(12) PATENT (11) Application No. AU 199951551 B2
(19) AUSTRALIAN PATENT OFFICE (10) Patent No. 764495

(54) Title
Method and system for maximising the range of stock management requirement profiles

(51)[6] International Patent Classification(s)
G06F 017/60

(21) Application No: 199951551 (22) Application Date: 1999.06.18

(87) WIPO No: WO99/67729

(30) Priority Data

(31) Number	(32) Date	(33) Country
60/089996	1998.06.19	US
09/204772	1998.12.03	US

(43) Publication Date : 2000.01.10
(43) Publication Journal Date : 2000.03.23
(44) Accepted Journal Date : 2003.08.21

(71) Applicant(s)
SAP Aktiengesellschaft

(72) Inventor(s)
Heinrich Braun

(74) Agent/Attorney
Griffith Hack,GPO Box 4164,SYDNEY NSW 2001

(56) Related Art
DE 19612652
US 5797113
US 5819232

S1551/99

PCT WELTORGANISATION FÜR GEISTIGES EIGENTUM
Internationales Büro

INTERNATIONALE ANMELDUNG VERÖFFENTLICHT NACH DEM VERTRAG ÜBER DIE INTERNATIONALE ZUSAMMENARBEIT AUF DEM GEBIET DES PATENTWESENS (PCT)

(51) Internationale Patentklassifikation 6 :
G06F 17/60

A1

(11) Internationale Veröffentlichungsnummer: **WO 99/67729**

(43) Internationales Veröffentlichungsdatum: 29. Dezember 1999 (29.12.99)

(21) Internationales Aktenzeichen: PCT/EP99/04229

(22) Internationales Anmeldedatum: 18. Juni 1999 (18.06.99)

(30) Prioritätsdaten:

60/089,996	19. Juni 1998 (19.06.98)	US
09/204,772	3. Dezember 1998 (03.12.98)	US

(71) Anmelder *(für alle Bestimmungsstaaten ausser US)*: SAP AKTIENGESELLSCHAFT [DE/DE]; Intellectual Property Abteilung, z. H. Herrn Dr. Harald Hagedorn, Postfach 14 61, D-69185 Walldorf (DE).

(72) Erfinder; und
(75) Erfinder/Anmelder *(nur für US)*: BRAUN, Heinrich [DE/DE]; Seewiesenäckerweg 35, D-76199 Karlsruhe (DE).

(81) Bestimmungsstaaten: AE, AL, AM, AT, AU, AZ, BA, BB, BG, BR, BY, CA, CH, CN, CU, CZ, DE, DK, EE, ES, FI, GB, GD, GE, GH, GM, HR, HU, ID, IL, IN, IS, JP, KE, KG, KP, KR, KZ, LC, LK, LR, LS, LT, LU, LV, MD, MG, MK, MN, MW, MX, NO, NZ, PL, PT, RO, RU, SD, SE, SG, SI, SK, SL, TJ, TM, TR, TT, UA, UG, US, UZ, VN, YU, ZA, ZW, ARIPO Patent (GH, GM, KE, LS, MW, SD, SL, SZ, UG, ZW), eurasisches Patent (AM, AZ, BY, KG, KZ, MD, RU, TJ, TM), europäisches Patent (AT, BE, CH, CY, DE, DK, ES, FI, FR, GB, GR, IE, IT, LU, MC, NL, PT, SE), OAPI Patent (BF, BJ, CF, CG, CI, CM, GA, GN, GW, ML, MR, NE, SN, TD, TG).

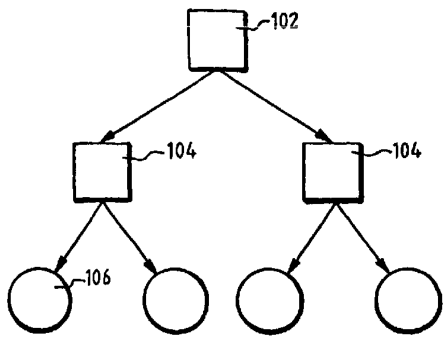
Veröffentlicht
Mit internationalem Recherchenbericht.
Vor Ablauf der für Änderungen der Ansprüche zugelassenen Frist; Veröffentlichung wird wiederholt falls Änderungen eintreffen.

(54) Title: METHOD AND SYSTEM FOR MAXIMISING THE RANGE OF STOCK MANAGEMENT REQUIREMENT PROFILES

(54) Bezeichnung: VERFAHREN UND SYSTEM ZUR MAXIMIERUNG DES BEREICHS VON ABDECKUNGSPROFILEN BEI DER BESTANDSVERWALTUNG

(57) Abstract

The invention relates to a method and system for carrying out stock management in a multilevel distribution chain by applying optimisation algorithms to a range profile built on a distribution network formulation and on different elements which are invoiced in the network. The optimisation problem is defined in a formal way and algorithms for maximising stock autonomy by means of requirement profiles are proposed. Said optimisation process allows transport costs to be reduced to a minimum and bottlenecks to be avoided in stocks. But should bottleneck occur, the system allocates priorities according to need and provides an optimum solution for monotonous product distribution. The inventive method comprises the following steps: initialisation of stock autonomy profile with a starting value; calculation of required entries to satisfy profile restrictions; creation of the most cost-effective flow for said needs by means of a minimal autonomy of the requirement profile until a solution is found.



Method and System for the Maximization of The Range of Coverage Profiles in Inventory Management

The present invention relates to a method for optimization of transportation planning, and in particular to a method for optimization of coverage profiles in a transportation network having a plurality of distribution levels by use of a computer-aided optimization algorithm, in particular to rapidly determine shipment and production schedules.

Background of the Invention:

A need for allocation of inventory and transportation resources arises in a broad range of industrial areas related to manufacturing and distribution.

A distribution procedure for optimally planning the material flow for goods from the manufacturer to the customer was published in DE 196 12 652. This procedure is performed using a central computer control room linked to data entry and data processing devices. In order to carry out requirements-related storage and distribution of goods tied to orders, for example, order lists that reflect the current goods requirements are accessed. By accessing the current lists of orders, predictions, or other pre-settings and requirements, goods production can be controlled according to current actual requirements without surplus production. Furthermore, the system can create a list of expected requirements by taking into account the number of sold or delivered goods and by using a statistical procedure that calculates the goods to be delivered in future on the basis of sales during the previous three years as stored in the central control room. Further parameters, beyond the number of goods required, that influence an optimal goods distribution, such as costs, storage capacity, number of distribution levels are not described in DE 196 12 952.

Allocation decisions are typically subject to limitations on equipment, time, cost, storage capacity and other parameters affecting the outcome of a distribution process, in particular a goods distribution process. As an example of the particular interest herein, there is a need to optimize the distribution and inventory levels of a supply chain where there are multiple production facilities with multiple distribution centers located strategically near customers.

Resource allocation decisions are typically subject to constraints on such allocations. Resources are always limited in overall availability, and, furthermore, the usefulness of a particular resource in some particular application may also be limited. For example, the traffic-carrying capacity of each individual link in a telecommunications system is limited, while the overall traffic offered to the communications system is also limited. Each particular allocation of resources can be associated with a "payoff," i.e. a cost of that allocation or an allocation benefit. The problem, then, is to allocate the resources so as to satisfy all of the constraints and, simultaneously, to maximize the payoff, i.e. minimize the disadvantages, e.g. costs, or maximize the advantages, e.g. benefits.

One method of representing such allocation decision problems is known as the linear programming model. Such a model consists of a number of linear relationships set forth in a matrix format, representing quantitatively the relationships among allocations, constraints and the results of the optimization process. In the linear relationships, there is provided the sum of constant coefficients multiplied by unknown allocation values. Such modeling of linear programming is accomplished in multidimensional space with multidimensional vectors providing a multidimensional figure, or polytope, wherein each facet on a surface thereof is bounded by equations defining relationships among allocated resources in the process. As an example, an optimal solution to the linear programming problem has been obtained by use of the simplex algorithm developed by George Dantzig in 1947, or by the Karmarkar algorithm. There are a number of optimization algorithms that are available to solve minimum-cost flow network problems. Network flow problems are a special case of linear programming.

Network flow algorithms have many applications in industrial planning problems. For example, they can be used in assignment, transportation, minimum-cost flow, shortest path, and maximum flow through network problems.

In assignment problems, there is a bipartite graph consisting of a set of productive nodes and consumptive nodes (for example, workers and tasks). The arcs between the workers and tasks are weighted by costs for assigning the worker with the task. The optimization problem in this scenario is the assignment of one task to each worker such that the overall costs are minimized. In transportation problems, there is a bipartite graph where the productive and consumptive nodes are plants and distribution centers. Each plant produces units and each distribution center has a demand for said units. The arcs between plants and the distribution centers are weighted by costs for transporting the units. In this case, the optimization problem is to minimize the transportation costs with the constraint that all demands in the distribution centers be fulfilled. The minimum-cost flow problem is merely the transportation problem with intermediate nodes in the network. Additionally, the arcs may have minimal and maximal capacity.

In the shortest path problem, you have a graph with positively weighted arcs. The optimization problem is to find the shortest path between two given nodes in the transportation network. The maximum flow through a network problem is similar to the transportation problem except that the arcs between the nodes have limited transportation capacities but no transportation costs. The optimization problem is to get a maximum flow transported through the network without violating the transport capacities.

However, in the prior art, network flow problems have been limited to networks with nodes and arcs between them, where flow is penalized linearly by transportation costs between them. There is a need for a network flow solution where the minimization of the transportation costs has a lower priority and where maximization of the range of coverage of profiles in the consumption nodes is of primary importance. This problem cannot be formulated as in the prior art as a linear optimization problem or even a network flow problem.

Brief Summary of the Invention:

The object of the present invention is to implement a new algorithm for the maximization of the range of coverage profiles in the distribution problem which arises in industrial production planning systems. Several algorithms designed to be applicable to this problem are proposed. The decision on which one is appropriate for a given distribution problem depends on the problem size and the maximum acceptable CPU-time for the computation. This invention proposes a new formulation of the network flow problem which takes into account different transport modes, calendar constraints, demand priorities, and fixed flows of production. The algorithmic function is then applied to this formulation.

The objective of the algorithm is to select the free variables of the range profile formulation such that, first, the range of coverage profiles is maximized and second, the transportation costs are minimized. The proposed algorithm can use any minimum-cost flow algorithm as a basic building block.

Detailed Description of the Invention:

In a first aspect the present invention provides a method for maximizing the range of coverage for coverage profiles using a computer program, including the following steps:

a. Connect to an online transaction processing system;

b. Read transaction data from the online transaction processing system; and

c. Enter the transaction data for the area of the coverage profile function.

d. Provide a range of coverage function for the coverage profile;

e. Initialize said range of coverage profile function with at least one starting value parameter;

f. Determine an optimal stock balance for starting the range of coverage function for the coverage profile. Calculation includes the following steps:

i. Use a minimum cost flow algorithm to identify an optimal transport solution to transport the necessary quantity of stock;
ii. Determine whether a solution has been found after the minimum cost flow algorithm for the area of the coverage profile function was used with the starting value;
iii. If no solution has been identified, incrementally reduce the start value and repeat the calculation of a solution until an optimal solution has been determined for the minimum cost flow algorithm;
g. If a solution has been identified, incrementally increase the start value and repeat the calculation of a solution until the calculation of a solution leads to the optimal solution for the minimum cost flow algorithm.

