July 1, 1958 S. LEES 2,841,332

TORSIONAL FOURIER TRANSFORMER

Filed June 22, 1955

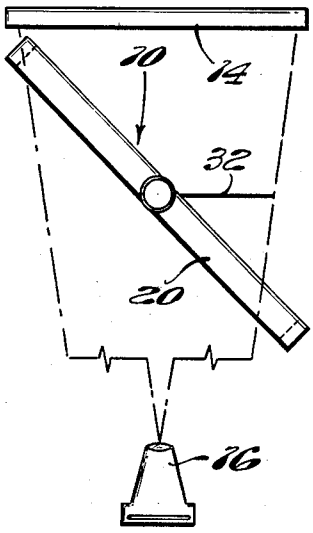


Fig. 1

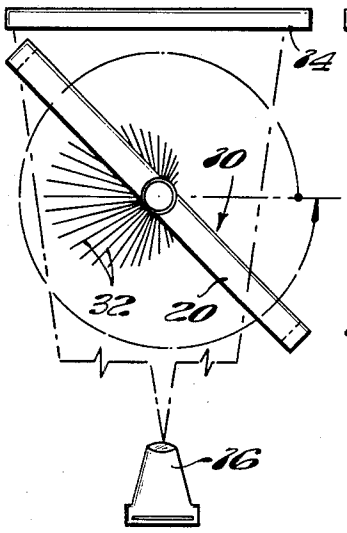


Fig. 3

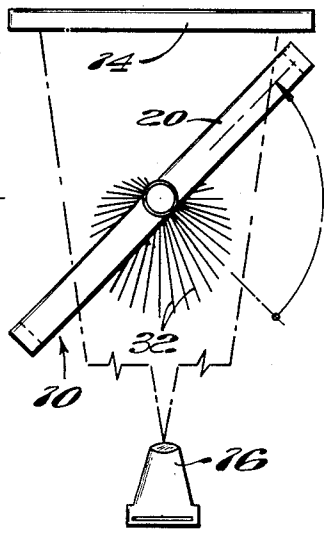


Fig. 5

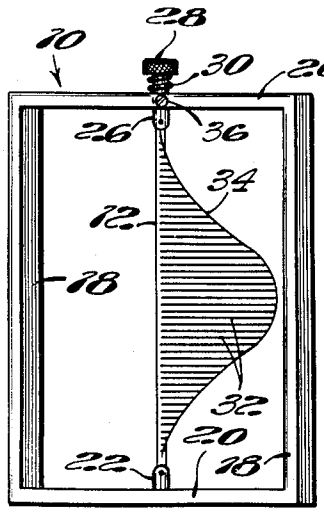


Fig. 2

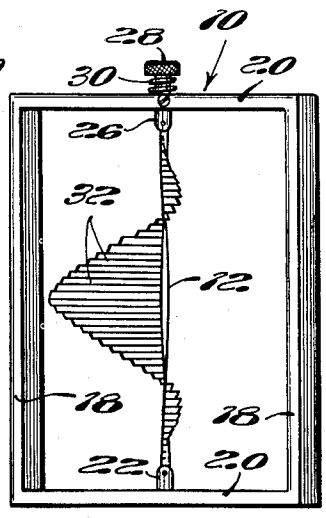
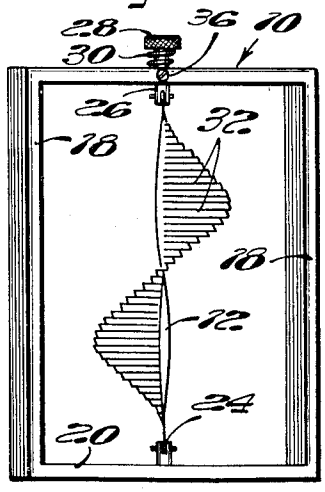


Fig. 4

Fig. 6



INVENTOR.
Sidney Lees
BY
George L. Greenfield
His Attorney

United States Patent Office

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TORSIONAL FOURIER TRANSFORMER

Sidney Lees, Newton, Mass.

