United States Patent

[11] 3,623,800

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Continuation of application Ser. No. 518,848, Jan. 5, 1966, which is a continuation-in-part of application Ser. No. 292,380, July 2, 1963, now Patent No. 3,239,967, dated Mar. 15, 1966.

[54] **OPHTHALMIC LENS OF CHANGING POWER**
5 Claims, 12 Drawing Figs.

[52] **U.S. Cl.** **351/169,** 351/176, 351/177
[51] **Int. Cl.** **G02c 7/06**
[50] **Field of Search** 351/169, 171, 176, 177

[56] **References Cited**

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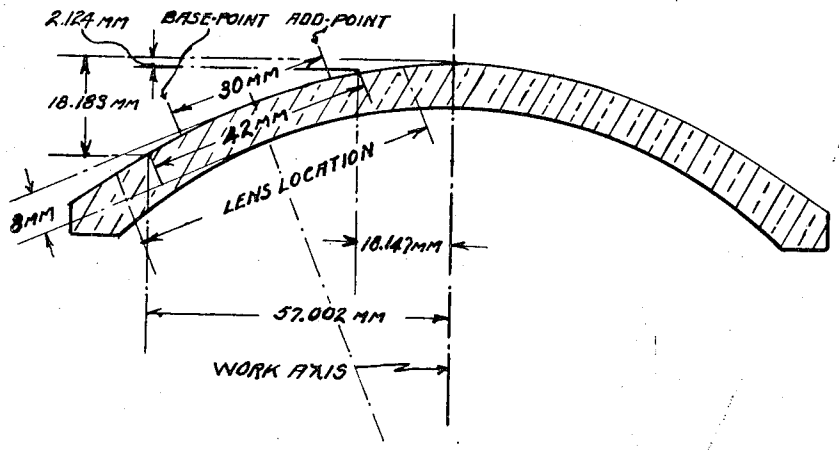
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ABSTRACT: An optical lens is provided, having a convex aspheric front surface useful for the correction of presbyopia. This front surface is a nonaxial portion of a convex surface of revolution, all meridian sections of which are identical elliptical arcs and all sections of this surface other than those sections normal to the axis of revolution being noncircular, the axis of revolution of said convex surface coinciding with a straight portion of the modified evolute of said elliptical arc. This front surface is characterized by having a substantially constant difference in principal curvatures at all points along all meridional sections providing a substantially constant astigmatism at all points outside the vertical principal meridian, while both principal curvatures along any elliptical arc meridian section change continuously and regularly by substantially equal amounts to provide an accelerating surface. This novel front surface is intended for use in a lens having a negatively curved spherocylindrical back surface optically coacting with the front surface and at least neutralizing the constant astigmatism of the front surface.



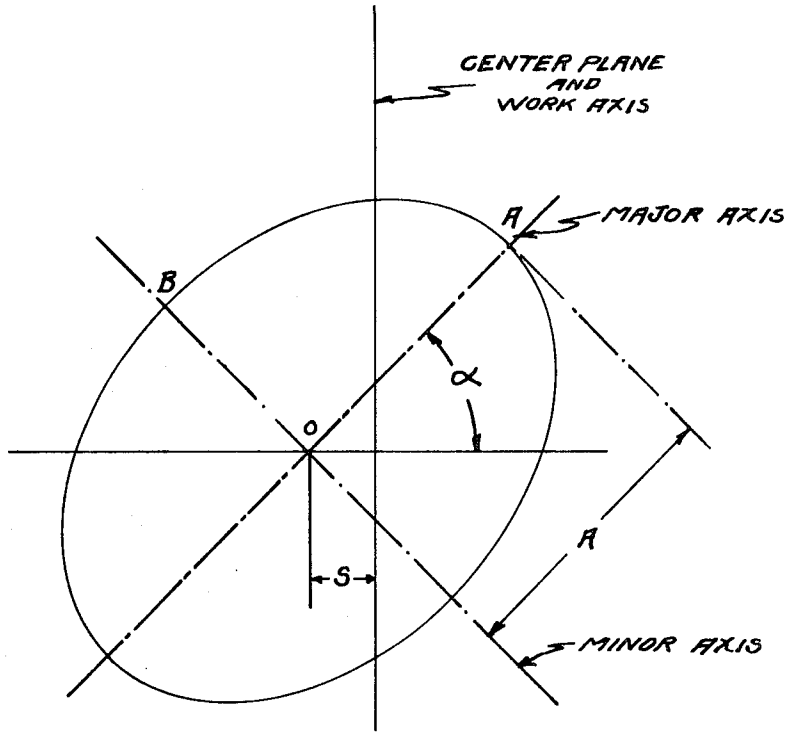


FIG. 1

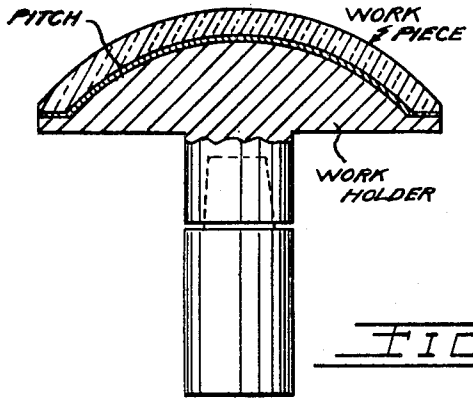


FIG. 2

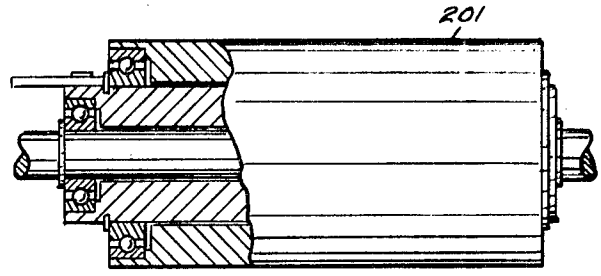


FIG. 5

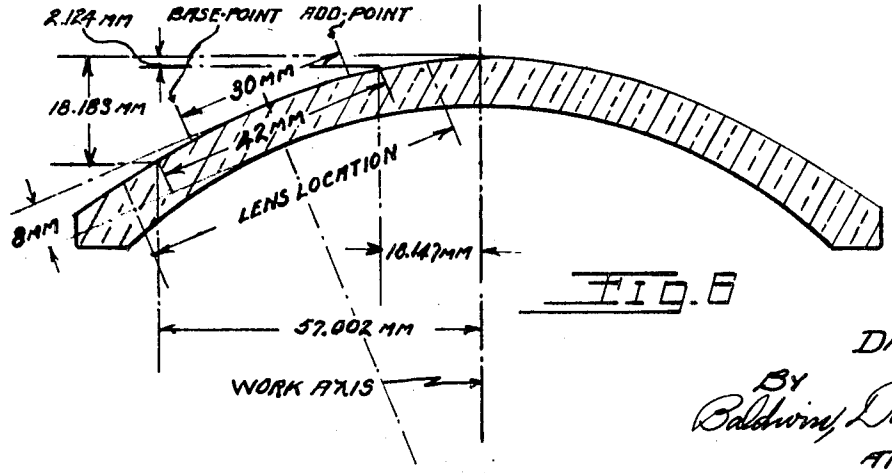


FIG. 6

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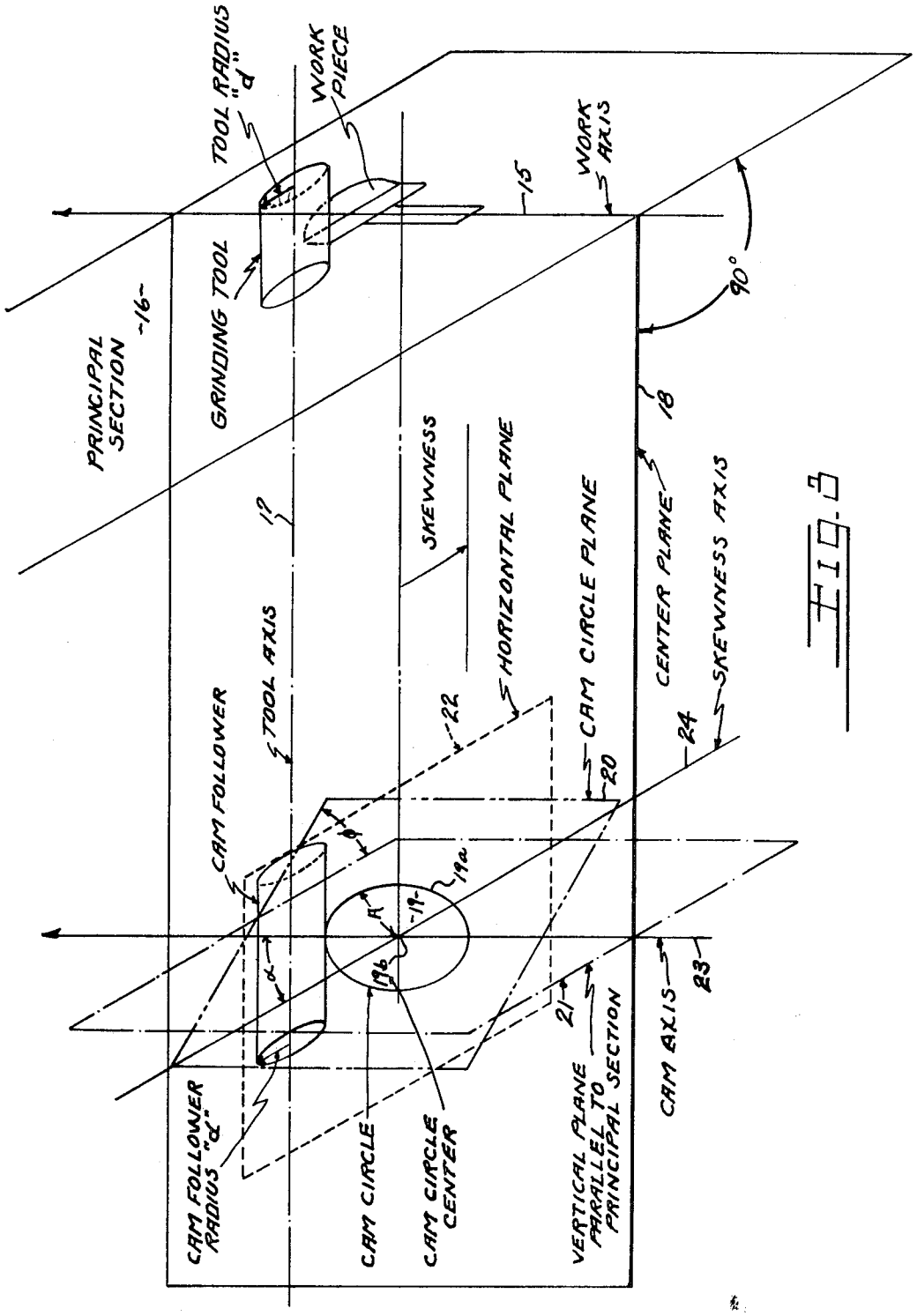


