a geometric progression toward the dead end of the filter. The increase of the pass bands widens the image impedance curves and the attenuation curves of the successive portions. The useful band is, however, the pass band of that portion of the filter having a minimum pass band. The remaining portions then have substantially no attenuation within the useful band and less attenuation just outside of the useful band than they would have if they were designed for the same band width, resulting in an overall uniform attenuation characteristic over the useful band, although the attenuation just outside the useful band is not as great as that obtained by the image impedance matching method of terminating the dead-end filter. This provides a gradual cutoff characteristic, which is sometimes desired.

Preferably, the pass band of each portion of the filter is proportioned as an exponential function of the number of the portion, the portions being numbered serially from the dead-end termination of the filter. The exponent of the function may be any number less than unity. Also, in the preferred embodiment of the invention, the above-mentioned portions are filter half-sections although in other embodiments the filter is tapered by whole-sections.

The novel features believed to be characteristic of the invention are set forth with particularity in the appended claims. The invention itself, however, both as to its organization and method of operation, together with other and further objects and advantages thereof, will best be understood by reference to the specification taken in connection with the accompanying drawings, in which Fig. 1 is a circuit diagram of a low-pass filter of exponential taper; Fig. 2 is a graph representing the impedance characteristics of the of the filter of Fig. 1; Fig. 3 is a circuit diagram of a filter which is tapered from section to section; Fig. 4 is a circuit diagram of a filter which is tapered by half-sections; Fig. 5 is a graph illustrating certain of the operating characteristics of the circuit of Fig. 4; Fig. 6 is a circuit diagram of a band-pass filter tapered by sections; Fig. 7 shows the transfer impedance characteristics of the circuit of Fig. 6; Fig. 8 is a circuit diagram of a band-pass amplifier embodying a practical circuit which is the equivalent of the circuit of Fig. 6; while Fig. 9 is a diagram of a simplified circuit which is the equivalent of that of Fig. 8.

Referring now to the drawings, Fig. 1 is a circuit diagram of a simple low-pass filter having a band width tapered exponentially along a plurality of filter half-sections. It has the general configuration of a confluent constant- k filter, but

the circuit values vary exponentially along the filter instead of being the same in each section. The pass bands of the sections increase through a large number of sections and the filter is terminated by a resistor R_0 . There is exact matching of the image impedance at zero frequency at all junctions and at the terminal resistor R_0 . Therefore, the ratio of capacitance to inductance is the same in all half-sections, while the product of capacitance and inductance in any section is inversely proportional to the pass band of that section. The half-sections may be numbered serially from the terminal resistor R_0 , the n th half-section being at the end of the filter remote from R_0 . The band width progressively increases from half-section to half-section toward the far end in the inverse order of the serial number. The pass bands of the filter portions and the resistance termination are so proportioned with regard to the capacitance across its input terminals that the response of the system is substantially uniform over the useful pass band of the system and is substantially the maximum that can be maintained by the input terminals over the range.

The filter of Fig. 1 thus comprises a pair of terminals 1, 2, across which is coupled condenser aC_n and the impedance across which is Z , the admittance looking into the n sections of the filter coupled to terminals 1, 2 being designated Y_n . The filter thus comprises only a part (C_n) of the capacitance across terminals 1, 2 as a terminal mid-element of the filter. Coupled to the pair of terminals 1, 2 is a series of filter portions terminated by the resistor, the conductance of which is R_0 . Specifically, the filter portions are filter half-sections and comprise series-inductance arms $L_n, L_n/a, L_n/a^2, L_n/a^3$, etc., and shunt-capacitance arms $C_n, C_n/a, C_n/a^2, C_n/a^3$, etc., in which a is the ratio of the pass bands of consecutive filter half-sections.

