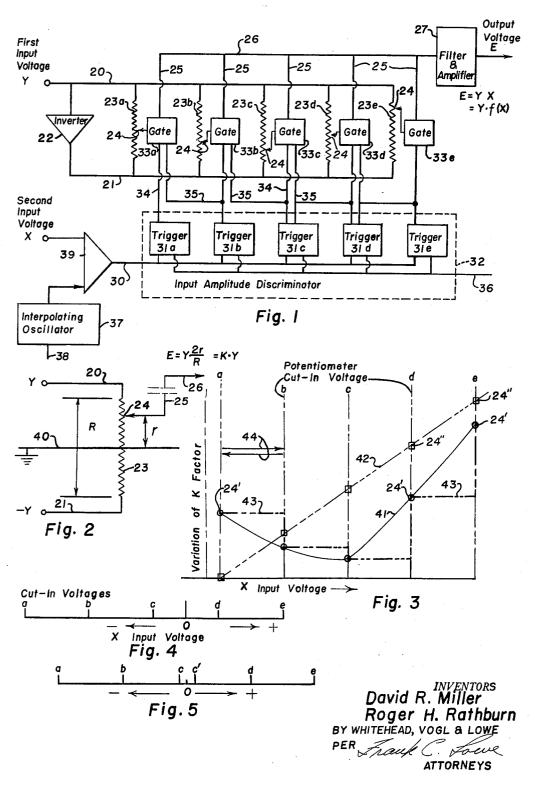
NON-LINEAR ELEMENT FOR AN ANALOG COMPUTER

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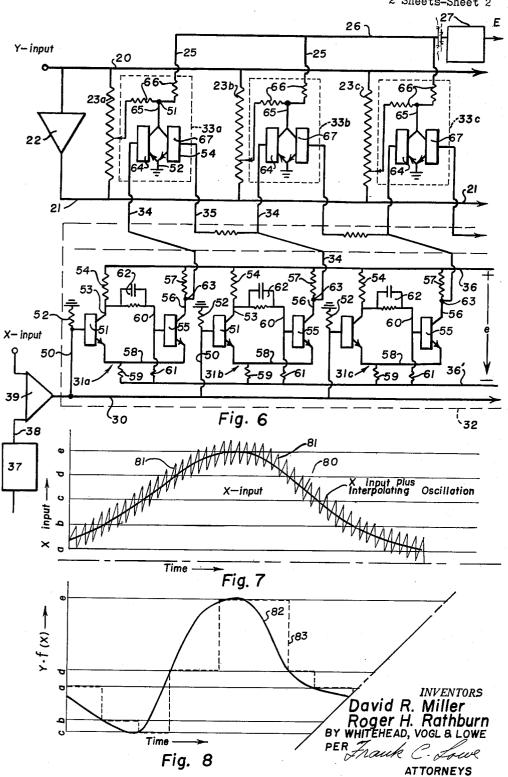
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NON-LINEAR ELEMENT FOR AN ANALOG COMPUTER

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3,217,151 NON-LINEAR ELEMENT FOR AN ANALOG COMPUTER

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This invention relates to analog systems and to the components therefor, and more particularly to analog computers and/or elements thereof which performs nonlinear operations such as multiplication and function generation. A primary object of the invention is to provide a novel and improved element which is adapted to be incorporated into electronic analog computers and which is capable of performing desired non-linear operations. As such the invention will be hereinafter referred to as a nonlinear element.

Another object of this invention is to provide a novel and improved non-linear element for an analog computer which combines the simplicity, stability and precision of a potentiometer with the speed and flexibility of electronic gating and triggering circuits.

Another object of the invention is to provide a novel and improved non-linear element for an analog computer which quickly discriminates the individual outputs of a group of potentiometers to generate the product of variable inputs or functions thereof to a high degree of analog 30 accuracy.

Another object of the invention is to provide a novel and improved non-linear element which is capable of accurately producing an output voltage which may represent the product of two varying input voltages, which is especially capable of representing the product of a first variable input voltage and a function of a second variable input voltage, and which further provides for an improved arrangement of circuits adapted to permit a quick and accurate setting of components to establish any desired arbitrary function of the second variable input voltage.

Another object of the invention is to provide a novel and improved non-linear element which is capable of discriminating the individual outputs of a plurality of potentiometers or other voltage sources to produce a product of two input variables or a product of an input variable and a function of another input variable, and which further incorporates an improved arrangement for receiving a selected oscillating wave at one input and filtering out its secondary effects at the output with a resulting accurate interpolation between the individual potentiometer or voltage source outputs.

