Aug. 18, 1970 J. JAKUBOWSKI 3,524,257

CONSTRUCTING ELLIPSES

Filed April 30, 1968 4 Sheets-Sheet 1

Fig. 1

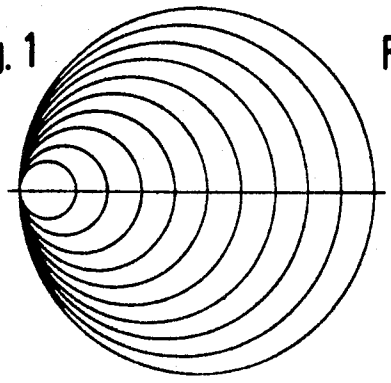


Fig. 2

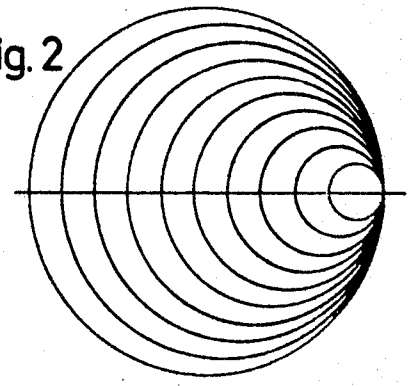


Fig. 3

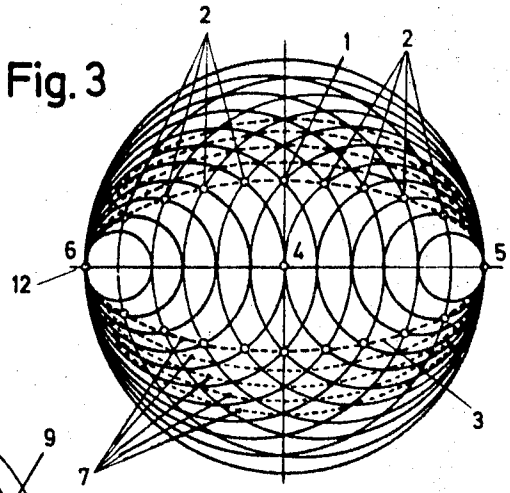


Fig. 4

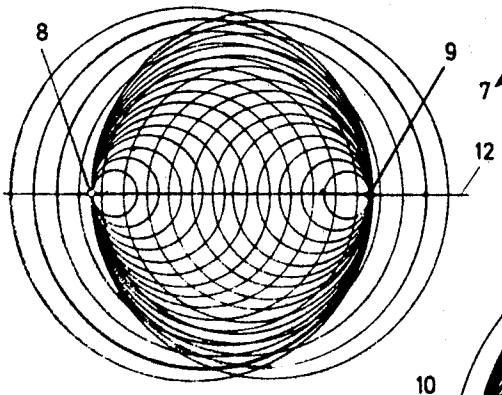
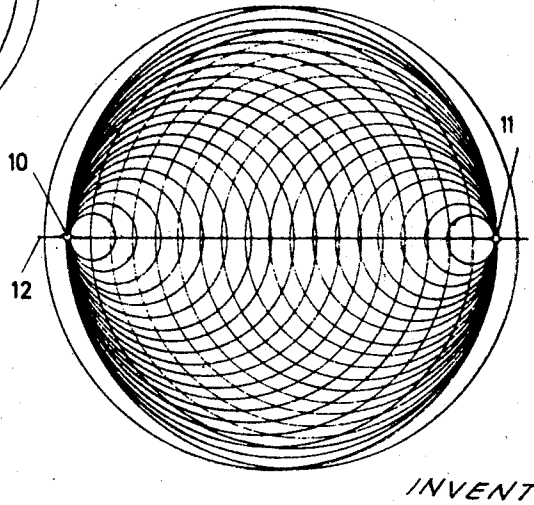


