

Sept. 3, 1929.

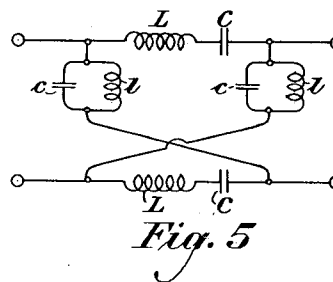
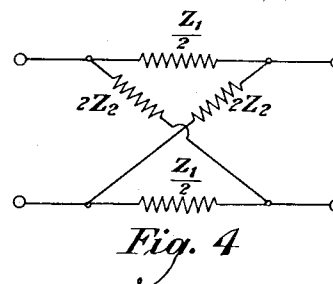
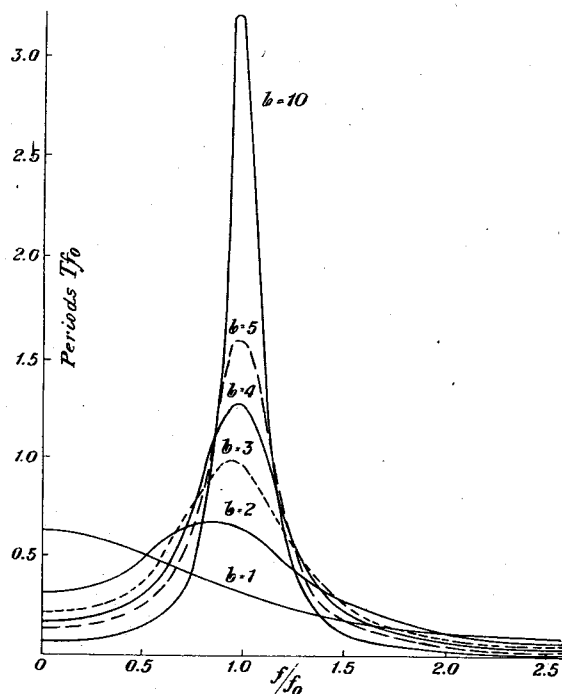
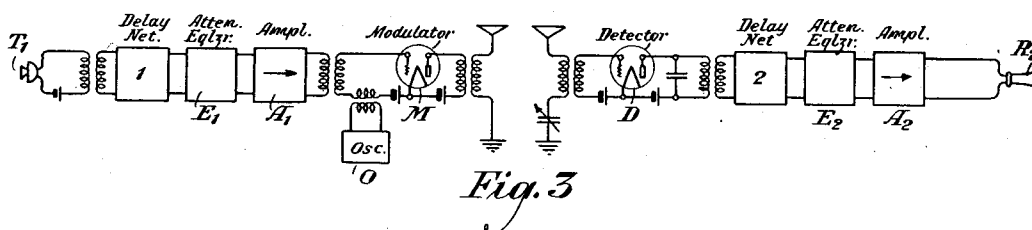
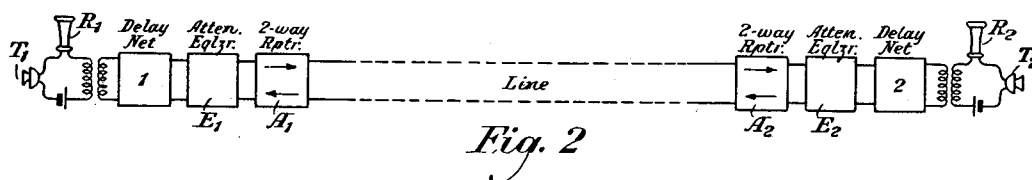
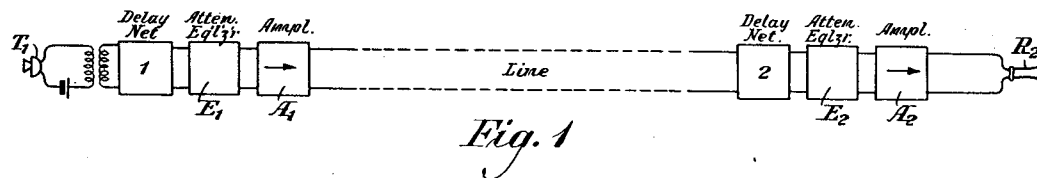
H. NYQUIST ET AL

1,726,578

SECRET TELEPHONE SYSTEM

Filed Feb. 21, 1928

3 Sheets-Sheet 1



INVENTORS
H. Nyquist and P. Mertz
BY *gr 408*
ATTORNEY

Sept. 3, 1929.

