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Cristal et al.

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[45] **July 10, 1973**

[54] **MICROWAVE AND UHF FILTERS USING DISCRETE HAIRPIN RESONATORS**

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[21] Appl. No.: **249,071**

[52] **U.S. Cl.** ..... **333/70 S,** 333/73, 333/84 M, 333/73 S

[51] **Int. Cl.** **H03h 7/10,** H03h 9/00

[58] **Field of Search** ..... 333/70, 70 S, 73, 333/73 S, 73 C, 84 M

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Attorney—Samuel Lindenberg et al.

[57] **ABSTRACT**

A new class of microwave and UHF filters is disclosed using only discrete hairpin resonators, with input and output lines either open circuited or short circuited at their ends, or using both discrete hairpin resonators and half-wave parallel-coupled line filter structures in hybrid filter configurations.

5 Claims, 12 Drawing Figures

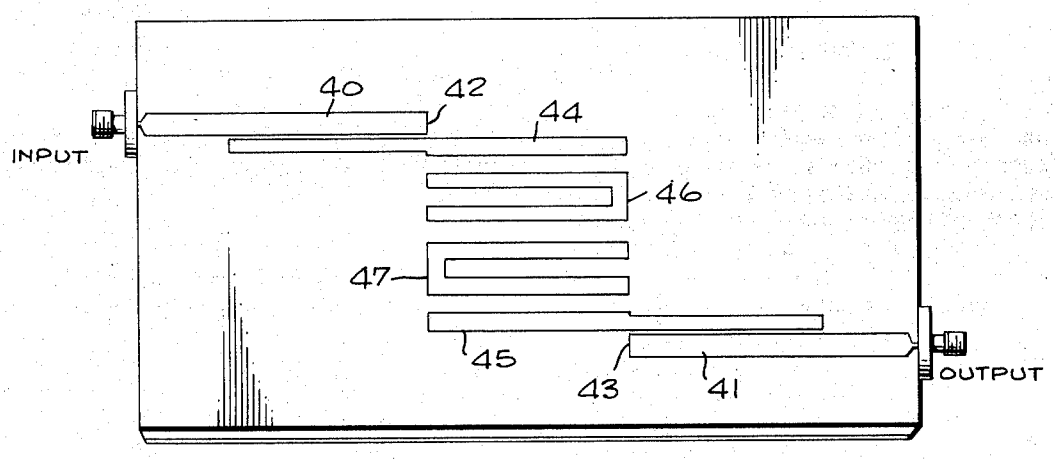


Fig. 1a

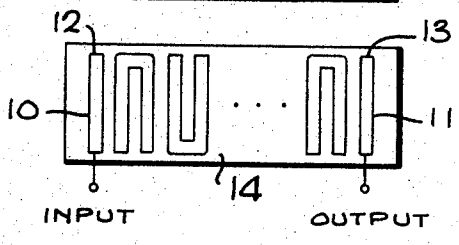


Fig. 1b

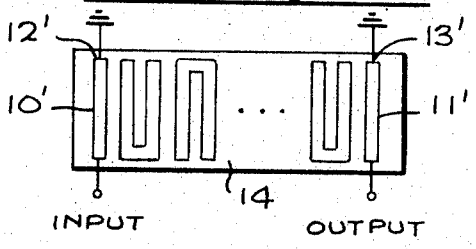


Fig. 2a

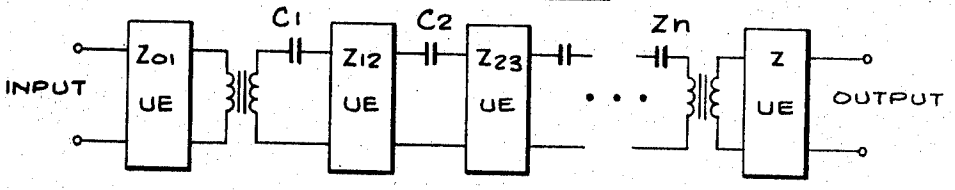


Fig. 2b

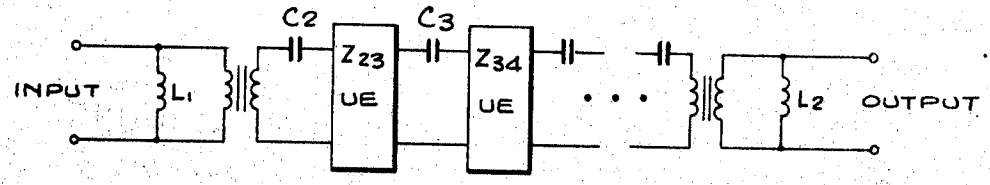


Fig. 3a

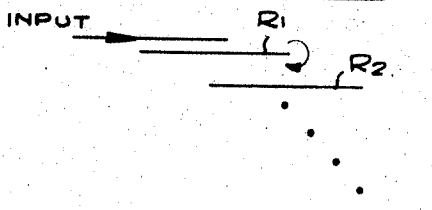


Fig. 3b

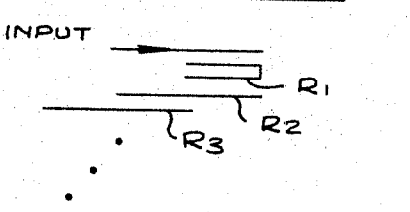


Fig. 4a

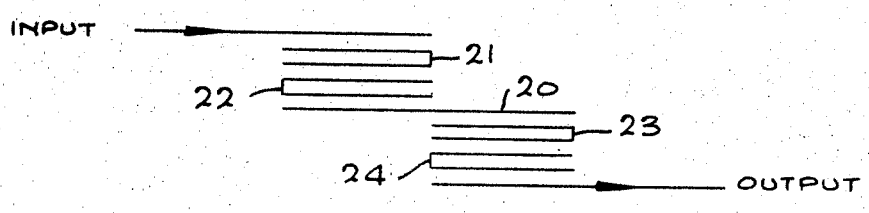


Fig. 4b

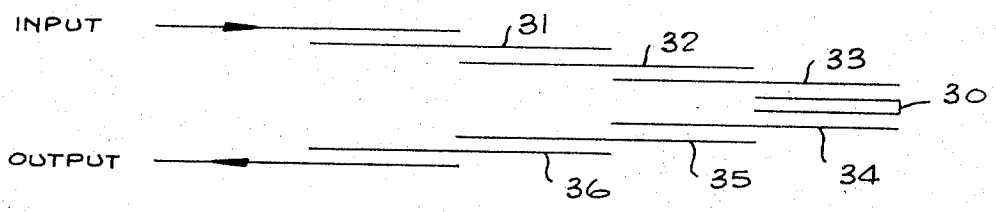


Fig. 5

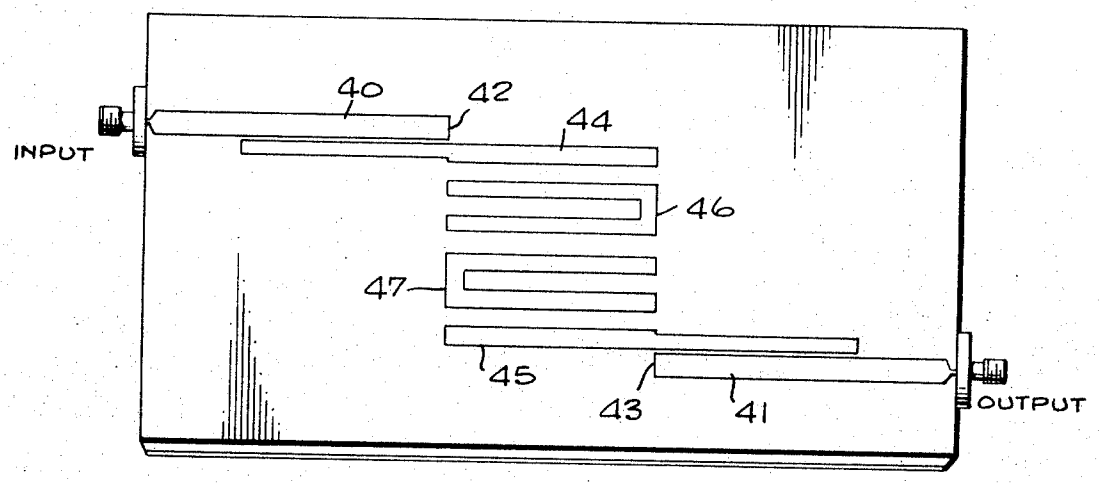


Fig. 6a

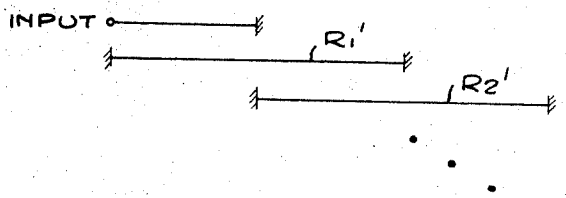
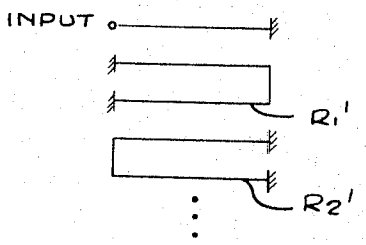


Fig. 6b



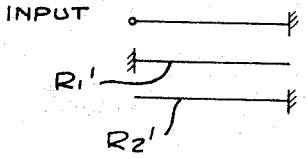


Fig. 6c

MICROWAVE AND UHF FILTERS USING DISCRETE HAIRPIN RESONATORS

