(12) INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(19) **World Intellectual Property Organization**
International Bureau

(43) **International Publication Date**
19 December 2024 (19.12.2024)

WIPO | PCT

(10) **International Publication Number**
WO 2024/255933 A1

(51) **International Patent Classification:**
G06Q 50/06 (2024.01) *H02J 3/00* (2006.01)
G06Q 10/04 (2023.01)

(21) **International Application Number:**
PCT/CZ2023/000027

(22) **International Filing Date:**
16 June 2023 (16.06.2023)

(25) **Filing Language:** English

(26) **Publication Language:** English

(71) **Applicant: ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE** [CZ/CZ]; Technická 2, 166 27 Praha (CZ).

(72) **Inventors: MAREČEK, Jakub**; Dr. Janského 432, 252 28 Černošice (CZ). **KUNGURTSEV, Vyacheslav**; Ve Smečkách 16, 11000 Praha 1 (CZ).

(74) **Agent: DANĚK, Vilém** et al.; Vinohradská 17, 120 00 Praha 2 (CZ).

(81) **Designated States** *(unless otherwise indicated, for every kind of national protection available)*: AE, AG, AL, AM, AO, AT, AU, AZ, BA, BB, BG, BH, BN, BR, BW, BY, BZ, CA, CH, CL, CN, CO, CR, CU, CV, CZ, DE, DJ, DK, DM, DO, DZ, EC, EE, EG, ES, FI, GB, GD, GE, GH, GM, GT, HN, HR, HU, ID, IL, IN, IQ, IR, IS, IT, JM, JO, JP, KE, KG, KH, KN, KP, KR, KW, KZ, LA, LC, LK, LR, LS, LU, LY, MA, MD, MG, MK, MN, MU, MW, MX, MY, MZ, NA, NG, NI, NO, NZ, OM, PA, PE, PG, PH, PL, PT, QA, RO, RS, RU, RW, SA, SC, SD, SE, SG, SK, SL, ST, SV, SY, TH, TJ, TM, TN, TR, TT, TZ, UA, UG, US, UZ, VC, VN, WS, ZA, ZM, ZW.

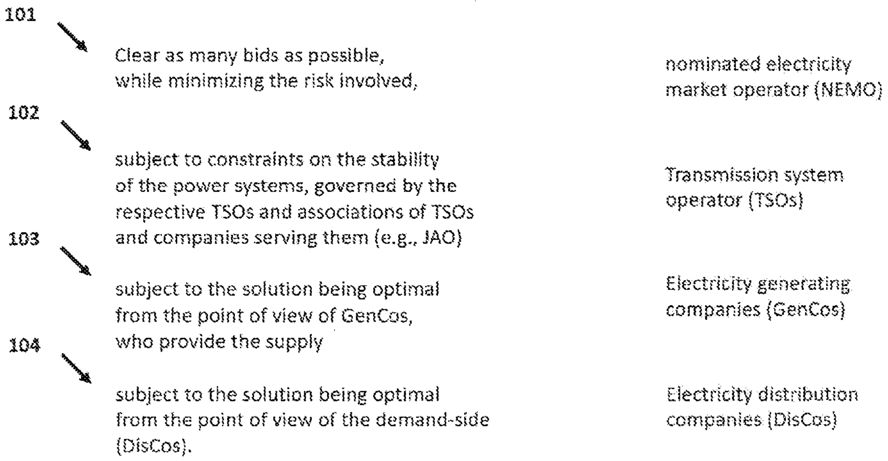
(84) **Designated States** *(unless otherwise indicated, for every kind of regional protection available)*: ARIPO (BW, CV, GH, GM, KE, LR, LS, MW, MZ, NA, RW, SC, SD, SL, ST, SZ, TZ, UG, ZM, ZW), Eurasian (AM, AZ, BY, KG, KZ, RU, TJ, TM), European (AL, AT, BE, BG, CH, CY, CZ, DE, DK, EE, ES, FI, FR, GB, GR, HR, HU, IE, IS, IT, LT, LU, LV, MC, ME, MK, MT, NL, NO, PL, PT, RO, RS, SE, SI, SK, SM, TR), OAPI (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, KM, ML, MR, NE, SN, TD, TG).