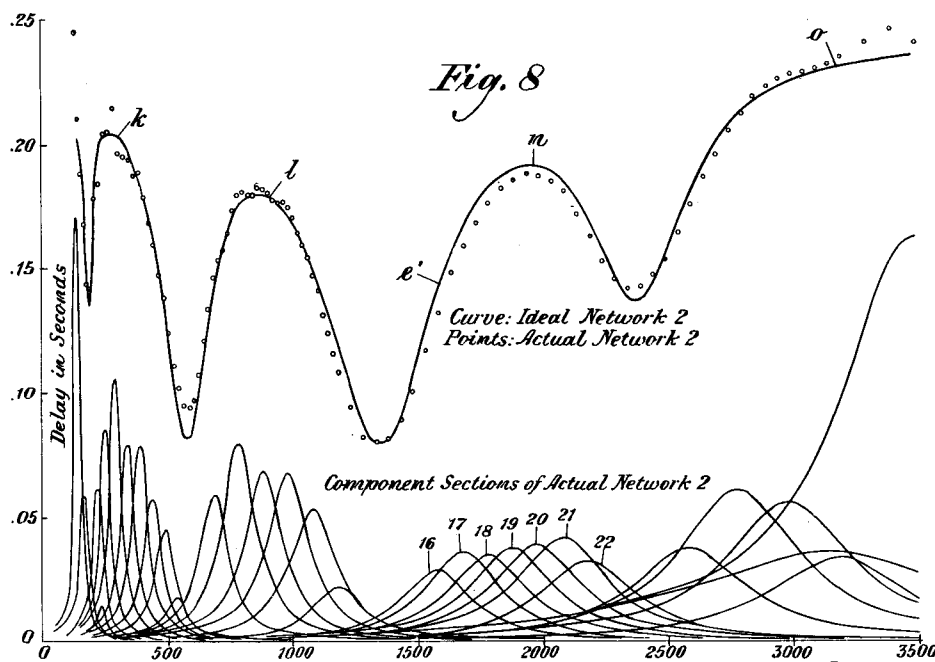
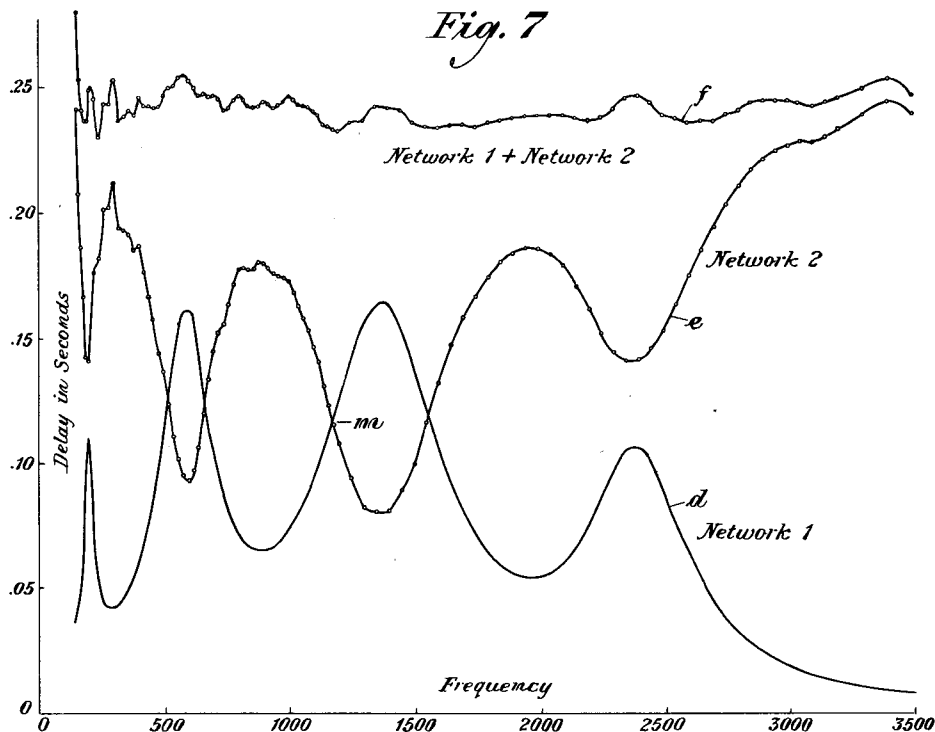
H. NYQUIST ET AL

1,726,578

SECRET TELEPHONE SYSTEM

Filed Feb. 21, 1928

3 Sheets-Sheet 2



INVENTORS
H. Nyquist & P. Mertz
BY *g v for*
ATTORNEY

Sept. 3, 1929.

H. NYQUIST ET AL

1,726,578

SECRET TELEPHONE SYSTEM

Filed Feb. 21, 1928

3 Sheets-Sheet 3

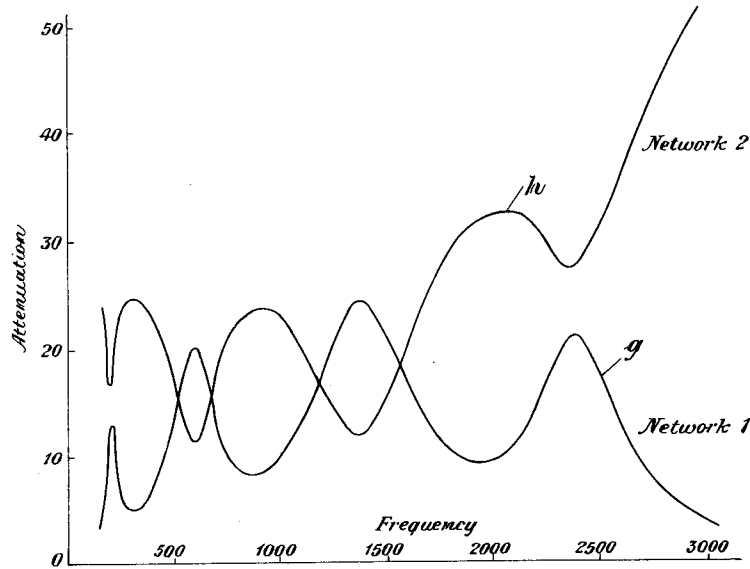


Fig. 9

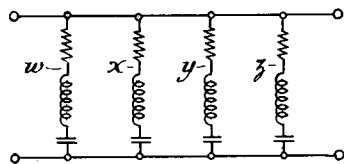


Fig. 10

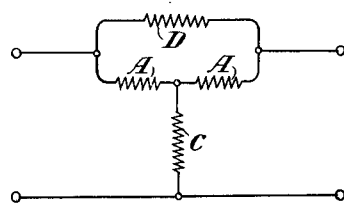


Fig. 12

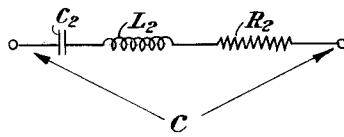


Fig. 14

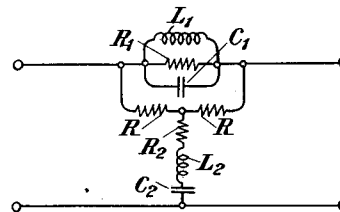


Fig. 11

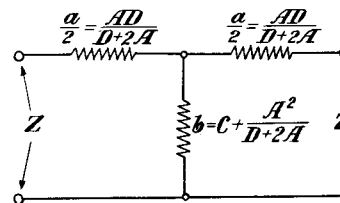


Fig. 13

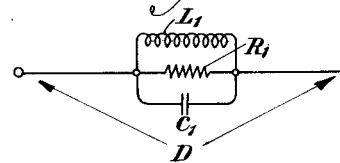


Fig. 15

INVENTORS
H. Nyquist & P. Mertz
BY
J. E. 402
ATTORNEY

UNITED STATES PATENT OFFICE.

HARRY NYQUIST, OF MILLBURN, NEW JERSEY, AND PIERRE MERTZ, OF BELLEROSE, MANOR, NEW YORK, ASSIGNORS TO AMERICAN TELEPHONE AND TELEGRAPH COMPANY, A CORPORATION OF NEW YORK.

SECRET TELEPHONE SYSTEM.

Application filed February 21, 1928. Serial No. 255,984.

This invention relates to signaling systems, and more particularly to methods and apparatus for attaining secrecy of transmission.

In the transmission of messages and particularly in telephony, it is desirable to transmit the message over the transmission medium in such a manner as to prevent the reception of the message by unauthorized persons, either by tapping the wire, in the case of line transmission, or by picking up the message through a radio receiver, in the case of radio transmission.