BACKGROUND OF THE DISCLOSURE

This invention relates to microwave or UHF filters, and more particularly to a new class of filters comprised of discrete hairpin resonators.

FIELD OF THE INVENTION

This invention relates to microwave or UHF filters, and more particularly to a new class of filters comprised of discrete hairpin resonators.

BACKGROUND OF THE INVENTION

The use of stripline and/or MIC (microwave-integrated-circuit) designs in microwave systems is often preferred to other manufacturing methods. Stripline and MIC have advantages with respect to size, weight, costs, and often reproducibility. MIC is particularly useful in hybrid integrated circuits. Circuits and systems constructed in stripline or MIC would preferably utilize geometries not requiring connections to ground, since this is quite difficult (or at least awkward) in most cases. Among the numerous transmission-line filter designs available, the half-wave parallel-coupled-line filter satisfies the above condition. Hairpin line structures also satisfy the above condition, although to date, theory and design equations for these structures as filters have been unavailable.

The image impedance and propagation constant for the infinite periodic hairpin line have been previously reported by J. T. Bolljohn and G. L. Matthaei in "A study of the Phase and Filter Properties of Arrays of Parallel Conductors Between Ground Planes," Proc. IRE, Vo. 50, pp. 299–311, March 1962. However, for finite length hairpin filters, neither exact nor approximate design equations have been reported prior to this invention.

OBJECTS AND SUMMARY OF THE INVENTION