Fig. 5



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Fig.6

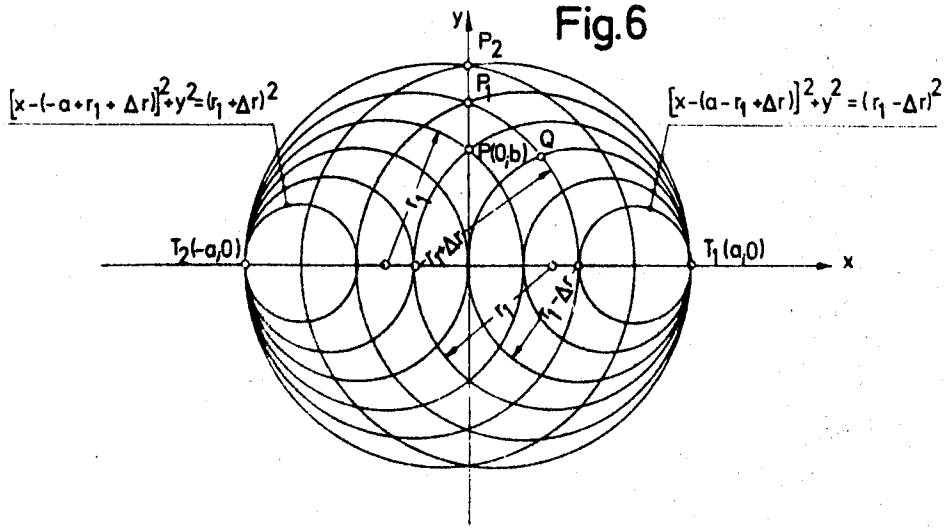


Fig.7

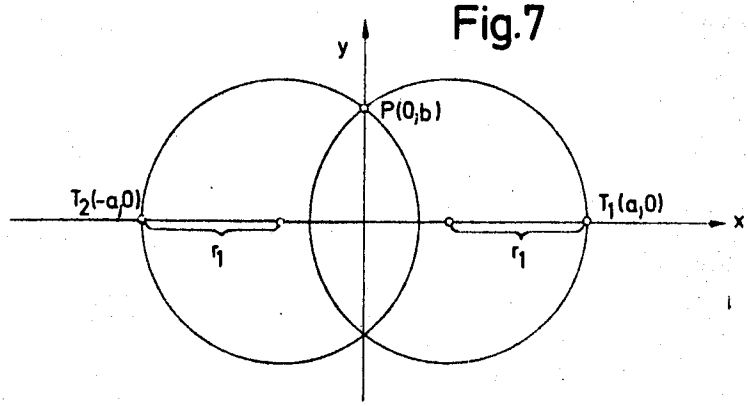
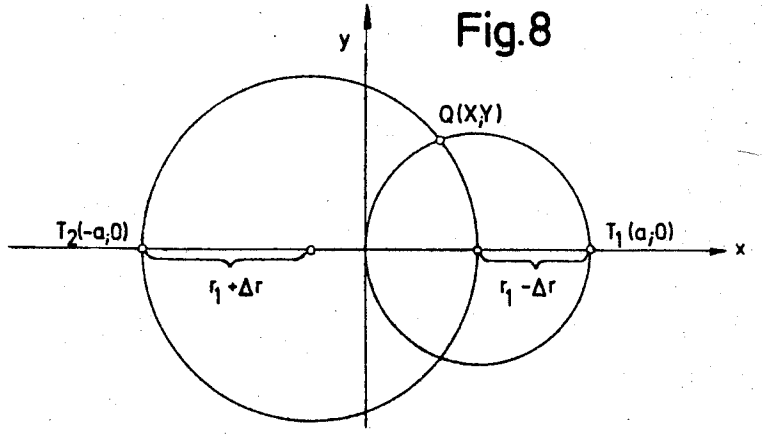