Yet further objects of the invention are to provide a novel and improved non-linear element for an analog computer which is simple to construct, easy to calibrate and use, and is reliable, stable and accurate.

With the foregoing and other objects in view, all of which more fully hereinafter appear, our invention comprises certain constructions, combinations and arrangements of circuits, and other parts and elements as hereinafter described, defined in the appended claims and illustrated in preferred embodiment in the accompanying drawing in which:

FIGURE 1 is a block diagram illustrating the preferred arrangement of groups of components of a non-linear element constructed in accordance with the principles of the invention, with the element including five of such groups as being representative of any number of groups which may be for use in the element.

FIGURE 2 is a circuit diagram of a fragmentary por-

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tion of a single potentiometer to illustrate the manner of obtaining a product in a four-quadrant operation.

FIGURE 3 is a two dimensional graph illustrating the manner in which the potentiometers may be set to obtain various functions of one input variable, hereinafter referred to as f(X).

FIGURE 4 is a one-dimensional graph illustrating one manner of arranging potentiometer cut-in voltages to secure regular cut-in points with uniform changes of the input variable X.

FIGURE 5 is a one-dimensional graph similar to FIG. 4 but illustrating another arrangement of potentiometer cut-in voltages to secure regular cut-in points with uniform changes of the input variable X, but with special arrangements for the condition where X becomes zero.

FIGURE 6 is a circuit diagram of a portion of the arrangement illustrated at FIG. 1, illustrating specifically selected and preferred circuits of certain components shown in FIG. 1 as blocks.

FIGURE 7 is a two dimensional graph with the ordinate representing voltages and the abscissa representing time, to illustrate the manner in which the invention operates.

FIGURE 8 is a two dimensional graph with the ordinate representing a modified voltage and the abscissa representing time, to illustrate further the manner in which the invention operates.

A practical and comprehensive treatment of the construction of direct current analog computers is set forth in the publication Electronic Analog Computers by Korn and Korn; McGraw-Hill, 1956 which discloses the basic elements of various computer elements that may be used in analog computers and the principles for combining the elements into composite analog computing machines to obtain the answers to various mathematical operations. As disclosed in chapter 6 of this publication, multiplication and function generation is possible by a number of means including potentiometers. Some of the other arrangements are very complex and expensive, especially when they employ pulsing to accomplish the operations or which rely upon cathode rays or diodes or the like as the primary elements of operation.

In many ways potentiometers are the most desirable elements as multipliers and function generators because of their mechanical simplicity and ruggedness, their freedom from microphonism and electrical noise, their accuracy of calibration, stability and low cost. The operation of a potentiometer is essentially to impose the voltage of one input variable to a resistance winding and to use the voltage of a second input variable to operate a servo-mechanism having a sweep-arm to move across wires and taps of the resistance winding in accordance with the variation of the second input. The output, the resistance-modified voltage of the input is then proportional to the product of the first input variable and a factor which is a function of the second input variable.

It becomes immediately apparent that a mechanical servo-mechanism is very limited in its speed of operation and is thus unsuitable for many applications. However, if it were not for this time factor, a potentiometer would be definitely preferable over other types of multipliers and function generators. It was with such factors in view that the present invention was conceived and developed, and the invention comprises, in essence, a potentiometer system operable by an electronic triggering, interlocking and gating arrangement which replaces a servo-mechanism and which is capable of high speed function generation at an accuracy heretofore not considered attainable with comparable elements.

With this arrangement it is easy to accurately set the potentiometer arms independently of each other to pro-

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duce a desired function as in the manner hereinafter described.

More especially, our non-linear element employs potentiometers or similar voltage sources for obtaining the product of two variables or the product of a first variable and function of a second variable. The element combines a group of potentiometers with a single pick-off arm

at each potentiometer. The two variable quantities to be operated upon will be supplied to the element as input voltages and will be hereinafter referred to as the X and Y inputs. The voltage output generated in the element will be designated as the E output, and it will be manifest that the E output will be proportional to the product Y.X inputs, and more especially to the product $Y \cdot f(X)$ inputs, where f(X) is 15 an arbitrary function determined by the several potentiometer arm settings. The Y input is adapted to control the voltage across the potentiometers and the X input is adapted to restrict the E output to selected potentiometers according to variation of the X input. An oscillating interpolating voltage is superimposed upon the X input to effect interpolation of values between the several potentiometer settings while other effects of this oscillating voltage on the E output are eliminated by a low-pass filter.

