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ACOUSTIC IMPEDANCE ELEMENT

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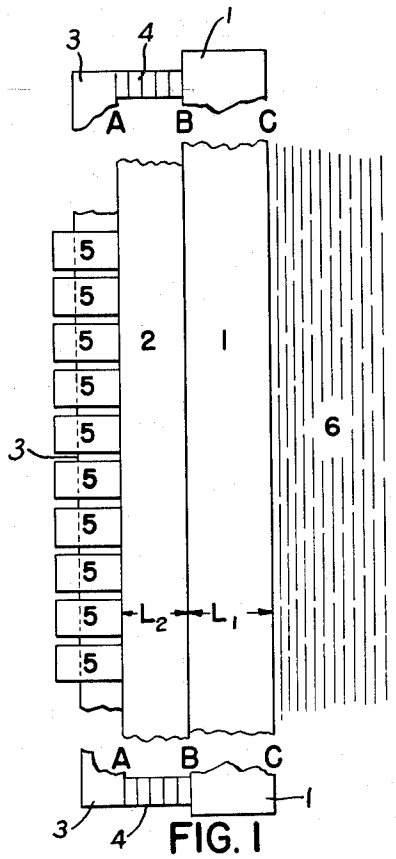


FIG. I

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ACOUSTIC IMPEDANCE ELEMENT

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The present invention relates to acoustic impedance elements intended to present a high impedance in a prescribed range of frequencies and to occupy a minimum of space. In particular, it relates to backing plates for electro-acoustic transducers having the above impedance characteristic and whose overall thickness is less than that of backing plates according to the prior art.

In the art of building transducers for converting electrical energy into sound or the reverse, it has been customary to mount the driving elements (such as magnetostrictive tubes or piezoelectric crystals) with one of their faces in contact with a "backing plate" of steel or other material having small elastic dissipation. When the transducer is to be used only over a narrow range of acoustic frequencies, and when the backing plate is in contact with air or other material of low impedance on its face opposite to the driving elements, it has in the past been common practice to make the thickness of such a backing plate substantially a quarter wave length at the frequencies used. It is well known that this dimension will cause the backing plate to present a high elastic or mechanical impedance to the driving element.

Especially where the frequency is comparatively low, the thickness and bulk of a quarter-wave length backing plate are undesirably great. Thus, a quarter-wave steel plate at 24 kc. would be 5.2 cm. thick. If it had an area of 1200 cm.^{2}, which is not unreasonable, it would weigh over 40 kg. or 90 lb. The importance of a construction which will reduce the thickness of the backing plate is at once apparent. The reduction is especially important in view of the fact that the backing plate must be housed in a substantial metal case and that, the thicker the plate, the larger and heavier must be the case.

It is shown in this specification that by the use of backing plates comprising layers of certain materials of dissimilar specific elastic impedance and arranged in certain orders, the total thickness of backing material may be substantially reduced. Liquids as well as solids may be used for some of the layers. For purposes of this application, backing plates, one or more of whose layers are of a liquid such as castor oil, are included in the general term "multiple-layer backing plate," since it will appear as the specification proceeds that the liquid layer is acoustically and elastically as much a part of the entire plate as the solid layer or layers.

Figure 1 illustrates an embodiment of my invention, comprising a two layer backing plate for an electro-acoustic transducer.

I now proceed to an exposition of the theory and principles of design of a backing plate comprising two layers of dissimilar acoustic impedance. In Figure 1, I illustrate schematically a backing plate having two layers designated 1 and 2, one face of the plate being in contact with the driving elements 5 and the other of its faces being in contact with a material 6 of negligible specific acoustic impedance, such as air.

It can be shown by means of conventional theory of elastic bodies that the specific acoustic impedance looking

toward the right from the interface BB, assuming negligible internal dissipation is:

$$Z_1 = j\rho_1 C_1 \tan K_1 L_1 \qquad (1)$$

where

$j=\sqrt{-1}$

ρ_1=density of material 1 in grams per cu. cm.

C_1=velocity of compressional waves in material 1 in cm. per sec.

$$K_1=\frac{2\pi f}{C_1}$$

f=frequency of vibration in cycles per second.

L_1=thickness of layer 1 in cm.

K_1L_1 is measured in radians.

Expression 1 assumes that the acoustic impedance of the air, viewed toward the right from the interface CC, is negligible compared with $\rho_1 C_1$, which is called the characteristic specific acoustic impedance of material 1.