In accordance with the present invention, this condition of secrecy is attained by passing the band of frequencies corresponding to the signal through a delay network at the sending end, so designed as to introduce different degrees of delay at the different frequencies involved in transmission. If the differences between the delays imposed on the various frequencies are sufficiently great, the message will be transmitted in such distorted form as to be unintelligible if received in the ordinary way. Before reaching the ultimate receiver, however, the band of waves corresponding to the signal is passed through another network which subjects the component frequencies to a delay which is complementary to that imposed at the sending end (i. e., such that the sum of the two delays is constant for all frequencies), so that there will be no distortion resulting from the networks. In the case of wire transmission, a person tapping the line, or, in the case of radio transmission, any one attempting to pick up the message by an ordinary radio receiver, will be unable to interpret the speech which is being transmitted if he does not have available a network having the same or very nearly the same delay frequency characteristic as that required at the receiving end of the system.

The invention will now be more fully understood from the following detailed description, when read in connection with the accompanying drawing, in which Figure 1 illustrates schematically the layout of a one-way wire telephone system employing the principles of the invention; Fig. 2 illustrates

the invention embodied in a two-way telephone transmission system; Fig. 3 illustrates the invention as embodied in a radio telephone system; Fig. 4 illustrates in schematic form the type of network section employed in making up the delay network in accordance with the invention; Fig. 5 illustrates a delay network section in greater detail; Fig. 6 shows a family of curves illustrating the principles in accordance with which the delay network is designed; Fig. 7 illustrates delay-frequency curves for the sending network, the receiving network, and the combination of the two in accordance with one possible design of the delay system; Fig. 8 is a series of curves illustrating how the complementary network may be designed; Fig. 9 shows curves illustrating the variation of the attenuation of the delay networks with frequency; Figs. 10 and 11 show types of networks which may be employed in correcting the variable attenuation introduced into a system by the variable delay networks; Figs. 12 and 13 illustrate schematically equivalent forms which the network of Fig. 11 may assume; while Figs. 14 and 15 illustrate in detail the resistance, capacity, and inductance elements making up certain component impedances of the network of Fig. 12.

Referring to Fig. 1, which shows the invention as applied to a one-way wire telephone system, a network 1 is introduced at the sending end of the system beyond the transmitter T_1 , this network being so designed as to have a delay which varies with frequency over a considerable range. At the receiving end a complementary network 2 is introduced ahead of the receiver R_2 , this network being so designed that it will introduce a delay for each frequency, which, when added to the delay introduced by the network 1, will give a constant over-all delay for all of the frequencies involved. Either network of course may be used at the sending end if the other is used at the receiving end. Possible delay frequency characteristics for these networks are illustrated in Fig. 7. The design of networks having a predetermined de-

lay frequency characteristic will be discussed in more detail later.

If desired, amplifiers A_1 and A_2 may be connected to the output sides of the delay networks to compensate for the transmission loss through the network. If the delay networks, as will usually be the case in practice, introduce a considerable variation of attenuation with frequency, this may be compensated for by associating with each delay network a corresponding attenuation equalizer network, as shown at E_1 and E_2 . These attenuation equalizer networks will be so designed that the over-all attenuation of the delay network with its corresponding attenuation equalizer network will be substantially constant with frequency. Where the attenuation equalizers are employed, the amplifiers A_1 and A_2 may be used to compensate for the losses introduced both by the delay networks and the attenuation equalizing networks. The design of the attenuation equalizer networks will be discussed more fully hereafter.

The application of the invention to a two-way telephone system is illustrated in Fig. 2. This arrangement differs from that of Fig. 1 principally in the fact that two-way repeaters are employed at the terminals to compensate for the losses introduced by the networks, instead of one-way amplifiers as in Fig. 1. Thus, during transmission from west to east, the two-way repeater A_1 employs its west-to-east amplifier unit to make up the loss introduced by the delay network 1 and the attenuation network E_1 , while the east-to-west amplifier unit of the same repeater compensates for the losses introduced by these networks during transmission in the opposite direction. The two-way repeater A_2 of course functions with respect to the delay network 2 and the equalizer network E_2 in a similar manner.

Fig. 3 illustrates the invention as applied to a radio transmission system. Here the delay network (and the attenuation equalizer, if employed) are inserted ahead of the modulator M at the sending end and beyond the detector D at the receiving end. There is of course the choice of introducing the variable delay to produce secrecy in either the voice current or the voice-modulated carrier current, but since in the latter case the design construction of the necessary delay-producing networks becomes practically impossible, the arrangement illustrated in Fig. 3 is preferable.

The minimum amount of difference between the delays at the various frequencies necessary to insure unintelligibility of the signals, may be estimated in the absence of direct experimental data. From a study by Crandall on speech sounds, published in the Bell System Technical Journal, vol. 14, page

586, for October, 1925, it appears unnecessary to confuse all possible voice sounds entirely in order to secure fairly complete unintelligibility. For example, if the larger part of the consonants are entirely confused and the vowels and remaining consonants are confused to an appreciable degree, the resulting sound will be, for all practicable purposes, unintelligible. It has been estimated, from the study by Crandall, above referred to, that most of the consonant sounds could be completely confused by differences in delay of the order of 0.1 second.

While the foregoing description has been given in terms of a telephone system, it will be understood that the same general method may be employed to obtain secrecy on practically any type of electrical communication systems where the successive signals are not too far apart, such, for example, as a telegraph, telautograph, or picture transmission system.

The phase changing networks used in connection with the present invention may comprise a plurality of filter sections, each section being of the so-called lattice type illustrated in Fig. 5. This type of filter is illustrated and described on page 19 of an article by Otto J. Zobel, entitled "Theory and design of uniform and composite electric wave filters," published in the Bell System Technical Journal, of January, 1923. In order that the methods of design of a phase-changing network of this type may be clear, it is desirable that certain fundamental principles underlying the lattice type of network be elucidated.

As will be seen from Fig. 5, the lattice section consists of two series resonant elements, each comprising an inductance L and capacity C in series and two shunt resonant elements, each comprising an inductance l and capacity c in parallel.

While the design formulæ for L , C , l , and c may be derived in accordance with various methods, it is convenient to express the values of these elements in terms of three parameters: f_0 , the common resonance frequency of the four tuned circuits; K , a constant equal to the characteristic impedance of the networks; and b , a parameter which is arbitrarily assigned the value $2\sqrt{\frac{L}{l}}$. This parameter b may be thought of physically as proportional to the energy stored at the resonance frequency in any one of the four tuned circuits. b may also be thought of as π times the delay at the resonance frequency expressed in cycles of such resonance frequency. If T represents the delay time this last relation may be expressed $\frac{b}{\pi} = T f_0$, this expression following from Equation (32) which will be derived hereinafter.