Referring more particularly to the drawing, the Y input is imposed on a circuit lead 20. To obtain a desired four quadrant operation wherein it is possible to multiply negative as well as positive quantities, the inverse of the Y input, a -Y voltage, is imposed on a parallel lead 21 as by an inverter 22. The inverter 22 is a conventional unit and may be of the type described in the reference publication, Electronic Analog Computers, at page 12, hence it need not be further described herein.

A group of potentiometers 23 shunt the leads 20 and 35 21 and each potentiometer includes a pick-off arm 24 and these potentiometer arms are set to define a selected function f(X) as in the manner hereinafter set forth. The number of potentiometers used will depend upon the desired spacing of intermediate points within the contemplated range of the f(X) designation, and as few as two or more than a hundred potentiometers may be used in the element. The five potentiometers illustrated at FIG. 1 and indicated sequentially as 23a through 23e exemplify a representative group that may be used in the description of the element. It is to be noted that sired results. FIG. 2 illustrates specifically the manner the resistance of each potentiometer is sufficiently high so that currents through it will not noticeably affect the voltage drop between the leads 20 and 21 and yet that this resistance is such as to permit a useable output from 50 the arm 24.

The voltage at each arm 24 is picked up by a gated lead 25 and the group of leads 25, from all of the potentiometers 23a to 23e, are connected to a common outerated by the several potentiometers. However, to produce a useable E output only one lead 25 can be operatively connected with the output line at a time so each lead 25 must be opened and closed by its gate means in When an interpolating 60 accordance with the X input. input is superimposed on the X-input, the resulting rapid shifting of the output from one lead 25 to another by the gate means will require a low-pass filter and amplifier arrangement 27 in the output line 26 to eliminate the secondary effects of the interpolating input and to obtain a correct output. The filter-amplifier 27 is a conventional unit and may be one of the types described in the reference publication, Electronic Analog Computers, as at table 3, page 417, hence it need not be further described.

A combined gate and interlocking means to circuit the Y input through selected potentiometer leads 25 at any given instant is operated by the X input. The X input is imposed on a lead 30 and trigger circuits or triggers

sequential arrangement in correlation with the arrangement of the potentiometers 23a to 23e. Each trigger 31 is set to operate at a selected voltage and the triggers, commencing with the first designated trigger 31a, are preferably set in sequence to operate at regular pre-determined increases of the X input so that in combination they form an X input amplitude discriminator 32.

In correlation with the potentiometer group 23a to 23e and the trigger group 31a to 31e, gates 33a to 33e are each interconnected into an appropriate lead 25 to control the current flow through said lead. Each gate is adapted to normally hold its lead 25 open and to cut off current flow. However, each gate is connected to its corresponding trigger by a closing circuit 34 and when this circuit 34 is energized by its trigger the gate closes the lead 25. Also, each gate is connected to the lead 34 of the trigger next-in-sequence by an interlock lead 35, the interlock lead 35 being a bifurcation of the next-in-sequence lead 34. Whenever the interlock lead 35 is energized, the gate connected to it operates to open its lead 25 and cut off the current flow even though the lead 34 of the gate is energized. For example, the gate 33a is connected to its trigger 31a by a closing lead 34 and is also connected to the next-in-sequence trigger 31b by an interlock lead 35. It is to be noted that while the gates 33 and triggers 31 are controlled by the X input, they are operated by a power source as from a lead 36 which is connected to the triggers 31a to 31e.

To complete this general arrangement, an interpolating 30 oscillator 37 adds an oscillating voltage to the X input voltage as from a source lead 38 and these voltages are combined by a summer 39 in the lead 30 before operating upon the triggers 31a to 31e. As hereinafter set forth, a sawtooth type of oscillation is especially desirable to obtain linear interpolation effects and an oscillator capable of producing this desired effect is described in the publication, Active Networks by Rideout, Prentice Hall 1954, at page 395, hence it need not be further described herein. Also, devices for summing voltages are well known and a summer 39 may be of the type described in the reference publication, Electronic Analog Computers, at page 13, hence it need not be further described herein.