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13 Claims. (Cl. 235—61)

This invention comprises a new and useful method and apparatus for obtaining the Fourier transform of an arbitrary function.

In its most general form the Fourier transform is defined by the equation:

$$(1) \qquad [FT]F(x)=\int_{-\alpha}^{\alpha} F(x)e^{-izx}dx$$

where j represents the complex quantity $\sqrt{-1}$. In practice the equation usually indicates a transformation from the time to the frequency domain and the relationship is defined by the following equation:

$$(2) \qquad [FT]F(t)=\int_{-\alpha}^{\alpha} F(t)e^{-iwt}dt$$

where t is time and w is frequency.

Computers now available and capable of yielding the transform of arbitrary functions are expensive. Although some of the computers are capable of determining the transform with a high degree of accuracy, they usually demand a considerable amount of labor from the operator.

The primary object of my invention is to provide a practical and inexpensive Fourier transformer. To accomplish this and other objects, my torsional Fourier transformer includes a rectangular frame which carries a torsion spring. The torsion spring is fixed at one end to the intermediate portion of one of the sides of the frame, and its free end is rotatably held by the opposite side. The end of the spring rotatably secured to the opposite side of the frame may be turned through any desired number of degrees by a knob carried on that end. Provision is made to retain the movable end of the spring in the position selected by the operator. A stiff piece of paper or cardboard is cemented to the spring and the function to be transformed is plotted on the paper with the spring serving as the time axis. Next, the paper is cut along the plotted curve and further cut into narrow strips perpendicular to the spring.

Before proceeding to a more detailed description of the illustrated embodiment of my invention, the torsional analogy of the device will be presented. The basis of the torsional Fourier transformer is the analogy between the physical angle of twist of a torsion spring and the kernel e^{-jwt}, which appears in the transform equation. Mathematically, the kernel indicates that a unit length is to be rotated through a variable angle in a complex plane. It is necessary to relate the physical variables of the spring to the mathematical variables w and t.

It is apparent that a shaft of uniform cross-section, fixed at one end, will have a constant angle of twist per unit length when the other end of the shaft is twisted. Thus, the angular displacement of any cross-section along the shaft is a linear function of two variables, i. e., the angle through which the free end of the shaft is twisted and the linear distance from the cross-section to the fixed end. By relating the variables w and t (frequency and time) to the angle of twist and the linear distance along

the shaft, respectively, the analogy of cross-section angular displacement to the kernel e^{-jwt} becomes evident.

However, the integrand appearing in the Fourier transform equation is the product of the kernel and the function $F(t)$. To introduce the function $F(t)$ into the apparatus, it is necessary to modify the cross-section of the spring in a manner analagous to the variation of the function. This must be accomplished without changing the linearity of twist per unit length of the spring. To do this requires the addition of material which imposes no additional resistance to sheer in a plane perpendicular to the axis of the spring, but which possesses rigidity in all planes containing the spring axis. This is accomplished in my device by adding the stiff paper or cardboard and plotting the function to be transformed on it. By cutting the paper into independent narrow strips separate from each other, the condition that there be no restraint in sheer is approximately achieved. The degree to which the approximation is satisfactory is dependent upon the width of the individual strips.