The specific impedance looking toward the right from the interface AA is:

$$Z_2=\rho_2 C_2 \frac{Z_1 \cos K_2L_2 + j\rho_2 C_2 \sin K_2L_2}{\rho_2 C_2 \cos K_2L_2 + jZ_1 \sin K_2L_2} \qquad (2)$$

In (2), the subscripts "2" refer to the material of layer 2. Their significance will be evident by analogy with the corresponding quantities for layer 1, defined above.

Substituting the right-hand side of Expression 1 for Z_1 in Expression 2 and dividing both numerator and denominator of the right-hand member of the resultant equation by $j\rho_2 C_2 \cos K_2L_2$, one obtains:

$$Z_2=\frac{\rho_1 C_1 \tan K_1L_1 + \rho_2 C_2 \tan K_2L_2}{1-\frac{\rho_1 C_1}{\rho_2 C_2} \tan K_1L_1 \tan K_2L_2} \qquad (3)$$

The impedance presented to the driving elements Z_2, is to be made infinite. This might be done in any of three ways:

(*a*) By making $L_2=0$ and L_1 an add number of quarter wave lengths at the driving frequency and in the material of layer 1, in which case tan K_1L_1 is infinite;

(*b*) by making $L_1=0$ and L_2 an odd number of quarter wave lengths, thereby making tan K_2L_2 infinite;

(*c*) by making the denominator of (3) zero.

If method (*a*) be adopted, we shall require a layer 1 which is at least a quarter wave length thick. If layer 1 be of steel, its thickness for a frequency of 24 kc. will be 5.2 cm., as pointed out above. In this case, the thickness of layer 2 must be zero.

If layer 2 be of castor oil a quarter wave length thick (method (*b*)), we shall have 1.56 cm. of castor oil.

Clearly, there is no advantage in giving to layer 1 or 2 a thickness of ¾, 5/4 or more wave lengths, because the magnitude of Z_2 would be about the same both in theory and practice, as it would be for the ¼ wave length layer, but the mass and bulk of the layer would be greater.

We now investigate the thicknesses of the layers necessary to make the denominator zero in (3). The use of this method constitutes the basis of the present invention. Let the steel be layer 1, next the air, and the castor oil be layer 2, next the driving units. Let the thickness of the steel layer L_1 be 0.337 cm. and that of the oil layer L_2 be 0.362 cm. Since the driving elements cannot be supported by oil, this example at first sight appears not very useful, but it is evident that such a construction could be embodied in a transducer if the driving elements were supported in a jig which held their sides. The jig could then be located with respect to a steel plate 0.337 cm. thick by shims so that there would be a 0.362 cm. oil layer between elements and steel. The jig 3 and the shims 4 are schematically shown in Fig. 1.

For steel (layer 1), $\rho_1 C_1 = 39 \times 10^5$ cgs. units, and $K_1 = 0.301$ cm.$^{-1}$. For castor oil (layer 2),

$$\rho_2 C_2 = 1.5 \times 10^5 \text{ cgs. units, and } K_2 = 1 \text{ cm.}^{-1}$$

Inserting the above values in the denominator of (3), it becomes

$$1 - \frac{39 \times 10^5}{1.5 \times 10^5} \tan (0.301 L_1) \tan (1 \times L_2)$$

which equals zero, as is desired.

The total thickness of backing plate is

$$0.337 + 0.362 = 0.699 \text{ cm.}$$

which is only 13.4% as great as that of the quarter-wave steel plate under method (*a*), and only 44.8% as thick as the quarter wave castor oil plate under method (*b*).

There are, of course, an infinite number of combinations of thickness of steel and oil layer which for a given frequency will make the denominator infinite in Equation 3. The particular values of 0.337 and 0.362 cm. respectively are those which will minimize the total thickness of backing plate. I now explain how these particular values are computed, restricting each of the values to those less than one-half wave length. If one plots (L_1+L_2) as a function of L_1 for a particular value of frequency (and thus of K_1 and K_2), one may in general hope for a minimum in (L_1+L_2). For certain combinations of materials, this minimum may be achieved only with 100% of one material and 0% of the other. However, as in the example chosen above, there may in some instances be a mimimum in (L_1+L_2) for finite thicknesses of both materials, indicating that particular values of L_1 and L_2 produce $Z_2 = \infty$ for (L_1+L_2) less than that with either layer above. If so, this offers the possibility of constructing backing plates thinner than heretofore in the art of transducer construction.