FIG. 3

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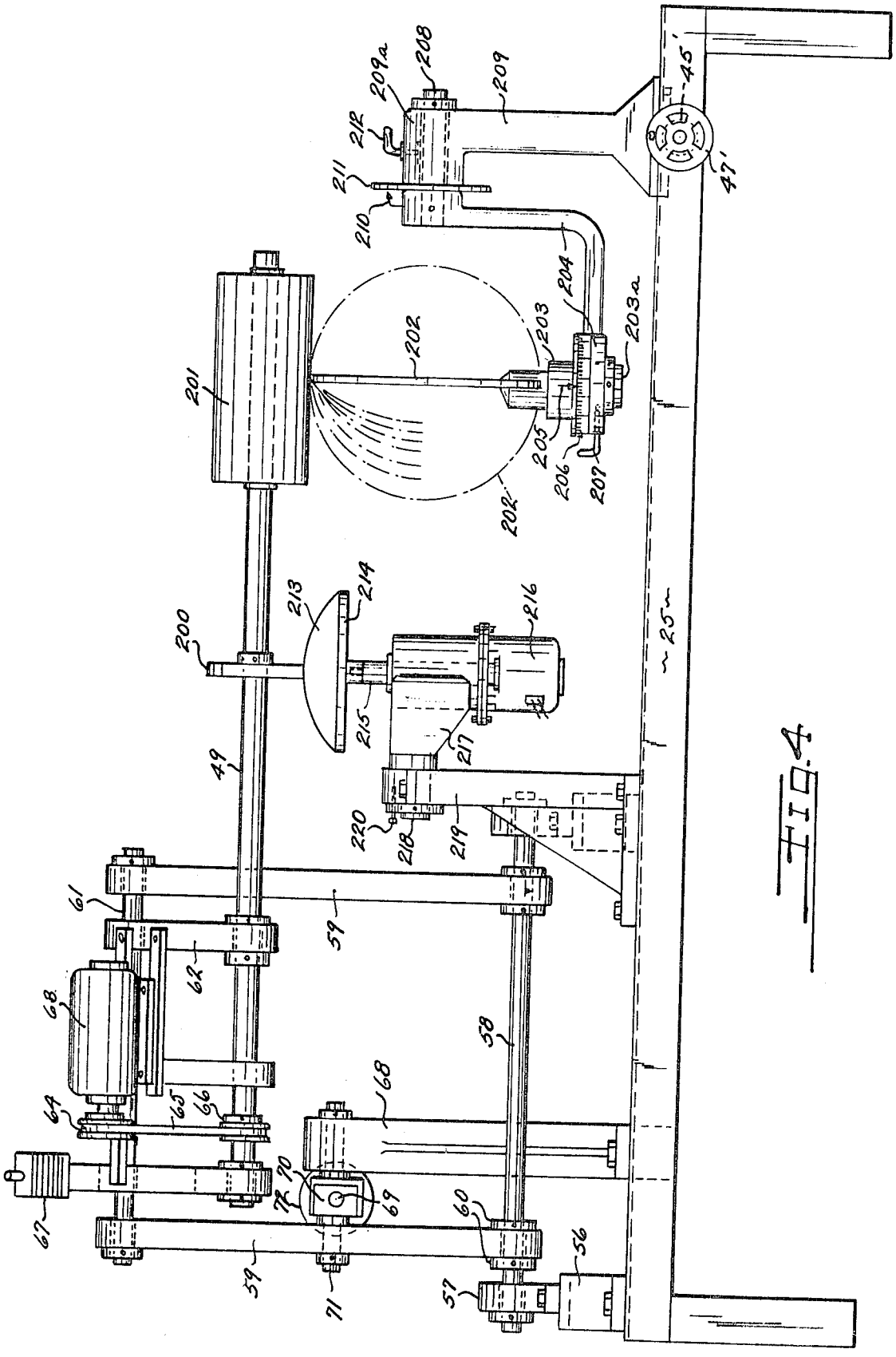

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209a
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FIG. 4