In terms of the cutoff frequency f_n of the n th half-section, that of the first half-section is:

$$f_1 = f_n a^{n-1} \quad (1)$$

and that of the k th half-section is:

$$f_k = f_n a^{n-k} \quad (2)$$

The n th half-section includes the elements C_n and L_n and in terms of these elements, the circuit values of the first section are:

$$C_1 = \frac{C_n}{a^{n-1}}; \quad L_1 = \frac{L_n}{a^{n-1}} \quad (3)$$

and the circuit values of the k th section are:

$$C_k = \frac{C_n}{a^{n-k}}; \quad L_k = \frac{L_n}{a^{n-k}} \quad (4)$$

The total capacitance and inductance of a large number of half-sections approach the limiting values:

$$C = \frac{a}{a-1} C_n; \quad L = \frac{a}{a-1} L_n \quad (5)$$

The effect of an infinite number of filter portions is closely approximated by a few filter portions at the end remote from the dead end, with the remainder of the total inductance and capacitance inserted in single lumps.

Fig. 2 shows the impedance and admittance characteristic of the filter of Fig. 1, assuming an infinite number of sections. The dotted curves are ideal curves based on exact matching and the full-line curves represent corresponding characteristics of the tapered filter of the invention.

Curve Y shows the admittance across terminals 1, 2. Curve Y_n shows the admittance characteristic looking through n half-sections, curve Z_{n-1} the relative impedance looking through $n-1$ half-sections, etc. All the curves have the same shape but the frequency pass bands are multiplied by the constant a from curve to curve. Adjacent curves are plotted in terms of inverse quantities, admittance, and impedance to give a family of similar curves, because each half-section inverts the impedance characteristic in addition to changing the scale of frequency. This inversion is carried over from the inversion of the image impedance by a half-section.

The curves of Fig. 2 are drawn approximately to show the general shape. They are susceptible of computation in terms of an infinite series, or as the limit approached by increasing the number of half-sections. They represent a new impedance function which is a continuous function of frequency approximating the usual discontinuous image impedance. Since it is continuous, it can be realized very closely in a finite network.

The iterative impedance of the filter with exponentially tapered band width depends not only on the image impedance of the individual sections, but also on their phase delay and attenuation. Therefore, different sections having the same image impedance do not present the same iterative impedance with tapered band width, although the difference may be small. Since this iterative impedance approximates the nonuniform image impedance of a confluent filter, a filter with tapered band width may be used instead of an m -derived filter to match a nonuniform image impedance of a confluent filter with a constant resistance. The approximation is closer the less the rate of taper, involving a filter of more sections.

A tapered band width filter is especially useful as a dead-end filter to secure maximum uniform impedance Z_0 across a given value of capacitance C_0 over a given band width of $\Delta\omega$. The figure of merit for a network for this purpose has the upper limiting value:

$$Z_0 C_0 \Delta\omega = 2 \quad (6)$$

which is approached by a conventional filter network of many sections. This upper limit may be approached with fewer sections by the use of a tapered band width filter.

The synthesis of this uniform impedance involves first setting up a network in which a constant- k mid-shunt image impedance is approximately realized and then connecting in parallel a constant- k mid-shunt arm. This procedure is fully described and claimed in the above-mentioned patents. Fig. 1 illustrates a variation of this method as applied to a filter of exponentially tapered band widths. The capacitance aC_n , to be connected across C_n is the mid-shunt arm of the $(n+1)$ th half-section in the series. That is, the mid-shunt arm aC_n is related in the same manner as that in which elements of like kind are proportioned in the successive filter portions of the tapered band width filter to which aC_n is coupled. The resulting impedance Z is built up across the total capacitance:

$$C_0 = C_n + C_{n+1} = C_n(1+a) \quad (7)$$

The resulting impedance is:

$$Z = \frac{1}{j\omega C_n a + Y_n} \quad (8)$$

This has a magnitude approximately constant

and equal to R_0 nearly up to the cutoff frequency:

$$f_{n+1} = \frac{f_n}{a} \quad (9)$$

The magnitude of the impedance decreases gradually in the cutoff region, as shown by the characteristic of the curve $Y=1/Z$ in Fig. 2.