L, C, l , and c may be expressed in terms of K, f_0 and b from the following relations:

$$LC = lc \quad (1)$$

$$K = \sqrt{\frac{L}{c}} = \sqrt{\frac{l}{C}} \quad (2)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{lc}} \quad (3)$$

$$b = 2\sqrt{\frac{L}{l}} = 2\sqrt{\frac{c}{C}} \quad (4)$$

Equation (1) expresses the fact that both the series resonant and shunt resonant tuned circuits in the network of Fig. 7 resonate at the same frequency. Formula (3) expresses the well known relation between the resonant frequency and the reactances of a tuned circuit. Equation (4) follows from the definition of b as above given, and Equation (2) may be derived as follows:

Consider a lattice network in which the series and shunt impedances have the values indicated in Fig. 4. Such a network as shown by Zobel in the paper published in the Bell System Technical Journal, above referred to, has a characteristic impedance which is constant at all frequencies. Let K be this characteristic impedance and assume that the network is so designed that K will be equal to the impedance of the circuit to which it is connected. By setting down equations in terms of the drops, impedances and currents for the different closed loops of the network, in accordance with Kirchhoff's laws and then solving, it will be found that

$$K = \sqrt{z_1 z_2} \quad (5)$$

(See Equation (23) of the Zobel paper above referred to).

Comparing Figs. 4 and 5, the impedance of a series element of a single section of the network may be expressed:

$$\frac{z_1}{2} = i\omega L + \frac{1}{i\omega C} \quad (6)$$

Similarly, the shunt element may be expressed:

$$2z_2 = \frac{i\omega l \frac{1}{i\omega c}}{i\omega l + \frac{1}{i\omega c}} = \frac{1}{\frac{i\omega l c}{l} + \frac{c}{i\omega l}} \quad (7)$$

From the relation expressed in Equation (1) it is evident that Equation (7) may be rewritten as follows:

$$2z_2 = \frac{1}{i\omega L \left(\frac{C}{l} \right) + \left(\frac{c}{L} \right) \frac{1}{i\omega C}} \quad (8)$$

Furthermore, since it follows from Equation (1) that

$$\frac{C}{l} = \frac{c}{L} \quad (9)$$

Equations (5), (6), and (8) may be combined in the following form:

$$K^2 = z_1 z_2 = \frac{l}{C} = \frac{L}{c} \quad (10)$$

Equation (10) obviously corresponds to Equation (2).

By simple algebraic transformation, the following design formulæ may be obtained from Equations (1), (2), (3), and (4):

$$l = \frac{K}{\pi b f_0} \quad (11)$$

$$c = \frac{b}{4\pi f_0 K} \quad (12)$$

$$L = \frac{b K}{4\pi f_0} \quad (13)$$

$$C = \frac{1}{\pi K b f_0} \quad (14)$$

From these design formulæ the several reactances making up each section of a delay changing network may be computed, once the parameters f_0 , K and b are known. In the design of a network to give any desired delay characteristic, K will in general be fixed by the impedance of the circuits between which the network is to be connected, while b and f_0 will be determined from the shape of the delay-frequency curve which it is desired to obtain.

In order to facilitate such a determination, it is convenient to take advantage of the fact that a family of curves may be plotted at different values of b between the delay and frequency for one section of the type of network shown in Fig. 5, such a family of curves being illustrated in Fig. 6. To make this family of curves perfectly general, it is most convenient, instead of plotting frequencies as abscissæ and delay time T as ordinates, to plot the ratio $\frac{f}{f_0}$ between each frequency f and the resonance frequency f_0 as abscissæ, and the product Tf_0 of the delay time and resonance frequency as ordinates. The resultant curve will then represent how the delay, expressed in periods or cycles of the resonance frequency of the tuned elements of the network, varies with frequency expressed in terms of the ratio of any given frequency to the resonance frequency.

It will be noted that the curves of Fig. 6 have peaks which increase with increasing values of b , and that these peaks occur at a

frequency in the neighborhood of the resonance frequency, the peak approaching the resonance frequency more closely as the value of b increases. It is also a fact that the shape of each curve is such that the total area under each curve of the family is the same, and this area for a single section of the network will be unity for each curve of the family, when the curves are plotted as above stated.

In utilizing these curves to design a network, a curve is chosen from the family whose shape is such as to simulate the desired delay characteristic. Since the curves each represent a different value of b , the curve thus chosen determines b , while f_0 , the resonance frequency, is determined by the peak frequency of the curve representing the desired delay characteristic. The number of sections necessary to give the desired delay may be obtained by determining the area beneath the curve of the desired delay characteristic and dividing this area by the area of the chosen curve for a single section of network. The effect of multiplying the sections is merely to correspondingly multiply the delay at each frequency without changing the essential character of the curve.

The formulæ in accordance with which the family of curves shown in Fig. 6 may be plotted, will now be derived. For a network section of the type indicated generally in Fig. 4, with the impedance values indicated thereon, the characteristic impedance will be given by Formula (5) above. Likewise, the propagation constant Γ may be expressed in accordance with the following formula:

$$\cosh \Gamma = 1 + \frac{2z_1}{4z_2 - z_1} \quad (15)$$

See Zobel article above cited.

It is desirable that the characteristic impedance shall be a real constant K and approximately the same as the impedance (resistance) of the elements with which the network is connected on the input and output sides. Assuming that the impedances z_1 and z_2 are dissipationless, that is, made of reactance elements only, then if z be taken as the reactance of the series element of a network having the desired characteristics, but whose characteristic impedance is equal to unity, the impedance of the corresponding series element for a network whose characteristic impedance is K may be expressed:

$$z_1 = Kz \quad (16)$$

By substituting this value of z_1 in Equation (5) we obtain, as the impedance of the corresponding shunt element,

$$z_2 = \frac{K}{z} \quad (17)$$

Substituting in Equation (15) the values given by Equations (16) and (17) it follows that:

$$\cosh \Gamma = 1 + \frac{2z}{z - z} \quad (18)$$

In accordance with the principles of hyperbolic trigonometry, the following well-known formula holds:

$$\tanh \frac{\Gamma}{2} = \sqrt{\frac{\cosh \Gamma - 1}{\cosh \Gamma + 1}} \quad (19)$$