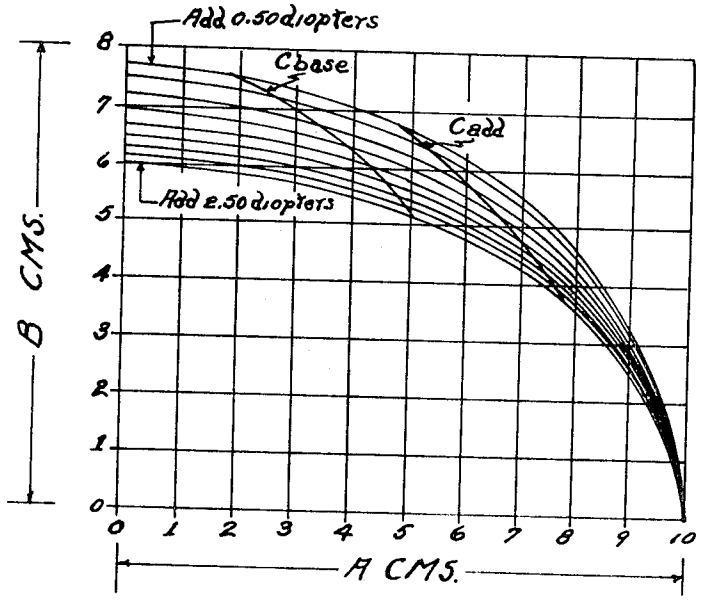


Fig. 7

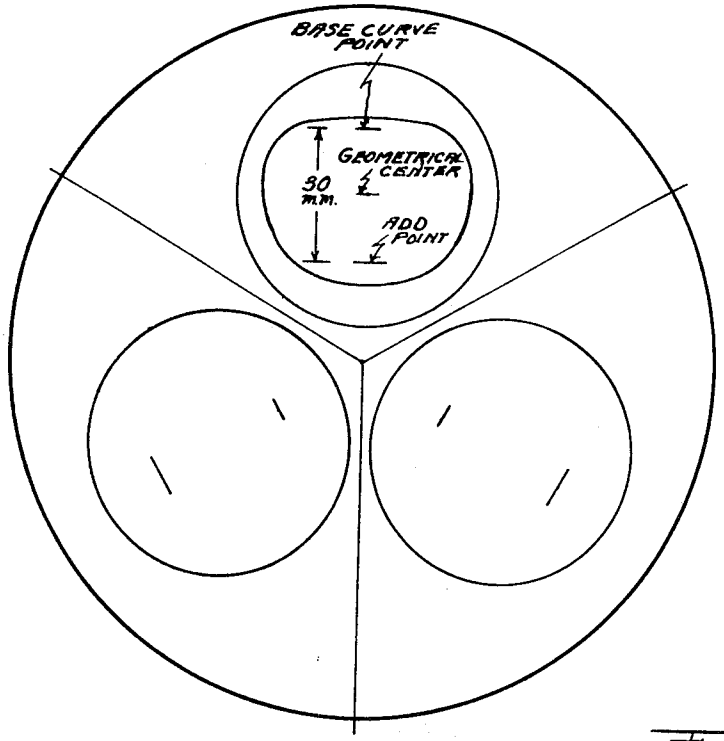


Fig. 10

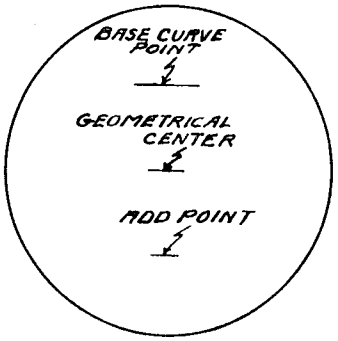


Fig. 11

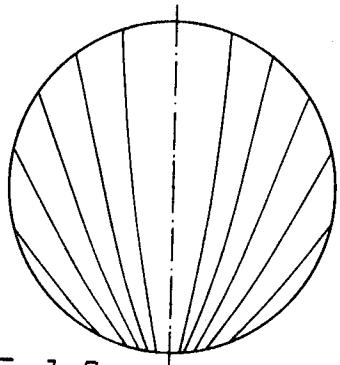


Fig. 12

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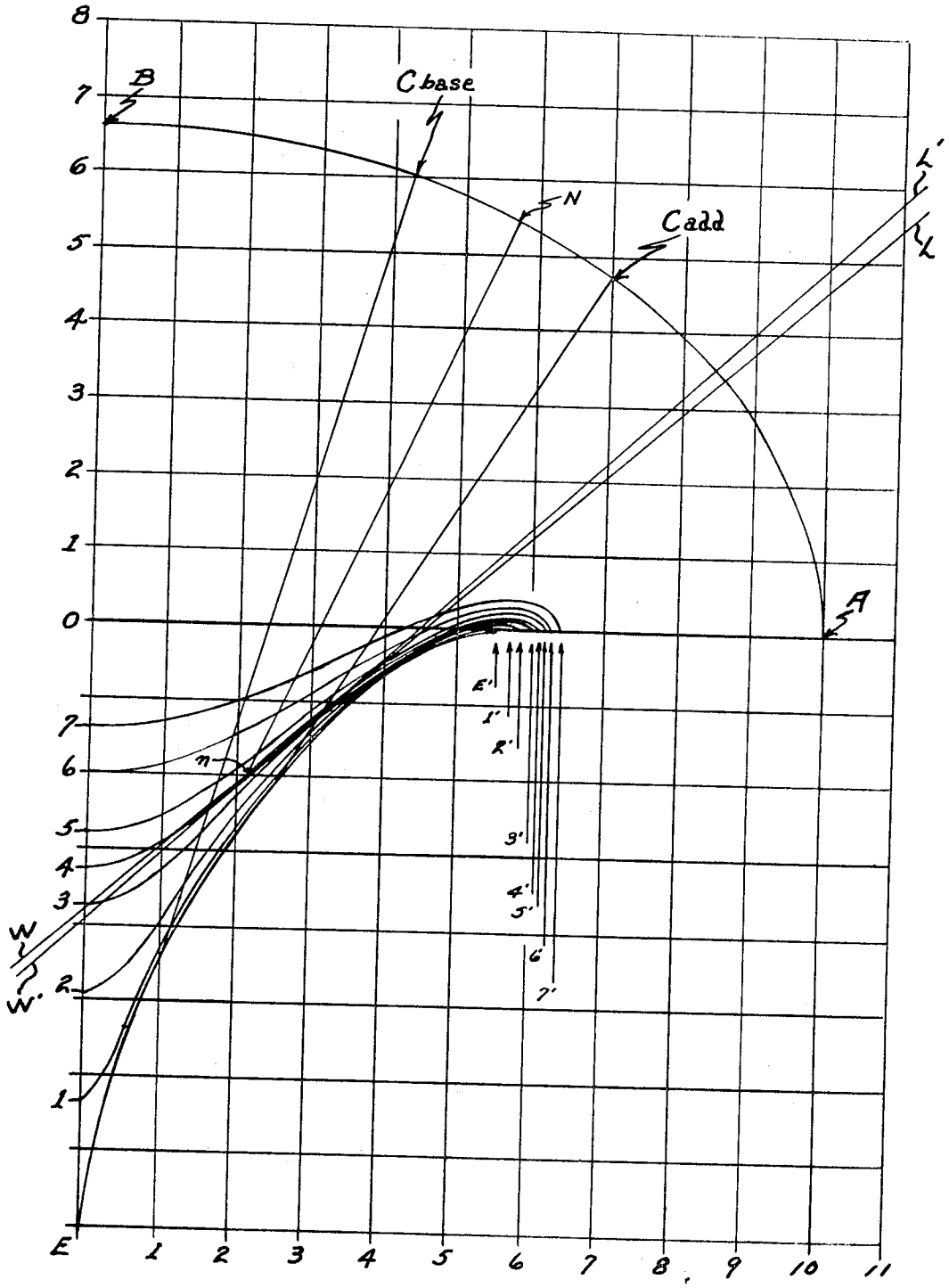


FIG. 8

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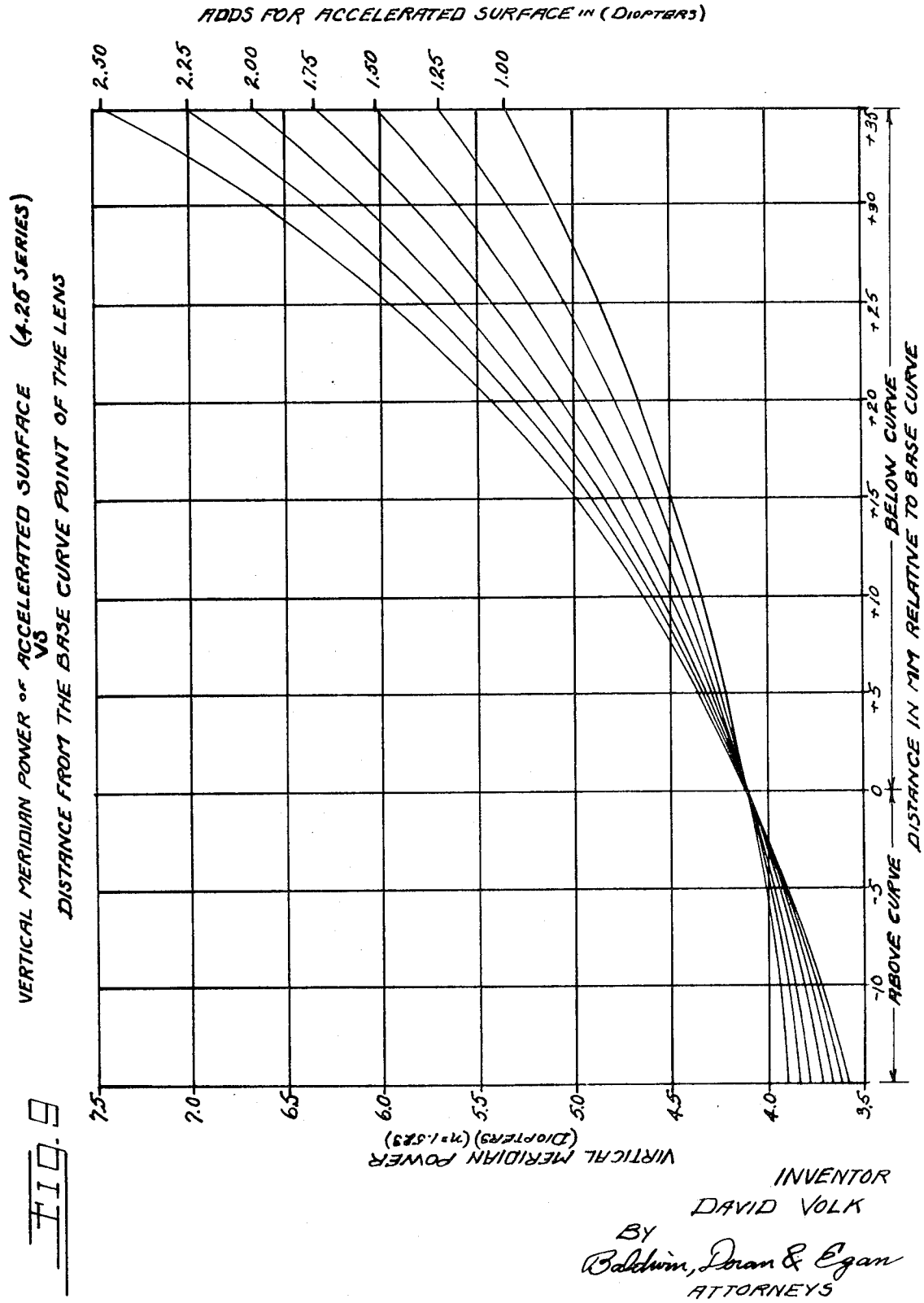


FIG. 9

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OPHTHALMIC LENS OF CHANGING POWER

This application is a streamline continuation of my application Ser. No. 518,848, filed Jan. 5, 1966 now abandoned, and which was a continuation-in-part of my copending application Ser. No. 292,380, filed July 2, 1963 for LENS SURFACE GENERATOR now U.S. Pat. No. 3,239,967, granted Mar. 15, 1966.

This invention relates to an improved ophthalmic lens primarily intended for the relief of presbyopia. Ordinarily the optical treatment of presbyopia is accomplished with either simple reading glasses, bifocals, or trifocals. In the lens of this invention, the change in optical power required to supplement the failing accommodation of the presbyopia is accomplished in a continuous and regular manner, without discontinuities in the field of vision through the lens, without localized distortion in the field. The design of the lens is such that, in level straightforward gaze at distant objects through the upper portion of the lens as it is worn in spectacles, vision is clear, and as objects are observed through lower and lower portions of the lens, they must be brought closer and closer to the wearer to be seen clearly.