An object of this invention is to provide a microwave or UHF filter comprised of discrete hairpin resonators.

Another object is to provide a microwave or UHF filter comprised of discrete hairpin resonators and half-wave parallel-coupled-line filters.

These and other objects of the invention are achieved by arrangements of conventional (straight) half-wave parallel-coupled open-circuited resonators, hereinafter referred to sometimes as a half-wave resonator, and hairpin resonators, wherein each hairpin resonator consists of a half-wave resonator folded in the center. These resonators are implemented in a stripline or MIC form and so disposed on a substrate in an array that capacitive coupling beyond nearest neighbors is usually negligible. Input and output coupling is through quarter-wave parallel-coupled lines which are open circuited in all cases excepting the case of an adjacent hairpin resonator with its open end opposite the input or output end of a quarter-wave parallel-coupled resonator; in that case the quarter-wave parallel-coupled line is short circuited.

BRIEF DESCRIPTION OF THE DRAWINGS

FIGS. **1***a* and **1***b* show schematically two types of hairpin-resonator filters.

FIGS. **2***a* and **2***b* show approximate equivalent circuits for the filters of respective FIGS. **1***a* and **1***b*.

FIGS. **3***a* and **3***b* illustrate a conceptual process for obtaining hairpin-resonator filters from parallel-coupled half-wave open-circuited resonator filters.

FIGS. **4***a* and **4***b* illustrate two hybrid hairpin-half-wave resonator filters.

FIG. **5** is a plan view of an experimental hybrid hairpin-half-wave resonator filter.