Declarations under Rule 4.17:
— *as to applicant's entitlement to apply for and be granted a patent (Rule 4.17(ii))*
— *of inventorship (Rule 4.17(iv))*

Published:
— *with international search report (Art. 21(3))*

(54) Title: BALANCING ELECTRIC POWER SYSTEMS USING MULTILEVEL OPTIMIZATION

Fig. 1



(57) **Abstract:** Present invention relates to a method of balancing electric power systems using multilevel optimization, wherein the method employs steps which shall primarily result in channeling power flows by clearing electricity markets, i.e., matching bids, close to real time. The method comprises following step of using a robust bi-level formulation, along with using a semidefinite-programming relaxation of the polynomial optimization problem to capture transmission constraints at the transmission-system operator (market operator) level and using extended formulation of the market-participant level to optimize run-time of the operation to effectively convexify the market-participant level.

WO 2024/255933 A1

Title: Balancing electric power systems using multilevel optimization

TECHNICAL FIELD

[0001] The invention relates to a method of balancing electric power systems using multilevel optimization. The method employs steps which shall primarily result in channeling power flows by clearing electricity markets, i.e., matching bids, close to real time. Furthermore, the invention relates to a device adapted to execute this method.

BACKGROUND ART

[0002] The problem of transmission system stability is one of the key aspects of reliable energy supply and the associated stability not only in key industries, but also to ensure the basic needs of all entities dependent on electricity supply.

[0003] There are many considerations, particularly with regard to the control of consumption points and the prediction of consumption in relation to trends in customer behavior over time, but these considerations can only be relevant if we are able to capture these trends in a sufficiently revealing quality and, in particular, in the rapidly changing market conditions in terms of newly installed energy sources caused, for example, by the shift away from fossil fuels on the one hand and by the rationalization of electricity production and other energy saving measures on the part of energy consumers on the other.

[0004] The introduction of further non-dispatchable renewable energy sources into the energy mix is challenging in terms of stabilization of the power systems, as it is not possible to predict the conditions for energy production in the short and sometimes also in the medium- and long-term. For example, we can mention the unclear predictions caused by weather conditions for wind power plants in the short term, or the amount of rainfall needed to maintain the required water level for cooling nuclear power plant units. The overproduction of electricity from renewable sources creates the need to store energy in corresponding reservoirs, such as batteries and/or dedicated storage facilities, as not all the energy can be used immediately. However, creating a sufficiently robust storage network poses significant financial and logistical challenges. Conversely, in the event of limited generation from renewables, it is usually necessary to bring in back-up sources of generation, usually using fossil fuels such as gas or coal.

[0005] For a number of reasons, deregulated electricity markets [12] are volatile [9]. There are a number of factors at play: intermittent renewable generation is inherently hard to control and traditional generation faces strict production constraints. Demand exhibits very limited [10] price-responsiveness (limited to demand-response management, which has numerous issues on its own) and is difficult to forecast. There are prohibitive costs associated with energy storage. There are highly non-convex transmission losses [17] involved in electricity transmission Mechanisms in place for balancing the market, especially in Europe [1, 34] are often fairly complex and convoluted, which influence the incentives of market participants.

[0006] Finally, markets for electricity are fragmented and the trading period (imbalance settlement period) is often rather long.

[0007] The present invention takes a more rational approach to balancing the power grid and focuses primarily on directing power flows where they are actually required, i.e., on demand. The advantage of this invention is that it can be implemented immediately without incurring financial costs significantly higher than those of shortening the trading interval. The use of the method according to the present invention optimizes the use of resources and, above all, leads to the stabilization of the energy network, since it does not create conditions for overproduction and can direct energy flows where they are actually required.