Since the results obtained by this method are only an approximation of the ideal, the relations are not critical. They may be modified by experiment if need be. Therefore, approximate, simple rules of design are useful. Fig. 3 is a circuit diagram of a tapered band width filter tapered in terms of full-series and full-shunt filter arms, instead of half-sections, as was done in the case of Fig. 1 and comprising only two full sections. The tapering factor a is applied in progressively increasing powers to the successive elements of the filter. The filter of Fig. 3 thus comprises a pair of terminals 1, 2, the impedance across which is Z , and two filter whole-sections, adjacent elements of the same type having been combined into a single element. The filter comprises series-inductance arms $2L_n/a$ and $2L_n/a^3$ and shunt-capacitance arms $2C_n$, $2C_n/a^2$, and C_n/a^4 , the filter being terminated in resistor R_0 . The performance of the resulting tapered filter is approximately that of Fig. 1, with uniform impedance across the terminal capacitance $2C_n$.

If only a few sections are to be used in a filter with tapered band width, the exponential taper may not be the optimum compromise. Also, there is the question of what the rate of taper should be for a filter of a given number of sections or half-sections. Fig. 4 is a circuit diagram of a tapered dead-end filter using another rule of taper for developing a uniform impedance over the useful frequency band across the terminal capacitance:

$$C_0 = C_4 + C_5 \quad (10)$$

This filter comprises four half-sections comprising mid-series inductance arms L_1 , L_2 , etc., and mid-shunt capacitance arms C_1 , C_2 , etc., and mid-shunt arm C_5 of a fifth half-section, numbered serially from the dead-end termination comprising resistor R_0 . The cutoff frequencies of the individual half-sections may likewise be determined as a similar function of their serial numbers to obtain an optimum relation between the cutoff frequencies.

The curves of Fig. 5 indicate the mid-shunt image admittance or the mid-series impedance characteristics of certain half-sections in the tapered filter of Fig. 4, curve o representing the uniform impedance or admittance of the terminal resistor, curve k representing the characteristic of any half-section of serial number k , and curve n the corresponding characteristic of the terminal half-section. It is desirable to distribute the mismatching of the image impedances equally among the several junctions of the filter so as to minimize its effect. This means that the intermediate curves k should fall between the extreme curves in geometric progression on the scale of ordinates. This is approximately realized over the majority of the useful band if the curves are uniformly spaced at the lower frequencies.

In order to secure this spacing, the pass bands of the filter sections should be tapered in the inverse ratio of the square root of their serial numbers:

$$f_i = f_n \sqrt{n}; \quad f_k = \frac{f_i}{\sqrt{k}} = f_n \sqrt{\frac{n}{k}} \quad (11)$$

In the low-pass example of Fig. 4, the circuit values for this relationship are:

$$C_k = C_n \sqrt{\frac{k}{n}}; L_k = L_n \sqrt{\frac{k}{n}} \quad (12)$$

In general, the circuit elements may be evaluated in each half-section according to its band width.

This method of design is equally applicable to band-pass filters by following the same rules.

- 10 The geometric mean frequency is maintained the same in all sections as the band width is tapered. There are well-known band-pass filters analogous to the low-pass filters of Figs. 1, 3, and 4. Fig. 6 is a circuit diagram of a band-pass example which cannot be divided into half-sections by the usual rules. The tapering of the filter of Fig. 6 is by whole-sections, of which two are shown, the series inductance of each section being designated by the subscript "s" in addition to the subscript relating to the serial number of its section and the parallel-connected inductance being correspondingly designated by the subscript "p". The third portion of the filter is a constant- k mid-shunt arm comprising parallel-connected inductance L_3 and capacitance C_3 . This filter is preferably designed by the square-root rule of tapering and then presents uniform impedance across the terminal capacitance C_3 over the useful band width. Also, it presents uniform transfer impedance between the two pairs of terminals designated.

The curves of Fig. 7 show the relative transfer impedance of a four-terminal network having the circuit of Fig. 6. The terminals 1, 2 and 3, 4 of the filter are indicated in Fig. 6 and are separated by one section of the filter at the end remote from the dead end. The curves are plotted in terms of the frequency departure from mid-band relative to the mean frequency of the band:

$$\omega_o = \frac{1}{2Z_o \sqrt{C_a C_b}} \quad (13)$$

in which C_a and C_b are, respectively, the total capacitance across the input terminals 1, 2 and across the output terminals 3, 4, and Z_o is the mid-band transfer impedance.