FIGS. **6***a*, **6***b* and **6***c* illustrate a conceptual process for folding a half-wave parallel-coupled-line filter into an intermediate "wicket filter" and then into an interdigital filter.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

Two types of hairpin-resonator filters are shown schematically in FIGS. **1***a* and **1***b*. The first type shown in FIG. **1***a* is characterized by having its input and output lines **10, 11,** open circuited at their ends **12, 13.** This type of filter has been found to yield practical impedance levels for narrow to approximately 25-percent bandwidths.

The second type shown in FIG. **1***b* is characterized by having its input and output lines **10′, 11′,** short-circuited at their ends **12′, 13′.** This type of filter has been found to yield practical impedance levels for bandwidths greater than 25 percent.

Theoretical background and design equations for filters of the first type are presented. Corresponding equations for filters of the second type can be developed in a strictly analogous manner. Also experimental data for several stripline and MIC filters of 5 to 20 percent bandwidths are given. For convenience only MIC structures are illustrated in FIGS. **1***a* and **1***b*, it being understood that the filter elements spaced from one ground plane by suitable dielectric material could be similarly spaced from a second ground plane on the opposite side for MIC structures.

Approximate equivalent circuits for the filters of FIGS. **1***a* and **1***b* have been derived and are presented in FIGS. **2** *a* and *b*, respectively, wherein: each capacitor C_i represents the open-circuited transmission line of characteristic impedance C_i; each indicator L_i represents the short-circuited transmission line of characteristic impedance L_i; and UE is the unit element of characteristic impedance $Z_{i,\ i+1}$. These circuits are based on the assumption that inductive coupling beyond immediately adjacent conductors is negligible. For cmmmonly used microwave-filter geometries using parallel-coupling-line arrays this assumption differs from, and is more difficult to satisfy than, the usual one of negligible capacitive coupling.

Physically the difference between inductive and capacitive coupling can be understood in terms of the basic definitions of the coupling parameters. In the case of the admittance (capacitance) parameters, the immediately adjacent conductors act as partial electric shields between a conductor and the next adjacent conductor. In the impedance-parameter (inductive) case, the "floating" conductors only slightly effect the tendency of magnetic field lines to spread out over the entire coupled array.

The reason for making the former rather than the latter assumption is the enormous simplification that results in the equivalent circuits for the two types of hairpin-line filters. The assumption of negligible capacitive coupling beyond immediately adjacent conductors leads to quite complex equivalent circuits that pre-

cludes any likelihood of achieving practical design procedures. Ultimately, since approximations are necessary in either approach, the practicality of either assumption must be judged on the basis of comparisons between theoretical and experimental results. Before leaving the subject of the approximate equivalent circuit, it is suitable to point out that a formulation for the analysis of hairpin-resonator filter response in terms of an equivalent circuit based only on immediately-adjacent-conductor capacitive coupling is straightforward. (See R. Sato and E. G. Cristal, "Simplified Analysis of Coupled Transmission Line Networks," IEEE Trans. on Microwave Theory *and* Techniques, Vol. MTT-18, pp. 122–131, March 1970). Any design synthesized through the use of the equations presented hereinafter could be analyzed through the more accurate formulation.

Conceptually, hairpin-resonator filters may be obtained by folding the resonators of parallel-coupled half-wave open-circuited resonator filters. FIGS. **3***a*, and **3***b* illustrate the process. The first of a plurality of half-wave resonators R_1, R_2 . . . R_n shown in FIG. **3***a* is folded into a hairpin, giving the hybrid hairpin half-wave resonator filter shown in FIG. **3***b*. Next, if desired, the second half-wave resonator may be folded, giving a second hairpin resonator and a new hybrid filter. Other half-wave resonators are indicated simply by dots. The process may be continued as many times as the designer wishes. It is evident that a large number of hybrid filter realizations are possible. In all cases the same design equations may be used, and the same electrical performance results.

FIGS. **4***a* and *b* illustrate two hybrid filters. The filter of FIG. **4***a* consists of one half-wave resonator **20** and four hairpin resonators, **21** to **24** while the filter of FIG. **4***b* consists of one hairpin resonator **30** and six half-wave resonators **31** to **36.** The hybrid structure is the most important aspect of the present invention. This is because the hybrid filter provides the designer with the greatest flexibility in shape factor, and does not allow a surface wave mode to propagate. A filter comprised of all hairpin resonators can propagate a surface wave mode at $2f_o$ in stripline, and below $2f_o$ in MIC structures, where f_o is the bandcenter frequency of the filter. Consequently, most hairpin filters will be of the hybrid type in order to surpress the surface wave caused spurious responses.