The continuous and regular change in power from the top to the bottom of the lens results from the combination of an ordinary toric or spherical back surface and a unique front surface, hereinafter termed the accelerating surface, which is a nonaxial portion of a surface of revolution designed such that meridian sections of said accelerating surface are elliptical arcs, said accelerating surface having the quality of substantially constant astigmatism at all points on the surface.

In the drawings

FIG. 1 is a diagrammatic showing of the parameters of an ellipse involved in the design and construction of the improved lens of this invention;

FIG. 2 is a central sectional view through a workpiece mounted on a work holder for carrying out the method of producing the lens of this invention;

FIG. 3 is a diagrammatic showing of a circular cam and follower in proper relation with a grinding tool and workpiece for carrying out the present invention and indicating the relationship between the various parts and values utilized in the description of this invention;

FIG. 4 is a side elevational view of a machine adapted to carry out the method of this invention in making the lens taught herein;

FIG. 5 is an enlarged side elevational view of a cam follower used in the machine of FIG. 4, the same being partly broken away in central section to show the friction reducing character thereof;

FIG. 6 is a central sectional view through the workpiece utilized in carrying out this invention and illustrating the location of the accelerating lens surface of this invention;

FIG. 7 is a diagram showing the location of the base and add points of this invention along various ellipses;

FIG. 8 is a diagram illustrating graphically how I determine the work line of this invention which is a modified evolute of the basic elliptic curve wherein the series of points representing the transmeridianal centers of curvatures fall on practically a straight line which practically minimizes any deviations from constant astigmatism in the utilized zone of the accelerated surface of this invention;

FIG. 9 is a diagram plotting, for one series of lenses, the vertical meridian power of the accelerating surface of this invention against the distance from the base curve point of the lens for surfaces having various adds at a fixed chord distance;

FIG. 10 is a top plan view of the workpiece of this invention illustrating the manner in which useful lenses are cut therefrom and marked for use in an ophthalmic laboratory;

FIG. 11 shows an elevational view of a lens of this invention marked for use in completing the patient's prescription; while

FIG. 12 is a flat mapping of the front surface of a lens of this invention showing how the meridian lines of the accelerating surface converge towards a point on the axis of revolution of the surface.

As will be disclosed in the description which follows, the elliptical arc plays the primary role in the design and manufacture of the accelerating surface of the lens of this invention, as follows:

1. The elliptical arc is readily obtained from the edge of an inclined circle.

2. By suitable inclinations of a circle of a given diameter, a series of elliptical arcs of the appropriate shape, useful for a series of ophthalmic lens accelerating surfaces, can be obtained.

3. A simple cam-following generator in which the circular edge of a right circular cylinder suitably inclined and positioned serves as the cam, is used to generate the accelerating lens surface of the lens of this invention. Said cam following generator is disclosed in my invention LENS SURFACE GENERATOR, Ser. No. 292,380, filed July 2, 1963, now U.S. Pat. No. 3,239,967, granted Mar. 15, 1966. An alternative method of generating the accelerating surface of the lens of this invention is disclosed in my invention LENS GENERATING METHOD, Ser. No. 218,601, filed Aug. 22, 1962, now U.S. Pat. No. 3,218,765, granted Nov. 23, 1965, and LENS GENERATING APPARATUS, Ser. No. 480,726, filed Aug. 18, 1965, now U.S. Pat. No. 3,267,617, granted Aug. 23, 1966, in which the suitably inclined and positioned circular edge of a circular cup wheel, fed into the work material, is used to generate the surface directly without the interposition of a cam.

The planar projection of a circle oblique with respect to the said plane is an ellipse, whose eccentricity, e, is given by the equation:

$$e = \sin \Phi \qquad (1)$$

where Φ is the inclination of the plane of the circle with respect to the plane of projection. The major axis of the elliptical projection of the inclined circle corresponds to that diameter of said circle which is parallel to the plane of projection. With the plane of projection vertical, the major axis of the elliptical projection and the corresponding diameter of the inclined circle, hereinafter called the cam axis, will form an angle with the horizontal which shall be defined as azimuth and will be symbolized by α.

The semimajor axis, OA, of length A, of the projected ellipse is equal to the radius of the inclined circle, hereinafter called the cam circle, and the semiminor axis, OB, of length B of the ellipse is equal to $A\cos\Phi$. The radius, r, at any point $P(a,b)$ along the ellipse, where a is the coordinate in the direction of the semimajor axis from the origin 0, at the geometrical center of the ellipse, and b is the coordinate in the direction of the semiminor axis, is given by the equivalent equations:

$$r_{P(a,b)} = \frac{(B^4a^2 + A^4b^2)^{3/2}}{A^4B^4} \qquad (2a)$$

$$r_{P(a,b)} = \frac{[(b^2 + a^2(1-e^2)^2]^{3/2}}{B^2(1-e^2)} \qquad (2b)$$

A second vertical plane, hereinafter called the center plane, intersects normally the vertical plane of projection, hereinafter called the principal section, the vertical line of intersection being defined as the work axis. For the generation of the accelerating surface of the lens of this invention, the center of the inclined cam circle on the inclined circular cam, hereinafter called the cam circle center, and the corresponding geometrical center of the elliptical projection of the inclined can circle, will be at a distance s from the center plane, of the skewness such that the center plane intersects the upper semimajor axis which is at angle α with the horizontal. In FIG. 1 I have shown the elliptical projection of the inclined cam circle, the semimajor axis of the projected ellipse, skewness, and azimuth.

In the generation of the accelerating surface by my method, there are only four variables; viz, A (radius of the cam circle),

Φ (inclination of the cam circle), α (azimuth of the cam axis), and *s* (skewness of the cam circle center). For the generation of a limited series of accelerating surfaces of the lens of this invention, one of the variables may be kept constant, while the other three are adjusted to predetermined values for each of the surface of the series. In a lens series later to be used for illustration, I have used, for generation, a circular cam with a cam circle of fixed radius A, each of the accelerating surfaces in the series being generated with adjusted values of Φ, α, and *s*.

In the generation of the accelerating surface of the lens of this invention, the workpiece, from which several identical lenses may be obtained, consists of a portion of a substantially spherical optical glass or plastic bowl, or a similar shaped bowl which has been molded to a shape such that its outer convex surface is approximately the same as the desired final shape. The workpiece is mounted by means of pitch or adhesive, or by mechanical means, pitch being preferred, to a work holder which in turn is attached by taper fit or by screw fit to a revolving axle or shaft in such a way that there is a common axis of revolution, the previously mentioned work axis, for the axle, work holder, and work. In FIG. 2 I have shown a typical workpiece, mounted by means of pitch to a typical work holder having a metal flange whose upper face is perpendicular to the axis of the work holder and whose outer circumference, concentric to the work axis, is equal to the outer circumference of the bowl-shaped workpiece. The general convexity of the work holder reduces the amount of pitch required for adherence of the workpiece to the work holder while the flange aids in the placing of the workpiece in a symmetrical position with respect to the work axis.

The cam-follower, which rolls freely along the inclined and skew cam circle, is an elongated right circular cylinder, freely rotatable by means of bearings, about a cylindrical shaft, the tool shaft, both the cam follower and the tool shaft being concentric to an axis, the tool axis (FIG. 4). Along the tool shaft is a circular grinding tool which has a cylindrical grinding edge embedded with diamond dust. The grinding edge, concentric to the tool axis, rigidly attached to the tool shaft and straddling the principal section, is of the same diameter as the cylindrical cam follower. By means of appropriate shafts and linkages, later described, the tool shaft and its axis are always maintained perpendicular to the principal section, as the cam follower rolls along the cam circle, with the tool shaft and attached grinding tool rapidly rotating about the tool axis.

In FIG. 3 I have shown the relationship between the various planes, axes, angles, and positions of the elements of the generator and workpiece, as described previously. Vertical center plane **18** and vertical principal section **16**, intersect normally in the vertical work axis **15**. A trace of the work holder and workpiece, symmetrical to the work axis, is shown in the principal section. Cam circle **19***a*, of radius A, is shown in cam circle plane **20**, which is inclined at angle Φ with respect to plane **21** which is parallel to principal section **16**. Cam axis **23** is shown at azimuth angle α with respect to horizontal plane **22** (line **24** is in plane **22**). Cam center **19***b* is adjustable in position along horizontal skewness axis **24**, though in FIG. **3** skewness is set at **0**. Elongated cam follower of radius "*d*," in contact with cam circle **19***a*, and grinding tool of radius "*d*," straddling the principal section, are both concentric to tool axis **17**, which is always maintained parallel to the center plane as the cam follower rolls along the cam circle.

In FIG. **4** I have shown the apparatus for generating the workpiece, which apparatus incorporates those features of FIG. **3** and its description.