Substituting from Equation (18), this reduces to:

$$\tanh \frac{\Gamma}{2} = \frac{z}{2} \quad (20)$$

In general, the propagation constant Γ may be put equal to $\alpha + i\beta$, where α is the real part of the attenuation and β is the phase shift constant. Since the structure for z is of reactance elements only, it follows that $\alpha = 0$, and hence Equation (20) may be written:

$$\tanh \frac{\Gamma}{2} = \tanh \frac{i\beta}{2} = \frac{z}{2} \quad (21)$$

Now, for a section of network of the type illustrated in Fig. 5, the element in series with the line is a series resonant circuit comprising an inductance L and a capacity C . Having in mind the relation expressed by Equation (16), the impedance of this element may be expressed as follows:

$$z_1 = \frac{Kz}{2} = i\omega L + \frac{1}{i\omega C} \quad (22)$$

This equation may be rewritten:

$$\frac{z}{2} = \frac{i\omega L}{K} + \frac{1}{i\omega KC} \quad (23)$$

Multiplying both numerator and denominator by $\frac{2}{K\sqrt{LC}}$ we have:

$$\frac{z}{2} = \frac{i\omega \frac{2}{K}\sqrt{\frac{L}{C}}}{2} + \frac{\frac{2}{K}\sqrt{\frac{L}{C}} \cdot \frac{1}{i2\omega}}{i2\omega} \quad (24)$$

But, from Equation (3) it follows that:

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (25)$$

where ω_0 is 2π times the resonant frequency. Also, from Equations (13) and (14) we have:

$$b = \frac{2}{K}\sqrt{\frac{L}{C}} \quad (26)$$

Substituting the values of Equations (25) and (26) in Equation (24) we obtain:

$$\frac{z}{2} = \frac{i\omega b}{2\omega_o} + \frac{b\omega_o}{i2\omega} \quad (27)$$

Substituting this value in Equation (21) we have:

$$\tanh \frac{i\beta}{2} = \frac{ib}{2} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right) \quad (28)$$

whence

$$\beta = 2 \tan^{-1} \frac{b}{2} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right) \quad (29)$$

It is approximately true that the delay in a circuit may be expressed:

$$T = \frac{d\beta}{d\omega} \quad (30)$$

By differentiating (29) and substituting in (30) we get:

$$T = \frac{\frac{b}{\omega_o} \left(1 + \frac{\omega_o^2}{\omega^2} \right)}{1 + \frac{b^2}{4} \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right)^2} \quad (31)$$

This may be rewritten:

$$Tf_o = \frac{b}{2\pi} \frac{1 + \frac{f_o^2}{f^2}}{1 + \frac{b^2}{4} \left(\frac{f}{f_o} - \frac{f_o}{f} \right)^2} \quad (32)$$

From Equation (32) the curves of Fig. 6 for the various values of b may be plotted between the delay in periods or cycles of the resonant frequency (Tf_o) and the ratio between frequency and resonant frequency ($\frac{f}{f_o}$).

In applying the foregoing principles to the design of delay networks such as illustrated at 1 and 2 in Figs. 1, 2, and 3, it is fairly clear that one of the two networks must be to a large degree arbitrary, the chief requirement being that the delay should vary within wide limits (for example, 0.1 seconds) throughout the range of voice frequency. This network, in the detailed design about to be described, will be designated as network 1, and as will be seen, the design for it will be comparatively simple. The other network hereinafter designated network 2 must be designed to be complementary to network 1. The design of this second network will obviously be much more difficult than that for network 1.

The delay-frequency curve for a design of network 1 is shown at d of Fig. 7. The network design to give this curve is obtained by superimposing groups of sections of the character shown in Fig. 5 with critical frequency f_o and characteristic b rather arbitrarily chosen for each group. In each group making

up one of the humps of the curve the sections are alike. The number of sections in each group corresponding to a hump of the curve was so chosen as to obtain a variation of about 0.1 second between maxima and minima. In this design all of the individual sections of the entire network may be of the same general character as the network section illustrated in Fig. 5, but the reactance elements making up a section of one group may have different electrical values from elements making up a section of another group.

The several groups of sections correspond to each of four critical frequencies corresponding to the humps of the curve d of Fig. 7. These frequencies are 200, 600, 1400, and 2400 cycles, respectively. The value of b assigned to the sections of each group, and the number of sections of each group necessary to make up the desired hump in the curve d , are given in the following table:

TABLE I.—Network 1.

f_o	b	No. of sections
200.....	10	5
600.....	5	52
1250.....	5	5
1400.....	5	132
2400.....	10	71
Total.....		265

It will be noted that in the above table a group of five sections appears, having a critical frequency of 1250 cycles. This group of sections does not correspond to a definite hump or peak in the curve d . After making up the z curve corresponding to d from the sections given in Table I, with this particular group of five sections omitted, and then making up a preliminary design of the corresponding complementary network, it was found that a depression occurred in the delay curves for both networks, in the neighborhood of 1250 cycles. In other words, the crossing corresponding to m of Fig. 7 for the curves corresponding to d and e in the original design, occurred at a delay value lower than the other crossings of these two curves. Additional sections could have been added to either network in the neighborhood of 1250 cycles to bring this crossing up to the desired delay. Inasmuch, however, as the peaks in the curve d were all rather sharp, it was preferred to add the five sections in question to network No. 1, as this tended to round out at least one of the peaks, thereby slightly increasing the range of frequencies in this region which would have a large delay.

In the original design of the network 1, as given in Table I, when the curve d was plotted it was found that the average delay was somewhat low for high and low frequencies as compared with intermediate frequencies. This is clearly shown by the curve

d , in which the peaks of the humps corresponding to 600 and 1400 cycles are much higher than the peaks of the humps corresponding to 200 in 2400 cycles. No attempt was made to change this as it represents an advantage in further extending the variations in delay.

As has already been stated, the design of a complementary network is a somewhat more complicated matter than that of the first network. The method of designing the complementary network 2, corresponding to network 1, is illustrated by the curves of Fig. 8. The full line curve e' is the delay frequency characteristic of the ideal network which would be exactly complementary to network 1. The actual network as finally designed has delay values for the different frequencies indicated by the dots or small circles in Fig. 8, as will be noted. These values closely approximate the ideal values indicated by the curve e' , and they are plotted in curve form at e of Fig. 7. The curve f of Fig. 7 represents the sum of the delay values indicated by the curves d and e and therefore represents the over-all delay introduced by the two networks 1 and 2 at the two frequencies. While theoretically the curve f should be a straight horizontal line, the slight irregularities or departures of the curve f from the ideal straight line are so small as to introduce a distortion which would not be noted by the hearer.