Fig.8



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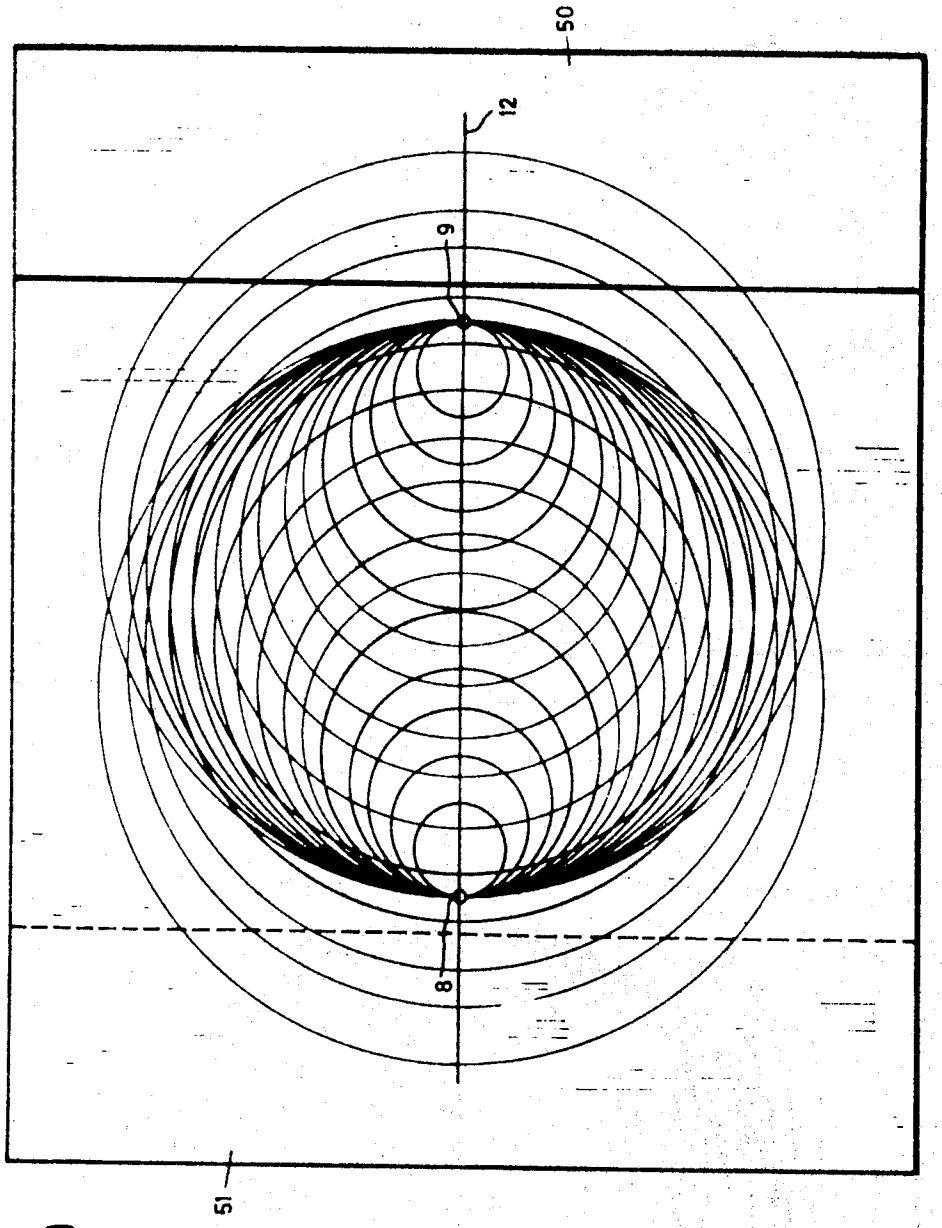