In order to obtain the criterion for the existence of this minimum in (L_1+L_2), we first write (L_1+L_2) in terms of L_1, introducing the condition that the denominator of (3) shall be zero:

$$L_1 + L_2 = L_1 + \frac{1}{K_2} \tan^{-1} \frac{\rho_2 C_2}{\rho_1 C_1 \tan K_1 L_1} \tag{4}$$

Setting the partial derivative of (4) with respect to L_1 equal to zero, and solving for L_1, we obtain:

$$L_1 = \frac{1}{K_1} \tan^{-1} \left[\frac{\frac{\rho_2 C_2}{\rho_1 C_1} \left(\frac{\rho_2 C_1}{\rho_1 C_2} - 1 \right)}{\frac{\rho_2 C_2}{\rho_1 C_1} - \frac{C_1}{C_2}} \right]^{\frac{1}{2}} \tag{5}$$

which gives either a maximum or a minimum value of L_1. One may determine which it is by any method conventional in such minimization problems.

In a similar way one may write the expression for (L_1+L_2) similar to (5) but in terms of L_2:

$$L_1 + L_2 = \frac{1}{K_1} \tan^{-1} \frac{\rho_2 C_2}{\rho_1 C_1 \tan K_2 L_2} + L_2 \tag{6}$$

Differentiating with respect to L_2 and solving for L_2, we obtain:

$$L_2 = \frac{1}{K_2} \tan^{-1} \left[\frac{\frac{\rho_2 C_2}{\rho_1 C_1} \left(\frac{\rho_2 C_2}{\rho_1 C_1} - \frac{C_1}{C_2} \right)}{\frac{\rho_2 C_1}{\rho_1 C_2} - 1} \right]^{\frac{1}{2}} \tag{7}$$

The Expressions 5 and 7 give criteria for the existence of a minimum (L_1+L_2) and the values of L_1 and L_2 required to produce this minimum. If the quantity in square brackets in (5) or in (7) is negative, then the argument of the corresponding arc tangent is imaginary, and there are no values of L_1 and L_2 which will produce a minimum in (L_1+L_2). It should be noted that the arc tangents are independent of frequency and depend only on the densities and propagation velocities in the two layers. The Expressions 5 and 7 give either a maximum

or a minimum in (L_1+L_2), as remarked above. It turns out that if a minimum exists for a given pair of materials arranged in a given sequence, then a maximum exists if the order of the sequence is interchanged. Thus, while it is possible to save considerable space and weight by proper choice of materials, and thicknesses (such as steel for layer 1 and castor oil for layer 2), it is also possible to suffer considerable disadvantage by using these same materials in the wrong order (as castor oil for layer 1 and steel for layer 2).

In order to illustrate this, consider further the rather hypothetical situation when the steel layer is next to the driving elements and the castor oil is next to the air. The criterion functions (5) and (7) yield 1.21 cm. and 4.86 cm. respectively for the thicknesses of the oil and steel layers. Thus, the total thickness is 6.07 cm. and it would be markedly better if one were to use a quarter-wave plate of either material and omit the other.

In order to see physically rather than mathematically how the backing plate presents an infinite impedance, one may consider the waves of particle velocity which exist in the plate when it is being driven by the driving elements. Initially, in a single-layer plate, a wave of compression is propagated in the plate from that one of its faces which is in contact with the driving elements toward the other face. In the case of the conventional single layer plate, the wave of velocity proceeds horizontally through the plate until it reaches the other face, where it is reflected. This face, being in contact with air which has an impedance nearly negligible in comparison with that of the plate, will reflect the incident wave with a negligible change in its amplitude and phase. The reflected wave will be propagated toward the driving elements. If the plate is a quarter wave length thick, the reflected wave will arrive at the driving elements 180° out of phase with the incident wave there, as in the case of the open organ pipe, familiar in elementary physics. The velocities there which are associated with the incident and reflected waves will neutralize each other almost completely, so that the total velocity is nearly zero. The acoustic impedance of the backing plate is the ratio between the driving force and the particle velocity. The latter being nearly zero for the quarter-wave plate, a finite force produces an exceedingly small velocity; i. e., the impedance of the plate is exceedingly high as viewed from the driving elements.

When the plate is made up of two or more layers of materials of dissimilar acoustic impedance, the situation is more complex. The layers are arranged in tandem horizontally, i. e., in the direction of wave propagation in the plate, since the waves travel horizontally toward the right or left. To produce a high impedance at the driving elements, one must still have nearly complete interference between incident and reflected waves of particle velocity. The complexity resides in the multiplicity of reflected waves which arrive at the driving elements: (*a*) one which arrives after reflection at the interface BB between layers, (*b*) a second which arrives after transmission through that interface, reflection at the air boundary at the right in Fig. 1, and subsequent transmission from right to left across the interface BB, (*c*) still others which have been reflected and re-reflected one or more times at the interface and the air boundary before finally returning to the driving elements. The phases and magnitudes of all these are of course dependent upon various parameters, namely the thicknesses, propagation velocities and characteristic impedances of the several layers. In view of the number of parameters over which one can exercise a choice, it is not surprising that it is in general possible to so choose the magnitudes of these parameters that the reflected waves neutralize the transmitted wave and so that the total thickness of backing plate is less than that required in a simple plate of one material. The choice might be arrived at by experiment or by "cut-and-try" calculation, which may sometimes be the easiest way in

case the number of layers is greater than two. In the case of two layers, however, it is not difficult to arrive at a proper design by direct calculation in the manner set forth above. Nevertheless, this disclosure is not limited to any particular design procedure, nor to any specific number of layers.