[0008] It is widely understood [24, 23] that the costs of balancing the market via auxilliary services grow sharply with the duration of the trading interval. Regulations, such as the Regulation (EU) 2019/943, hence suggest shortening the trading interval “functioning of the intraday market to provide the possibility for market participants to balance themselves as closely as possible to real time". The current trading interval of 1 hour is set to decrease to 15 minutes throughout Europe, for instance. While this has significant potential in improving efficiency in theory, the mechanisms to realize efficient market clearing in practice are not available, thus the capacity for moving much closer to real time remains elusive.

[0009] Mathematically, the process of market balance in power systems is aided by the modeling of the process as an equilibrium problem, based on standard economic models of market clearing, see, e.g., the classic [16]. A model is formed using all participants demands, behavior, and the availability of the generation at a given moment in time, and solved for its optimal operation while satisfying physical constraints and demand. At a lower level, one may wish to model the such market clearing [20, 36, 5, 4, e.g.] as a bi-level optimization problem. Often, the lower-level models market participants (DSOs and GenCos) and the upper-level models electricity market operator (NEMO) or a transmission system operator (TSO), or the other way round. In many cases, the models governing the structure of the power flow can be faithfully given as systems of polynomials, resulting in a polynomial optimization problem [17] which, for tractability, is convexified using semidefinite programming [27].

[0010] These techniques are unworkable in the contemporary context of rising volatility and uncertainty in power production, together with the inherent unresponsiveness as well as fundamental uncertainty in the forecasting of demand. In particular, obtaining a solution to a static equilibrium problem is the wrong tool for the job. With the shortening of the trading interval, methods that inherently consider power systems dynamics, consider equilibrium as a moving time-dependent (and as such, largely a theoretical) entity, and incorporate robustness within its modeling and operation are necessary in the next generation of technical operation of power system distribution.

Challenges

[0011] There are three complications to implementing market–clearing mechanisms in trading electric power. First stems from the fact that the market participant's level is non-convex. The second stems from the fact that considering the transmission constraints is non-convex. Finally, the third stems from the fact that a multi-level problem with convex problems at each level may be non-convex.

a. First, there are various combinatorial structures at the market participant's level. Notably, economies of scale, start-up and/or shut-down costs, indivisibilities or minimum bid requirements, as well as the need to supply bids in combinatorial auctions make the problem non-convex [38, 31, 22, 11, 35, 28].

b. Second, it is understood [30, 18, 7, 6] that the TSO's problem needs to consider transmission constraints. The transmission constraints [29] can be formulated as multi-variate polynomial constraints [17], for which in isolation there are globally convergent methods, but for the combination of polynomial constraints in one level and the combinatorial constraints in the other level of a bi-level problem, only heuristic methods [18, 7, 8, 6, 42, 37, 26] are available. In particular, it was not known whether the solutions are globally optimal for the bi-level problem with polynomial constraints and integer decision variables. (We note that many studies [31, 42, 26, e.g.] focus on the location-marginal prices with side-payments known as "uplifts", which are common in the USA, but less so in Europe.)

c. Third, it is understood [16] that even for a bilevel optimization problem with a linear programming problem at each level (i.e., convex problems at each level), the overall bilevel problem is non-convex without further assumptions.

[0012] This invention provides the first methods for the problem, considering both of the challenges above, for three different ways of modelling power-systems dynamics and correspondingly the transmission constraints. Additionally, formal guarantees of global convergence could be derived.

[0013] The closest piece of related work in the academic literature is [40], which does not consider the robustness of the market clearing, considers only a basic model of swing dynamics at the TSO level, assumes convexity at the market-participant level, which is not realistic. Furthermore, it guarantees only local convergence to an arbitrarily bad solution.