Fig.9

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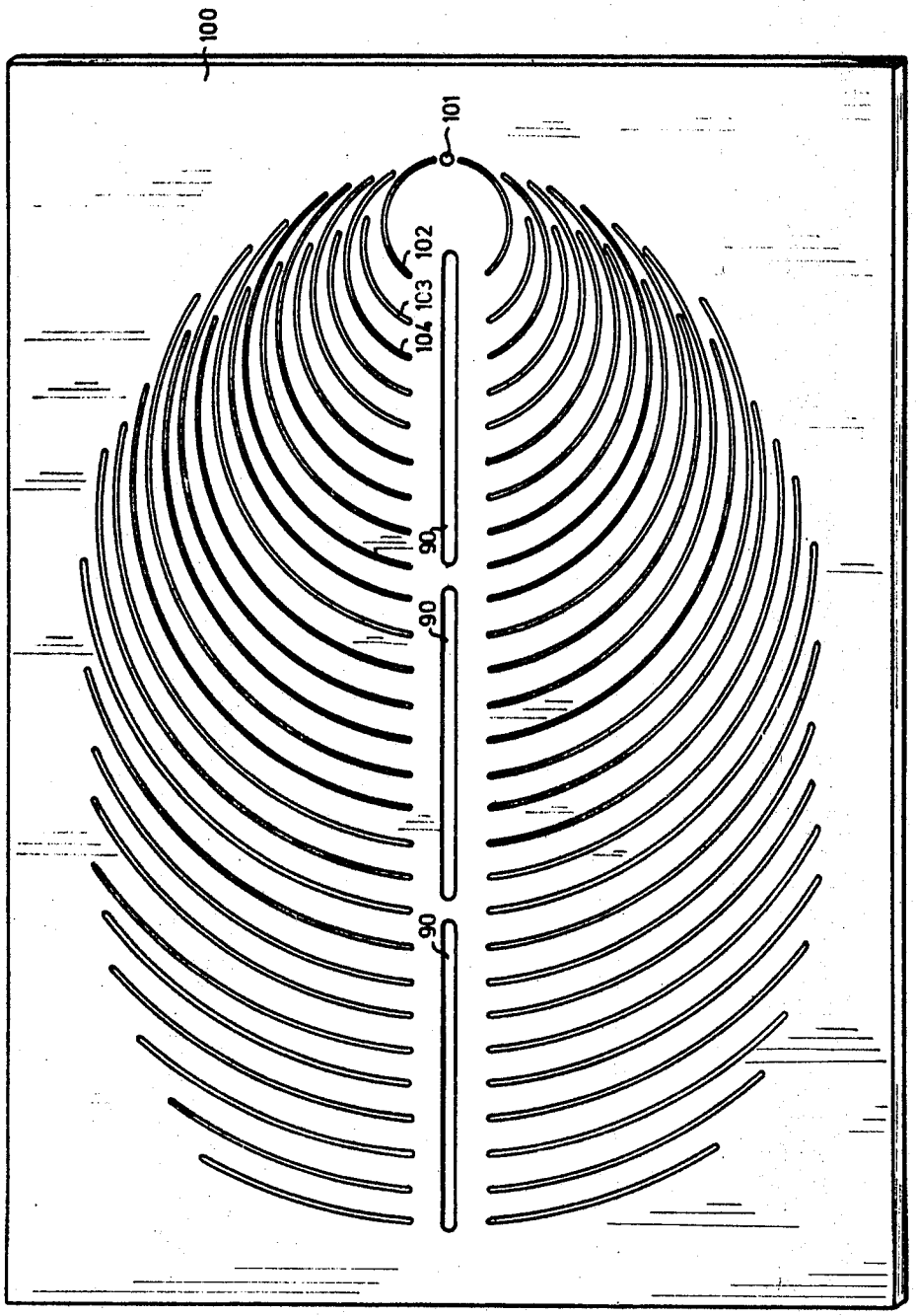


Fig.10

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CONSTRUCTING ELLIPSES

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Int. Cl. B43l 5/00
U.S. Cl. 33—1 9 Claims

ABSTRACT OF THE DISCLOSURE

The specification describes a method and apparatus for constructing ellipses in which two families of circular curves, of which each family has a common tangent point, has circular curves increasing arithmetically in diameter, and corresponds as regards curve diameters to the other family of curves. The two families of curves are caused to intersect with their centers of curvature and tangent points on a common line and the points of intersection corresponding to an ellipse are registered.