It will be evident to those skilled in the theory of acoustic and elastic behavior that the mathematical and physical principles here enunciated have a close relationship to fields other than the design of backing plates. For example, it may be desired to construct a composite bar whose mechanical impedance in longitudinal vibration and at some frequency one wishes to be very high, and whose overall length is to be a minimum. A similar problem might be the design of an air column of high acoustic impedance and of minimal length, the column being formed by two or more rigid walled metal tubes of different cross-sectional areas and joined in tandem. Thus, the composite acoustic or mechanical impedance might have any of a wide variety of forms other than a flat plate. My invention comprises all such forms, the embodiment of Fig. 1 being intended to be illustrative and not restrictive.

The term acoustic, as used herein, includes all mechanical vibrations in gases, liquids or solids at frequencies below, in and above the audible range of frequencies.

Having described my invention, I claim:

1. In an electro-acoustic transducer, at least one driving element and a backing plate comprising structure defining a plurality of layers disposed in tandem in a direction parallel to that of acoustic wave propagation of prescribed frequency and wave length, the thickness of each of said layers measured parallel to the direction of wave propagation and the characteristic impedances of each of said layers being so interrelated that the acoustic impedance of said backing plate at said frequency as viewed from the driving element of said transducer is a maximum; the overall thickness of said backing plate measured in a direction parallel to that of wave propagation being substantially less than one fourth of said wave length; the surface of said plate remote from said driving element being in contact with an acoustic medium which represents a negligible impedance to said surface.

2. In an electro-acoustic transducer, a driving element; and a backing plate comprising structure defining three layers disposed in tandem in a direction parallel to that of acoustic wave propagation of prescribed frequency and wave length; the terminating layer of said plate remote from said driving element having a characteristic impedance which is negligible in comparison with that of the layer adjacent to it, the thickness of the remaining two layers in a direction parallel to that of wave propagation and the characteristic impedances of said layers being so interrelated that the acoustic impedance of said backing plate at said frequency as viewed from said driving element is a maximum; the overall thickness of said remaining two layers measured in a direction parallel to that of wave propagation being substantially less than one fourth of said wave length.

3. An acoustic impedance element, comprising: structure confining a plurality of layers disposed in tandem in a direction approximately parallel to the direction of an acoustic wave propagation of prescribed wave length and frequency; said layers being arranged in interface contact with each other so that said element presents a first terminating face and a second terminating face; the dimensions of each of said layers measured in said direction and the characteristic impedance of each of said layers being so inter-related that the acoustic impedance of said element at said frequency when viewed from said first face is at a maximum; the overall dimension of said element measured in said direction being substantially less than one fourth of said wave length; said second face being in contact with an acoustic medium which presents negligible impedance thereto.

4. In an electro-acoustic transducer adapted for operation with an acoustic wave propagation of prescribed wave length; a driving element; a backing plate having an overall thickness substantially less than one fourth of said wave length, said plate comprising: structure confining an inner layer positioned so that one face of said inner layer is in operating relationship with said driving element; and an outer layer positioned so that one face of said outer layer is in operating relationship with the other face of said inner layer; the other face of said outer layer being exposed to a medium of negligible acoustic impedance; said inner layer comprising a liquid having a specific elastic impedance; said outer layer comprising solid material having a dissimilar specific elastic impedance.

5. An acoustic impedance device comprising a pair of layers of acoustic material disposed in face to face contact in tandem in a direction substantially parallel to the direction of propagation of an acoustic wave of predetermined frequency f, and predetermined wavelength, said layers having a total thickness less than one quarter of said wavelength and characterized by the relation

$$1=\frac{\rho_1 C_1}{\rho_2 C_2}\tan K_1L_1 \tan K_2L_2$$

where

ρ_1 and ρ_2 are the densities of the respective layers

C_1 and C_2 are the velocities of compressional waves in respective layers,

$$K_1=\frac{2\pi f}{C_1}$$

and

$$K_2=\frac{2\pi f}{C_2}$$

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