The equivalent circuit for the first type (FIG. **1***a*) of filter shown in FIG. **2***a* is topologically dual to the well-known interdigital filter. Hence, existing design tables and approximate design procedures may be adapted for hairpin resonator filter designs. Space does not allow for the detailed development of the design equations; however, the essential design procedures are summarized below. Presented first are approximate design equations that were developed by modifying those given in "New Design Equations for a Class of Microwave Filters," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-19, pp. 486–490, May 1971, by E. G. Cristal. Theoretically, hairpin resonator filters designed from these equations will have precisely controlled bandwidth in all cases, and nearly equal ripple responses in Chebyshev cases.

First to be defined are the set of prototype filter parameters and auxiliary equations given in the following Table 1.

TABLE 1

AUXILIARY EQUATIONS AND PARAMETER DEFINITIONS

N = Order of Low-pass Prototype Filter

g_i = Low-pass Prototype Filter Element Values (= 0, 1, 2, . . . $N + 1$)

ω_1' = Low-pass Prototype Radian Cutoff Frequency

w = Fractional Bandwidth of the Microwave Filter

$w = 2[(f_2 - f_1)/(f_2 + f_1)] = (f_2 - f_1)/f_o$

where f_1 and f_2 are the lower and the upper band-edge frequencies of the microwave filter, and f_o is bandcenter

$\theta_1 = \pi/2\ [1 - w/2]$

$\tau = \frac{1}{2} \tan \theta_1$

$G_i = 1/\sqrt{\omega_1 g_{i-1} g_i}$ for $i = 1$ and $N + 1$

$G_i = 1/\omega_1' \sqrt{g_{i-1} g_i}$ for $i = 2, 3, \ldots N$

cp = hairpin coupling in dB

$k = 10^{-|cp/20|}$

h = Arbitrary positive dimensionless parameter usually less than **1**, which controls the immittance level in the filter interior

For $i = 1$ and $N + 1$	For $i = 2, 3, \ldots N$
$A_{11}^{(i)} = 1$	$A_{11}^{(i)} = h\tau$
$A_{12}^{(i)} = \sqrt{hG_i}$	$A_{12}^{(i)} = hG_i \sin \theta_1$
$A_{22}^{(i)} = h[G_i^2 + \tau]$	

N is the order of the low-pass prototype filter, having a cut-off frequency of ω_1', and normalized element values g_i. Tables of g_i values are available that give Chebyshev, Butterworth, and other types of responses in *Design of Microwave Filters, Impedance - Matching Networks, Coupling Structures* (New York: McGraw Hill, 1964) by G. O. Matthael, et al., and *Network Analysis and Synthesis* (New York: McGraw Hill, 1962) by L. Weinberg. The parameter "cp" in Table 1 is the coupling between lines constituting a single hairpin and not between adjacent hairpin resonators. Once the cp factor is chosen, the initial synthesized design will consist of hairpin resonators all having the same cp values. However, a simple method is presented later for obtaining arbitrary and different values of cp for each resonator, if desired.

A hairpin-line filter based on an N order low-pass prototype filter will consist of 2N+2 parallel-coupled-lines. The array of 2N+2 coupled lines is completely defined in terms of its inductance matrix which, for the assumed coupling conditions, can be written as

Inductance matrix normalized to

$$Z_A v^{-1} = \begin{bmatrix} L_{11} & L_{12} & 0 & 0 & \cdots & 0 \\ L_{12} & L_{22} & L_{23} & & & \\ 0 & L_{23} & L_{33} & L_{34} & 0 & \cdots 0 \\ \cdot & & L_{34} & L_{44} & L_{45} & \\ \cdot & & & \cdot & & \\ \cdot & & & & & \cdot\ L_{2N+2,2N+2} \end{bmatrix} \quad (1)$$

where

v = Velocity of propagation in the medium

$L_{ij} = l_{ij}/(Z_A v^{-1})$

l_{ij} = Self or mutual inductance per unit length of the i and j^{th} conductors

Z_A = Source impedance

In terms of the Auxiliary equations given in Table 1, the L_{ij} are given by Table 2.