[0014] Among the patent documents that show some relevance in relation to the subject invention, we can mention for example document US8977524B2, which relates to a method for approximating an optimal powerflow of a smart electric power grid, which includes providing a cost function that models a smart electric power grid having buses connected by branches, deriving a set of linear equations that minimize the cost function subject to constraints from an expression of an extremum of the cost function with respect to all arguments, reducing a dimension of the linear equations by solving for a subset of the linear equations, re-organizing the reduced dimension linear equations into primal and dual parts, and decomposing the re-organized reduced dimensional linear equations into two systems of block matrix equations which can be solved by a series of back substitutions. This is inferior to the proposed invention, as it does not consider the combinatorial nature of bids and the need to clear markets using those.

[0015] Further document US6775597B1 relates to a Security-Constrained Optimal Power Flow (SCOPF) process employing a quadratic programming (QP) primal-dual interior point (IP) solution method. The IP method efficiently solves practical SCOPF problems involving large numbers of contingencies and controls in preventive and preventive/corrective operating modes. An EMS system is described incorporating the inventive SCOPF process. This is inferior to the proposed invention, as it does not consider the combinatorial nature of bids and the need to clear markets using those.

[0016] Another document US9953117B2 relates to a method for solving a two-stage non-linear stochastic formulation for the economic dispatch problem under renewable-generation uncertainty. Certain generation decisions are made only in the first stage and fixed for the subsequent (second) stage, where the actual renewable generation is realized. The uncertainty in renewable output is captured by a finite number of scenarios.

Any resulting supply-demand mis-match must then be alleviated using high marginal-cost power sources that can be tapped in short time frames. The solution implements two outer approximation algorithms to solve this nonconvex optimization problem to optimality including the application of a decomposition approach derived from the Alternating Direction Method of Multipliers (ADMM) algorithm. This is inferior to the proposed invention, as it does not consider the combinatorial nature of bids and the need to clear markets using those.

[0017] The document US9912153B2 relates to a method for controlling the ratio between injected and extracted electric energy in an electric energy supply grid with a number of grid participants, which are selected from a group including producers, consumers, and storage devices, with at least two of the group being included. A grid state variable is used as a control variable, the value of said variable depending on the ratio between inserted and extracted electric energy and being ascertainable from the grid by the grid participants. The invention is characterized by a number of grid participants ascertain the grid state variable from the grid and use said variable at least indirectly to control the grid in a decentralized manner based on a respective specific grid participant behavior. This is inferior to the proposed invention, as it does not consider non-convexities of the problem, such as the combinatorial nature of bids and non-convex nature of AC transmission constraints.

[0018] Finally, we can also mention the document US10317970B2, which relates to distributed optimal power flow processes for unbalanced radial distribution networks. One embodiment includes a node controller including a distributed power control application; a plurality of node operating parameters describing the operating parameter of a node in an unbalanced network; wherein the processor is configured by the distributed power control application to: send node operating parameters to nodes in the set of at least one node; receive operating parameters from the nodes in the set of at least one node; calculate a plurality of updated node operating parameters using an iterative process to determine updated node operating parameters using the node operating parameters that describe the operating parameters of the node, and the operating parameters of the set of at least one node, where each iteration in the iterative process involves evaluation of a subproblem; and adjust node operating parameters. This is inferior to the proposed invention, as it is restricted to radial feeders, and does not consider market clearing.

SUMMARY OF THE INVENTION

[0019] Present invention relates to a method of balancing electric power systems using multilevel optimization, wherein the method employs three main components in each of three embodiments to address the challenges represented by the complex problem of balancing the grid by clearing markets near real time.

Embodiment 1

[0020] In one embodiment of the present invention, the method comprises following steps:

a. using a robust bi-level formulation [14], which extends previous work on bi-level polynomial optimization [21]; this makes it possible to consider both sides of the market as a bi-level optimization problem [16], while additionally considering the worst-case realization of certain random variables (e.g., production limits and demand) within some uncertainty sets around their nominal values. This makes it possible to produce solutions robust against all realizations of uncertainty within the uncertainty sets. Technically, this adds one or two more levels to the multi-level optimization problem.