The FIGS. 2 and 5 and 7 and 8 illustrate more specifically the manner in which the components of the nonin which the components of the non-linear element are adapted to function to obtain the desired results. FIG. 2 illustrates specifically the manner of establishing a product relationship with a single potentiometer 23. This potentiometer is essentially a resistor shunting the leads 20 and 21 which carry the Y input and -Y input. At the midpoint of this potentiometer 23 the voltage will be zero as indicated by the ground line 40. It is immediately eviput line 26 which receives the E output voltage gen- 55 dent that the voltage of the arm-contact point 24 is proportional to the Y input voltage and to a resistance factor which is related to the resistance r from the ground 40 to the arm 24, and the total resistance R of the potentiometer. While the relationship may be expressed as

$E=2r/R\cdot Y$

a more simple relationship is expressed as E = KY where K is a selected resistance factor less than 1.

The K factor is determined by the location of the po-65 tentiometer arm 24 and will differ in each of the potentiometers and will change from one potentiometer to the next in accordance with a desired functional relationship. This is illustrated at FIG. 3 which shows a two dimensional graph wherein the ordinate represents the K factor and the abscissa represents X input voltage. The potentiometer cut-in voltages as determined by the selected sequentially-increasing settings of the triggers 31a to 31e are indicated by the gradient lines a to e. An arbitrary predetermined function of the K factor with respect to 31a to 31e are individually connected to this lead in a 75 voltage may be then represented as by a curve 41 and the

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proper location of the potentiometer arms 24 of the several potentiometers may be easily determined and are represented by the points 24' on the curve 41. Since the X input voltage controls the cut-in of the potentiometers it is immediately apparent that the K factors define a function of the X input and that operation of the non-linear element will produce an E output which is substantially proportional to the product $Y \cdot f(X)$ inputs. It is also apparent that when the K factors are set for a uniform progression as at points 24', to form the straight-line curve 42, the E output will be substantially proportional to the produce Y and X inputs, and thus direct multiplication will be possible.

Without interpolation, however, a true function generation or product cannot be obtained because a finite voltage step is required between the potentiometer cut-in points for such variations of the X input cannot be sensed by the apparatus. Without interpolation, the functional relationship obtained will be in a stepping pattern such as the indicated dashed curve 43, which steps from one point 24' to another. An oscillation superimposed onto the X input is thus desirable and a sawtooth type oscillation by the component 37 is most desirable in order to provide a linear interpolation between the potentiometer cut-in points a to e.

The amplitude of this oscillation, indicated as 44, is at least or only very slightly less than the voltage change between adjacent cut-in points such as a and b and it may be substantially greater in special applications. The frequency of this oscillation must also be substan- 30 tially greater than any frequency attainable by the X input but at the same time substantially less than operational speed of the triggers 31 and gates 33. A desirable operative condition is attained when the X input frequency is in the range of 103 cycles per second by 35 providing an oscillation frequency in the range of 105 cycles per second. These conditions are easily obtained when the triggers 31 and gates 33 are transistor-actuated circuits as hereinafter described. However, other frequencies of operation can be easily established by a 40 skilled designer to accomplish specific and predetermined purposes.

Where four quadrant operation is necessary, the zero voltage of the X input is often an important factor for then the potentiometer cut-in voltages of the sequence 45 of units will vary from minus quantities to positive quantities, as from minus 100 volts to plus 100 volts. It is generally desirable to balance the location of zero voltage between two potentiometer cut-in voltages, as between the indicated voltages c and d at FIG. 4 which shows 50 one manner of locating the cut-in voltages with respect to the X input voltage. Another mode of arranging the cut-in voltages of the potentiometers is illustrated at FIG. 5 where one potentiometer c is set to cut in at a minus voltage slightly less than zero and another poten- 55 tiometer c' is set to cut in at a positive voltage slightly greater than zero. With an X input of zero the oscillating interpolating voltages will then oscillate between these cut-in voltages c and c' to provide for a zero result at the E output.

A preferred circuit arrangement of the triggers and gates is illustrated at FIG. 6 in which transistors are used to accomplish the triggering and gating functions. Each trigger is substantially of a type known as a cathode-coupled binary and described in the publication, Pulse 65 and Digital Circuits by Millman and Taub, McGraw-Hill 1956, FIGS. 5–17 at page 165. In the present arrangement however, the X input is diverted from the lead 30 through a lead 50 and to the base of a transistor 51, and a grounded resistor 52 forms a bias for the lead 50. 70 The power source includes leads 36 and 36' and the leads 36 having a positive voltage is connected to the collector of the transistor 51 by a lead 53 which includes a resistor 54. The collector of a second transistor 55 is also connected to the power lead 36 by a lead 56 which 75

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includes a resistor 57. The emitters of the transistors are connected to the negative branch of the power lead 36' by a common bifurcated lead 58 which includes a suitable resistor 59. A feedback loop 60 connected with the power lead 36' and with the lead 53 between the transistor 51 and the resistor 54. The loop includes a suitable resistor 61 and paralleled resistor-capacitor 62, and to complete the trigger the lead 60 connects with the base of the transistor 55. By appropriate balancing of the bias resistance 52 and the other resistances and capacitances, a voltage is obtained on the lead 56 at a take-off point 63 adjacent to the transistor 55 whenever the X input on lead 50 increases to a selected voltage.