TABLE 2

DESIGN EQUATIONS FOR HAIRPIN-LINE FILTERS

$$L_{pp} = [A_{22}^{(i)} + A_{11}^{(i+1)}2(1-k)] \quad \left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad \begin{matrix} p = 2i \\ i = 1, 2, \ldots N \end{matrix}$$

$$L_{p+1,p+1} = L_{pp}$$

$$L_{11} = A_{11}^{(1)}$$

$$L_{2N+2,2N+2} = A_{11}^{(N+1)}$$

$$L_{p,p+1} = A_{12}^{(i)} \quad \left\{\begin{matrix} p = 2i-1 \\ i = 1, 2 \ldots (N+1) \end{matrix}\right.$$

$$L_{p,p+1} = kL_{pp} \quad \left\{\begin{matrix} p = 2 \\ i = 1, 2, \ldots N \end{matrix}\right.$$

Having obtained the normalized inductance matrix from Table 2, the noralized capacitance matrix may be computed from the matrix equation

$$[C] = [L]^{-1} \tag{2}$$

where

$$C_{ij} = c_{ij}/(v^{-1}\, Y_A)$$

c_{ij} = Self or mutual capacitance per unit length of the i and j^{th} conductors

$$Y_A = 1/Z_A$$

The normalized parameters C_{ij} may be converted into the convenient form c_{ij}/ϵ by the equation

$$c_{ij}/\epsilon = (376.7/\sqrt{\epsilon_r})\; Y_A\; C_{ij} \tag{3}$$

where

ϵ_r = Effective relative dielectric constant of the medium

$\epsilon = \epsilon_0\epsilon_r$

ϵ_0 = Permittivity of free space

From the numerical values for c_{ij}/ϵ the physical dimensions of the coupled lines may be determined from various design charts.

If the L matrix is sparse, as is the case here, the C matrix will generally be dense. (It may even represent a nonphysical system.) However, only the diagonal and the upper and lower sub-diagonal entries are of significance. These will be referred to as principal terms or principal entries. Other entries (referred to as nonprincipal) will generally be small in comparison, at least for narrowband filters, and may be neglected. However, as the filter bandwidth is increased, the nonprincipal terms of the C matrix will also increase. Since it is known that the capacitance matrix is approximately sparse for the planar-array filter configurations ordinarily used at microwave frequencies, checking the nonprincipal terms is one way of estimating when the filter responses may begin to deviate appreciably from the theoretically expected behavior.

As noted previously, the equations in Table 2 give designs consisting of hairpin resonators, all of which have the same coupling. However, a simple transformation may be performed that allows modification of the coupling between two lines forming a hairpin, and by which hybrid hairpin-half-wave resonator filters may be designed.

The detailed theory for hairpin-resonator filters of the type under discussion reveals that their responses are invariant, provided that the expression

$$L_{ii} - 2L_{i,i+1} + L_{(i-1),i+1} \text{ for } i = 2, 4, 6, \ldots (2N) \tag{4}$$

remains constant. In addition, physical realizability requires

$$L_{ii} - L_{i,i-1} - L_{i,i+1} > 0 \text{ for all } i\,. \tag{5}$$

The L_{ij} parameters in Eq. (4) correspond to the self and mutual inductance between lines constituting a hairpin resonator. Thus, it is evident that there is considerable redundancy and flexibility in determining suitable numerical values for the hairpin resonators. A matrix transformation that satisfies Eq. (4) and whose effect on the filter geometry is easily visualized, is easily programmed on a computer. That matrix is as follows:

$$\begin{bmatrix} L_{11} & L_{12} & 0 & 0 & . & . & 0 \\ L_{12} & L_{22}+\Delta_{22} & L_{23}+\Delta_{22} & 0 & . & . & 0 \\ 0 & L_{23}+\Delta_{22} & L_{33}+\Delta_{22} & L_{34} & . & . & 0 \\ . & & L_{34} & L_{44}+\Delta_{44} & L_{45}+\Delta_{44} & \ldots & \\ . & & & & & \ldots & \\ . & & & & & . & \\ 0 & & & & & & L_{2N+2,2N+2} \end{bmatrix}$$

Note that the even-numbered rows have the increment Δ_{ii} added to the appropriate L_{ij} values in a way that satisfies Eq. (4). Thus, all values Δ_{ii} not violating Eq. (5) give physically realizable inductance matrices corresponding to equivalent hairpin-resonator filters.