In a second aspect the present invention provides a computer program including several instructions to maximize the range of coverage of coverage profiles, where the various instructions contain instructions that, when carried out by a processor, ensure that the processor performs the following steps:
a. Connect to an online transaction processing system;
b. Read transaction data from the online transaction processing system;
c. Enter the transaction data in the range of coverage function for the coverage profile;
d. Initialize the range of coverage function with a starting value;
e. Calculate the stock balance required to start the range of coverage function for the coverage profile;
f. Calculate an optimal solution. Calculation includes the following steps:
i. Use a minimum cost flow algorithm to identify an optimal transport solution to transport the necessary quantity of stock;
ii. Determine whether a solution has been found after the minimum cost flow algorithm for the area of the coverage profile function was used with the starting value;

iii. If no solution has been identified, incrementally reduce the start value and repeat the calculation of a solution until an optimal solution for the minimum cost flow algorithm has been determined;

g. If a solution has been identified, incrementally increase the start value and repeat the calculation of a solution until the calculation of a solution leads to the optimal solution for the minimum cost flow algorithm.

In a third aspect the present invention provides a System for maximizing the range of coverage of coverage profiles, including:

a. A means to connect to an online transaction processing system;

b. A means to read transaction data from the online transaction processing system;

c. A means to enter the transaction data for the area of the coverage profile function;

d. A means to initialize the warehouse range of coverage function with a starting value;

e. A means to calculate the stock balance required to start the warehouse range of coverage for the coverage profile;

f. A means to calculate an optimal solution, including:

i. A means to use a minimum cost flow algorithm to identify an optimal transport solution to transport the necessary quantity of stock;

ii. A means to determine whether a solution has been found after the minimum cost flow algorithm for the area of the coverage profile function was used with the starting value;

iii. A means to incrementally reduce the start value, if no solution has been identified, and repeat the calculation of a solution until an optimal solution for the minimum cost flow algorithm has been determined;

g. A means to incrementally increase the start value if a solution has been identified and repeat the calculation of a solution until the calculation of a solution until the calculation of a solution results in the optimal solution for the minimum cost flow algorithm.

In a fourth aspect the present invention provides a computer-readable medium carrying a computer program according to the second aspect of the invention.

The present invention presents a planning system for optimizing the flow of goods in complex distribution networks using a new, efficient algorithm. First, the formulation of the planning system is discussed, then the new algorithms to be applied to this formulation are introduced. The starting point in building the scenario in which the algorithm is to be applied, is the assumption that the production volume to be distributed for the time period under consideration is already fixed, and that demand data is also known. The distribution algorithm described here is mainly intended to react to disturbances in the production process or to changes in the distribution process on short notice. The system is well suited for products which are produced in large quantities and for which demand strongly fluctuates. The optimization process presented here attempts to minimize transportation costs and to avoid bottlenecks by forward planning (push distribution). If bottlenecks develop anyway, an attempt at an optimal response (fair share distribution) is made by prioritizing demand.

The basic structure of a system for distributing goods to which this invention may be applied, consists of three elements: manufacturing plants, central warehouses and distribution centers.

Brief Description of the Drawings:

The invention is explained in more detail below with the aid of exemplary embodiments illustrated in figures in which:

Figure 1 is a diagram of a simple distribution network.

Figure 2 is a diagram of a complex distribution network.

Figure 3 is a diagram of the assignment problem as a bipartite graph.

Figure 4 is a diagram of an equalized distribution network.

Figure 5 is a diagram of production and demand nodes with various possible assignments.

Figure 6 depicts a production plant with supply and demand.

Figure 7 is a diagram of a distribution network with several modes of transportation.

Figure 8 is a diagram of an optimized distribution network with several modes of transportation.

Figure 9 is a flowchart of both possible and optimized transport routes.

Figure 10 is another example of possible and optimized transport routes.

Figure 11 depicts the influence of prohibited periods on the possible transportation options in a distribution network.

Figure 12 is a diagram of a simple distribution network with time values on the arcs.

Figure 13 is a diagram of the production and demand profiles of a distribution network.

Figure 14 is a chart of a global range profile for one possible assignment.

Figure 15 is a chart of a global range profile for another possible assignment.

Figure 16 is an example of a monotone optimal transportation plan.

Figure 17 depicts the search for the maximum monotone range profile.

FIG. 1 depicts the simplest case of a distribution network. In manufacturing plant **102**, goods can be produced utilizing production facilities with a limited availability. These goods are then placed into intermediate storage **104**, (e.g. in warehouses). In order to guarantee short routes to customers, and to guarantee fast and cost-efficient delivery, intermediate storage facilities, or distribution centers **106**, are required. The intermediate storage facilities (intermediate nodes) or the central warehouses **104**, help to achieve larger volume of transportation. The use of central warehouses **104** leads to longer delivery times, but also to lower transport costs. If necessary, a distribution center may also receive a direct delivery. This particular illustration only reflects a schematic structure. In practical operation, network structures may reach much larger dimensions with a large number of plants, warehouses, and distribution centers. As shown in **FIG 2** these network structures comprise multiple manufacturing plants **202**, central warehouses **204**, and distribution centers **206**.

The objective of the distribution problem may be summarized as follows. In the typical supply chain, there are sources, sinks, and a transportation network. The sources are plants for which there is a planned production profile. The sinks are the distribution centers for which there is a demand profile (customer orders or forecast demand). The transportation network consists of arcs with transportation costs and time values between the intermediate nodes. The present invention searches for the optimal assignment of the production (dispersed over space=locations and time) to the demand (also dispersed over space=locations and time) such that the range of coverage profiles is maximized. Two cases are distinguished as mentioned above. In the "Push" case, there are more products than demand, and the products are distributed such that the range of coverage of each increases uniformly. In the "Fair Share" case, there are less products than demand, and the products are distributed such that the more important customers are prioritized and customers of equal priority are treated similarly. Specifically, the system is not allowed to serve customer demand with lower priority before servicing customer demand with higher priority. In the "Push" case, the minimum of the range of coverage profiles is applied to all distribution centers. For the "Fair Share" case, each priority class is considered sequentially and coverage is maximized to serve those high priority demands. Furthermore, if different modes of transportation, transportation paths and sources of supply are available, the entire distribution plan should be optimized toward the lowest possible overall cost.

In the approach to solving the above-described distribution problem, the transportation problem, or assignment problem, plays a central role. The basic assignment problem is defined as follows: in a bipartite graph with an equal number of left and right nodes, each node on the left side should be assigned to a node on the right side so that each node has exactly one partner. The resulting connection ("arc") between two nodes has certain costs. The resulting overall costs should be minimal. **FIG. 3** describes the assignment problem as a graph. Arrow **302** represents a possible assignment of source **304** on the left side to sink **306** on the right side. The costs of assignments are written in as values **308** of the arcs. The classic transportation problem is distinct from the assignment problem in that one node on the left side (now called "producer") may be assigned to any number of nodes on the right side (now called "consumers"). The problem to be solved is how to find suitable distribution of materials between the nodes such that the so-called inventory range is maximized. This means maximizing the minimum inventory across all nodes and periods. In addition the solution with the lowest transportation cost should be selected among the possible solutions (solution group). The range of a warehouse is defined as the number of subsequent periods (including the current period), for which the current inventory suffices. A range of 1 thus means that the inventory will cover exactly the demand of a single, i.e. the current period.

The development of the problem to which the proposed algorithms should be applied is now described in further detail.

THE CLASSICAL CASE

In the simplest case, consider a simple bipartite network with production plants (PP) on the one side and distribution centers (distribution center) on the other side. In this classical case the deliveries are always made from the plants to the distribution centers and not in the opposite direction. A PP can supply any distribution center. **FIG. 4** shows an example with two plants **402, 404** and two centers **406, 408**. Normally, one will have substantially more distribution centers than production plants.

A production plant i in period t has a production rate $p_i(t)$. In distribution center j there is a customer demand of $o_j(t)$. The initial inventory of all plants and centers is zero. When the range is set for 1 period (that is, the inventory of a distribution center in a certain period must only suffice for exactly this period), the following situation results: the demand of a distribution center j in period t can be covered by all productions which can be delivered in a timely manner to the distribution center (that is no later than period t). Therefore, in order to cover this demand, the production of all plants i in all periods $t' \leq t-T_{ij}$ (where T_{ij} is transport time) could be used. The transport times in this first example are set uniformly at one period. In transporting one unit of material, a transport cost of C_{ij} is incurred. **FIG. 5** shows the situation in our network where the window is extended to three subsequent periods (thereby introducing the additional variable of time into the network computation). The number of periods under consideration will be called the horizon. In this new scenario, every node **502** in the illustration now stands for one day's production or, equivalently, one day's demand. The arrows **504** no longer correspond directly to the transportation paths, but instead, correspond to the logical assignment between a certain unit of production and a very specific unit of demand. Every arc therefore can be clearly identified by the source and target place (i and j) and by the start and end time (t_i and t_j). The assignment of a certain amount of material from a node on the left side to a node on the right side is called the flow on the arc. The flow creates costs in the amount of the corresponding transportation costs C_{ij} in the distribution network.

As one can see, this is a classical, simplified transportation problem as it is known from operations research. As such, it can be solved through a standard approach from operations research (for example, the MODI-algorithm, as known in the prior art). Since the actual optimizer works on the level of the transportation problem, the transportation network will be called the *optimization network* from now on, and the optimizer will be called the *transport optimizer*. The nodes in the distribution net are referred to as *distribution network nodes*, the nodes in the optimization network as *optimization network nodes*, when both are referred to. The nodes in the optimization network are divided into *sources* and *sinks*.

This particular case of a distribution network is, however, simpler because, here, production and demand of the optimizing net correspond exactly to those in the distribution network for a single period. When the range is increased to multiple time periods, this is no longer the case. The exact transfer of these values from the distribution net into the optimizing net will be described in the next section.

FROM THE DISTRIBUTION PROBLEM TO THE TRANSPORTATION PROBLEM IN THE CLASSICAL CASE.

In the prior section, the distribution problem was projected onto the transportation problem for a fixed range of one period. Now, the goal is to generalize the projection for any possible range. Since the values of the *sinks* in the *optimizing network* already depart from the demand in the distribution net by a range of more than one period and since the values of the sources can differ from the production values in a distribution network, we further distinguish the data of the optimizing network specifically from the original data in the distribution net. In the optimizing network, we will, in the future, always speak of supply and demand - the terms production and order will always refer to the values in the distribution net. The supply in period t of a node i in the optimizing net is defined to be $s_i(t)$, and the demand is defined to be $d_i(t)$.