In Fig. 7, curve a is the idealized curve for a large number of sections; curve b is computed for the particular filter circuit of Fig. 6 in which the cutoff frequencies, in terms of departure from mid-band, are in the proportion:

$$f_1:f_2:f_3 = 1:\frac{2}{1+\sqrt{5}}:\frac{2}{1+\sqrt{5}} = 1:\frac{1}{1.62}:\frac{1}{1.62} \quad (14)$$

$$\frac{f_1}{f_o} = \frac{1+\sqrt{5}}{2} \sqrt{1+\sqrt{5}} = 2.91$$

$$\frac{f_2}{f_o} = \frac{f_3}{f_o} = \sqrt{1+\sqrt{5}} = 1.80 \quad (15)$$

Curve c represents the characteristic for the filter of Fig. 6 tapered in accordance with the square-root rule:

$$f_1:f_2:f_3 = 1:\frac{1}{\sqrt{2}}:\frac{1}{\sqrt{3}} = 1:\frac{1}{1.41}:\frac{1}{1.73}$$

$$\frac{f_1}{f_o} = 2.76; \frac{f_2}{f_o} = 1.94; \frac{f_3}{f_o} = 1.58 \quad (16)$$

Curve d represents the filter characteristic for untapered band width. The square-root rule gives a good practical compromise.

The terminals of the dead-end filters above described have been indicated. It will be understood, however, that the circuits may be used having a single terminal circuit or having a plu-

rality of terminal circuits in the manner fully described in the above-mentioned patents.

Fig. 8 is a circuit diagram of a band-pass amplifier using a practical circuit equivalent to the filter of Fig. 6 for coupling the output circuit of a vacuum tube 10 to the input circuit of a vacuum tube 11. This filter comprises three resonant circuits $L_a, C_a; L_b', L_b'', C_b$; and L_c, C_c . Condensers C_a and C_b have been shown by dotted lines to indicate that they may be comprised in whole or part of the inherent capacitance of the circuit in which they are connected. Since no coupling is desired between tuned circuit L_a, C_a and tuned circuit L_c, C_c , the circuit L_b', L_b'', C_b has its inductance split into two parts L_b' and L_b'' , coupled respectively with L_a and L_c of the first and third circuits. If parts L_b' and L_b'' were equal, the coefficient of coupling k' between inductances L_a and L_b' would be much less than the coefficient of coupling k'' between inductances L_b'' and L_c . These coefficients are equalized by making L_b' less than L_b'' . The coil arrangement shown has each pair of coupled coils on one of two parallel coil forms 12 and 13. Zero coupling between inductances L_a and L_c is procured by separating the coil forms a suitable distance and so disposing them that their line of centers and their axes make an angle of about 55 degrees. Coils L_b' and L_b'' are at least approximately uncoupled so that they can be moved relative to coils L_a and L_c , respectively, for adjusting the coupling between successive circuits without appreciable change of the total inductance of the circuit L_b', L_b'', C_b . The dead-end circuit may have any convenient value of impedance so that L_c may be identical to L_b' . The shunt condenser C_c and resistor R_c are proportioned accordingly. All three circuits are individually resonant at or near the mid-band frequency of the system.

In the design of the circuit of Fig. 8 to meet the critical conditions which give curve b of Fig. 7, the following formulae are followed; they are applicable when the band width is much less than the mean frequency of the band. First, each capacitance C_a and C_b is determined; then each inductance is computed to resonate each circuit at the mean frequency ω_m :

$$L_a = \frac{1}{C_a \omega_m^2}; L_b = \frac{1}{C_b \omega_m^2} \quad (17)$$

The inductance of the second circuit is divided in such a ratio as to secure equal coefficients of coupling k' and k'' :

$$L_b' = \frac{1}{2+\sqrt{5}} L_b; L_b'' = \frac{1+\sqrt{5}}{2+\sqrt{5}} L_b \quad (18)$$