If $F(t)$ is a real function, the plane of the spring and the plotted curve is a real plane, and the plane perpendicular to the real plane and which contains the spring represents the imaginary plane. If the movable end of the spring is then rotated, projected areas will be seen in both the real and imaginary planes, and those areas result in the two components of the desired frequency function. The governing equation for the process is:

$$(3) \qquad [FT]_{\text{approx.}} F(t)=\sum_{i=1}^{n} F(t)[\cos(wi\Delta t)-j\sin(wi\Delta t)]\Delta t$$

These relationships will be better understood and appreciated from the following detailed description of a preferred embodiment of my invention, selected for purposes of illustration and shown in the accompanying drawing in which:

Figure 1 is a plan view of a Fourier transformer embodying my invention,

Figure 2 is an elevation view of a portion of the transformer illustrated in Figure 1,

Figure 3 is a plan view of the transformer illustrated in Figure 1 with the free end of the spring rotated through 360°,

Figure 4 is an elevation view of a portion of the transformer positioned as shown in Figure 3,

Figure 5 is a plan view of the transformer illustrated in Figure 1 and showing parts of the apparatus in a third position, and

Figure 6 is an elevation view of a portion of the transformer positioned as shown in Figure 5.

The embodiment of my invention illustrated in the drawings includes in its general organization a frame **10** carrying a torsion spring **12**, a light box **14** and a camera **16**.

The frame **10** which may be made of any desired material comprises a pair of vertical channels **18** and a pair of horizontal channels **20**. A post **22** having a vertical slot **24** is secured to the midportion of the lower horizontal member **20** and receives one end of the spring **12**. The spring **12** extends vertically across the frame and is anchored to a shaft **26** rotatably mounted on the midportion of the upper horizontal channel of the frame. The lower portion of the shaft **26** is slotted to receive the upper end of the spring **12** in the same manner as the post **22**, and the upper end of the shaft **26** carries a knob **28**.

The torsion spring **12** may take any form, that is, it may be flat or coiled. Essentially, any spring, which has the characteristic that the angle of twist per unit length is constant, may be employed. A compression spring **30** surrounding the shaft **26** and bearing against the knob **28**

and the upper horizontal channel 20 prevents the spring 12 from sagging. It is important, however, that the coil spring 30 not have sufficient strength to retain the original length of the spring 12 when the upper end of the spring is twisted. If the compression spring 30 were of sufficient strength to preserve the original length, stresses in the spring 12 resulting from its being twisted would no longer be pure sheer; therefore, the necessary linearity between angle of twist and length from the fixed end would be destroyed.

The spring 12 carries a number of thin strips 32 of a length determined by the curve 34 of the plotted function $F(t)$. The function actually plotted on the strips in the illustrated embodiment is a displaced cosine pulse. In practice, the sheet of material from which the individual strips are formed must have sufficient rigidity so that they will remain perpendicular to the spring axis. Corrugated paper and cardboard have proved satisfactory. Obviously, countless other materials may be used; therefore, the breadth of this invention is not to be limited to those specific materials.

In Figure 4, the knob 28 fixed to the shaft 26 is shown rotated through 360°. A set screw 36 passing through the upper horizontal channel of the frame 10 may be tightened to bear against the shaft 26 to retain it in the turned position against the action of the spring. Because the spring 12 is twisted, the function defined by the length of the strips 32 has projected areas in both the original plane of the plotted function and the plane which lies normal to the original plane and contains the spring 12. Since the original function (a displaced cosine pulse), represented by the length of the strips 32, is a real function, the function shown in Figure 2 lies in the real plane. The plane perpendicular to the real plane and which contains the shaft axis represents an imaginary plane. This corresponds to the Cartesian coordinate system of complex numbers.

The camera 16 is provided to obtain a physical representation of the projected areas of the function in both the real and imaginary planes.

In Figure 3, the camera and the frame 10 are positioned for the photograph of the projected area in the real plane. To obtain the physical representation of the projected area in the imaginary plane, the frame 10 is rotated through 90°. The apparatus is shown in that position in Figure 5. Figures 4 and 6 illustrate what the camera records as the projected areas of the function in the real and imaginary planes, respectively. The third equation set forth in the introduction governs this process.

To complete the integration, the photographs may be cut along the projected curves of the function, and the paper may then be weighed. The light box 14 aids the photographing of the projected areas of the strips 32 by producing a silhouette of the strips in front of the camera. Obviously, it is a convenience which often may be unnecessary.