Definition of range of coverage

The target inventory $L_i(t)$ in period t for given range $R(t)$ results from the orders of the subsequent periods (the safety stock $m_i(t)$ is not yet taken into account at this point):

$$L_i(t) = \sum_{\tau=t+1}^{t+\lfloor R(t) \rfloor} o_i(\tau) + (R(t) - \lfloor R(t) \rfloor) o_i(t+\lceil R(t) \rceil)$$

Alternative Definition / Generalization of range of coverage

The general statement of the range of coverage (storage range) can be alternatively written to include a maximization of the multiplicator of the safety stock:

$$L_i(t) = o_i(t) + R(t) * \text{safety stock}$$

or in general, for any monotone increasing function f with

$$L_i(t) = f(R(t))$$

For the sake of clarity we will consider only the first definition of range of coverage in the following.

In the case of a distribution center, the demand $d_i(t)$ is dependent on the difference of the target inventory $L_i(t) - L_i(t-1)$ (which in turn depends on the range $R(t)$) and the customer order $o_i(t)$). Demand $d_i(t)$ gives:

Distribution Centers: $d_i(t) = o_i(t) + L_i(t) - L_i(t-1)$

[The conditions: $d_i(t) \geq 0 \Leftrightarrow R(t) \geq R(t-1) - 1$ should be met]

The supply $s_i(t)$ of a plant in this simple model corresponds to real production. A plant normally has no demand of its own, that is, it does not have a target range to be reached. The supply of the plant $s_i(t)$ gives:

$$s_i(t) = p_i(t)$$

Now the transportation problem as shown in the prior section can be defined. In **FIG. 5**, we only have to replace $p_i(t)$ with $s_i(t)$ and the $o_i(t)$ with $d_i(t)$.

<u>FROM THE DISTRIBUTION PROBLEM TO THE TRANSPORTATION PROBLEM IN THE GENERAL CASE.</u>

The models for projecting the transportation problem shown so far are limited to the essential values. Now a model is created where safety stocks and orders of several priority classes are considered:

$$L_i(t) = m_j(t+R(t)) + \sum_{\tau=t+1}^{t+R(t)} \sum_{P \le P_{forecast}} o_j^P(\tau) \quad \text{\{generalization for non integer } R(t) \text{ see above\}}$$

Remark: If we take also intransits $tr_i(t)$ (fixed deliveries) into account, then the demand reduces to

$$d_i(t) = o_i(t) - tr_i(t) + L_i(t) - L_i(t-1)$$

Whenever the intransits are large enough this demand may, if possible, become negative, i.e. this node changes to a supply node with positive supply:

$$s_i(t) = -(\ o_i(t) - tr_i(t) + L_i(t) - L_i(t-1)\ )$$

That means that a distribution center may become a supply node (instead of a demand node) at a particular time t whenever the fixed delivery $tr_i(t)$ is large enough. In this case such a supply of a distribution center i at a particular time t can be assigned to any demand of another node in respect of the transportation time. Especially, the transportation time and costs to the future demands of the same distribution center i are set to zero. Initial inventory of distribution centers may be modeled by fixed deliveries $tr_i(0)$ at the beginning of the planning horizon t = 0. **FIG. 6** shows a small network with one manufacturing plant **602** and two distribution centers **604, 606**. In distribution center 1, for example, there is an oversupply in period 1 due to a very large initial inventory. This can now be assigned to the demand of period two **608** or to the demand of distribution center 2.

DIFFERENT TRANSPORTATION POSSIBILITIES

In the prior section, a very simple network with only one transportation means between two nodes was considered. We will now examine the change which results in the process of finding a solution when different transportation methods are available. Different transportation methods do not necessarily mean different transportation means, it may also refer to alternatives in the path (for example in a case of truck transports). The longer path must be worth it, that is, it only makes sense if it incurs lower transportation costs.

FIG. 7 illustrates a distribution network with several possible means of transportation such as by truck **702** or plane **704**. Different arcs correspond in this case to different means of transportation. The links are always marked with the means of transportation used and with the time **706** of transportation (in periods). For projection onto the optimizing network, one must be aware that every arc is correlated to a maximum transportation duration. For example, for the assignment of a supply from period 1 to a demand in period 2 only a transportation means may be selected which requires no more than one period for transportation. In addition, it makes sense to select not just a feasible possibility, but the best possibility. One can then make the fundamental assumption that if a slower transportation means is selected, it is also the cheaper one. In principle, this addition thus only changes the cost of assignment in the corresponding transportation problem. The linkage to the transportation problem is projected as shown in **FIG. 8**, which displays an optimization net where there are several modes of transport. The projection of the transportation costs has been expanded by the different utilized possibilities of transportation.

TRANSPORTATION OVER INTERMEDIATE STORAGE FACILITIES.

This section first considers a case of no more than one possibility of transportation for each direct link between two nodes. Then we consider the change which arises from permitting several alternatives for such a link.

INTERMEDIATE STORAGE FACILITIES IN A CASE OF ONE POSSIBILITY OF TRANSPORTATION PER LINK.

Up to this point, we only considered a two stage distribution network. The consumers (in this case the distribution centers) are directly supplied by the manufacturing plants. Often, distribution networks also use central intermediate storage facilities. By introducing these facilities, the transportation paths to the distribution centers can now, in part, be bundled together. Deliveries can also be bundled and thus possibly made less expensive. The central warehouse acts as a pass-through station in order to assist in delivery optimization. Therefore, one can directly assign the production of the plants to the distribution centers. For the duration of the overall transport (via the intermediate storage facility) the sum of the transport time to the central warehouse and from there to the distribution center may be used. Time needed to move in and out of the facilities, if necessary, is already included in the transportation time. Since the priority of the method according to the invention is to maximize the range of the coverage profiles of the warehouses, the earliest possible deliveries and therefore short transportation times are advantageous. For this reason, preference is always given to the fastest path between two nodes even if it is not necessarily the most direct path. If one enters the transportation times into a graph in order to then receive the shortest overall transportation time between two randomly chosen nodes one must also calculate the transitive closure of the graph. **FIG. 9** shows an example (on top the given net is projected, below the transitive closure that belongs to it) with the shortest transportation time between two nodes.

In first flow chart **902**, the various transportation routes and times are shown. To get from node **904** to node **906**, it takes three periods. However, to get from node **904** to node **906** through node **908**, it only takes two periods. Taking this into account, flowchart **910** is created, showing the minimum times needed to get between the nodes.

If one has calculated the duration of the fastest path in this manner one then calculates the costs that go with it. These are composed, analogously to the times, of the sum of the costs for each part of the path. If there are several fast paths, the costs of the most efficient and/or most favourable among them are utilized.

INTERMEDIATE STORAGE FACILITIES IN A CASE OF ALTERNATIVE POSSIBILITIES OF TRANSPORTATION FOR EACH LINK.

As explained before, one can also consider with the method according to the invention alternative arcs in the distribution network. In transport over intermediate nodes the question arises which arc should be used for the different segments. If one permits all combinations, then in a case of several intermediate nodes, one very quickly arrives at many possibilities of transportation between the final nodes. Upon closer examination, however, one will find that not every one of these arc combinations makes sense. Normally, for example, one would not use an express connection in a first segment and a slow connection in a second. For this reason, the following method has been selected for entering the connections. First, one specifies the number of the maximum permissible transportation modes in the net. The transportation modes here correspond to different speed classes (for example regular and express). Now, if a direct connection is to be defined between two nodes, then for each transportation mode (i.e. speed, class, etc.) one states the cost and duration. The optimizer then calculates the best path as described in the last section separately for each class. This data is then used in the optimization network. In paths with several segments, the transportation modes are not mixed together. If one nevertheless would like to permit only one possibility of transport on a given link, one provides the same data for all transportation modes. **FIG. 10** shows an example. On the top of the drawing is the provided network as is, and on the bottom of the drawing the network as created by the optimizer is shown. Path **1002** shows manufacturing center **1004**, central warehouse **1006**, and distribution center **1008**. It displays route **1010** via train which takes 1 period, route **1012** via truck which takes 3 periods, routes **1014, 1016** via plane each of which takes 1 period. The arcs of the normal mode are shown with dots, the arcs of the express modes are shown with lines. Between nodes **1006** and **1008** there exists only one real connection, which is transportation by plane. Path **1018** shows the paths compressed by the optimizer between the manufacturing center **1004**, central warehouse **1006**, and distribution center **1008**.

PROHIBITED PERIODS

Now we will discuss implementation of prohibited arcs. Every transportation connection and every distribution node can be provided with a list of prohibited periods. Prohibited periods are windows of time such as holidays on which transportation rests or on which no goods can be accepted or given out. We recall the classical case described above where every node in the optimization net stands for one point of origin *i*, a starting time t_i and a target place *j* and a target particular time t_j of a certain material assignment. In order to take holidays into account every arc is pushed into the future until t_i falls on a permitted period for giving out goods at node *i* (that is, a common delta of time is added to both the start time and target time). After this, t_j is also shifted until the sums of the permitted transportation periods between t_i and t_j are sufficient for the corresponding transport and until t_j falls on a permitted period for accepting goods at node *j*. **FIG. 11** shows an optimizing network with a horizon of six periods **1102, 1104, 1106, 1108, 1110, 1112**. The nodes are marked with the distribution network node number *i* and the corresponding period number t_i. The prohibited periods are in each case shown shaded in gray. In period two, therefore, there is a prohibition against accepting goods in distribution net node 1. Delivering products from node 2 in period 5 is prohibited. Transportation is prohibited in period 3 **1114**. The described distribution of the prohibited periods is of course an extreme case. In most cases the holiday will at least affect the areas receiving goods and giving out goods at the same time.

THE SEARCH FOR THE MAXIMUM RANGE PROFILE.

The aim of this invention is to supply the distribution centers of the demand nodes with the available material such that the range profile of the inventory is maximized over all warehouses. The task, therefore, is to find a suitable assignment from the supply nodes to the demand nodes. So far we have seen how one can determine an assignment to a given range profile which meets the prior stated conditions (minimal transportation costs taking into account holidays etc.). This is done by a projection of a transportation problem and solution of the same utilizing an efficient transportation algorithm, for example, the MODI-algorithm, which can serve as the basis for the enhanced algorithm proposed by the present invention. In the following we define the problem of maximizing the range of coverage.

MAXIMIZING THE RANGE OF COVERAGE FOR OVERSUPPLY

Oversupply means that the planned-for customer demand can be completely met. The transport optimizer thus has found a solution for a minimum range profile of one period across all distribution centers and points in time. Now the range profile must be maximized. At first glance the only option is to try out different profiles with the given possibilities and to gradually work one's way toward the optimum, e.g. a trial and error method. Assumptions about the existing supply and demand situation which do not restrict the general validity of the process excessively can, however, greatly speed up the search. However, one must accept the risk of not always finding an optimal solution. Before we go into more detail, we should answer the question: What is to be understood by a maximum range profile?

WHAT DOES A MAXIMUM RANGE PROFILE LOOK LIKE?

If one wants to describe the range of all distribution centers in the network, one would need to describe the range profile for each individual distribution center. However, because the goal is to even out the ranges between the warehouses at a given particular time as much as possible it will suffice to speak of only one global range profile which is valid as a lower boundary value for each distribution center. Stated more clearly, the global profile $R(t)$ is the minimum for the local profile $R_i(t)$ over node i. The local profiles may if possible at times lie above the global profile; they may, however, never lie below it: $R(t) = min_i\ (R_i(t))$.