There are in existence a number of devices for constructing ellipses. The simplest form of device is a fixed stencil but this has the disadvantage that only ellipses or parts of ellipses which have already been cut in the stencil can be produced.

A further known apparatus is a trammel which is steplessly adjustable so that an infinitely large number of different ellipses can be produced. However the construction is complicated and therefore so expensive that the market is comparatively limited.

There is also the method using a piece of string anchored at both ends which has the advantage of simplicity and cheapness but with it there is the drawback that extensive trial and error is necessary before an ellipse of the desired dimensions can be constructed.

One object of the present invention is to provide a method and apparatus for drawing ellipses which is more satisfactory than the above-mentioned methods in apparatus.

A further object of the invention is to provide a steplessly adjustable cheap form of ellipse constructing apparatus with which ellipses can evenly be constructed with a pre-set major axis.

A still further object of the invention is to provide an ellipse constructing apparatus which is in the form of simple draughtsman's stencils and is nevertheless steplessly adjustable.

The present invention consists in a method of constructing an ellipse using two families of circular curves, the curves in each family having arithmetically increasing diameters, a common tangent point, and corresponding to the diameters of the other family of curves, the method comprising the superposing of one family of curves onto the other with the centers of curvature of all circles on a single line, one family of curves being 180° offset from the other, and registering a series of points of intersection between curves corresponding to an ellipse. The tangent points can be virtual.

The present invention also consists in an apparatus for constructing an ellipse comprising two relatively movable parts each carrying a family of circular curves on it, the curves in each family having arithmetically increasing diameters with a common tangent point and the diameters of the curves corresponding to those of the other part, the parts being capable of being placed so that the families of curves are superposed with the centers of curvature of the circular curves on a common axis and

the families of curves offset by 180° in relation to each other so that a series of points of intersection between curves corresponding to an ellipse can be registered.

Preferably one of the parts is transparent. The parts can be in the form of sheets, for instance transparent sheet. Both parts can be in the form of transparent sheets of plastic foil. One of the parts can be in the form of a stencil adapted for use with a writing instrument for transferring points of intersection onto an underlay. Such a stencil can have slots corresponding to the circular curves, the slots extending symmetrically to each side of an axis passing through all centers of curvature.

The invention will now be described with reference to the accompanying drawings.

FIG. 1 shows a family of circular curves which have a real tangent point, that is to say a point at which the circular curves touch and through which a common tangent could be drawn. The circular curves in each family increase arithmetically in diameter.

FIG. 2 shows a family of identical circular curves offset through 180° in relation to the curves shown in FIG. 1.

FIG. 3 shows the series of points of intersection produced by superposition of the two families of curves shown in FIGS. 1 and 2, the lines running through the centers of the curves in each family also being superposed.

FIG. 4 shows the superposition of two circular curve families in accordance with FIGS. 1 and 2 with a shorter main or connecting axis than is the case in FIG. 3.

FIG. 5 shows the superposition of the two curve families for producing ellipses with a larger main axis than is the case with FIG. 4.

FIGS. 6, 7, and 8 show curves required for proving that the geometrical construction in accordance with the invention produces ellipses.

FIG. 9 shows two superposed sheets of which each is provided with a family of curves similar to FIGS. 1 and 2.

FIG. 10 shows a stencil having circular curves in accordance with FIG. 1.

FIGS. 1 and 2 show two families of circular curves which only differ in that one is offset about 180° in relation to the other. In both families of curves the radii increase arithmetically.

FIG. 3 shows the superposition of two families of circular curves of the type shown in FIGS. 1 and 2 though there are more circular curves than in the families of FIGS. 1 and 2. The two families of curves are superposed in such a manner that the two axes of symmetry connecting the center points of the circles, shown in FIGS. 1 and 2, are superposed. The common axis of symmetry of the families of curves is denoted by reference numeral **12** in FIG. 3.