The gate closing circuit lead 34 is connected at point 63 and it extends to the base of a PNP type transistor 64 of the gate 33. The collector of this transistor 64 is connected by a lead 65 to the potentiometer take-off lead 25 and the emitter of the transistor is to ground so that whenever the circuit between the collector and emitter is closed the lead 25 is grounded. Suitable isolating resistors 66 are interposed in the lead 25 at each side of the connection to control the current flow through lead 25 to ground or to the output lead 26. The transistor 64 is operative to normally close the lead 65 to ground and the voltage change in lead 34 is effective to open this transistor.

The interlock lead 35 which is a bifurcation of the lead 34 of the next-in-sequence trigger connects with the base of a second transistor 67 of the NPN type in the gate with the collector and emitter of this transistor is connected to the lead 65 and to ground in the same manner as the transistor 64. However, this transistor normally holds the circuit open and closes whenever a voltage change is applied to the lead 35. It follows that whenever the X input increases sufficiently to activate one trigger, for example trigger 31b, the voltage change in the lead 34 opens the transistor 64 of gate 33b and the circuit 25 is then connected to input lead 26. At the same time the interlock lead 35 from the trigger 31b, which is connected to gate 33a, closes the transistor 67 of gate 33a to cut off the circuit 25 of that gate from the input lead 26. As the X input voltage increases, the trigger 31c is activated to cut in the circuit 25 of the gate $3\overline{3}c$ and cut out the circuit 25 of the gate 33b. It is to be noted that these cutting in and cutting out operations will occur with great rapidity, depending upon the frequency of the interpolating oscillation.

The results obtainable with the apparatus herein described are illustrated by the curves at FIGS. 7 and 8. The ordinate of the graph at FIG. 7 represents voltage variation, with the respective potentiometer cut-in points a to e being located at their respective voltages. The abscissa represents time. Curve 80 is representative of a portion of a cycle of the X input voltage while curve 81 represents the X input voltage modified by the interpolating oscillation.

The graph at FIG. 8 represents the product $Y \cdot f(X)$ as the ordinate and time as the abscissa to correspond with the curves of FIG. 7. The curve 82 represents the product $Y \cdot f(X)$ when Y remains constant and the variation of f(X) is according to the variation of the X input substantially as indicated at FIG. 7 and where f(X) is established substantially by the relationship of the curve 41 at FIG. 3. The effect of omitting the interpolation oscillation is shown by the stepped-formed curve 83 illustrated in broken lines. It is immediately obvious that a high degree of accuracy can be obtained by the proper spacing of a sufficient number of potentiometers in the element.

We have now described our invention in considerable detail and it is immediately obvious that others skilled in the art can devise and design other similar and equivalent elements and circuits which are within the spirit and scope of our invention, hence we desire our protection to be limited, not by the constructions and circuits

7 illustrated and described, but only by the proper scope of the appended claims.

We claim:

1. A non-linear element adapted to generate an output voltage proportional to the product of a first input voltage and a function of a second input voltage, and comprising, a plurality of potentiometers adapted to individually receive said first input voltage, and output line having a plurality of output leads with each lead connecting with a potentiometer arm, a gate means in each output lead having a control lead and an interlock lead and being adapted to normally hold its output lead open but to close the output lead responsive to a voltage change in the control lead and to open the output lead responsive to a voltage change in the interlock lead, and a plurality of 15 triggers adapted to individually receive said second input voltage and with a trigger being connected to each control lead, said triggers being set to operate to produce a voltage change in their control leads in a sequential order responsive to selected incremental voltage increases of said second input and with each gate-interlock lead being connected wth the control lead of the next-in-sequence trigger, whereby to permit only one gate means to close its output lead at a time as the said second input voltage varies to operate the triggers in said sequential order.