As was the case before, we will in the future only speak of the range profile (which refers to all distribution centers). Intuitively, one would think the maximum range profile to be that profile which is at a maximum in each period. However, this is not the case. The maximum range in a certain period may depart significantly from the range of another period which precedes it.

FIG. 12 shows an example network with two manufacturing plants and two distribution centers. On the arcs the corresponding transportation times are written in. The associated production and demand profiles are shown in **FIG. 13.** The example is structured according to the simple model of a distribution network and does not include any safety stocks, initial inventories or fixed deliveries. All the demand classes are of equal priority, and there is only one transportation mode. For the following consideration of the range profile this simple model is always used. The maximum possible global range in a certain period is the range which both distribution centers can reach simultaneously in that period. In period two it has a value of three. This value is reached only then when one assigns the production of the first period from plant **1202** (40 units) in equal parts to the distribution centers. Plant **1202** has no influence on the range in the second period because the material from this plant **1202** can be delivered no earlier than the third period. In this assignment, a range of one automatically emerges in the fourth period for distribution center 1 since this distribution center is only reached by plant 2 in the fifth period and since the initial production of manufacturing plant 1 is already "used up." **FIG. 14** shows the maximum global range for this first assignment. The minimum inventories over both centers **1206, 1208** at the onset of each period are shown in brackets. An infinite range always results when the inventory is sufficient at least for all following periods since for periods beyond the horizon the demand of zero units is assumed.

If one now would like to raise the global range in period 4 from 1 to 2 the following assignment results. Of the 40 units produced by plant **1202** in period 1, 30 units are assigned to distribution center **1206** and 10 units to distribution center **1208**. Of the 40 units of the first period in plant **1204**, one assigns at least 20 units to distribution center **1208**. With a suitable distribution of the remaining production of plant **1202** (for example in equal parts to both centers **1206, 1208**) the global range shown in **FIG. 15** is reached.

This assignment has the additional advantage that the minimum of the global range has increased over all periods from 1 to 2. This example demonstrates that it is not always beneficial to build up the greatest possible global range at the onset. Sometimes this destroys the possibility of evening out a minimum of the global range at a later point. This process of evening out the inventory is desirable in every case in order to achieve an overall good profile. The range profiles in Fig. 14 and Fig. 15 must thus be ranked such that the range profile in Fig. 15 is evaluated as better than the profile in Fig. 14. It is not sufficient to compare only the global minimum of the profiles (as one might think) since the other periods in which a range may lie barely above the minimum are not considered in a comparison. We therefore define the following ranking for the global range profile:

<u>Ranking for the global range profile:</u> The following applies to two global range profiles R^1 and R^2:

$$R^1 > R^2 :\leftrightarrow min\{R^1(t) \backslash R^1(t) \neq R^2(t)\} > min\{R^2(t) / R^1(t) \neq R^2(t)\}$$

To demonstrate the completeness of this ranking, one must also demonstrate that, if two elements cannot be compared (the profiles have the same minimum at different points) there is always an element which is larger than both the other elements.

<u>THE MONOTONE RANGE PROFILE</u>

It is very time-consuming to find the optimal range profile with the known methods (trial and error methods), if for example the following method is used: beginning with a minimum uniform range of 1 at every particular time, one increases the range in period 1 by a small delta. If a solution is found, then one increases the range in period 2 also and so forth. Once all the periods are done, the process begins again with the first period. If the range cannot be increased in a certain period, one excludes that period from the increases in the further iterations. The time complexity of this approach is bounded by $O(R_{max}/R_{min} \cdot H)$ calls of the transport optimizer, which may if possible be too time consuming with R_{max} being defined as the optimal range which may occur in the period, R_{min} being defined as the precision, or granularity, with which this range is to be determined, and H standing for the horizon. In this case, the optimizer always must optimize the data of the entire horizon.

For this reason according to the invention we consider first the monotone range of coverage profiles.

In a second step we enhance this approach to non monotone ranges of coverage profiles. Since in a monotone increasing function, the minimum always lies at the beginning, the profile which is the maximum one among the monotone increasing profiles, is the first (with the smallest t) to reach a larger value than the other profiles. A transportation plan which is calculated on the basis of the maximum monotone range profile is very good, however, it is unfortunately not always optimal. An example will demonstrate this. The table in **FIG. 16** shows a possible situation in a distribution network with one plant and two distribution centers. The transportation time uniformly is one period.

The maximum monotone range profile R^M **1602** has a value of 1 in every period (except for the last period since for demand of periods beyond the horizon, the value 0 is assumed) since no larger range is possible in period 5. The table shows the plan which emerges from assigning the supply to the demand in the optimization net and when delivery starts immediately. At Z^M **1604,** for example, the production of 30 units in the first period is assigned to the first 3 demands of distribution center **1206** (see Fig. 12). If all 3 assigned deliveries commence immediately (that is, in this first production period) then a total transport of 30 units takes place to distribution center **1206**. The transportation plan Z^M **1604** is sufficient for the maximum monotone range profile; but it creates an actual global range (that is, the minimum of the actual ranges of the distribution centers) of R'^M, **1606** which is smaller than the maximum possible range R **1608** (in the case of assignment Z). In other words, plan Z^M **1604** is very good for the maximum monotone range (it is *monotone* optimal) but is not fully optimal.

An optimal transportation plan for the first period is produced as follows: the assignment of supply of the first period should not have a negative effect on the range in all periods. The following assumption can substantially reduce the effort of such a search according to this plan. In order to obtain the monotone optimal transportation plan of the first period it is sufficient to restrict the search for the maximum range profile to such profiles which are uneven only in the initial range, that is, at $t \le max_{i,j}\{T_{ij}\}$ with T_{ij} transportation time between nodes i and j. One need only increase the range profile up to the period of the maximum transportation time; in all following periods the profile may have a constant value.

If one does not consider the entire horizon H in establishing the optimization network but only the periods $t \le H_{short}$ up to a particular time $H_{short} < H$, that is, if one only considers supply $s_i(t)$ and demand $d_i(t)$ when $t \le H_{short}$, the optimization network then yields a maximum range profile R_{max}^{short} which in every period is at least as large as the maximum profile R_{max} over the entire horizon:

$$\forall t \le H_{short} : R_{max}^{short}(t) \ge R_{max}(t).$$

RAPID RANGE PROFILE SEARCH

Now that we have defined the short horizon, let's shorten it to one period. The resulting optimization network (which only makes sense if it is possible to execute the transport in one period) is very small and the arc flows can be calculated very rapidly. The maximum range in period 1 can be located by a binary search. In the next step, the second period is added and an attempt is made to locate a maximum range profile over both periods. In doing so, we maintain the range profile from the first step (which so far applies only to the first period) and we initialize the second period with the value of the prior period (in this case the first). If we find a solution for this profile then the range of the second period can be maximized again using a binary search, etc. If for a short horizon, H_{short}, range $R(H_{short})$ of $R(H_{short}-1)$ is taken over, that is, if no solution is found, then the range in period H_{short} is lowered by one period. The resulting profile is always kept monotone, that is, in all periods $t < H_{short}$ the range is set to $R(t) = min\{R(t),R(H_{short}\}$. Now, at the latest, a solution must be possible since a range of $R(H_{short}) = R(H_{short}-1)-1$ results from range $R(H_{short}-1)$ in period H_{short_3}. Now we again attempt to increase the value of this and the following periods, beginning with the first period in which a range was changed during this step. The horizon under consideration remains H_{short}. If one finally has reached $T_{max} = max_{ij}\{T_{ijj}\}$ with the short horizon then in the next step all subsequent periods are treated as a single period. The horizon under consideration is expanded to the overall horizon and all following periods are set to the value $R(H_{short})$. After this step the procedure is completed. The maximum monotone profile has been located. It increases up to point T_{max} and later remains constant.

FIG. 17 shows an example of the method according to the invention. Step 4 **1702** shows a step backward. The range in period 4 (time interval) cannot be maintained at the value of period 3 and therefore it is reduced to a value of 1.25. All prior periods with a larger range are reduced to this range. Thereafter, one again beging to increase the range of the affected period. In every step the value found for the range in the last period of the horizon currently under consideration, that is, $R(H_{short})$ constitutes an upper limit for the range which can be achieved in this period. Since the maximum monotone profile also tolerates transportation plans which make it impossible to achieve a general maximum profile (for example in this section) one can use these boundary values in order to make visible a possible mistake in the calculated monotone profile and possibly react to it interactively.

If Opt(H) is the runtime of the transport optimizer for horizon H then the run time of the worst scenario is:

$$O(log(R_{max}/R_{min}) \cdot T_{max} \cdot Opt(H_{max}))$$

R_{max} is the maximum range which can occur in a period, R_{min} is the precision (granularity) with which this range can be determined, T_{max} is the maximum transportation time between two nodes (with the slowest transportation mode) and H_{total} is the entire horizon (for the overall run time including the run time for the transportation optimizer).

In practical application it turned out that a step backward was rarely necessary. If one takes as point of departure that bottlenecks are more likely to occur at the beginning and that production volume overall slightly exceeds demand volume (push-distribution) then in the last step (in which all periods at T_{max} are incorporated into the optimization net) no step back takes place and the transport optimizer is only called up $log(R_{max}/R_{min}$ for the entire horizon. Then, taken together with the effort for the initial range, the following run time results:

$$O(log(R_{max}/R_{min}) \cdot (T_{max} \times Opt(T_{max}) + Opt(H_{total})))$$

The method according to the invention will quickly generate a monotone optimal transportation plan for the first period.

INSUFFICIENT SUPPLY

The prior section assumed that production always suffices to cover all demands in a timely manner. This of course is not always the case. If there is insufficient supply then the demand situation must be lightened in a suitable manner. This can be done in one of two ways: first, the demand itself can be reduced, that is, a few customer orders are only partially or not at all fulfilled. Second, a delay in the completion of the orders is permitted. Also one can combine both variants. A combination would make sense especially when insufficient supply is not merely a short term issue, that is, when production generally lies below demand. In this situation, delays without demand reductions would continue to accumulate.

REDUCTION IN DEMAND

Let's now turn to the first case (no delay). In order to reduce the demand in a suitable fashion we introduce different priorities for customer orders. An order $o_i^P(t)$ of distribution net node i in period t is now additionally marked with its priority P. The unimportant priority classes (i.e., orders in those classes) are cut back until the volume of material at hand suffices to satisfy the remaining orders on time. We use the same approach as above. The integer part $\lfloor R(t) \rfloor$ specifies the number of priority classes, whose orders are completely fulfilled. The rational part $R(t)-\lfloor R(t) \rfloor$ is the part fulfilled from the next priority class. For example we could define 4 priority classes:

1. P_{high}: Demands of customer with high priority.
2. P_{normal}: Demands of customer with normal priority.
3. P_{low}: Demands of customer with low priority.
4. $P_{forecast}$: Additional demand which is only forecast but not yet ordered by a customer

A range of coverage R(t) = 2.7 would mean that we fulfill all demands for high and normal prioritized customers at time t. For the low prioritized customer we fulfill only 70%.