If one starts, for example, at the point of intersection of two circles with the same radii **1** and then connects all the diagonally opposite points of intersection **2** until a closed curve results, one obtains an ellipse **3**, which is shown in the drawing by means of dotted lines with a semi-minor axis extending from point **1** to point **4** and with a semi-major which extends a distance equal to the spacing between points **4** and **5**. The major axis of the ellipse **3** extends between points **5** and **6**.

For any randomly chosen major axis (the distance between points **5** and **6**) there is formed, in the above manner, a series of ellipses **7** with different minor axes; some of these elipses **7** are shown in broken or dotted lines in FIG. 3.

If the distance between points **5** and **6** is changed by parallel movement of the families of circular curves along the axis of symmetry **12**, for example by moving sheets on which the two families of curves are respectively drawn in accordance with FIGS. 1 and 2, the upper sheet at least being transparent, it is possible to obtain fur-

ther series of ellipses with different major axes. An example of such a series of ellipses with two different major axes are shown in FIGS. 4 and 5. In FIG. 4 the major axis of the various ellipses, which are also constructed as is the case in FIG. 3, lies between points **8** and **9**. In FIG. 5 the major axis of the family of ellipses, which can also be constructed as is the case in FIG. 3, lies between points **10** and **11**.

The proof that the above-described geometrical loci of points represent true ellipses is as follows: it is first assumed that there are two families of curves in accordance with the equations

$$[x-(-a+r_1+\Delta r)]^2+y^2=(r_1+\Delta r)^2 \quad (1)$$

$$[x-(a-r_1+\Delta r)]^2+y^2=(r_1-\Delta r)^2 \quad (2)$$

in which

a=abscissa of the common point of a family of curves
r_1=initial radius of both families of curves
Δr=constant increment or decrement in radius (family parameter).

In accordance with the value of Δr and owing to the crossing of the two families of curves a more or less dense series of points of intersection is obtained (FIG. 6).

In this series of points of intersection it is possible to see the geometrical locus of points which consists of a selected intersection point P of a pair of circular curves with the same radii r_1 and of all further points of intersection Q of neighbouring pairs of circular curves in which the radius of a circle belonging to such a pair of curves is increased by the amount Δr and the radius of the other circular curves is decreased by the same amount, Δr.

The equation of this geometrical point locus can be derived by eliminating the parameters r_1 and Δr from Equations 1 and 2.

To do this the relationship between r_1, a, and b is first found from Equation 1 or Equation 2 by replacement of the coordinates of the point of intersection P in FIG. 7 with a pair of circular curves having the same radii equal to $r_1(\Delta r=0)$,

$$r_1=\frac{a^2+b^2}{2a} \quad (3)$$

Furthermore by putting the coordinates of a point of intersection Q of circular curves in accordance with FIG. 8 into Equations 1 and 2 and eliminating the ordinates, the family parameter Δr is expressed in accordance with the abscissa X of the intersection point Q as follows

$$\Delta r=\frac{a-r_1}{a}\cdot X \quad (4)$$

From (3) and (4) we then obtain

$$\Delta r=\frac{a^2-b^2}{2a^2}X \quad (5)$$

By then putting Δr in accordance with Equation 5 and r_1 in accordance with Equation 3 for example into Equation 1, one then obtains the sought-after equation of the geometrical locus of points

$$\left[X-a+\frac{a^2+b^2}{2a}-\frac{a^2-b^2}{2a^2}X\right]^2+Y^2=\left(\frac{a^2+b^2}{2a}-\frac{a^2-b^2}{2a^2}X\right)^2$$

which after shortening and re-arrangement finally gives the equation of an ellipse

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2}=1$$

in which, as is known, a and b respectively denote the semi-major and semi-minor axes. If one chooses a different starting point, for example P_1 or P_2 in FIG. 6 one obtains in each case an ellipse with a different semi-axis b.