Referring to FIG. **4**, a pair of seats **56** are bolted to the base **25** and bearings **57** are bolted to their respective seats and a shaft **58** is rotatably mounted in the bearings. Parallel links **59** are rotatably mounted on shaft **58** and held in position by collars **60**. At their upper ends, links **59** support shaft **61** which in turn carries a pair of parallel links **62** extending at an angle to the links **59**. These links are freely rotatable on shaft **61**. At their outer ends, links **62** mount shaft **49** for rotation therein by means of motor **63** which is here shown as mounted on the links **62** although the motor might be connected in any other suitable position in order to provide its driving function. As here shown, the motor drives a pulley **64** which through belt **65** drives a pulley **66** rigidly fixed to shaft **49**. Preferably, a counter weight is provided at **67** so as to substantially balance the weights on opposite sides of the shaft **61**. To accurately control the position of shaft **41**, cam follower **201** and grinding wheel **200**, a bracket **68** is fixed to the base **25** and carries at its upper end a rotatably mounted screw **69** which has a threaded connection with a nut **70** which is mounted by a trunnion **71** in one of the links **59**. Rotation of the screw **69** by means of handwheel **72** then varies the angular position of the links **59** with respect to the base **25** so as to carry the cam follower **48** in any desired direction as it rolls across the cam **202**. Thus, energization of motor **63** will cause shaft **49** and grinding wheel **200** to rotate rapidly as the grinding wheel follows the pattern provided by the cam follower **201** rolling over the cam **202**. For further details of this machine one may refer to my copending application Ser. No. 292,380, filed July 2, 1963, now U.S. Pat. No. 3,239,967, granted Mar. 15, 1966.

The drive shaft **49** has a grinding wheel **200** fixed to turn with the shaft in a position closer to the driving mechanism. Farther to the right, a cam roller **201** is mounted on shaft **49**, as shown in FIG. **5**. Cylindrical cam follower **201** of the same outside diameter as grinding wheel **200** rolls on cam **202**. This circular cam is fixed to a manipulating hub **203** having a cylindrical extension **203***a* which fits rotatably in a suitable socket in an arm **204** so that the cam **202** may be rotated about a diametrical axis, this adjustment being read by pointer **205** on indicia **206** and the adjustment held as desired by a thumb screw **207**. The arm **204** is bent at right angles and fixed to a trunnion pin **208**, the central axis of which lies on a projected diameter through the circular cam **202**. Trunnion pin **208** is mounted in a suitable bearing **209***a* at the upper end of bracket **209** which in turn is secured to the base **25**. The arm **204** may be oscillated about the trunnion pin **208** and this position is read by means of a pointer **210** which moves with arm **204** across indicia carried by a plate **211** which is fixed to the bracket **209**. This position is held by means of a thumb screw **212**. In this form of the invention, the inclination Φ is read directly on indicia **206** and the azimuth α is read directly on the indicia at **211**.

A skewness adjustment is provided in connection with the bracket **209**. The bracket **209** is movable in ways **45'** by means of a screw manipulated by handwheel **47'** so as to move the bracket **209** crosswise of the base **25** or at right angles to a vertical plane passing through the axis of shaft **49'**.

The workpiece to be shaped, indicated at **213**, is mounted in a work holder **214** which in turn is connected to vertical shaft **215** rotatable by means of a motor **216**. This whole work holding device is held in a bracket **217** which has a stub shaft **218** preferably rotatably mounted in a bracket **219** fixed to the base **25**. A pin **220** is provided to hold the work in the position with shaft **215** vertical as shown in FIG. **4** or, alternatively, to rotate the same 90° and hold the shaft **215** in a generally horizontal position.

Consider the surface generated on the workpiece with the following adjustment of the variables:

A=100 mm.
Φ=44.027°
α=40.349°
s=26.663 mm.

After adjusting the work holder so that the uppermost portion of the workpiece is slightly above the level of the cam circle where it would be intersected by the center plane, the cam follower is caused to roll slowly along that portion of the circular edge of the cam which is on the same side of the center plane as the cam circle center, while the grinding tool, rotating rapidly about the tool axis, contacts the workpiece, rotating rapidly about the work axis, removing material from the work as the cam follower moves along the cam, until the entire surface of the workpiece has been generated.

The surface generated, shown in section in FIG. 6, will be spindle shaped with an apical cusp. Within the broad surface area contained within a zone limited by parallel planes perpendicular to the work axis at 2.124 mm. and 18.183 mm. from the apical cusp of the surface, which surface area is more than 42 mm. in chord length along a meridian between the two said planes, the meridianal refracting power of a glass surface of index of refraction 1.523 increases continuously and regularly from 3.872 diopters at the lower level to 5.008 diopters at the upper level of the zone, while the transmeridianal power at any level between the limits of the zone is always 1.000±0.011 diopters greater than the meridianal power at that level. Table 1 contains data on this surface, showing the meridianal and transmeridianal powers and their differences at several points along a meridian. Since all meridians in a surface of revolution are identical, it is obvious that the entire surface within the aforementioned zone is of substantially constant astigmatism. The ± 0.011 diopter difference from the designated value of 1.000 diopters is well within the tolerances accepted in standard ophthalmic practice.

In the lens of this invention, a range of accelerating front surfaces can be readily designed to an accuracy comparable to that of the above example. However, in order to systemize the production of series of accelerating surfaces, so that a single circular cam may serve for the production of several surfaces in a series, and so that the accelerating surfaces so produced will be compatible with the usual standard graded powers of the back surfaces, I have set an upper limit of tolerances of ±0.04 diopters for the accuracy of the accelerating front surface constant astigmatism, a value which is comparable to the tolerances of standard ophthalmic lens practice.

In the determination of the actual values of the adjustable variables for a series of lenses, the following factors must be considered:

1. The radius of the cam circle must be such that with each setting of inclination, the projected ellipse must contain the required curvature values at specific chord distances.

2. The values of azimuth and skewness must be so determined that the transmeridianal power of the generated accelerating surface will be greater than the meridianal power thereof, within the given tolerances, by a predetermined amount, for those portions of the workpiece from which the spectacle lens in obtained, and in particular for those portions of the finished spectacle lens which are used in straightforward vision at eye level and thence downward to a portion of the lens used for near work such as reading. The vertical distance referred to above is about 30 mm., but, as in the example of the generated surface in which astigmatism of the accelerating surface was maintained well within tolerances for more than 42 mm., azimuth and skewness for the lens series of this invention have been adjusted so that accelerating surface astigmatism is within the tolerance for a chord length of more than 40 mm.

For a given cam circle of radius A, each successive increase in inclination of the cam circle (for a series of lenses) results in the projected ellipse having a shortened semiminor axis and a consequent longer radius of curvature at the end of the minor axis, r_m, and a shortened radius of curvature at the end of the semimajor axis, r_M. Between the two extreme values of radius of curvature along the ellipse, the radius of curvature changes continuously and progressively from r_m to r_M according to equation (2*a*, *b*). Once inclination has reached a sufficiently large value, then the minimum curvature at the end of the minor axis will be equal to some preassigned value C_{base}. With each increase in Φ, the preassigned value C_{base} will be at a greater distance from the minor axis of each successive ellipse. The intervals of Φ are such that at each successive value of Φ, the difference in curvature between C_{base} and C_{add}, where C_{base} and C_{add} represent two points on the ellipse separated by a chord distance of 30 mm., will value an increase in refractive power by a definite increment, 0.25 diopters for example, where n=1.523. Illustrating the above is FIG. 7 which is a drawing of a series of elliptical arcs, each formed with A constant at 100 mm. but Φ differing. Also shown in FIG. 7, by means of two short curved lines intersecting the elliptical arcs, are the position of C_{base} and C_{add} on each elliptical arc, with the dioptric power of each C_{base} point equal to 4.10 diopters, when n=1.523, while for each successive pair of C_{base} and C_{add} points, the power difference, hereinafter termed the add, increases by 0.25 diopters, with the range of adds extending from 0.50 to 2.50 diopters. The values of Φ and the coordinates for each pair of C_{base} and C_{add} points on each elliptical arc, and the adds for each pair of said points, are listed in table 2.

For any given elliptical curve, normals to the ellipse form an envelope, the evolute of the ellipse, which is convex towards that portion of the ellipse from whose normals it is formed. The curvature R at any point along the ellipse is the reciprocal of the distance r along the normal from said point on the ellipse to its point of tangency to the evolute, $1/r$.

In the discussions following, it is to be assumed that the optical material is ophthalmic crown glass of n=1.523. Since refracting power of a refracting surface in air is $R(n-1)$, where r is measured in meters, it will be convenient to discuss the refracting surfaces in terms of units of curvature, it being understood that one unit of curvature will result in a refracting power of 0.523 diopters, so that 0.50 diopters of refracting power will result when the curvature is 0.956 curvature units.

If a fixed increment of curvature units is added to the value of the curvature at each point along the ellipse, the reciprocals of the sums, measured in meters along the normals from said points of the ellipse towards the evolute, is the locus of a new curve related to the evolute, which new curve will hereinafter be termed in the specification and claims as the modified evolute. Said modified evolute will differ from the evolute in that it falls between the evolute and the ellipse, and that it has a point of inflection, so that it is both concave and convex towards the elliptical arc from which it was derived. By adding a series of fixed increments of curvature to the curvature values of the ellipse, a series of modified evolutes can be obtained, each differing somewhat from the adjacent modified evolute.