Our definition according to the invention of maximal range of coverage profile guarantees that no demand of higher priority is fulfilled at the expense of a demand of lower priority.

DELIVERY WITH DELAY

In the above considerations no delayed delivery was allowed. That means whenever an order could not be delivered in time it was not delivered at all. Example: If we have a demand of high priority for t=1 and a demand of low priority for t=2 and given that the first supply arrives at t=2 then the demand with low priority will get all supply, but the demand with high priority nothing.

This problem is solved according to the invention by introducing additional arcs with reduced transportation time: Delivering with delay D is equivalent to reducing the corresponding transportation time by D.

For these additional arcs we have to define the costs as follows:

$$penalty = \Delta^2 * 2C_{max}$$

Δ represents the delay and C_{max} the maximum cost of the arcs without a delay.

The term 2 C_{max} guarantees that a delay cannot be prevented by a swap, i.e. exchanging the assignments

$s_i(t_i) \rightarrow d_j(t_j)$ and $s_{i'}(t_{i'}) \rightarrow d_{j'}(t_{j'})$

with $s_i(t_i) \rightarrow d_{j'}(t_{j'})$ and $s_{i'}(t_{i'}) \rightarrow d_j(t_j)$

If there are no delays in the exchanged assignments then their costs are $\leq 2C_{max} < \Delta^2 + 2C_{max}$ and thus cheaper than the assignments without a delay. Therefore, the optimizer will never produce a solution with a delay which could be prevented by a swap.

Similarly, the term Δ^2 guarantees that a long delay cannot not be shortened by a swap.

DESCRIPTION OF THE PROJECTION OF THE DEPLOYMENT PROBLEM INTO THE DISTRIBUTION PROBLEM.

In this section the illustration of the distribution problems shown so far with regard to the transportation problem will be summarized one more time in mathematical form.

CASE 1: OVERSUPPLY

PRIOR CALCULATION

One first must calculate production and demand of the optimization network:

Distribution Centers:

$$D_j(t) = \sum_{P} o_j^P(t) - tr_j(t) + L_j(t-1) - L_j(t)\}$$

If $D_i(t) > 0$

Then $d_i(t) = D_i(t)$ *{demand node}*

Else $s_i(t) = -D_i(t)$ *{supply node}*

with target warehouse inventory of the distribution centers:

$$L_j(t) = (1 - R(t) - \lfloor R(t) \rfloor) \cdot L'_i(t, \lfloor R(t) \rfloor) + (R(t) - \lfloor R(t) \rfloor) \cdot L'_i(t, \lceil R(t) \rceil)$$

$$L'_j(t,r) = m_j(t+r) + \sum_{\tau=t+1}^{t+r} \sum_{P} o_j^P(\tau)$$

Manufacturing plants:

$$s_i(0) = p_i(0) + L^{initial}$$

$$\forall t > 0:\ s_i(t) = P_i(t)$$

THE OPTIMIZATION PROBLEM

Minimize:

transportation costs =

$$\sum_{i,j} - \left(\sum_{t_j - t_i \geq T_{ij}^{D_{fastest}}} y_{it_i jt_j} \cdot C_{ij}^{D_{fastest}} + \sum_{D < D_{slowest}} \sum_{T_{ij}^{D} \leq t_j - t_i < T_{ij}^{slower(D)}} y_{it_i jt_j} \cdot C_{ij}^{D} + \sum_{t_j - t_i \geq T_{ij}^{D_{slowest}}} y_{it_i jt_j} \cdot C_{ij}^{D_{slowest}} \right)$$

With the following auxilliary conditions:

Constrained supply: $\forall i : s_i(t_i) = \sum_{j,t_j} y_{it_i jt_j}$

Constrained demand: $\forall j : d_j(t_i) = \sum_{i,t_i} y_{it_i jt_j}$

CASE 2A: REDUCTION OF DEMAND IN A CASE OF INSUFFICIENT SUPPLY

PRIOR CALCULATION

Production and demand of the optimization net must again be calculated first:

Distribution Centers:

$$D_j(t) = \sum_{P \leq P\lfloor R(t) \rfloor} o_j^P(t) - tr_j(t) + L_j(t-1) - L_j(t)\}$$

If $D_i(t) > 0$

Then $d_i(t) = D_i(t)$ *{demand node}*

Else $s_i(t) = -D_i(t)$ *{supply node}*

with target warehouse inventory of the distribution centers:

$$L_j(t) = (1 - R(t) - \lfloor R(t) \rfloor) \cdot L'_i(t, \lfloor R(t) \rfloor) + (R(t) - \lfloor R(t) \rfloor) \cdot L'_i(t, \lceil R(t) \rceil)$$

$$L'_j(t,r) = \sum_{\tau = t+1}^{t+r} \sum_{P \leq P\lfloor R(t) \rfloor} o_j^P(\tau)$$

Remark: The integer part $\lfloor R(t) \rfloor$ specifies the number of priority classes, whose orders are completely fulfilled. The rational part $R(t) - \lfloor R(t) \rfloor$ is the part fulfilled from the next priority class.

Manufacturing plants:

$$s_i(0) = p(0) + L^{initial}$$

$$\forall t>: \ s_i(t) = P_i(t)$$

THE OPTIMIZATION PROBLEM

Minimize:

transportation costs =

$$\sum_{i,j}\left(\sum_{t_j - t_i \geq T_{ij}^{D_{fastest}}} y_{it_i jt_j} \cdot C_{ij}^{D_{fastest}} + \sum_{D<D_{slowest}} \quad \sum_{T_{ij}^{D} \leq t_j - t_i < T_{ij}^{slower(D)}} y_{it_i jt_j} \cdot C_{ij}^{D} + \sum_{t_j - t_i \geq T_{ij}^{D_{slowest}}} y_{it_i jt_j} \cdot C_{ij}^{D_{slowest}}\right)$$

With the following auxiliary conditions:

Constrained supply: $\forall i : s_i(t_i) = \sum_{j,t_j} y_{it_i jt_j}$

Constrained demand: $\forall j : d_j(t_i) = \sum_{i,t_i} y_{it_i jt_j}$

CASE 2B: DELAY WITH UNDERSUPPLY

PRIOR CALCULATION

Again production and demand of optimization net must be calculated first in the same manner as above, only the optimization function changes by penalizing the arcs with delay. We may also limit the delay by introducing only those additional arcs with an acceptable delay.

THE OPTIMIZATION PROBLEM

Minimize:

Cost = transportation cost + delay penalty

Transportation cost =

$$\sum_{i,j}-\left(\sum_{t_j - t_i \geq T_{ij}^{D_{fastest}}} y_{it_i jt_j} \cdot C_{ij}^{D_{slowest}} + \sum_{D<D_{slowest}} \quad \sum_{T_{ij}^{D} \leq t_j - t_i < T_{ij}^{slower(D)}} y_{it_i jt_j} \wedge.\wedge C_{ij}^{D} + \sum_{t_j - t_i \geq T_{ij}^{D_{slowest}}} y_{it_i jt_j} \cdot C_{ij}^{D_{slowest}}\right)$$

Delay penalty:

$$\sum_{i,j}\left(\sum_{t_j - t_i < T_{ij}^{D_{fastest}}} \left(t_i + T_{ij}^{D_{fastest}} - t_j\right)^2 \cdot 2C_{max} \cdot y_{it_ijt_j}\right)$$

With the following auxiliary conditions:

Constrained supply: $\forall i : s_i(t_i) = \sum_{j,t_j} y_{it_ijt_j}$

Constrained demand: $\forall j : d_j(t_i) = \sum_{i,t_i} y_{it_ijt_j}$

Maximal delay constraint:

$$\forall i, t_i, j, t_j : \; y_{it_ijt_j} \; defined \; \Leftrightarrow \; t_i + T_{ij}^{D_{fastest}} - t_j \leq A_{jt_j}$$

<u>VARIABLES AND CONSTANTS</u>

Free decision variables:

$y_{it_ijt_j}$ = Part of the supply of plant i at time t_i which is assigned to the demand of distribution center j at time t_j

Bound variables (to be maximized with constrained decision $R(t)$)

$R(t)$ = Range of coverage of all distribution centers at time t

$L_j(t)$ = Target inventory in distribution center j at time t (depending on range of coverage $R(t)$)

$s_i(t)$ = Supply of node i (=plant) at time t

$d_j(t)$ = Demand for node j (=distribution center) at time t

Constants:

$P_i(t)$ = Production rate in production plant i at particular time t

$tr_i(t)$ = Previously fixed supply (in transit) in distribution center or central warehouse j at particular time t

$L_i^{initial}$ = $tr_i(0)$ = Initial inventory in production plant i or distribution center i

P	=	Demand priority P, $P \in \{P_{high}, . . . , P_{forecast}\} = \{1, 2, 3, . . .\}$ with $P_{high} = 1 < . . . < P_{forecast}$
$o_j^P(t)$	=	order (=aggregated demand) for priority class P for distribution center j at time t
$m_j(t)$	=	Safety stock in distribution center j at time t
D	=	Transportation mode $D \in \{slowest, . . ., fastest\}$
C_{ij}^D	=	Transport cost for transport of a material unit with transportation mode D between nodes i and j
T_{ij}^D	=	Transportation time for the transport of a material unit with transportation mode D between nodes i and j
$D_{fastest}$	=	Fastest transportation mode: $\forall D, i, j: T_{ij}^{Dfastest} \leq T_{ij}^D$
$D_{slowest}$	=	Slowest transportation mode: $\forall D, i, j: T_{ij}^{Dslowest}$
$slower(D)$	=	Next slowest transportation mode after transportation mode D : $\neg \exists D' : [\forall i, j : T_{ij}^D \leq T_{ij}^{D'} < T_{ij}^{slower(D)}]$
A_{jt_j}	=	Maximum permitted delay of a demand from distribution center j at particular time t_j

ALGORITHMS

Now that the formulation of the problem is complete, the algorithm to be applied will be discussed. The basic idea of the algorithm is to mutate incrementally the range of the coverage profile $R(t)$ until the optimal profile is reached. The basic structure of the algorithm comprises the following steps:

1. Initialize the range of coverage profile $R(t)$ (e.g. $R(t)=1$).
2. Compute the necessary demands $d_j(t)$ in the demand nodes in order to fulfill profile constraints.
3. Construct the cheapest flow for these demands using any minimum cost flow algorithm.
4. If no solution is found, then reduce the range of the coverage profile $R(t)$, otherwise, increase the range of coverage profile $R(t)$.
5. If the optimal solution is not found, go back to step 2.