The transformation

$$\Delta_{ii} = -L_{i,i+1} \quad i = 2, 4, \ldots (2N) \tag{6}$$

is particularly useful, for in this case no inductive (and capacitive) coupling exists between lines originally constituting a hairpin resonator. In other words, Eq. (6) transforms the hairpin resonator into a half-wave parallel-coupled-line resonator. Repeated application of Eq. (6) permits the design of hybrid hairpin-half-wave filter structures. The following example will serve as an illustration. The original L matrix for the filter shown in FIG. 5 with a hairpin-resonator coupling of 12 dB (cp = 12 dB), is given in the upper half of Table 3.

TABLE 3

NORMALIZED L VALUES FOR EXAMPLE HYBRID FILTER OF FIG. 5

NORMALIZED I	L-MATRIX L(I,I)	L(I,I+1)
1	1.00000	0.30031
2	1.40641	0.08874
3	1.40641	0.08303
4	1.35828	0.08570
5	1.35828	0.06571
6	1.35828	0.08570
7	1.35828	0.08303
8	1.40641	0.08874
9	1.40641	0.30031
10	1.00000	

NORMALIZED I	L-MATRIX L(I,I)	L(I,I+1)
1	1.00000	0.30031
2	1.31767	0.00000
3	1.31767	0.08303
4	1.35828	0.08570
5	1.35828	0.06571
6	1.35828	0.08570
7	1.35828	0.08303
8	1.31767	0.00000
9	1.31767	0.30031
10	1.00000	

The physical realization of the filter for this matrix would be an all-hairpin resonator filter like that in FIG. 1(*a*). By letting

$$\Delta_{88} = \Delta_{22} = -0.08874\,, \tag{7}$$

the results given in the lower half of Table 3 are obtained. This leads to the hybrid configuration of FIG. 5 comprised of input and output lines **40** and **41**, open circuited at their ends **42** and **43** and coupled to respective half-wave resonators **44** and **45**, and two hairpin resonators **46** and **47**.

The use of the preceding transformation allows, in a straightforward way, the application of exact design tables for interdigital filters to the design of hybrid hairpin-half-wave resonator filters. In order to present the method concisely we summarize below some needed facts concerning the design of interdigital filters.

An interdigital filter may be conceptualized as the result of folding the half-wave parallel-coupled line filter shown in FIG. 6(*a*) comprised of half-wave resonators R_1', R_2'. . . , short-circuited at both ends into the intermediate "wicket filter" shown in FIG. 6(*b*). Then, since the grounded ends of the wickets are physically close and at the same potential, they may be connected, yielding the interdigital form given in FIG. 6(*c*). Although the original justification for this folding procedure was based on physical considerations, the entire process has been rigorously verified mathematically by Sato, et al., supra. The wicket filter and the hairpin-resonator filter are electrical duals. Consequently, if the interdigital filter in FIG. 6(*c*) can be converted back to the wicket form in FIG. 6(*b*), the principle of duality may be applied to achieve a hairpin-resonator filter design.

It is well known that the following capacitance matrix can uniquely represent an interdigital filter of the form shown in FIG. 6(*c*).

$$\begin{bmatrix} C_{11} & -C_{12} & 0 & \cdots & 0 \\ -C_{12} & C_{22} & -C_{23} & & \\ 0 & -C_{23} & C_{33} & & \\ \cdot & & & \cdot & \\ \cdot & & & & \\ 0 & & & & C_{N+1,N+1} \end{bmatrix}$$

Interdigital Filter Capacitance Matrix

Tables of c_{ij}'s are available that yield optimum Chebyshev and maximally flat response, and it can be proved that the above $(N+1) \times (N+1)$ capacitance matrix corresponding to an interdigital filter can be expanded into the following $(2N+2) \times (2N+2)$ matrix, with the latter corresponding to a wicket filter in which there is no coupling between any two lines constituting a wicket.