In seeking to obtain the curve e' of Fig. 8, the humps in the characteristic were treated, as far as possible, separately. The method of treatment may be illustrated by the hump having a maximum at about 1900 cycles. The top of this hump is evidently too broad and the sides too steep to be handled by a single type or design of section of the character shown in Fig. 5. It was therefore decided to approximate the result by a series of groups of sections, each group having a sharp characteristic ($b=10$) and the successive groups having resonant or characteristic frequencies 100 cycles apart throughout the range of the hump.

The approximate numbers of sections of each of the five groups corresponding to the frequencies ranging from 1700 to 2100 cycles, were arrived at by making the rough computed sums of the ordinates equal to the height of the hump of the curve e' . For example, the height of the hump of the curve e' in the neighborhood of 1900 cycles is about .19 second. Considering the curves 17, 18, 19, 20, and 21, which are the curves of the five principal groups of sections, which, when added together, are to give the curve approximating the hump of e' at about 1900 cycles, it will be seen that the ordinate of the curve e' at 1900 cycles is made up of the peak value of the curve 19 plus the delay represented by each of the curves 17, 18, 20, and

21 at this frequency. The exact contribution of each of the curves other than 19 to the final result could not be originally predicted. It was therefore assumed for preliminary computation that each of the five curves 17 to 21, inclusive, would have the same number of sections, and would have approximately the same peak value. It was also assumed that the contribution of each of the curves to the ordinate of the curve e' at 1900 cycles, was that of its peak value.

Now, from Fig. 6, a section having the parameter $b=10$ would have a delay Tf_0 in periods per second at its peak of 3.5. For the critical frequency ($f_0=1900$ cycles) the delay T would then be .00168 seconds for each section. Dividing this into .19 seconds, the desired delay at 1900 cycles, as given by the curve e' , the total number of sections would be 115.

This total of 115 sections was first assumed to be equally divided among the groups corresponding to the five different frequencies from 1700 cycles to 2100 cycles, so there would be 23 sections for each group. Curves corresponding to 17 to 21, inclusive, were then plotted on the assumption that there were twenty-three sections entering into the delay of each curve, and the resultant of the five curves was then plotted. It was at once discovered that this preliminary design made the resultant curve too high at the left of the frequency 1900 cycles and too low at the right. Accordingly, the number of sections for the groups corresponding to 1700 and 1800 cycles was reduced and the number of sections for the groups corresponding to 2000 and 2100 cycles was increased. The final design was worked out at 19 sections for the groups corresponding to 1700 and 1800 cycles, 22 sections for 1900 cycles, 24 sections for 2000 cycles, and 27 sections for 2100 cycles. This gave a fair approximation of the top of the hump of the curve e' in the neighborhood of 1900 cycles. It was then necessary to add a somewhat smaller number of sections for each side of the hump at the frequencies 1600 and 2200 cycles, as shown by the curves 16 and 22.

A similar procedure was followed with each of the other humps. On adding the results for the different humps together, however, it was found that the sections on the high frequency side of one hump would affect the delay on the low frequency side of the next hump. These numbers, therefore, had to be revised until after several trials satisfactory results were obtained. For the entire design the individual sections were all of the type having the value $b=10$ but the critical frequency of course increased for each successive group from left to right in Fig. 8. The frequency spacing of the critical frequencies of the several groups was made somewhat lower at the lower frequencies than

100 cycles and somewhat higher than this value at the upper frequencies. The final design in terms of critical frequency parameter b and number of sections for each group is given in the following table:

TABLE II.—Network 2.

f_o	b	No. of sections	f_o	b	No. of sections
10	150	10	Hump "n"	1600	10
	175	10		1700	10
	220	10		1800	10
	240	10		1900	10
	260	10		2000	10
	300	10		2100	10
	350	10		2200	10
	400	10		2600	10
	450	10		2800	10
	500	10		3000	10
15	550	10	Hump "o"	3200	10
	700	10		3200	5
	800	10		3500	10
	900	10			
	1000	10			
	1100	10			
	1200	10			
20			Total		

In the final design, as represented in the above table, it was also found necessary to add sections at miscellaneous places in the frequency range. The basis on which these were added was to examine the delay curve f for the two networks combined. Wherever there were appreciable depressions in this curve, sections were added. For the final design the number of sections of each characteristic frequency was revised up or down according to whether the curve of total delay was low or high at this frequency. The resultant design, as above given, is not perfect as is evidenced by the irregularities of curve f of Fig. 7, but these deviations should not appreciably diminish the equality of voice transmission.

Network sections of the type shown in Fig. 5, when non-dissipative, give no attenuation whatever throughout an infinite frequency range, from zero up. These network sections are also of constant characteristic impedance (pure resistance) throughout the entire frequency range, so that terminal reflection effects on the attenuation can be eliminated. If these ideal conditions could be obtained, it would be unnecessary to provide attenuation equalizing networks such as indicated in Fig. 1 at E_1 and E_2 .

In practice, however, it is impossible to make the networks non-dissipative with practical coils and condensers in which there is a dissipation which increases gradually with frequency. Under these conditions the attenuation will be proportional to the delay, but the ratio of proportionality will gradually increase with frequency. In the present networks in which the number of sections is very large, the variation with frequency in this attenuation will be appreciable. The variation of the attenuation with frequency for the networks 1 and 2, as above designed, is shown at g and h of Fig. 9. In practice the

attenuation curves would be obtained accurately by actual measurements of the networks 1 and 2. The curves g and h , however, as shown in Fig. 9, represent computed or theoretical estimates of the attenuation values.

Where the attenuation varies, as shown by curves g and h , it is obviously necessary to correct for the attenuation distortion of each network if the energy levels for the various frequencies are to be kept equal in the line or other transmission medium. Accordingly, attenuation equalizing networks, such as E_1 and E_2 , are provided as shown in Figs. 1, 2, and 3. Various methods of designing these networks so as to make the desired attenuation correction without giving further appreciable phase distortion, will now be briefly described.

For example, the attenuation network may be made up of a series of tuned circuits such as w , x , y , and z , shunted across the line conductors as indicated in Fig. 10. The tuned circuits are adjusted to resonate at the frequencies corresponding to minimum attenuation in the network to be corrected. For example, in the case of the network 1, whose attenuation is indicated by the curve g of Fig. 9, such minima occur at approximately 300, 900, 2000, and 3000 cycles. The resonant frequency of each tuned circuit fixes the product of the inductance by the capacity. The ratio of the inductance to the capacity is adjusted to vary the height of the attenuation peak, the greater this ratio the less being the height of the peak. The resistance or damping of the tuned circuit can then be adjusted to vary the sharpness of the peak. These various adjustments may be made by a cut and try method until the desired equalization of attenuation is effected.