Step 2 is the same for all algorithms as described in the sections above:

$$d_j(t) = \sum_P o_j^P(t) - tr_j(t) + L_j(t-1) - L_j(t)$$

with $L_j(t) = (1 - R(t) - \lfloor R(t) \rfloor) \cdot L'_i(t, \lfloor R(t) \rfloor) + (R(t) - \lfloor R(t) \rfloor) \cdot L'_i(t, \lceil R(t) \rceil)$

$$L'_j(t,r) = m_j(t+r) + \sum_{\tau=t+1}^{t+r} \sum_P o_j^P(\tau)$$

Various alternatives according to the invention to step 4 are proposed in the following section. The full problem is first discussed, i.e. the efficient construction of a maximal range of coverage profiles. This algorithm can be significantly speeded up by the restriction to a monotone range of coverage profiles which will be described later. A fast algorithm for monotone profiles for the maximization of the range of coverage is defined below.

A. MAXIMAL RANGE OF COVERAGE PROFILE

For constructing the maximal (integer) range of coverage profile, the actual profile is enlarged iteratively by 1 whenever this is possible for each time step sweeping several times over the whole planning horizon.

Algorithm for step 4

```
While Profile_enlarged do begin
     {Sweep over the planning horizon}
     enlarged = false;

     for all t (planning horizon do begin
     R(t):=R(t)+1;                              {enlarge R}
     Generate min_cost_flow problem F(R) for profile R(t)
     {=step 2 (see above)}
     If min_cost_flow (F(R)) solvable
        then enlarged = true
        else R(t):=R(t)-1{reset};
        end;
     end;
```

The generalization for enlarging the profile by a smaller step width $\Delta < 1$ instead of 1 gives:

Algorithm of step 4 with step width Δ for profile *R*

```
  While Profile_enlarged do begin
     {Sweep over the planning horizon}
     enlarged = false;

     for all t ≤ planning horizon do begin
     R(t):=R(t)+Δ;                              {enlarge R}
     Generate min_cost_flow problem F(R) for profile R(t)
     {=step 2 (see above)}
     if min_cost_flow (F(R)) solvable
```

```
      then enlarged = true
      else R(t):=R(t)-Δ                    {reset};
   end;
end;
```

B. <u>MAXIMAL MONOTONE RANGE OF COVERAGE PROFILES</u>

The basic idea is to solve the problem by induction:

- Initialize with an empty profile
- Construct a solution for t time steps and enhance this solution to one for $t+1$ time steps.
- Iterate until t = Planning horizon.

<u>Algorithm for step 4</u>

```
Initialize R(t):     ∀t:R(t)=0
t=t0; {Planning Start}
While t < planning horizon do begin
{Induction Step}
R(t+1)=R(t);
Generate min_cost_flow problem F(R) for profile
R(t)                          {=step2 (see above)}

If min_cost_flow (F(R)) solvable
Then Begin                          {enlarge R(t+1)}
   Repeat
         R(t+1):=R(t+1)+1;
         Generate min_cost_flow problem F(R) for
         profile R(t)
   Until min_cost flow (F(R)) not solvable
   R(t+1):=R(t+1)-1                       {reset}
   End
Else                                      {lower R}
   Repeat
   R^max=R(t+1)
   t_R = min{t \ R(t) = R^max - 1};
   ∀t'>t_R: R(t') = R^max - 1;
```

```
    Generate min_cost_flow problem F(R) for profile
            R(t)
    Until min_cost_flow (F(R)) solvable
```

<u>Algorithm for step 4-monotone range of coverage profiles with step width Δ</u>

The generalization for enlarging the profile by a smaller step Δ<1 instead of 1 gives:

```
Initialize R(t):        ∀t:R(t)=0
                        t=t0; {Planning Start}
While t < planning horizon do begin
    {Induction Step}
R(t+1)=R(t);
Generate min_cost_flow problem F(R) for profile R(t)
                        {= step 2 (see above)}

If min_cost_flow (F(R)) solvable
Then Begin                          {enlarge R(t+1)}
    Repeat
      R(t+1):=R(t+1)+ Δ;
      Generate min_cost_flow problem F(R) for profile
        R(t)
    Until min_cost_flow (F(R)) not solvable
    R(t+1):=R(t+1)- Δ                       {reset}
    End
Else                                        {lower R}
    Repeat
            R^max: =R(t+1);
            t_R=min{t' / R(t')>R^max - Δ}
            ∀t'≥t_R: R(t')=R^max - Δ;
            Generate min_cost_flow problem F(R) for
            profile R(t)
    Until min_cost_flow (F(R)) solvable
```

C. EFFICIENCY IMPROVEMENT FOR ROLLING PLANNING SCHEME

For the rolling planning scenario the planner needs only to know, where to ship the production for start time t_0, i.e. the shipment planning $y_{it_0\ jt_j}$ for time t_0 should be extendable to an optimal planning, however, the shipment planning for the following time steps may be not optimal. These steps will be corrected according to the invention by moving the planning window.

In the following we restrict the profiles on the following type: Even until the maximal transportation time t_{max} and constant after t_{max}. As long as this monotone profile is not solvable we lower the last constant function part.

Algorithm for step 4-rolling planning scheme

Given the maximal transportation time i.e. t_{max}

Compute the maximal monotone range of coverage profile $R^{max}(t)$ within the planning window [planning_start, planning_start+ t_{max}];

Initialize $R(t)$:{constant continuation of R}

$\forall t \leq t_{max}: R(t) = R^{max}(t)$

$\forall t > t_{max}; R(t) = R^{max}(t_{max})$;

$R^{max} := R^{max}(t_{max})$; {adjust last constant part of R}

Generate min_cost_flow problem $F(R)$ for profile $R(t)$

While min_cost_flow$(F(R))$ not solvable do begin

$t_R = min\{t \mid R(t) \geq R^{max} - 1\}$

$\forall t \geq t_R: R(t) = R^{max} - 1$;

$R^{max} = R^{max} - 1$

Generate min_cost_flow problem *F(R)* for profile *R(t)*

End;

GENERALIZATIONS

For constructing non-integer capacity profiles we have to limit the step width Δ of the possible values for range of coverage profile *R(t)*. When the step width Δ is small, there are many iterations necessary until the maximal profile is found. In the following sections we describe how the search for maximal monotone profiles can be speeded up by a binary search.

1. BINARY SEARCH FOR MAXIMAL MONOTONE PROFILES

In the algorithm for maximal monotone range of coverage profiles with step width Δ we consider two parts "enlarge *R(t)*" and "lower *R(t)*". In both cases we speed up the search for the optimal tuning of the last constant part of the capacity profile using a binary search. We describe in the following a straightforward augmentation of the algorithm according to the invention by a binary search. Alternative variants are omitted for the sake of clarity (e.g. selecting the step width δ of the binary search with values learned by experience instead of the smallest possible value $\delta=\Delta$)

ALGORITHM FOR STEP 4-MONOTONE RANGE OF COVERAGE PROFILES WITH STEP WIDTH Δ (BINARY SEARCH)

```
Initialize R(t):    ∀t:R(t)=0
                    t=t0; {Planning Start}
While t < planning horizon do begin
     {Induction Step}
     R(t+1)=R(t);
```

```
Generate min_cost_flow problem F(R) for profile
R(t)                                    {=step 2}

If min_cost_flow (F(R)) solvable

Then Begin                    {enlarge R(t+1)}
δ:=Δ;
Repeat               {binary search-enlarge δ}
     R(t+1):=R(t+1)+δ
     δ:=2*δ;
     Generate min_cost_flow problem F(R) for profile R(t)
Until min_cost_flow (F(R)) not solvable

Repeat               {binary search-lower δ}
     R(t+1):=R(t+1)+δ;
     δ:=δ/2;
     Generate min_cost_flow problem F(R) for profile R(t)
     If min_cost_flow (F(R)) solvable
          Then δ:=|δ|
          Else δ:=-|δ|
Until |δ|=Δ;
If min_cost_flow (F(R)) not solvable
   Then R(t+1):=R(t+1)-δ
End

Else                                    {lower R}
δ:=Δ;
Repeat               {binary search-enlarge δ}
     R^max:=R(t+1);
     δ:=2*δ;
     t_R=min(t'|R(t')≥R^max-δ)
     ∀t'≥t_R:R(t')=R^max-δ;
     Generate min_cost_flow problem F(R) for profile R(t)
Until min_cost_flow (F(R)) solvable
```

```
Repeat                {binary search-lower δ}
        R^max:=R(t+1);
        δ:=δ/2;
        t_R=min(t'/R(t')≥R^max + δ}
        ∀t'≥t_R:R(t')=R^max+ δ;
        Generate min_cost_flow problem F(R) for profile R(t)
        If min_cost_flow (F(R)) solvable

            Then δ:=/δ/
            Else δ:=-/δ/

Until /δ/ = Δ;
If min_cost_flow (F(R) not solvable
      Then ∀t'≥t_R: R(t') = R^max - δ;
End
```

2. BINARY SEARCH FOR ROLLING PLANNING SCHEME

The enhancement of the algorithm for the rolling planning scheme is similar to those of the preceding section. Since it is not necessary to consider the enlargement of the range of the coverage profile, it suffices to insert the augmentation of the second part of that algorithm ("lower R").

ALGORITHM FOR STEP 4 - ROLLING PLANNING SCHEME

Given the maximal transportation time t_{max}

Use binary search process to compute the maximal monotone range of coverage profile $R^{max}(t)$ within the planning window [planning__start, planning start+t_{max}]

Initialize R(t):

$\forall t \leq t_{max}$: $R(t)=R^{max}(t)$

$t>t_{max}$: $R(t)=R^{max}(t_{max})$;

$R^{max}:=R'^{max}(t_{max})$

Generate min_cost_flow problem F(R) for profile R(t) {=step2}

Repeat {binary search enlarge δ}

$R^{max}:=R(t_{max});$

$\delta:=2*\delta;$

$t_R=min\{t'/R(t')\geq R^{max}-\delta\}$

$\forall t'\geq t_R:R(t')=R^{max}-\delta;$

Generate min_cost_flow problem $F(R)$ for profile $R(t)$

Until min_cost flow $(F(R))$ solvable

Repeat {binary search stop width-lower δ}

$R^{max}:=R(t_{max});$

$\delta:=\delta/2;$

$t_R=min\{t'/R(t')\geq(R^{max}+\delta\}$

$\forall t'\geq t_R:R(t')=R^{max}+\delta;$

Generate min_cost_flow problem $F(R)$ for profile $R(t)$

Until min_cost flow $(F(R))$ solvable

Then $\delta:=/\delta/$

Else $\delta:+\_/\delta/$

Until $/\delta/=\Delta;$

If min_cost_flow $(F(R))$ not solvable

Then $\forall t\geq t_{max}$: $R(t): = R^{max}-\Delta$

End

3. A FAST ALGORITHM FOR A NON-MONOTONOUS RANGE OF COVERAGE PROFILE

We have already described an algorithm for the maximization of the non-monotonous range of coverage profiles (see section A above). This approach can be very slow, if we use a fine step width Δ. In the following we propose an algorithm using the algorithm for the monotone range of coverage profiles (see section B above) as basic building block. The basic idea is to solve the problem recursively:

1. Generate a maximal monotone range of coverage profile R^m.

2. Reduce the problem by fixing the parts of this profile which have to be the same for the maximal range of coverage profile *R*.