For convenience, L_b' and L_c are made alike, which then determines C_c :

$$L_c = L_b'; C_c = \frac{1}{L_c \omega_m^2} \quad (19)$$

The coupling and damping determine the band width. By reference to curve b of Fig. 7, the value of ω_o or f_o is chosen, which is about half the band width. From this value the coupling and damping are computed:

$$k' = k'' = \frac{2\omega_o}{\omega_m} \sqrt{2+\sqrt{5}} = 2.06 \frac{2\omega_o}{\omega_m} \quad (20)$$

$$G_c = 2\omega_o C_c \sqrt{4+2\sqrt{5}} = 2.91 (2\omega_o C_c) \quad (21)$$

The resulting transfer impedance at mid-band is:

$$Z_o = \frac{1}{2\omega_o \sqrt{C_a C_b}} \quad (22)$$

If the band width is comparable with the mean

frequency of the band so that the approximate formulae fail, the circuit is designed from filter formulae starting with Fig. 6. The geometric mean frequency is the same in all sections and the image impedance is matched at each junction at that frequency. Any desired rule of tapering may be employed to determine the relative band widths of the filter section. The dead-end circuit C_c , L_c may be utilized also to couple the signal to another tube in an auxiliary channel of the system.

Fig. 9 is a circuit diagram of a simplified arrangement equivalent to the circuit of Fig. 8. The circuit of Fig. 9 requires less coupling between the windings of the system and requires fewer coils, but has less freedom in adjusting the coupling and adjusting the individual circuits. Elements which are identical in the two figures have been given similar reference numerals. The following formulae are applicable to the circuit of Fig. 9 if the band width is much less than the mean frequency of the system. The formulae for L_a and L_b are the same as Equation 17 above:

$$L_b'' = L_c = (1 + \sqrt{5}) L_b \left(\frac{2\omega_o}{\omega_m} \right)^2 = 3.24 L_b \left(\frac{2\omega_o}{\omega_m} \right)^2 \quad (23)$$

$$L_b' = L_b - L_b''; \quad k' = \frac{2\omega_o}{\omega_m} \sqrt{\frac{L_b}{L_b'}} \quad (24)$$

$$C_c = \frac{1}{L_c \omega_m^2} = \frac{C_b}{1 + \sqrt{5}} \left(\frac{\omega_m}{2\omega_o} \right)^2 = 0.308 C_b \left(\frac{\omega_m}{2\omega_o} \right)^2 \quad (25)$$

The formulae for G_c and Z_o are the same as Equations 21 and 22 above.

It will be understood that the square-root rule of tapering generally gives an approximation for the critical conditions of a uniform impedance over a wide frequency band. If a wider band is desired at the expense of uniformity of impedance, this rule may be compromised by utilizing the cube-root or even the fourth-root in the taper formulae, the results then being more like the untapered filter. That is to say, instead of proportioning the filter portions so that their band widths vary inversely to the square-root of the serial number of filter portions from the dead end of the filter, the band widths of the filter portions may be proportioned to vary inversely in accordance with a greater root of the serial number of filter portions.

While there have been described what are at present considered to be the preferred embodiments of this invention, it will be obvious to those skilled in the art that various changes and modifications may be made therein without departing from the invention, and it is, therefore, aimed in the appended claims to cover all such changes and modifications as fall within the true spirit and scope of the invention.

What is claimed is:

1. A signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial reactance tending to render the response of said system nonuniform over said range, a multisection dead-end filter coupled to said pair of terminals, said filter comprising only part of said reactance as a terminal, mid-element of said filter, said filter comprising portions the band widths of which increase progressively toward said dead end, a resistance termination for said filter at said dead end matching the image impedance thereof over the useful pass band, the pass bands of said filter portions and said resistance termination being so proportioned with respect to said reactance that the response of said

system is substantially uniform over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