An arrangement involving parallel tuned circuits, such as illustrated in Fig. 10, has the disadvantage that any adjustment of one tuned circuit produces some effect on the adjacent tuned circuits. For this reason it is sometimes preferable to make up the attenuation equalizing network of a plurality of sections of the type illustrated in Fig. 11, one section being built up for each minimum point in the attenuation curve. As will be noted, each section includes a series resonant and a shunt resonant circuit and the two tuned circuits in each section are made resonant at the same frequency. By a proper proportioning of the parts the characteristic impedance of the section may be made a constant pure resistance for all frequencies. Under these circumstances it is of course possible to shift the frequency adjustment and the attenuation of the section without producing any change in the attenuation of adjacent sections. The design of networks of the type of Fig. 11 to accomplish the desired attenuation compensation may be understood

from the following theoretical considerations.

Let us consider a network consisting of impedances A, A, C and D, connected as shown in Fig. 12. It will be seen that this network corresponds to that of Fig. 11, A representing the resistance R and C and D representing the impedances of the shunt and series tuned circuits, respectively.

A network such as that of Fig. 12 has been shown by K. S. Johnson in his book "Transmission Circuits for Telephonic Communication," page 282, to be equivalent to a simple T-network of the type shown in Fig. 13, with shunt impedance b , and series impedances $\frac{a}{2}$ on either side of the shunt impedance, if the following relations hold:

$$\frac{a}{2} = \frac{AD}{D+2A} \quad (33)$$

$$b = C + \frac{A^2}{D+2A} \quad (34)$$

If the network be assumed to be terminated at the distant end in its characteristic

impedance Z then the characteristic impedance Z, looking into the network from the other end, will have the following value:

$$Z = \frac{a}{2} + \frac{b\left(\frac{a}{2} + Z\right)}{b + \left(\frac{a}{2} + Z\right)} \quad (35)$$

This reduces to:

$$Z^2 = a\left[b + \frac{1}{4}a\right] \quad (36)$$

Now, if the values of a and b , as given by Equations (33) and (34) are substituted in Equation (36) we have:

$$Z^2 = \frac{AD}{D+2A} \left[\frac{2CD+4AC+2A^2+AD}{D+2A} \right] \quad (37)$$

Now, let us assume that the following relation exists between the series and shunt impedances of the network of Fig. 13:

$$CD = A^2 \quad (38)$$

Substituting the value of C as given by Equation (38) in Equation (37) we have:

$$Z^2 = \frac{AD}{D+2A} \left[\frac{2A^2 + \frac{4A^3}{D} + 2A^2 + AD}{D+2A} \right] = A^2 \quad (39)$$

If now, we make A in Fig. 12 a pure resistance R (as indeed it is in its prototype of Fig. 11) we get:

$$Z^2 = R^2 = CD \quad (40)$$

From the above equation it is obvious that if the elements A, C, and D are related to each other in accordance with Equation (38) the impedance of the network will be a pure resistance and constant at all frequencies.

Now let us determine how the inductance, resistance and capacity elements of the tuned circuits, comprising impedances C and D of Fig. 12, must be related. Let us assume that the shunt impedance C is made up of the capacity C_2 , inductance L_2 , and resistance R_2 , as shown in Fig. 14, and that the series impedance D of Fig. 12 is made up of an inductance L_1 , resistance R_1 , and capacity C_1 , as shown in Fig. 15. It is obvious from Fig. 14 that if the shunt impedance C be constructed as a series resonant tuned circuit, its impedance may be expressed as follows:

$$C = R_2 + i\left(L_2\omega - \frac{1}{C_2\omega}\right) \quad (41)$$

Likewise, if the series elements D be an anti-resonant tuned circuit, as shown in Fig. 15, the admittance of the anti-resonant element may be expressed as follows:

$$\frac{1}{D} = \frac{1}{R_1} + i\left(C_1\omega - \frac{1}{L_1\omega}\right) \quad (42)$$

In order to satisfy Equation (40), let us assume that the following relations hold:

$$R_1R_2 = \frac{L_2}{C_1} = \frac{C_2}{L_1} = R^2 \quad (43)$$

Equation (42) may then be written:

$$\frac{1}{D} = \frac{1}{R^2} \left[R_2 + i\left(L_2\omega - \frac{1}{C_2\omega}\right) \right] = \frac{C}{R^2} \quad (44)$$

From Equations (41) and (44) we get:

$$CD = R^2$$

which is Equation (40).

It follows, therefore, that if the resistances, capacities, and inductances of the network are adjusted to vary the attenuation, they must be so adjusted as to maintain the relations given by Equation (43). If this is not done the impedance of the network will not be a pure resistance and constant with frequency.

The important feature of the network is its propagation constant as this determines the attenuation introduced by the network. From Equation (4), page 123, of the book by K. S. Johnson, above referred to, the propagation constant Γ for the network of Fig. 13 (and hence for its prototype of Fig. 11) may be expressed as follows:

$$\Gamma = \log \frac{\frac{a}{2} + b + \sqrt{ab + \frac{a^2}{4}}}{b} \quad (45)$$

From this it follows that the attenuation e^r will have the following value:

$$b = 2A^2 \frac{(D+A)}{D(A+2A)} \quad (47)$$

$$e^r = \frac{\frac{a}{2} + b + \sqrt{ab + \frac{a^2}{4}}}{b} \quad (46)$$

Substituting the value of C as given by Equation (38) in Equation (34) we have:

$$e^r = \frac{\frac{AD}{D+2A} + 2A^2 \frac{(D+A)}{D(D+2A)} + \sqrt{\frac{2AD}{D+2A} \cdot 2A^2 \cdot \frac{(D+A)}{D(D+2A)} + \left(\frac{AD}{D+2A}\right)^2}}{2A^2 \frac{(D+A)}{D(D+2A)}} \quad (48)$$

Whence

$$e^r = 1 + \frac{D}{A} = 1 + \frac{D}{R} \quad (49)$$

In view of Equation (40) Equation (49) may be written:

$$e^r = 1 + \frac{A}{C} = 1 + \frac{R}{C} \quad (50)$$

From Equation (50) it is evident that in the network of Fig. 11 the resistance elements R may remain fixed, these elements being determined by the desired characteristic impedance, in accordance with equation (40). The resonant frequency of the tuned circuit corresponding to impedance C is determined by the point in the curve g or h , as the case may be, where minimum attenuation between any two successive humps is to be compensated. This fixes the product of the elements C_2 and L_2 . By adjusting the ratio of the inductance L_2 to the capacity C_2 , the attenuation of the network of Fig. 11 at the resonant frequency may be given the desired value necessary to compensate the attenuation at that frequency of the circuit to be corrected, and by adjusting the resistance R_2 the attenuation at adjacent frequencies may be varied. In other words, as shown by Equation (50) the impedance C of the circuit shown in Fig. 14 may be varied to produce any desired attenuation without varying the resistance elements R or the characteristic impedance of the section. The values of the elements L_1 , R_1 , and C_1 of the tuned circuit comprising the series impedance D of the section are at once determined by the relation expressed in Equation (43).