$$\begin{bmatrix} C_{11} & -C_{12} & 0 & & \cdot & 0 \\ -C_{12} & a_{22}C_{22} & 0 & & & \cdot \\ 0 & 0 & (1-a_{22})C_{22} & -C_{23} & & \cdot \\ 0 & & -C_{23} & a_{33}C_{33} & 0 & \cdot \\ \cdot & & & 0 & & \cdot \\ 0 & \longleftarrow & \text{Row } 2N+2 & \longrightarrow & & C_{N+1,N+1} \end{bmatrix}$$

Wicket Filter Capacitance Matrix

Furthermore, the responses of the wicket and interdigital filters will be identical. The parameters a_{ii} in this matrix are real, but at this intermediate point need not satisfy any other conditions. Of course, if the wicket filter itself were to be realized, then each row of the capacitance matrix would have to satisfy Eq. (5) with L replaced by C.

At this point it is evident how to proceed. The C_{ij} of this last matrix are to be replaced by L_{ij} (with suitable normalization), so that the matrix formally represents a half-wave parallel-coupled-line filter with open-circuited resonators. Next, the transformations depicted in the matrix preceding Eq. (6) are performed to obtain the particular hybrid hairpin-half-wave resonator filter desired. The L matrix is then inverted to obtain the C matrix, which is then realized physically.

A second transformation that is useful is a special form of inductance congruence transformation very similar to capacitance matrix transformations described by R. J. Wenzel, "Theoretical and Practical Applications of Capacitance Matrix Transformations to TEM Network Design," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-14, pp. 635–647, Dec. 1966. The effect of this transformation is to scale the impedance level at specific points within the filter, without changing its transfer response. The transformation as applied to interior sections of the filter is as follows.

$$\overbrace{\quad}^{N}$$

$$N\begin{Bmatrix}\rightarrow\\ \rightarrow\end{Bmatrix}\begin{bmatrix} L_{11} & L_{12} & 0 & 0 & \cdots & 0 \\ L_{12} & L_{22} & L_{23} & 0 & & 0 \\ 0 & L_{23} & L_{33} & L_{34} & & \\ \cdot & 0 & L_{34} & \cdot & & \\ \cdot & \cdot & & & \cdot & \\ \cdot & \cdot & & & & \\ 0 & 0 & & & & \cdot \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} L_{11} & NL_{12} & 0 & 0 & \cdots & 0 \\ NL_{12} & N^2L_{22} & N_2L_{23} & & & \\ 0 & N^2L_{23} & N^2L_{33} & NL_{34} & & \\ \cdot & 0 & NL_{34} & \cdot & & \\ \cdot & \cdot & & & \cdot & \\ \cdot & \cdot & & & & \\ 0 & 0 & & & & \cdot \end{bmatrix}$$

Note that the transformation must be performed on paired-rows and paired columns, in distinction to conventional capacitance-matrix transformations. The allowed pairs of rows and columns are

Row-Columns i and $i+1$, for $i = 2, 4, \ldots 2N$ (8)

The only exceptions to this rule are when transforming the input and output impedances. In order to modify these or, equivalently, to change the terminations, Row-Column 1 (or $2N+2$) would be multiplied by a suitable constant—say, M. This would modify the input (or output) impedance of the filter by the factor M^2.

The congruence transformation and the transformation of preceding Eq. (6) may be carried out in any sequence or be intermixed. Both transformations are particularly useful when programmed for an interactive time-share computer terminal.

What is claimed is:

1. A structure selected from stripline or microwave-integrated-circuit forms for use as a finite filter comprised of: a plurality of half-wave, parallel-coupled and open-circuited resonators disposed in parallel in an array, at least one of said resonators being folded at the center to form a discrete hairpin resonator, each of said resonators having negligible coupling to all other resonators in said array excepting immediately adjacent resonators and the end of each resonator being opposite the center of adjacent resonators in said array and unconnected to any other resonator; an input line parallel to one of said resonators at one end of said array; and an output line parallel to one of said resonators at the other end of said array.

2. A structure as defined in claim **1** wherein each of said input and output lines is open circuited.

3. A structure as defined in claim **1** wherein each of

said input and output lines is short circuited.

4. A structure as defined in claim 1 wherein only some of said resonators are folded at the center to form a hybrid array of hairpin and half-wave resonators.

5. A structure as defined in claim 1 wherein all of said resonators are half-wave parallel-coupled open circuited resonators folded at the center to form an array of hairpin resonators.

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