2. In the element defined in claim 1, means for producing an oscillating voltage operable at a frequency substantially greater than the normal frequency of said first and second inputs and at an amplitude substantially at least as great as the said incremental voltage changes of said second input and means for adding said oscillating voltage to said second input voltage whereby to interpolate the voltages of said second input between said selected in-

cremental voltage increases.

means in said output line adapted to bypass the secondary effects of said oscillating voltage.

4. In the element defined in claim 2, said oscillating voltage being in the frequency range of 105 cycles per second when the frequency of said input voltages is in the 40 range of 103 cycles per second.

- 5. A gating apparatus adapted to connect an output line with individual voltage sources in a sequential manner and according to the variations of a voltage input control, and including, in combination therewith, a lead from the out- 45 put line to each voltage source, a gate in each output lead having a control lead and an interlock lead extending therefrom and being adapted to normally open the output lead but to close the same by a voltage change on the control lead and to open the same by a voltage change on 50 the interlock lead, and a trigger connected with each control lead and with said input control adapted to produce a voltage change in the control lead at a selected input control voltage, wherein said triggers are set to operate in a sequence at selected incremental increases of the input 55 control voltage and each interlock lead is connected with the control lead of the next-in-sequence trigger.
- 6. In the apparatus defined in claim 1, wherein each gate includes an NPN type and a PNP type transistor shunting the output lead to ground and the output lead 60 includes isolating resistors adjacent to the transistors.
- 7. In the apparatus defined in claim 1, wherein each trigger is substantially a cathode-coupled binary.
- 8. A non-linear element adapted to generate an output voltage proportional to the product of a first input voltage 65 and a function of a second input voltage, and comprising, a plurality of potentiometer means, having voltage source arms, adapted to individually receive said first input voltage and output line having a plurality of output leads with each lead connecting with a voltage source arm, a gate 70 means in each output lead having a control lead and being adapted to normally hold its output lead open but to close the output lead responsive to a voltage change in the lead, an interlock means associated with the control means adapted to open the output lead responsive to a voltage 75

change in the interlock means, and a plurality of triggers adapted to individually receive said second input voltage and with a trigger being connected to each circuit control lead, said triggers beng set to operate to produce a voltage change in their control means in a sequential order responsive to selected incremental voltage increases of said second input and with each gate-interlock lead being connected with the control means lead of the next-in-sequence trigger, whereby to permit only one gate means to close its output lead at a time as the said second input voltage varies to operate the triggers in said sequential order.

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9. A gating apparatus adapted to connect an output line with dependent voltage sources which vary in proportion to a first independent continuously variable voltage input source, in a sequential manner and according to voltage variations of a second continuously variable voltage input line, which varies through a selected range, and including in combination therewith, an output lead from the output line to each dependent voltage source, a gate means in each output lead having a control lead extending therefrom adapted to normally hold the output lead open to close the output lead responsive to the impressing of a control voltage in the control lead and to reopen the output lead responsive to the releasing of the control voltage, and a discrminator means in said second voltage input line connecting with the control leads adapted to impress control voltage on one selected control lead at a time, wherein said discriminator means includes a trigger means connecting with each control means and arranged in a selected sequential order and with each trigger means therein being adapted to operate through selected incremental portions of the voltage range of said second voltage input line, and an interlock means between the sequentially set trigger means adapted to per-3. In the element defined in claim 2, a low-pass filter 35 mit the operation of only one trigger means at a time.

10. A non-linear element adapted to generate an output voltage proportional to the product of a first continuously variable input voltage and a function of a second continuously variable input voltage which varies through a selected range and comprising, in combination:

(a) a plurality of potentiometer means having voltage source arms adapted to individually receive said first input voltage;

(b) an output line having a plurality of output leads with each lead connecting with a voltage source

(c) a gate means including a control lead in each output lead adapted to normally hold the output lead open but to close the lead responsive to a voltage change in the control lead; and

(d) a discriminator means connecting with the control leads and with said second input and being adapted to produce a voltage change in one gate control lead at a time and to shift the voltage change from one gate control lead to another responsive to and in correlation with the change of voltage of said second input, and wherein the voltage source arm settings and the discriminator means are sequentially correlated to operate the potentiometer gate means in accordance with selected incremental portions of the voltage range of said second input and the voltage source arms are set to produce an output voltage in their sequential order which is proportional to the product of the first said input and a function of the incremental changes of said second input, and wherein said discriminator means include a plurality of trigger means adapted to individually receive said second input with a trigger being connected to each circuit control lead, and said discriminator means and gate means including an interlock means between sequentially set trigger means, whereby to permit closing of the output lead with only one gate at a time.

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