3. If not the whole profile is fixed, then go back to step 1 (and solve the remaining part).

For step 2 we have to select the time steps *t* where

$R^m(t) < R^m(t+1) \Rightarrow R^m(t) = R(t)$

i.e. the strongly monotone increasing parts of the maximal monotone range of coverage profile can be fixed. Moreover, the end of the planning horizon can be fixed also:

$R^m(planning\_end) = R(planning\_end)$

Therefore, we can reduce the planning window at least by 1 (possibly by more, when we can reduce also some fixed strongly monotone increasing parts of R^m). Thus it is guaranteed that the above loop will be repeated at most *T* steps where *T=planning_end-planning_start* (number of time steps in the planning window), i.e. the time complexity is bounded by $T*O(R^m)$ where $O(R^m)$ is the time complexity of the algorithm for the maximal monotone range of coverage profile. The algorithmic complexity increases advantageously at most by a factor *T* compared to monotone profiles, which also makes it possible at all to calculate extensive range of coverage profiles using traditional computers.

Claims:

1. A method for maximizing the range of coverage for coverage profiles using a computer program, including the following steps:

a. Connect to an online transaction processing system;

b. Read transaction data from the online transaction processing system; and

c. Enter the transaction data for the area of the coverage profile function.

d. Provide a range of coverage function for the coverage profile;

e. Initialize said range of coverage profile function with at least one starting value parameter;

f. Determine an optimal stock balance for starting the range of coverage function for the coverage profile. Calculation includes the following steps:

i. Use a minimum cost flow algorithm to identify an optimal transport solution to transport the necessary quantity of stock;

ii. Determine whether a solution has been found after the minimum cost flow algorithm for the area of the coverage profile function was used with the starting value;

iii. If no solution has been identified, incrementally reduce the start value and repeat the calculation of a solution until an optimal solution has been determined for the minimum cost flow algorithm;

g. If a solution has been identified, incrementally increase the start value and repeat the calculation of a solution until the calculation of a solution leads to the optimal solution for the minimum cost flow algorithm.

2 A computer program including several instructions to maximize the range of coverage of coverage profiles, where the various instructions contain instructions that, when carried out by a processor, ensure that the processor performs the following steps:

a. Connect to an online transaction processing system;

b. Read transaction data from the online transaction processing system;

c. Enter the transaction data in the range of coverage function for the coverage profile;

d. Initialize the range of coverage function with a starting value;

e. Calculate the stock balance required to start the range of coverage function for the coverage profile;

f. Calculate an optimal solution. Calculation includes the following steps:

i. Use a minimum cost flow algorithm to identify an optimal transport solution to transport the necessary quantity of stock;

ii. Determine whether a solution has been found after the minimum cost flow algorithm for the area of the coverage profile function was used with the starting value;

iii. If no solution has been identified, incrementally reduce the start value and repeat the calculation of a solution until an optimal solution for the minimum cost flow algorithm has been determined;

g. If a solution has been identified, incrementally increase the start value and repeat the calculation of a solution until the calculation of a solution leads to the optimal solution for the minimum cost flow algorithm.

3. A System for maximizing the range of coverage of coverage profiles, including:

a. A means to connect to an online transaction processing system;
b. A means to read transaction data from the online transaction processing system;
c. A means to enter the transaction data for the area of the coverage profile function;
d. A means to initialize the warehouse range of coverage function with a starting value;
e. A means to calculate the stock balance required to start the warehouse range of coverage for the coverage profile;
f. A means to calculate an optimal solution, including:
  i. A means to use a minimum cost flow algorithm to identify an optimal transport solution to transport the necessary quantity of stock;
  ii. A means to determine whether a solution has been found after the minimum cost flow algorithm for the area of the coverage profile function was used with the starting value;
  iii. A means to incrementally reduce the start value, if no solution has been identified, and repeat the calculation of a solution until an optimal solution for the minimum cost flow algorithm has been determined;

g. A means to incrementally increase the start value if a solution has been identified and repeat the calculation of a solution until the calculation of a solution results in the optimal solution for the minimum cost flow algorithm.

4. A computer-readable medium carrying a computer program according to claim 2.

5. A method for maximising the range of coverage for coverage profiles substantially as described herein with reference to the accompanying drawings.

6. A computer program to maximise the range of coverage profiles substantially as described herein with reference to the accompanying drawings.

7. A system for maximising the range of coverage of coverage profiles substantially as described herein with reference to the accompanying drawings.

Dated this 20th day of June 2003

SAP AKTIENGESELLSCHAFT

By their Patent Attorneys

GRIFFITH HACK

FIG.1

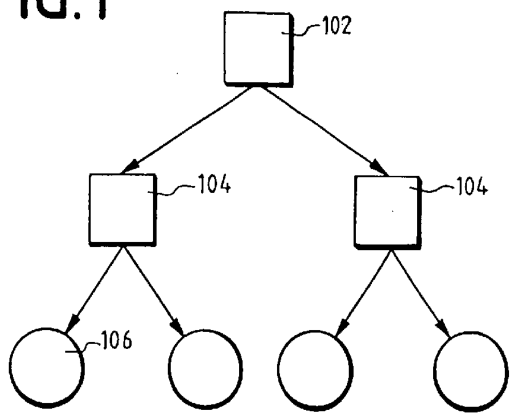


FIG.2

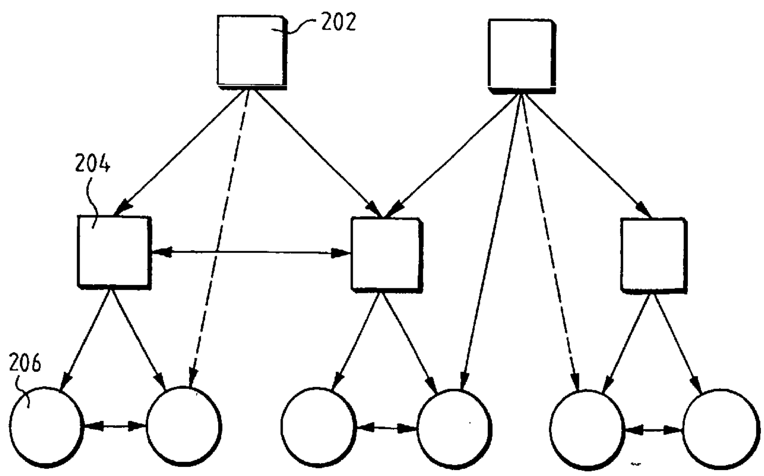


FIG.3

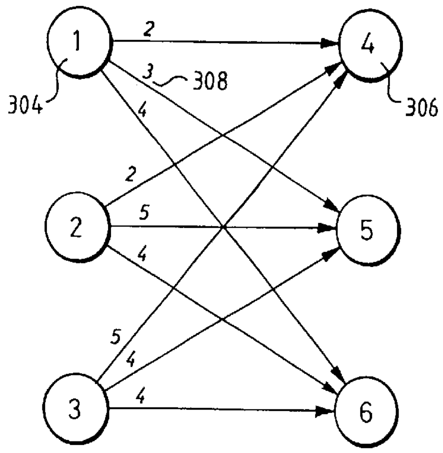


FIG.4

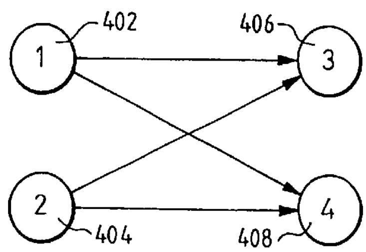


FIG.5

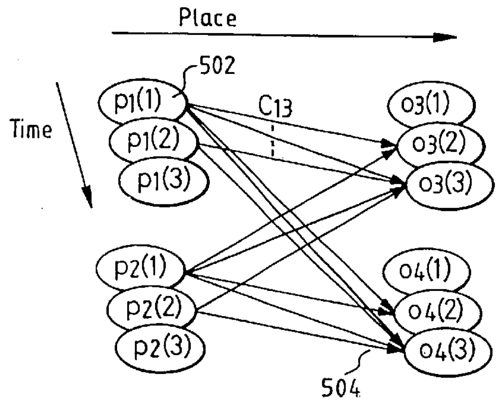

Place
Time
502
p1(1)
p1(2)
p1(3)
C13
o3(1)
o3(2)
o3(3)
p2(1)
p2(2)
p2(3)
o4(1)
o4(2)
o4(3)
504

FIG.6

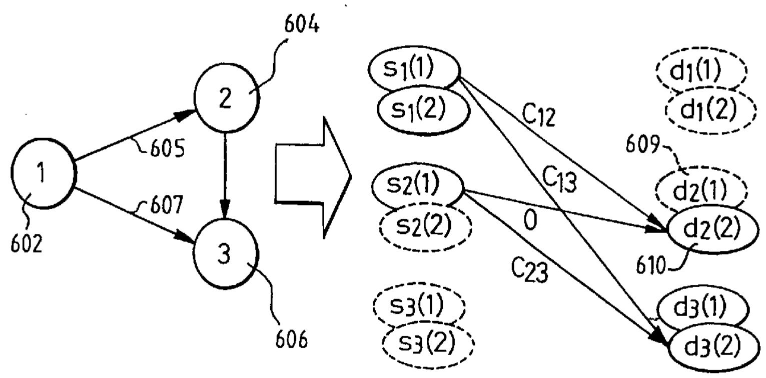

604
1
2
3
605
607
602
606
s1(1)
s1(2)
s2(1)
s2(2)
s3(1)
s3(2)
C12
C13
0
C23
d1(1)
d1(2)
609
d2(1)
d2(2)
610
d3(1)
d3(2)

FIG.7

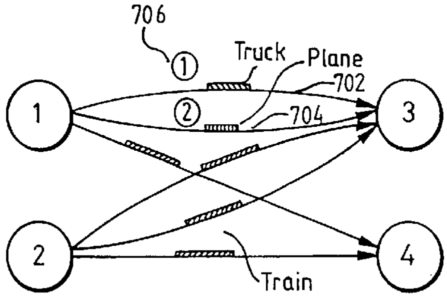

706
①
②
Truck
Plane
702
704
1
2
3
4
Train

FIG.8

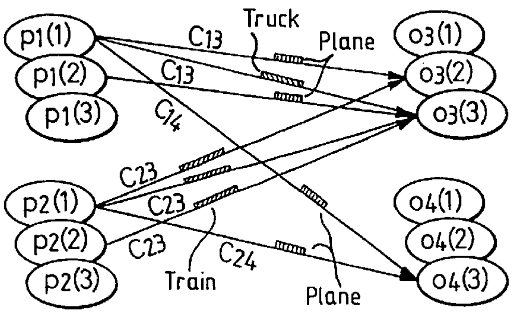

p1(1)
p1(2)
p1(3)
p2(1)
p2(2)
p2(3)
o3(1)
o3(2)
o3(3)
o4(1)
o4(2)
o4(3)
C13
C13
C14
C23
C23
C23
C24
Truck
Plane
Train
Plane