Having described in detail the embodiment of my invention its operation will now be described. A plot of the function to be transformed is first made on the paper which is cut into the narrow strips 32. For best results, these strips should be of equal width. Next, the strips are mounted on the spring 12, one end of which is fixed to the post 22 on the lower horizontal channel 20 of the frame 10. Initially, the frame is positioned so that the plane of the strips 32 is perpendicular to the line of sight between the light box and the camera mounted some distance away. The first picture is taken with the spring 12 untwisted, representing a zero value of frequency. After an angle of twist representing the desired frequency value is introduced into the spring by turning the knob 28, pictures are taken both with the frame in its initial position (the position illustrated in Figure 3), and after it is rotated through 90° (the position shown in Figure 5). The silhouettes of the function $F(t)$ represent the two components of the transformed function. Integration of these areas is then accomplished by cutting out the silhouettes on the photographic paper and weighing each of them on a balance. The weight of the photograph of the projected area of the function in the real plane represents the real part of the governing equation set forth above, while the weight of the paper used to record the projected area of the function in the imaginary plane represents the imaginary portion of that equation.

To obtain the inverse Fourier transform by my torsional device, the function is plotted, either in terms of frequency or a nondimensional frequency ratio, and the material upon which the function is plotted is cut into narrow strips in the same manner as described above. Various values of time t are introduced by means of shaft rotation, and integration of the projected areas is achieved by weighing the photographic paper.

From the foregoing description of one embodiment of my invention, those skilled in the art will appreciate that a number of modifications may be made thereof without departing from the spirit of my invention. For example, many other methods may be employed to perform the integration. Therefore, I do not intend to limit the scope of my invention by the foregoing detailed description, but rather it is my intention that its breadth be limited by the appended claims and their equivalents.

What I claim as new and desire to secure by Letters Patent of the United States is:

1. A method of obtaining the Fourier transform of a function comprising plotting the function on a sheet of material, twisting uniformly throughout its length the sheet of material containing the plotted function along the time axis of the function, obtaining projections of the area of the plotted function in the original plane of the function and in a second plane normal to the original plane and containing the time axis, and measuring the areas of the projections.

2. A method of obtaining a physical representation of a Fourier transform of an arbitrary function comprising the steps of plotting the function on a sheet of material having a constant angle of twist per unit length when one end is twisted, fixing one end of the sheet of material on the axis of the function, twisting the other end of the sheet about the axis of the function through a number of degrees proportional to the desired value of the domain to which the function is to be transformed, obtaining projections of the area of the plotted function in the original plane of that function and in a second plane containing the axis of the function and normal to the original plane, and measuring the areas of the projections.

3. Apparatus for obtaining the Fourier transform of a function comprising an elongated torsion spring, a sheet of material secured to the spring, a function to be transformed plotted on the sheet of material with the axis of the spring corresponding to the time axis of the function, means for fixing one end of the spring, means for retaining the other end of the spring in any selected position, said sheet of material being cut into narrow strips perpendicular to the axis of the spring, and means for obtaining a physical representation of the plotted function both in the original plane of the function and in a second plane containing the axis of the spring and normal to the original plane.

4. Apparatus for obtaining the Fourier transform of a function comprising a frame, a spring anchored at one end to one side of the frame, said spring extending across the frame with its other end rotatably secured to the opposite side of the frame, means for holding the said other end of the spring in any selected position, a sheet of material secured to the spring within the frame, a mathematical function to be transformed plotted on the sheet with the spring serving as the time axis, and means for recording the projected areas of the plotted function

in the original plane of the function and in a second plane containing the spring and normal to the original plane.

5. A method of obtaining a physical representation of a Fourier transform of an arbitrary function comprising the steps of plotting the function to be transformed on material having a constant angle of twist per unit length when one end of the material is fixed and the other end is twisted, fixing one end of the material carrying the plotted function on the axis of the function, twisting the other end of the axis of the function through a number of degrees proportional to the value of the domain to which the function is to be transformed, and recording the projected areas of the function in both the original plane of the function and in a second plane containing the axis of the function and normal to the original plane.