In FIG. 8 I have drawn elliptical arc BA, one of the elliptical arcs of FIG. 7 and table 2, for which A=10, Φ=48.072, and for which the add is 1.50 diopters. Also drawn is the evolute EE' for arc BA, and a series of modified evolutes, 1, 1', through 7, 7', each modified evolute being separated from the adjacent modified evolute by 0.956 curvature units or 0.50 diopters, with the exception of 4, 4' which is 0.478 curvature units, 0.25 diopters, from 3, 3' and 5, 5'. Also drawn are the normals to the elliptic arc from the points C_{base} and C_{add} to the evolute E, E'. By inspection it can be seen that modified evolutes 1, 1', 2, 2', and 3, 3', between the two said normals, are generally convex towards said elliptical arc segment C_{base}, and C_{add}, while modified evolutes 5, 5', 6, 6', and 7, 7' are generally concave towards said elliptical arc segment, and modified evolute 4, 4' is practically a straight line between the two said normals, and beyond. This is the "straight portion of the modified evolute" referred to in the claims. Through that portion of modified evolute 4, 4' between and beyond the two said normals, and tangent to 4, 4' at its point of inflection n, I have drawn a straight line WL, hereinafter termed the work line, said work line thus representing the locus of those points, the reciprocals of whose distances along the normals from arc segment C_{base}, C_{add} is greater, by practically a constant, than the corresponding values of curvature along said arc segment, and beyond. Note the normal to the ellipse Nn lies approximately half way between the normals at C_{base} and C_{add}.

For the purpose of this example, the modified evolutes have been drawn no closer than 0.478 curvature units, but the coordinates for a series of modified evolutes of any desired degree of closeness may be easily determined by modern computer techniques. To the data so determined for each series of coordinates of a portion of a modified evolute, corresponding to a specified portion of an elliptical arc, one may apply the method of least squares and find that modified evolute most

closely approaching a straight line in the required portion, while said straight line, from which the sums of the squared deviations of the points in the modified evolute are minimized, would be the optimum work line. However, from the practical standpoint such precision in the determination of the work line is not required, in view of the tolerances previously specified, and graphical methods such as represented in FIG. 8 have been utilized with the desired accuracy.

Again referring to FIG. 8, the slope m of the work line, with respect to the major axis of the ellipse, can be determined quite simply from the graph by selecting any two points (b', a'), (b, a) on the modified evolute through which the work line passes, and applying the formula:

$$m=(b'=b)/(a'a) \qquad (3)$$

By applying the coordinates of one of the two points and slope m to the standard equation of a line:

$$b=ma+x \qquad (4)$$

the value of the constant x, where x is the value of the ordinate b when $a=0$, is obtained.

One may also obtain the value of x simply and directly by reading its value on the ordinate where the extended work line WL crosses the ordinate, while the slope of WL is accurately determined by means of equation (3) utilizing the coordinates of any two widely separated points along WL.

If the ellipse is oriented so that the work line WL is vertical, then the angle α (see FIG. 3) which the major axis of the ellipse makes with a horizontal plane, which is the angle between the work line WL and the minor axis of the ellipse, OB in FIG. 8, is ($90-\tan^{-1}$m.), and skewness $s=x \sin \alpha$. In the example of FIG. 8, α is 49.87° and s=29.05 mm. In the generation of the work surface by the methods and apparatus described previously, the position and orientation of the elliptical projection on the principal section, of the inclined cam circle of radius A and inclination Φ, is such that the work line coincides with the work axis when azimuth is set at α and skewness is set at s. When this is done, the transmeridianal radius of curvature, at any point on the generated surface, within the zone containing the base and add points, will be greater, within the given tolerances, than the meridianal curvature at said point, by that amount of curvature units which has been added to that of the meridianal curvature in producing the utilized modified evolute and associated work line. This is true since the transmeridianal curvature of any point on a surface of revolution is the reciprocal of the distance along the normal from said point to the axis of revolution of the surface.

In the above example, generating the work surface with α=49.87° and s=29.05 mm. results in a substantially constant astigmatism of 1.75 diopters in the desired zone of the generated surface. In the present state of the art of ophthalmic spectacle lens grinding by prescription shops, the grinding tools are designed for refracting power at one-eighth diopter intervals. Furthermore, the grinding tools are designed for refractive material of index of refraction 1.530 whereas the ophthalmic crown glass used has an index of refraction of 1.523. Since standard prescription shop tools are to be used for the processing of the back surface of the lens of this invention, the dioptric power of the astigmatism of the accelerating surface must be compensated for lens thickness and the actual dioptric power which the tool grinds on the back surface. These factors of specific power intervals, lens thickness, and the index of refraction of the optical material, have been taken into account in the design of the accelerating surface, so that the actual dioptric power of the astigmatism of the accelerating surface is slightly less than the named value, by the compensating amount, the named values being at the one-eighth and one-fourth diopter intervals. The increment of curvature units added to those of the elliptical arc in producing the modified evolute of FIG. 8 is such that the dioptric power of the astigmatism of the generated accelerating surface in the desired zone, will be the required compensated value for each accelerated surface of the lens series of this invention.

In FIG. 8, I have drawn line W'L', which is the actual work line for the compensated accelerating surface, the modified evolute corresponding to W'L' being obtained by adding 3.25 curvature units to those of the elliptical arc, so that the dioptric power of the compensated front surface astigmatism is actually about 1.70 diopters. The portion of the modified evolute used for the determination of work line W'L', for the required portion of the elliptical arc, is practically coincident with a straight line, so that the va lue of the astigmatism of the accelerating surface, at all points within the required zone, will be well within the specified tolerances of ± 0.04 diopters.

Ophthalmic spectacle lenses of multifocal type may be supplied to the prescription shop in one or more forms: (1) They may be completed on both sides with standard surfaces and be of average center thickness and of relatively large diameter or area, so that all that is required for the filling of a prescription for glasses is the shaping of the lens by the removal of excess peripheral glass so that it may be used in the spectacle frame. Such lenses are called finished-uncut, and they have spherical powers only; and (2) They may be completed on both sides with standard surfaces and be of excessive thickness and relatively large diameter, so that the operations in the prescription shop may include the process of generating, grinding, and polishing one of the surfaces for the filling of a prescription, thereby reducing the thickness of the lens to conventional thickness, in addition to the other procedures of shaping the lens for the spectacle frame. Such lenses are called semifinished, since one of the surfaces must be refinished, as described above, by the prescription shop.

In the production of the finished-uncut and semifinished ophthalmic lens of this invention for commercial use, the accelerated surface is standardized in terms of 4.25, 6.25, 8.25, etc., diopter base curve series, wherein the base curve designation refers to the compensated vertical meridian power at a single point along the vertical meridian, said point having been referred to previously as the base curve point. The standardization of the accelerating surface into such base curve series has been done for the purpose of minimizing such aberrations as are manifested in oblique gaze through the periphery of the finished spectacle lens, in a manner analogous to the standardization of commercial finished-uncut and semifinished bifocals and trifocals into base curve series, wherein such base curve designations refer to the compensated front surface dioptric power, the compensation taking into account the range of lens refractive powers for which a base curve is used, lens curvatures, refractive index of the lens material used in relationship to the refractive index for which the tools were designed, and center thickness of the finished lens.

In the finished-uncut lens of this invention, the specific compensated base curve accelerating surface which is used for a specific lens, is in accordance with the dioptric power of the lens for light rays through the lens at both the base curve point and the add point. In general, negative power lenses require the weaker base curves, while positive power lenses require the stronger base curves. Since the average curvature of the total accelerating surface increases as the add increases, with the average curvature at the base curve point also being increased, the range of lens refractive powers for which a specific base curve series is recommended is, in general, shifted toward the less negative or more positive powers as the adds are increased from the weakest to the strongest.

For the processing of the semifinished lens of this invention, the prescription shop is instructed as to what base curve lens is appropriate for the powers of the prescription, taking into account the effects of prescription cylinder power and axis, and the add, so that when the processing of the lens is completed, aberrations will be minimized in oblique gaze through the lens.

Referring to FIG. 7 and table 2, the series of elliptical arcs shown are an example of the curves which may be used in a 4.25 diopter compensated series of lens accelerating surfaces. The base curve point is found along the vertical elliptical meridian in the upper part of the lens, and the additional

power corresponding to the added positive spherical power required for the correction of presbyopia is at a chord distance across the vertical elliptical meridian 30 mm. from the base curve point. In each of the accelerated lens surfaces above the base curve point, the meridianal dioptric power of a point on the vertical elliptical meridian is less than that of the base curve point, while for a point on the vertical elliptical meridian below the add point, the dioptric power of the accelerated surface in the meridianal direction is greater than that of the add point. In FIG. 9, I have shown graphically the meridianal dioptric power at various chord distances along the elliptical vertical meridian for seven of the nine accelerated surfaces of the 4.25 diopter compensated series. The compensated power at the base curve point is shown as about 4.10 diopters. The amount of power added to that of the base curve point at any chord distance from the base curve point, for each of the seven vertical meridians shown, can be obtained from FIG. 9.