It will of course be understood that additional sections may be designed to equalize the attenuation for other minimum points of the curves of Fig. 9. The ultimate design of an attenuation equalizing network of any of the types described of course involves, to a certain degree, cut and try methods.

It will also be obvious that the general principles herein disclosed may be embodied in many other organizations widely different from those illustrated, without departing

If now the values of $\frac{a}{2}$ and b , as given by Equations (33) and (47), respectively, be substituted in Equation (46) we have:

from the spirit of the invention as defined in the following claims.

What is claimed is:

1. The method of secret transmission which consists in passing all of the component frequencies of a signal band together through a static network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, and transmitting the band of frequencies as distorted by the resulting phase shift over a transmission medium.

2. The method of secret transmission which consists in passing all of the component frequencies of a signal band together through a static network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, transmitting the band of frequencies as distorted by the resulting phase shift over a transmission medium, receiving the transmitted band, and restoring the component frequencies to substantially their normal phase relations.

3. The method of secret transmission which consists in passing all of the component frequencies of a signal band together through a static network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, transmitting the band of frequencies as distorted by the resulting phase shift over a transmission medium, receiving the transmitted band, and passing the band of frequencies through a static network having a delay-frequency characteristic complementary to that to which the frequencies were subjected before transmission.

4. The method of secret transmission which consists in subjecting the component frequencies of a signal band to different degrees of delay before applying them to a transmission medium, and compensating for the variable attenuation to which the band of frequencies is subjected as a result of the delay shift.

5. The method of secret transmission which consists in subjecting the component frequencies of a signal band to different degrees of delay before applying them to a

transmission medium, compensating for the variable attenuation to which the band of frequencies is subjected as a result of the delay shift, transmitting the band so modified over a transmission medium, subjecting the frequencies of the received band to degrees of delay complementary to those to which the frequencies were subjected before application to the transmission medium, and compensating for the variable attenuation to which the band of frequencies is subjected as a result of the complementary delay shift.

6. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a static network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, said network being interposed between said means and said medium, and means to transmit all of the frequencies of said band together through all of the elements of said network.

7. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a static network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, said network being interposed between said means and said medium, means to transmit all of the frequencies of said band together through all of the elements of said network, and means at a receiving point to restore to substantially normal phase relation the component frequencies of said band as shifted in phase by said network.

8. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a static network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, said network being interposed between said means and said medium, means to transmit all of the frequencies of said band together through all of the elements of said network, and a network at a receiving point, said network having a delay-frequency characteristic substantially complementary to that to which the frequencies were subjected before transmission.

9. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, said network being interposed between said means and said medium, and an attenuation network associated with said delay network and having an attenuation-frequency characteristic substantially complementary to that of the delay network.

10. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, said net-

work being interposed between said means and said medium, an attenuation network associated with said delay network and having an attenuation-frequency characteristic substantially complementary to that of the delay network, means at a receiving point to restore to normal phase relation the component frequencies of said band as shifted in phase by said band, and means associated with said last mentioned means to substantially equalize the attenuation of the frequencies of the received band.

11. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a network having a delay-frequency characteristic such that the delay varies with frequency in an arbitrary manner, said network being interposed between said means and said medium, an attenuation network associated with said delay network and having an attenuation-frequency characteristic substantially complementary to that of the delay network, a second delay network at a receiving point, said second network having a delay-frequency characteristic substantially complementary to that to which the frequencies were subjected before transmission, and an attenuation network associated with said second delay network and having an attenuation-frequency characteristic substantially complementary to that of said second delay network.

12. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a static delay network interposed between said means and said medium, said network having a delay-frequency characteristic such that the delay increases and diminishes successively a number of times over the range of said signal band, and means to transmit all of the frequencies of said band together through all of the elements of said network.

13. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a static delay network interposed between said means and said medium, said network having a delay-frequency characteristic such that the delay increases and diminishes successively a number of times over the range of said signal band, the variation in delay of said network being in the neighborhood of 0.1 second, and means to transmit all of the frequencies of said band together through all of the elements of said network.

14. In a secrecy system, means to produce a band of signal frequencies, a transmission medium, a static delay network interposed between said means and said medium, said network having a delay-frequency characteristic such that the delay increases and diminishes successively a number of times over the range of said signal band, the variation in delay of said network being not substantially less than 0.1 second, and means to transmit

all of the frequencies of said band together through all of the elements of said network.

15. In a secrecy system, means to produce a band of signal frequencies, a transmission
5 medium, a static delay network interposed between said means and said medium, said network having a delay-frequency characteristic such that the delay increases and diminishes successively a number of times
10 over the range of said signal band, means to transmit all of the frequencies of said band together through all of the elements of said network, and a second delay network at a receiving point, said second network having
15 a delay-frequency characteristic substantially complementary to that of said first mentioned delay network.

16. In a secrecy system, means to produce a band of signal frequencies, a transmission
20 medium, a static delay network interposed between said means and said medium, said network having a delay-frequency characteristic such that the delay increases and diminishes successively a number of times over the
25 range of said signal band, the variation in delay of said network being in the neighborhood of 0.1 second, means to transmit all of the frequencies of said band together through

all of the elements of said network, and a second delay network at a receiving point, 30 said second network having a delay-frequency characteristic substantially complementary to that of said first mentioned delay network.

17. In a secrecy system, means to produce a band of signal frequencies, a transmission 35 medium, a static delay network interposed between said means and said medium, said network having a delay-frequency characteristic such that the delay increases and diminishes successively a number of times over the 40 range of said signal band, the variation in delay of said network being not substantially less than 0.1 second, means to transmit all of the frequencies of said band together through all of the elements of said network, and a 45 second delay network at a receiving point, said second network having a delay-frequency characteristic substantially complementary to that of said first mentioned delay network. 50

In testimony whereof, we have signed our names to this specification this 18th day of February 1928.

HARRY NYQUIST.
PIERRE MERTZ.