b. using a semidefinite-programming relaxation of the polynomial optimization problem [17] to capture transmission constraints at the transmission-system operator (market operator) level. In general, modelling the transmission constraints makes it possible to obtain market clearing that is feasible with respect to a given model of the power systems dynamics. This improves over the current trial-and-error approach, wherein the market is cleared, the feasibility is simulated by a simulator of power-systems dynamics, and if not feasible, the market clearing needs to be adjusted. In particular, the use of the semidefinite programming allows for a very efficient consideration of the transmission constraints in the alternating-current model.

c. using a convexification of the market-participant level to allow for efficient execution. In general, this reformulates non-convex problem at the market-participant level to allow for efficient optimization methods. In particular, the option for the convexification considered in this embodiment is the use of the so-called extended formulation [32, 25]. The so-called extended formulation is an equivalent, convex higher-dimensional formulation of the non-convex problem capturing the operational constraints of the market participant (e.g., active power output within a range, ramping constraints in the so-called unit commitment problem). While the higher-dimensional formulation may have an exponentially higher number of variables or inequalities, compared to the traditional "compact" formulations (e.g., of unit commitment), these have recently been shown [25, Table 5] to be competitive with the best compact formulations in terms of run-time.

Embodiment 2

[0021] In an advantageous embodiment of present invention, one replaces the convexification of step c of [21] with the so-called semidefinite programming (SDP) relaxation [3]. Thus, the method steps a and b of the embodiment 1 are employed and the method step c is replaced with the following wording of step c so that the method according to embodiment 2 comprises steps:

a. using a robust bi-level formulation [14], which extends previous work on bi-level polynomial optimization [21]. The advantages are the same as in previous embodiment.

b. using a semidefinite-programming relaxation of the polynomial optimization problem [17] to capture transmission constraints at the transmission-system operator (market operator) level. The advantages are the same as in previous embodiment.

c. using the so-called semidefinite programming (SDP) relaxation [3] of the market-participant level as the convexification of the market-participant level to ensure tractability, as in the availability of solutions with a reasonable runtime. The feasible set of the SDP relaxation is still semi-algebraic, thus the methods in the robust bi-level formulation in [14] still apply, but now lend themselves to being achievable with under reasonable demands for computing power. This, again, can be seen as convexification at the market-participant level.

This is advantageous in that the semidefinite program may be easier to solve than the extended formulation of Embodiment 1.

Embodiment 3

[0022] In an advantageous embodiment of present invention, we shall now consider the *power-system dynamics* i.e., time-varying process, from first principles, rather than seeking a static equilibrium market clearing solution. (See par. 0031 for details.) This changes the inherent methodology to be consistent with the rising level of volatility relative to the trading interval. To this end, we apply two mathematical tools in development for proper, robust and efficient operation.

The method according to this embodiment advantageously comprises any of following steps, and preferably all those steps, wherein:

a. Aside of evaluating a present state of uncertainty as in Embodiments 1 and 2, the method employs defining the projected uncertainty moving forward in time, preferably followed by cross verifying the uncertainty between the nonconvex polynomial formulation using ellipsoidal tube methods as used in robust model predictive control [19] and SDP techniques [13] tailored specifically to polynomial systems [41].

b. Instead of considering transmission constraints at a specific point in time using a specific nominal values or using specific uncertainty sets by solving a static SDP problems relaxing the polynomial optimization as in Embodiments 1 and 2, the method employs using a trajectory of states of the dynamics, by incorporating time and the predicted operation in the future by solving optimal control problems using time-dependent SDPs. Time-dependent SDPs or time-varying polynomial optimization problems and SDPs were introduced formally in [2], and novel algorithms that carefully use solution estimates at a current time to quickly obtain ones at the next time interval are in extensive development.

c. Finally, the method employs step of combining the continuous time scale and discrete time scales and analyzing stability of the related systems [33]. This is advantageous in that the solvers for time-varying semidefinite program may provide guarantees as to the ability to track the trajectory of optimal solutions.