In table 3 I have listed the values of the adjustable variables for the production of the sample 4.25 diopter compensated series of the accelerating lens surfaces so far described, along with the nominal power of the astigmatism of the accelerating surface and the actual dioptric power of said astigmatism. It is obvious that by applying the same principles and method of this invention already described, other 4.25 diopter series may be designed in which, for example, the chord separation of the base curve and add points on the accelerated surface might be 25 mm. or 20 mm. instead of the 30 mm. described for the sample series. Referring to FIG. 9 and the curve labeled "2.00 add," the power difference between the base curve point and a point at a chord distance of 25 mm. is about 1.50 diopters. The same lens might well be considered one of a series in which the base curve and add points are separated by a chord distance of 25 mm., in this case the add being 1.50 diopters. Other accelerating lens surfaces may be designed, according to this invention, to complete a series of nine lens surfaces, for adds from 0.50 to 2.50 diopters, all having a chord separation of 25 mm. between the base curve and add points.

After completing the generation of a workpiece by the method described, there remain many small pits and scratches which must be removed prior to polishing the surface. For the removal of these scratches and pits, I use the invention of my copending U.S. Pat. application, LENS GRINDING APPARATUS, Ser. No. 337,514, filed Nov. 14, 1962, which utilizes a grinding tool having a slitted flexible sheet metal backed by a resilient material such as sponge rubber. The previously generated workpiece is mounted on a vertical spindle and caused to rotate about the work axis. The grinding tool with its slitted flexible grinding surface is caused to oscillate along a meridian of the rotating work, while a slurry of fine grinding compound is continuously fed to the work surface, and grinding is continued until the pits and scratches are removed.

The ground surface can then be polished with a sheet of nylon or cotton cloth which conforms to and completely covers a sector of the surface from the apex to the periphery, said cloth being oscillated across the surface in a substantially meridianal direction, while the workpiece is rotating about the work axis. Cerium oxide suspension is continuously fed to the work surface during the polishing operation.

After the workpiece is polished, it is removed from the work holder by chilling both the work holder and the workpiece, which causes the workpiece to separate from the pitch.

For the production of semifinished lenses, the inner surface of the finished workpiece is then ground and polished to a precise standard negative curvature, each specific base curve and add requiring a specific negative inner, or back, surface. The radius of curvature of the negative surface is 8 mm. less than that of the transmeridianal radius of curvature at a point on the generated surface corresponding to the midpoint of the chord between the base curve point and the add point. As will be later described, said point on the generated workpiece will be at the geometrical center of the accelerating lens surface; and the normal to said point, which is also the normal to a standard negative spherical back surface, will be defined as the optic axis of the lens. The individual lenses may then be cut out of the bowl. An workpiece method of completing the lens is to first cut the finished workpiece into circular discs or wedge-shaped sections and then grind and polish the back surface to a precise standard concave, or negative, spherical surface.

In FIG. 10 I have shown in a diagrammatic top view the location of three lenses on the surface of a workpiece finished as described above. Each lens is 58 mm. in diameter, a size convenient for use by prescription shops, of both the finished-uncut and the semifinished lens. In one of the lens locations, I have drawn the outline of a typical finished ophthalmic spectacle lens which includes within its confines the zone of substantially constant astigmatism on the accelerating lens surface, previously referred to, and have also drawn a meridian line A' through the middle of the lens, said meridian line to be the vertical meridian line of the finished prescription lens when it is inserted into a spectacle frame. The three lenses are then cut out of the workpiece with a circularly cylindrical diamond edged saw, the inner spherical surface having been ground and polished previously as described. With the precise spherical back surface the lenses can be tested optically through various points on the accelerating surface, thus providing a means for measuring the accelerating lens surface for both quality and power. The 8 mm. center thickness of the semifinished lenses is sufficient thickness for generating and grinding the back surface by prescription shops, when modifying the lens to meet the specifications of a prescription.

The finished-uncut lens of this invention, which is supplied to the prescription shops with a toric surface on the back, as will be later described, is marked on its accelerating surface with a thin line of waterproof ink along a meridian which bisects the front surface into symmetrical halves. Short lines crossing the marked vertical meridian B' in FIG. 11 are used to identify the base curve point, the geometrical center of the lens, and the add point. The lens so marked is packaged and the carton labeled to identify power of the lens through the base curve point, and the add. The semifinished lens of this invention is marked on the accelerating surface in the same manner as the finished-uncut lens. The lens so marked is packaged and the carton labeled to identify the base curve, the add, and the dioptic power of the astigmatism of the accelerating surface. FIG. 11 is a drawing of a lens marked as described above.

Semifinished lenses of different base curves and different adds are supplied to the prescription shop for the incorporation of a patient's distance prescription and for the additional refractive power required for the correction of presbyopia. In a semifinished lens with the appropriate accelerating surface which lens is to be modified by the prescription shop to incorporate a patient's prescription, the astigmatism resulting from the excess of transmeridianal refracting power of the accelerated surface over the meridianal refracting power, at any point along the marked vertical meridian, must be neutralized at the back surface by cylindrical refracting power, the axis of said cylindrical refracting power being vertical when its power is negative. In the incorporation of patient's distance prescription into the lens of this invention, said astigmatism neutralizing cylindrical refracting power must be combined with the cylindrical refracting power of the patient's prescription, and the resulting sphero-cylindrical refracting power be combined with the spherical refracting power of the patient's prescription. In those instances in which the patient's prescription is for spherical power and add only, the prescription shop may choose to grind the required back toric surface instead of utilizing a finished-uncut lens. As an example, consider the following prescription: −1.00 diopter sphere, add +1.50 diopters. Reference to table 3 shows that the 4.25 diopter base curve, 1.50 diopter add, semifinished lens has a nominal front surface astigmatism of 1.75 diopters. The neutralizing cylindrical refracting power is therefore −1.75 diopters and the back surface is ground with a toric tool which grinds curves resulting in refractive power of −5.25 diopters in the vertical

meridian and −7.00 diopters in the horizontal meridian, as calculated for optical material of n=1.53 . The power of the lens through the base curve point will then be −1.00 diopter and through the add point, +0.50 diopters. It is to be understood that a lens finished by the factory, in the manner thus described, is the finished-uncut lens of this invention.

Now consider a prescription similar to the one above except that the prescription calls for cylindrical refractive power in addition to the spherical refractive power. For example, consider the following prescription: −1.00 diopter sphere combined with −1.75 diopter cylinder axis 60°, add 1.50 diopters. The semifinished lens is a 4.25 diopter base curve, 1.50 diopter add lens requiring −1.75 diopters of cylindrical refracting power axis 90° at the back surface for neutralizing the astigmatism produced by the accelerating front surface along the vertical meridian. The angle γ between the neutralizing cylinder and the prescription cylinder is 30°. The resultant cylinder C is obtained by the following formula:

$$C=(A^2+B^2+2AB\cos 2\gamma)^{1/2} \quad (5)$$

where A is the power of the neutralizing cylinder and B is the power of the prescription cylinder, so that $C=-3.03$ diopters, which, to the nearest one-eighth diopter interval, is −3.00 diopters. The sphere power, D, resulting from combining the neutralizing cylinder and the prescription cylinder is: $D=A+B-C/2$ (6) so that $D=-0.24$ diopters, which to the nearest one-eighth diopter interval, is −0.25 diopters. The total sphere power required is that of the patients's prescription plus the sphere power resulting from the combination of the cylinders and is −1.25 diopters. The angle β between the vertical meridian and the axis of the resultant cylinder is obtained by the following formula:

$$\beta=(\text{arc sin}B/C)\,[\sin(180-2\gamma)]/2 \quad (7)$$

so that β=15°, with the resultant cylinder axis falling between the axis of the neutralizing cylinder and that of the prescription cylinder, so that the axis of the resultant cylinder is 75°. The base curve of the lens being 4.25 diopters (compensated) the powers ground on the back surface are then −5.50 in the 75° meridian and −8.50 diopters in the 165° meridian as calculated for n=1.53. The powers at the base curve point of the finished lens will then be that of the patient's distance prescription and the powers at the add point will be that of the patient's distance prescription plus the add, within the tolerances permitted for ophthalmic prescription lenses, which is the order of ±0.06 diopters.

Along the vertical meridian, the lens will have the astigmatism correction in the amount and at the axis called for in the prescription. On either side of the vertical meridian, there will be astigmatism other than that called for in the prescription and required for the correction of the patient's ocular astigmatism. This said other astigmatism results from the fact that the principal directions of the cylindrical component of the back surface for neutralizing the astigmatism due to the accelerating surface of the front surface can coincide with the principal directions of the accelerating surface only along one meridian line, the vertical meridian. Consider the vertical meridian section of the accelerating surface, and its containing plane, which I have called the principal plane. Now consider the neutralizing cylindrical component of the back surface with its axis in the said principal plane, with said axis parallel to a plane tangent to the vertical meridian section of the accelerating surface at the geometrical center of the lens. The normal to the tangent plane at its point of contact with the accelerating surface, which is also perpendicular to the back surface, coinciding with an equatorial axis of symmetry of the back surface, is analogous to the optic axis of an ordinary lens, and only at the normal common to both surfaces, can the principal directions of the accelerating surface and those of the neutralizing cylinder component of the back surface coincide. Elsewhere along the vertical meridian line of the accelerating surface, above and below the normal common to both surfaces, the principal directions of the accelerating surface of the front surface and those of the neutralizing cylinder component of the back surface, are close, but do not coincide, and appropriate bending or coflexure of the two surfaces are used to minimize lens aberrations in the completed lens along the vertical meridian, in a manner analogous to the bending or coflexure of ordinary "corrected curve" ophthalmic spectacle lenses. This aspect of minimizing aberrations has been discussed earlier in relation to the use of multiple base curves for minimizing aberrations. On either side of the vertical meridian line, meridian lines of the accelerating surface, shown diagrammatically as a flat mapping in FIG. 12, converge towards a point on the axis of revolution of the surface, with a resulting tilting of their containing planes and obliquity of their principal directions with respect to those of the neutralizing cylinder component of the back surface which are substantially vertical and horizontal. The optical effect of combining the cylinder component of the accelerating front surface, whose principal directions at any point on the accelerating surface, become progressively oblique and tilted with increasing distance from the vertical meridian line, with the neutralizing cylinder component of the back surface, whose principal directions remain substantially vertical and horizontal, is astigmatism, with principal direction in the 45° and 135° meridians, which increases progressively in proportion to the distance laterally from the vertical meridian and to the rate of change in refractive power along the vertical principal meridian at the specified level.