FIG. 9 shows an an embodiment of the invention in which reference numeral **50** denotes a planar base or sheet on which a family of circular curves is drawn. On the sheet **50** there is a further sheet or base **51** which is also provided with the above-mentioned family of circular curves. Reference numeral **12** denotes the whole axis of symmetry. By movement of the two sheets **50** and **51** in relation to each other it is possible to arrange for different major axes for different families of ellipses. The less the distance between the radii of the bases **50** and **51**, the smaller the distance between the points of intersection and the more accurate it is to construct the desired ellipse, the two bases or sheets **50** and **51** can be transparent or preferably at least the upper sheet **51** is transparent.

The manner of operation with an opaque sheet **50** and a transparent sheet **51** is as follows: The axes of symmetry **12** of the two families of circular curves offset from each other by 180° are brought so that one lies exactly on top of the other. By parallel displacement between the two sheets **50**, **51** along the axis of symmetry **12** the distance between the points **8** and **9**, which form the main vertices of the ellipse, is altered in order to make the major axis of the required length. Then the sheets are fixed together, for example by means of removable sticky tape, at two positions. The ellipses required are then registered, for example by copying onto drawings, plates or the like. They can also be photo-copied or registered point by point and then drawn by means of a standard draughtsman's set of curves.

The method in accordance with the invention for producing and drawing ellipses of any required size represents a substantial saving in time and the apparatus used is cheap. The apparatus can consist simply of two printed sheets, for example as shown in FIG. 9. In practice it is found that it is sufficient to purchase the upper transparent sheet **51** since it is possible to produce a cheap photo-copy for the lower sheet required.

FIG. 10 shows a stencil **100** for the construction of ellipses. It is rectangular and is provided with a series or family of circular curves represented by slots such as **102**, **103**, **104**. The tangent point is for most curves virtual that is to say they do not extend as far as it. At the point corresponding to the tangent point there is a hole **101** passing through the stencil. Along the axis **12** of symmetry of the family of curves there are longitudinal slots **90**. Preferably the stencil **100** is made of transparent synthetic resin.

For the construction of ellipses one can for example use two stencils **100**, with curves similar to the curves marked on sheets shown in FIG. 9. The stencils are laid on top of one another so that the individual points can easily be marked or otherwise transferred onto any desired object. It is also possible to use a stencil together with another transparent or opaque sheet having a family of curves marked on it and then to transfer the ellipse points onto an object.

I claim:

1. An apparatus for constructing an ellipse comprising two relatively movable parts each carrying a family of circular curves on it, the curves in each family having arithmetically increasing diameters with a common tangent point, the diameters of the curves corresponding to those of the other part, the parts being capable of being placed so that the families of curves are superposed with the centers of curvature of the circular curves on a common axis and the families of curves offset by 180° in relation to each other so that a series of points of intersection between curves corresponding to an ellipse can be registered.

2. An apparatus in accordance with claim **1** in which one of the parts is transparent.

3. An apparatus in accordance with claim **1** in which the parts are in the form of sheets.

4. An apparatus in accordance with claim 1 in which both parts are in the form of transparent sheets.

5. An apparatus in accordance with claim 1 in which both the parts are in the form of plastic foil.

6. An apparatus in accordance with claim 1 in which at least one of the parts is in the form of a stencil adapted for use with a writing instrument for transferring points of intersection onto an underlay.

7. An apparatus in accordance with claim 6 in which the stencil has slots corresponding to the circular curves, the slots lying symmetrically to each side of an axis passing through all centers of curvature.

8. An apparatus in accordance with claim 6 in which the stencil is in the form of transparent synthetic resin.

9. An apparatus in accordance with claim 1 in which the tangent points are virtual.

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WILLIAM D. MARTIN, JR., Primary Examiner

U.S. Cl. X.R.

33—174; 35—34