[0023] Furthermore, the invention relates to a device for balancing electric power systems using multi-level optimization, comprising a network interface, a memory containing a plurality of operating parameters describing the dynamics of an electric power system; and a plurality of participant operating parameters describing operating parameters for a set of at least one participant selected from the group consisting of producers and consumers and storage providers; a processor, configured to: receive operating parameters from the participants; upon receipt of operating parameters, calculating a plurality of updated operating parameters using an iterative process to determine the updated operating parameters as values of an optimizer using other operating parameters as coefficients in a multi-level optimization problems; and immediately upon calculating, sending participant operating parameters based on the calculated plurality of updated operating parameters.

[0024] Advantageously, the participants comprise of operators of generators (genco). Each generator may have its operational constraints described, such as outages planned and unplanned, the lower and upper limits on active and reactive power outputs, which may be time-varying, and limits on so-called ramping, i.e., the rate at which the active and reactive power outputs may change.

[0025] Advantageously, the operating parameters received from the participants comprise of distributional forecasts of production and bids, which have not been matched so far. Each distributional forecast of production may be a sequence of bi-variate distributions on the support given by the lower and upper limits on active and reactive power outputs, indexed by time steps in the future, which suggests the probability of producing a particular active and reactive power at a given time. Each distributional forecast of bids may be a sequence of bi-variate distributions on the support given by the lower and upper limits on active and reactive power demand, indexed by time steps in the future, which suggests the probability of demanding a particular active and reactive power at a given time.

[0026] Advantageously, the updated operating parameters comprise of information on the bids, which have been cleared. For supply-side bids, these include the active and reactive power outputs. For demand-side bids, these include the active and reactive power demand. These quantities may be suggested by their nominal values, while knowing that there is uncertainty as suggested by the distributional forecasts of [25].

[0027] Advantageously, the updated operating parameters are obtained using multi-level optimization with robustness properties. The robust multi-level optimization may extend the robust bi-level formulation of [14].

REFERENCES CITED IN THE DESCRIPTION

[0028] Non-patent literature cited in the description:

- [1] Commission regulation (eu) 2017/2195 of 23 november 2017 establishing a guideline on electricity balancing. Official Journal of the European Union, L 312/6, 28.11.2017.
- [2] A. A. Ahmadi and B. E. Khadir. Time-varying semidefinite programs. arXiv preprint rXiv:1808.03994, 2018.
- [3] M. Ashraphijuo S. Fattahi J. Lavaei and A. Atamtürk. A strong semidefinite programming relaxation of the unit commitment problem. In 2016 IEEE 55th Conference on Decision and Control (CDC), pages 694--701, 2016.
- [4] D. Aussel, P. Bendotti, and M. Pištěk. Nash equilibrium in a pay-as-bid electricity market: Part 1--existence and characterization. Optimization, 66(6):1013--1025, 2017.
- [5] D. Aussel, P. Bendotti, and M. Pištěk. Nash equilibrium in a pay-as-bid electricity market part 2-best response of a producer. Optimization, 66(6):1027--1053, 2017.