2. A signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial capacitance tending to render the response of said system nonuniform over said range, a multisection dead-end filter coupled to said pair of terminals, said filter comprising only part of said capacitance as a terminal mid-shunt element of said filter, said filter comprising portions the band widths of which increase progressively toward said dead end, a resistance termination for said filter at said dead end matching the image impedance thereof over the useful pass band, the band widths of said filter portions and said resistance termination being so proportioned with respect to said capacitance that the response of said system is substantially uniform over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

3. A signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial reactance tending to render the response of said system nonuniform over said range, a multisection dead-end filter coupled to said pair of terminals, said filter comprising only part of said reactance as a terminal mid-element of said filter, said filter comprising a series of half-sections the band widths of which increase toward said dead end, a resistance termination for said filter at said dead end matching the image impedance of said filter over the useful band width, the band widths of said half-sections and said resistance termination being so proportioned with respect to said reactance that the response of said system is substantially uniform over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

4. A signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial reactance tending to render the response of said system nonuniform over said range, a multisection dead-end filter coupled to said pair of terminals, said filter comprising only part of said reactance as a terminal mid-element of said filter, said filter comprising filter whole-sections the band widths of which increase toward said dead end, a resistance termination for said filter at said dead end matching the image impedance of said filter over the useful band width, the band widths of said filter whole-sections and said resistance termination being so proportioned with respect to said reactance that the response of said system is substantially uniform over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

5. A signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial reactance tending to render the response of said system nonuniform over said range, a multisection dead-end filter coupled to said pair of terminals, said filter comprising only part of said reactance as a terminal mid-element of said filter, said filter comprising portions the band widths of which vary exponentially toward said dead end, said mid-element bearing an exponen-

tial relation to elements of like kind in said portions similar to that between said elements of like kind, and a resistance termination for said filter at said dead end matching the image impedance of said filter over the useful band width, whereby the response of said system is substantially uniform over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

6. A signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial reactance tending to render the response of said system nonuniform over said range, a multisection dead-end filter coupled to said pair of terminals, said filter comprising only part of said reactance as a terminal mid-element of said filter, said filter comprising portions the band widths of which vary inversely in accordance with the square-root of the serial numbers of the portions counted from said dead end, said mid-element bearing a relation to elements of like kind in said filter portions similar to that between said elements of like kind, and a resistance termination matching the image impedance of said filter at said dead end, whereby the response of said system is substantially uniform over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

7. A signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial reactance tending to render the response of said system nonuniform over said range, a multisection dead-end filter coupled to said pair of terminals, said filter comprising only part of said reactance as a terminal mid-element of said filter, said filter comprising portions, the band widths of which vary inversely as a root of an order higher than the square-root of the serial numbers of the portions counted from said dead end, said mid-element bearing a relation to elements of like kind in said filter portions similar to that between said elements of like kind, a resistance termination matching the image impedance of said filter at said dead end, whereby the response of said system is substantially uni-

form over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

8. A band-pass signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial reactance tending to render the response of said system non-uniform over said range, a multisection band-pass dead-end filter coupled to said pair of terminals, said filter comprising only part of said reactance as a terminal mid-element of said filter, said filter comprising portions including a mid-series arm having inductance and a mid-shunt arm comprising a parallel resonant circuit, the band widths of said portions increasing toward said dead end, a resistance termination for said filter at said dead end matching the image impedance of said filter over the useful band width, the band widths of said filter portions and said resistance termination being so proportioned with respect to said reactance that the response of said system is substantially uniform over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

9. A low-pass signal-translating system for operation over a range of frequencies comprising at least one pair of terminals between which there is substantial reactance tending to limit the response of said system over said range, a multisection low-pass dead-end filter coupled to said pair of terminals, said filter comprising only part of said reactance as a terminal mid-element of said filter, said filter comprising portions including series inductance and shunt capacitance, the band widths of said portions increasing toward said dead end, a resistance termination for said filter at said dead end matching the image impedance of said filter over the useful band width, the band widths of said filter portions and said resistance termination being so proportioned with respect to said reactance that the response of said system is substantially uniform over the useful pass band of said system and is substantially the maximum that can be maintained between said pair of terminals over said range.

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