6. Apparatus for obtaining a physical representation of the Fourier transform of a mathematical function comprising a frame, a torsional device having a constant angle of twist per unit length when one end of the device is twisted fixed at one end to the frame, a shaft rotatably mounted on the opposite side of the frame, the other end of the torsional device being secured to the shaft, a plurality of narrow strips mounted on the spring with their long dimensions perpendicular to the spring, the length of said strips representing the function to be transformed, and means for recording the projected area of the strips in the original plane of the strips and in a second plane containing the spring and normal to the original plane after the shaft has been twisted through a number of degrees proportional to the value of the domain to which the function is to be transformed.

7. Apparatus for obtaining a physical representation of the Fourier transform of a mathematical function comprising a frame, a torsion spring having a constant angle of twist per unit length when one end is twisted fixed to one side of the frame, the other end of the torsional spring being rotatably secured to the opposite side of the frame, means for twisting the said other end of the spring through any desired number of degrees, a sheet of rigid material cut into narrow strips and secured to the spring, the width of the sheet from the spring representing the function to be transformed, and means for recording the projected area of the sheet in the original plane of the sheet and in a second plane perpendicular to original plane and containing the spring after the said other end of the spring has been turned through a number of degrees proportional to the value of the domain to which the function is to be transformed.

8. A method of obtaining a Fourier transformation of a function comprising the steps of fixing one end of a torsion spring having a constant angle of twist per unit length, securing to the spring a sheet of material having a plotted function to be transformed so that the axis of the spring coincides with the axis of the function, twisting the other end of the spring about its axis through a number of degrees proportional to the desired value of the domain to which the function is to be transformed, and recording the projected areas of the sheet in two mutually perpendicular planes which contain the axis of the spring.

9. A method of obtaining the sine and cosine functions of the kernel in the Fourier transform equation comprising the steps of fixing one end of a flat torsion spring, securing to the spring a sheet of material having a plotted function to be transformed so that the axis of the spring coincides with the axis of the function, twisting the other end of the spring so that it has a constant angle of twist per unit length, and recording the projected areas of the spring in the original plane of the spring and in a second plane perpendicular to the original plane and containing the spring axis.

10. Apparatus for obtaining a physical representation of the sine and cosine functions of the kernel in the Fourier transform equation comprising a flat torsion spring having a constant angle of twist per unit length, a sheet of material secured to the spring, a mathematical function to be transformed plotted on the sheet with the spring serving as the axis of the function, means for anchoring one end of the spring, means for rotating the other end of the spring about the center line of the spring, and means for recording the projected areas of the spring in the original plane of the spring and in a second plane perpendicular to the original plane and containing the center line of the spring after the said other end of the spring has been turned through a number of degrees.

11. A method of obtaining the inverse Fourier transform of a function comprising the steps of plotting the function to be transformed on a sheet of material, twisting the sheet of material with a constant angle of twist per unit length along the frequency axis of the plotted function, obtaining projections of the plotted function in the real and imaginary planes of the function and measuring the area of the projections.

12. A method of obtaining the Fourier transform of a function comprising the steps of plotting the function to be transformed on a sheet of material, twisting the sheet of material with a constant angle of twist per unit length along the axis of the plotted function, obtaining projections of the plotted function in the real and imaginary planes of the function, and measuring the areas of the projections.

13. Apparatus of the class described comprising torsional spring means, the function to be transformed plotted on the spring means with the axis of the spring means corresponding to the axis of the function, means for fixing one end of the spring means, means for retaining the other end of the spring means in any selected position, and means for obtaining a physical representation of the plotted function both in the original plane of the function and in a second plane containing the axis of the spring means and normal to the original plane.

No references cited.