- [6] D. Aussel, M. Cervinka, and M. Marechal. Deregulated electricity markets with thermal losses and production bounds: models and optimality conditions. RAIRO-Operations research, 50(1):19--38, 2016.
- [7] D. Aussel, R. Correa, and M. Marechal. Electricity spot market with transmission losses. Journal of Industrial & Management Optimization, 9(2):275, 2013.
- [8] M. Bjørndal and K. Jörnsten. Equilibrium prices supported by dual price functions in markets with non-convexities. European Journal of Operational Research, 190(3):768--789, 2008.
- [9] S. Borenstein. The trouble with electricity markets: understanding california's restructuring disaster. Journal of economic perspectives, 16(1):191--211, 2002.
- [10] S. Borenstein. To what electricity price do consumers respond? residential demand elasticity under increasing-block pricing. Preliminary Draft April, 30:95, 2009.
- [11] S. Borenstein. The private and public economics of renewable electricity generation. Journal of Economic Perspectives, 26(1):67--92, 2012.
- [12] S. Borenstein and J. Bushnell. The us electricity industry after 20 years of restructuring. Annu. Rev. Econ., 7(1):437--463, 2015.
- [13] H. Choi P. J. Seiler and S. V. Dhople. Propagating uncertainty in power-system dae models with semidefinite programming. IEEE Transactions on Power Systems, 32(4):3146--3156, 2017.
- [14] T. D. Chuong and V. Jeyakumar. Finding robust global optimal values of bilevel polynomial programs with uncertain linear constraints. Journal of Optimization Theory and Applications, 173(2):683--703, 2017.
- [15] C. De Persis and S. Grammatico. Continuous-time integral dynamics for a class of aggregative games with coupling constraints. IEEE Transactions on Automatic Control, 65(5):2171--2176, 2019.
- [16] S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, and C. Ruiz. Complementarity modeling in energy markets, volume 180. Springer Science \& Business Media, 2012.
- [17] B. Ghaddar, J. Marecek, and M. Mevissen. Optimal power flow as a polynomial optimization problem. IEEE Transactions on Power Systems, 31(1):539--546, 2015.

- [18] R. Henrion, J. Outrata, and T. Surowiec. Analysis of m-stationary points to an epec modeling oligopolistic competition in an electricity spot market. ESAIM: Control, Optimisation and Calculus of Variations, 18(2):295--317, 2012.
- [19] B. Houska and M. E. Villanueva. Robust optimization for mpc. In Handbook of Model Predictive Control, pages 413--443. Springer, 2019.
- [20] X. Hu and D. Ralph. Using epecs to model bilevel games in restructured electricity markets with locational prices. Operations research, 55(5):809--827, 2007.
- [21] V. Jeyakumar J. B. Lasserre G. Li, and T. S. Pham. Convergent Semidefinite Programming Relaxations for Global Bilevel Polynomial Optimization Problems. SIAM J. Optimization, 2018. Also, arXiv:1506.02099.
- [22] P. Joskow and J. Tirole. Reliability and competitive electricity markets. The Rand Journal of Economics, 38(1):60--84, 2007.
- [23] T. Kerci, J. Giraldo, and F. Milano. Analysis of the impact of sub-hourly unit commitment on power system dynamics. International Journal of Electrical Power & Energy Systems, 119:105819, 2020.
- [24] T. Kerci and F. Milano. Sensitivity analysis of the interaction between power system dynamics and unit commitment. In 2019 IEEE Milan PowerTech, pages 1--6, 2019.
- [25] B. Knueven, J. Ostrowski, and J.-P. Watson. A novel matching formulation for startup costs in unit commitment. Mathematical Programming Computation, pages 1--24, 2020.
- [26] X. Kuang, A. J. Lamadrid, and L. F. Zuluaga. Pricing in non-convex markets with quadratic deliverability costs. Energy Economics 80:123--131, 2019.
- [27] J.-B. Lasserre. Moments, positive polynomials and their applications, volume 1. World Scientific, 2010.
- [28] G. Liberopoulos and P. Andrianesis. Critical review of pricing schemes in markets with non-convex costs. Operations Research, 64(1):17--31, 2016.
- [29] J. D. Molina and H. Rudnick. Transmission of electric energy: A bibliographic review. IEEE Latin America Transactions, 8(3):245--258, 2010.