The amount of said astigmatism, V, in diopters, for light rays through the lens through a given point on the accelerating surface, said point being at a given distance, h, in meters, lateral to the vertical elliptical meridian at the level of point P(a,b) is

$$V=\frac{6(n-1)(A^2-B^2)(AB)^6hab}{(A^4b^2+B^4a^2)^3} \quad (8)$$

where n is the index of refraction of the optical material, A and B are the semimajor and semiminor axes, respectively, of the ellipse whose arc is that of the vertical meridian of the accelerated surface, and a and b are the coordinates of the given point P(a, b) on the elliptical arc.

TABLE 1

Distance from apex of generated surface (mm.)	Distance from axis of symmetry (mm.)	Meridianal power (M) (diopters)	Transmeridianal power (T) (diopters)	Difference (T−M) (diopters)	Deviation of (T−M) from 1.00 diopter
2.124	18.147	5.0080	5.9982	0.990	−0.010
4.981	28.841	4.5615	5.5664	1.005	0.005
8.688	38.814	4.2426	5.2491	1.006	0.006
13.112	48.177	4.0196	5.0307	1.011	0.011
18.183	57.002	3.8721	4.8620	0.990	−0.010

The said other astigmatism which is present everywhere in the lens of this invention, except along the vertical meridian, has the effect of blurring vision in lateral gaze through the lens, but the amount of blurring is of sufficiently small magnitude near the vertical meridian, that clear and useful vision can be obtained through the lens along and in the immediate vicinity of the vertical meridian. Vision through lateral portions of the lens is useful, though not as clear as through the lens along the vertical meridian.

Although the lens of this invention has been described as made of glass, it is to be understood that it may be made of any other useful optical material, such as optical plastic.

The sample 4.25 diopter series used as an example is by way of illustration only. Other 4.25 diopter series, as well as other base curve series, may be designed having chord separations

of the base curve point and the add point other than 30 mm., and in which the compensated dioptric power of the base curve point is slightly different for each of the various adds. The use of a single cam for the production of a series is convenient and economical, though more than one cam, or a series of cams, may need to be used for some series.

TABLE 2

Add (diopters)	ϕ (inclination, degrees)	C_{base} a, mm.	C_{base} b, mm.	C_{add} a, mm.	C_{add} b, mm.
.50	39.40	18.74	75.90	47.70	67.91
.75	41.71	26.79	71.92	55.17	62.26
1.00	43.92	33.70	67.82	61.51	56.79
1.25	46.04	38.80	63.98	66.30	51.97
1.50	48.07	42.78	60.40	69.92	47.77
1.75	49.87	45.82	57.29	72.74	44.22
2.00	51.33	48.08	54.79	74.85	41.44
2.25	52.63	49.78	52.65	76.51	39.09
2.50	53.85	51.36	50.61	78.04	36.88

TABLE 3

Add	A, mm.	ϕ (inclination) degrees	α (azimuth) degrees	s (skewness mm.	Front surface astigmatism Nominal diopters	Front surface astigmatism Actual diopters
.50	100	39.40	34.12	21.35	.62	.60
.75	100	41.71	39.16	23.82	.87	.85
1.00	100	43.92	43.04	26.65	1.12	1.09
1.25	100	46.04	47.40	27.54	1.50	1.46
1.50	100	48.07	48.49	29.35	1.75	1.70
1.75	100	49.87	53.61	28.98	2.25	2.19
2.00	100	51.33	54.99	29.77	2.50	2.43
2.25	100	52.63	56.66	30.15	2.75	2.68
2.50	100	53.85	58.13	30.40	3.00	2.92

What is claimed is:

1. An optical lens of transparent optical material, useful for the correction of presbyopia, having a convex aspheric front surface which is a nonaxial portion of a surface of revolution having an apical cusp, all meridian sections of said convex lens surface being identical elliptical arcs with the least curved portion of the elliptical arc in the upper portion of the lens, all sections of said convex lens surface other than those sections normal to the axis of revolution being noncircular, the axis of revolution of said convex surface coinciding with a straight portion of a modified evolute of said elliptical arc determined by first determining a series of modified evolutes by adding for each modified evolute to be determined a different fixed increment of curvature units to the value of the curvature at each point along said ellipse and using the reciprocals of said sums at each point as distances measured along normals from each of said points of said ellipse toward said modified evolute as the locus of each said modified evolute, then taking a base curve point on said ellipse corresponding to the vertical elliptical meridian in the upper part of the lens, for far vision, and selecting an add point in the lower part of the lens, for reading, corresponding to the added positive spherical power for the correction of a wearer's presbyopia at a chord distance of predetermined length across said vertical elliptical meridian from said base curve point, then drawing normals from said base curve point and from said add point through all of said modified evolutes, and finally selecting that modified evolute which is substantially a straight line between said two last named normals, so that said surface is characterized by having a substantially constant difference in principal curvatures at all points along all meridian sections providing a substantially constant astigmatism at all points on said surface, with the transmeridional power being greater than the meridional power, while both principal curvatures along any elliptical arc meridian section change continuously and regularly and by substantially equal amounts to provide an accelerating surface, said lens having a negatively curved back surface optically coacting with said front surface, there being a common normal to both surfaces through the geometrical center of the front aspheric surface and through the midpoint of said chord, said back surface at least neutralizing said constant astigmatism of the front surface at the common normal, and the difference in dioptric powers between said base curve and add points being the added power required for the correction of the wearer's presbyopia.

2. A lens as defined in claim 1 wherein said lens back surface is negatively curved with a radius of curvature approximately 8 mm. less than the transmeridional radius of curvature of the aspheric convex surface at its geometrical center, the thickness of the lens along the common normal being approximately 8 mm.

3. A lens as defined in claim 1 wherein said back surface is toric and negatively curved in both principal meridians, said normal coinciding with an equatorial axis of symmetry of the toric surface, the principal directions of the back toric surface and the front aspheric surface coinciding along the common normal to both surfaces, the astigmatism of the back surface neutralizing that resulting from the front surface at the common normal, there being a minimum of unneutralized astigmatism away from the common normal for light rays in a plane containing the common normal and a meridian of the front aspheric surface and a principal meridian of the back surface, there being astigmatism for light rays through the lens at points lateral to said plane with principal directions at 45° and 135° to said plane, the amount of said astigmatism increasing with increasing lateral distance of said points from said plane, the amount of astigmatism, V, in diopters for a given distance h, in meters from said plane, at the level of point $P(a, b)$ along the elliptical arc meridian section of the front aspheric surface contained in said plane, being:

$$V=\frac{6(n-1)\,(A^2-B^2)\,(AB)^6\,hab}{(A^4b^2+B^4a^2)^3}$$

where a is the coordinate of the point $P(a, b)$ on the elliptical arc in the direction of the semimajor axis of the ellipse, OA, of length A, and b is the coordinate of the point $P(a, b)$ on the elliptical arc in the direction of the semiminor axis of the ellipse, OB, of length B, all distances being measured in meters, the origin of the Cartesian coordinates being the point 0 at the geometrical center of the ellipse, said lens being of usual spectacle lens thickness.

4. A lens as defined in claim 1 wherein said back surface is toric and negatively curved in both principal meridians, said normal coinciding with an equatorial axis of symmetry of the toric surface, the principal directions of the back toric surface and the front aspheric surface being noncoinciding along the common normal to both surfaces, the toric back surface being mathematically resolvable into two components: (1) an astigmatic component neutralizing that of the front aspheric surface at the common normal and approximately neutralizing the astigmatism for points along the meridian section of the front aspheric surface intersected by the common normal, and (2) a sphero-cylindrical component in which, when the meridian of the front aspheric surface which contains the normal to said surface at its geometrical center is oriented vertically in front of the eye, the cylinder portion and its axis found in this component is that of a patient's prescription, and the spherical portions of said sphero-cylindrical component is that of the patient's prescription to which has been added the negative of the dioptric power at the base curve point, said lens being of usual spectacle lens thickness.

5. A lens as defined in claim 1 wherein the excess of said transmeridional over said meridional power at a given level is 1.00 diopter.

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