FIG.9

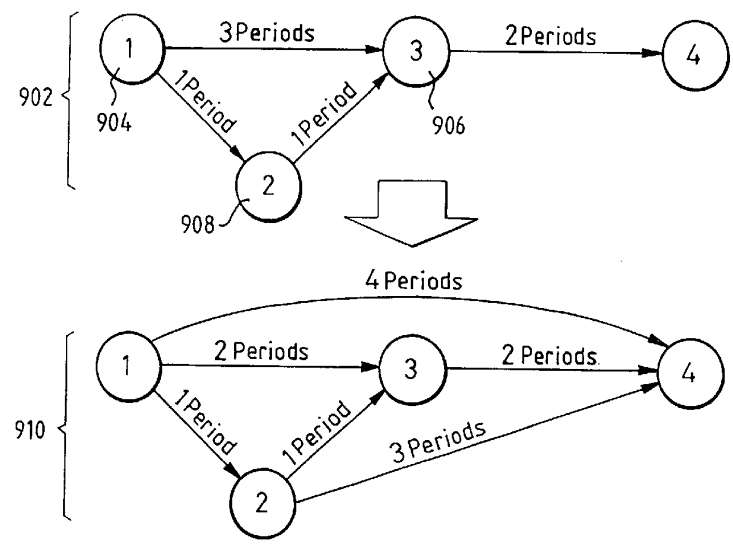

902
1
3 Periods
3
2 Periods
4
1 Period
1 Period
904
906
2
908
4 Periods
910
1
2 Periods
3
2 Periods
4
1 Period
1 Period
3 Periods
2

FIG.10

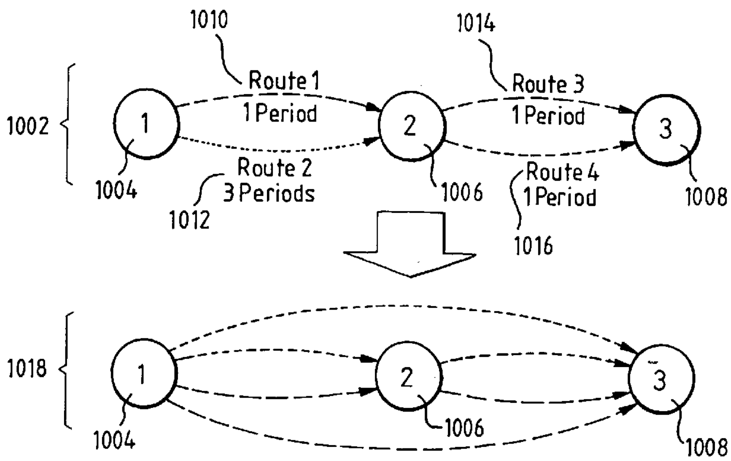

1010
1014
Route 1
1 Period
Route 3
1 Period
1002
1
2
3
Route 2
3 Periods
Route 4
1 Period
1004
1012
1006
1008
1016
1018
1
2
3
1004
1006
1008

FIG.11

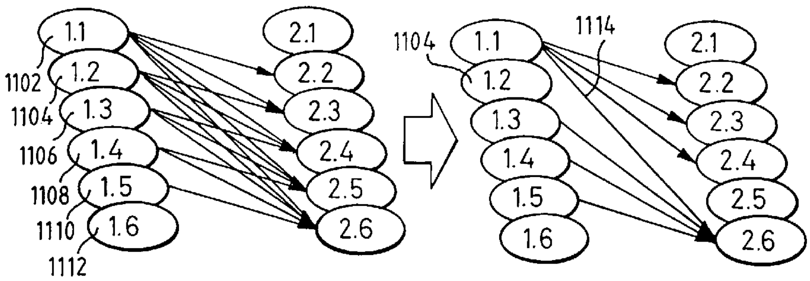

1.1
1.2
1.3
1.4
1.5
1.6
1102
1104
1106
1108
1110
1112
2.1
2.2
2.3
2.4
2.5
2.6
1114

FIG.12

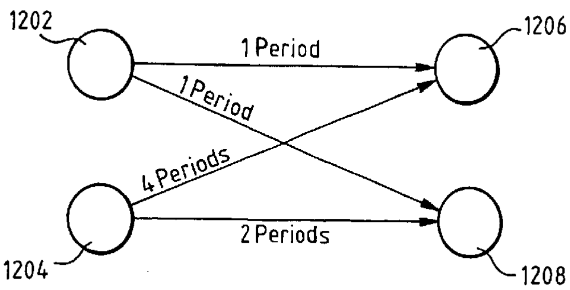

1202
1204
1206
1208
1 Period
1 Period
4 Periods
2 Periods

FIG.13

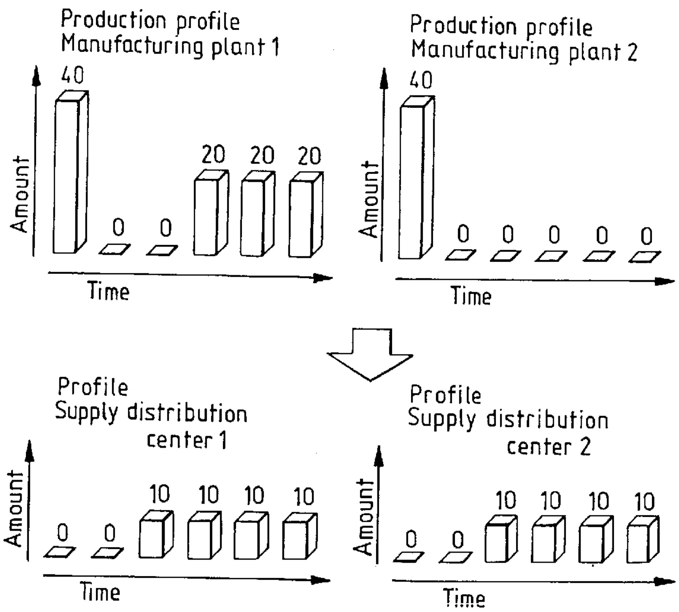

Production profile
Manufacturing plant 1
40
0
0
20
20
20
Amount
Time
Production profile
Manufacturing plant 2
40
0
0
0
0
0
Amount
Time
Profile
Supply distribution
center 1
0
0
10
10
10
10
Amount
Time
Profile
Supply distribution
center 2
0
0
10
10
10
10
Amount
Time

FIG.14

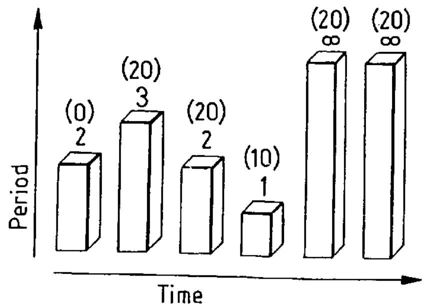

(0)
2
(20)
3
(20)
2
(10)
1
(20)
∞
(20)
∞
Period
Time

FIG.15

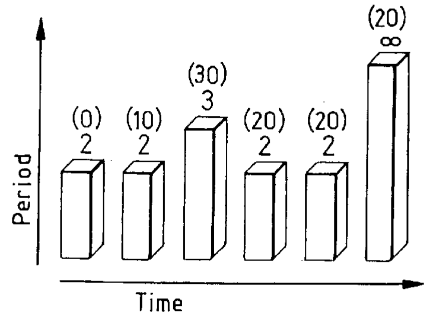


FIG.16

Period	1	2	3	4	5	6
M.P.	30	10	10	10	20	0
D.C.1	0	0	10	10	10	10
D.C.2	0	0	10	10	10	10
1604 Z^M	30 to DC1 0 to DC2	0 to DC1 10 to DC2	0 to DC1 10 to DC2	0 to DC1 10 to DC2	10 to DC1 10 to DC2	0 to DC1 0 to DC2
Z	15 to DC1 15 to DC2	5 to DC1 5 to DC2	5 to DC1 5 to DC2	5 to DC1 5 to DC2	10 to DC1 10 to DC2	0 to DC1 0 to DC2
1602 R^M	1	1	1	1	1	∞
1606 R'^M	2	1	1	1	1	∞
1608 R	2	2.5	2	1.5	1	∞

FIG.17

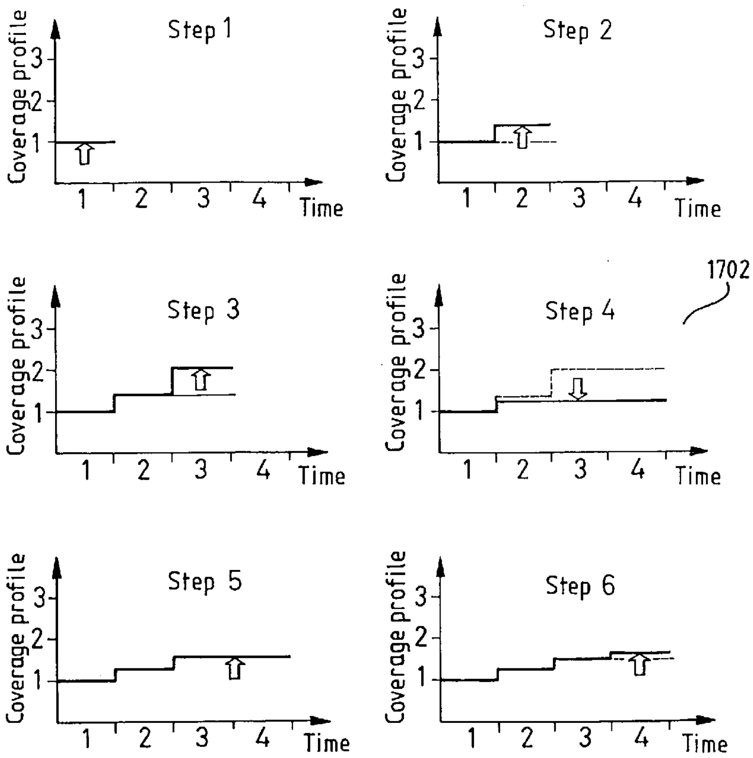

Step 1
Coverage profile
3
2
1
1
2
3
4
Time
Step 2
Step 3
Step 4
1702
Step 5
Step 6

FIG.18

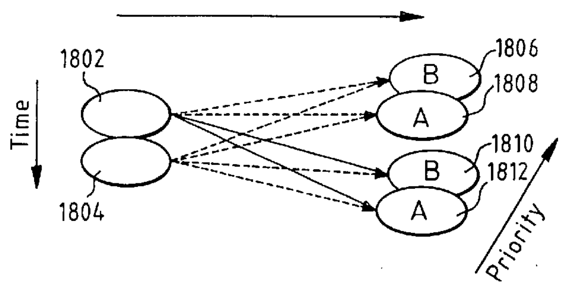
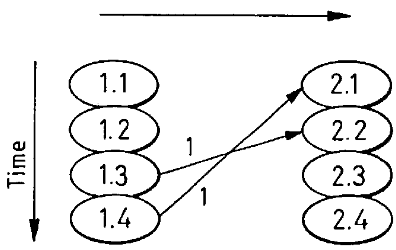

1802
1804
Time
B
A
B
A
1806
1808
1810
1812
Priority

FIG.19


Time
1.1
1.2
1.3
1.4
2.1
2.2
2.3
2.4
1
1