- [30] A. L. Motto, F. D. Galiana, A. J. Conejo, and M. Huneault. On walrasian equilibrium for pool-based electricity markets. IEEE Transactions on Power Systems, 17(3):774--781, 2002.
- [31] R. P. O'Neill, P. M. Sotkiewicz, B. F. Hobbs, M. H. Rothkopf, and W. R. Stewart Jr. Efficient market-clearing prices in markets with nonconvexities. European journal of operational research, 164(1):269--285, 2005.
- [32] Y. Pochet and L. A. Wolsey. Production planning by mixed integer programming. Springer Science & Business Media, 2006.
- [33] C. Pötzsche, S. Siegmund, and F. Wirth. A spectral characterization of exponential stability for linear time-invariant systems on time scales. 2001.
- [34] F. Röben. Comparison of european power balancing markets-barriers to integration. In 2018 15th International Conference on the European Energy Market (EEM), pages 1--6. IEEE, 2018.
- [35] C. Ruiz, A. J. Conejo, and S. A. Gabriel. Pricing non-convexities in an electricity pool. IEEE Transactions on Power Systems, 27(3):1334--1342, 2012.
- [36] C. Ruiz, A. J. Conejo, and Y. Smeers. Equilibria in an oligopolistic electricity pool with stepwise offer curves. IEEE Transactions on Power Systems, 27(2):752--761, 2011.
- [37] H. Shavandi, M. Pirnia, and J. D. Fuller. Extended opportunity cost model to find near equilibrium electricity prices under non-convexities. Applied Energy, 240:251--264, 2019.
- [38] R. M. Starr. Quasi-equilibria in markets with non-convex preferences. Econometrica: journal of the Econometric Society, pages 25--38, 1969.
- [39] T. Stegink, A. Cherukuri, C. De Persis, A. van der Schaft, and J. Cortés. Frequency-driven market mechanisms for optimal dispatch in power networks. arXiv preprint rXiv:1801.00137, 2017.
- [40] T. Stegink A. Cherukuri C. De Persis A. Van Der Schaft and J. Cortes. Stable interconnection of continuous-time price-bidding mechanisms with power network dynamics. In 2018 Power Systems Computation Conference (PSCC), pages 1--6, 2018.

- [41] B. Xue, M. Fränzle, and N. Zhan. Inner-approximating reachable sets for polynomial systems with time-varying uncertainties. IEEE Transactions on Automatic Control, 65(4):1468--1483, 2019.
- [42] I. Zoltowska. Demand shifting bids in energy auction with non-convexities and transmission con straints. Energy Economics, 53:17--27, 2016.

BRIEF DESCRIPTION OF THE DRAWINGS

[0029] Figure 1 presents a high-level overview of the multi-level optimization view of the problem of clearing a market for electricity. The nominated electricity market operator clears the market, while clearing as many bids as possible, minimizing the risks involved, or both.

[0030] Figure 2 presents two examples of the constraints that model power systems dynamics, which underlies the stability of the power system in a transmission system, as imposed by the transmission system operator. This refers to advantageous step of defining the projected uncertainty moving forward in time as described in Embodiment 3. On the left (**201**), it presents an overview of a rather detailed model power systems dynamics model, which considers: (a) inertia of the rotating machinery, where the kinetic energy of the rotating mass is proportional to its moment of inertia and the square of its angular velocity, or more sophisticated ones; (b) the operations of local feedback controller of the rotating machinery, known as governors or droop controllers. These act on the steam valve of the turbine, for instance. Their gain is known as the "droop characteristic", and can be seen as the fraction of capacity that the governor deploys in steady state in response to a given frequency deviation. (c) alternating-current model of the power flows. On the right (**202-203**) it presents an easier model, restricting the problem to stationary states of the alternating-current model of the power flows. Details are provided in the following paragraphs.

[0031] In the model of **201**, which refers to advantageous step of defining the projected uncertainty moving forward in time as described in Embodiment 3, we consider: (a) generators modelled by differential equations of the two-axis synchronous generator model and stator equations. (b) differential equations for IEEE Type DC-1 exciter associated with each generator. (c) alternating-current equations for the power flow. The usual notation includes: S_G is the set of generator buses, \delta_i is the rotor angle at generator i \in S_G, \omega_i is the rotor speed at generator i \in S_G, T_{Mi} is the mechanical power output of generator i \in S_G , M_i is the inertia constant of generator i \in S_G, D_i is the damping torque coefficient of generator i \in S_G, E_{fdi} is the excitation output voltage, V_i is the bus i voltage magnitude, V_{Ri} is the voltage regulator output, V_{refi} is the reference voltage, R_{Fi} is the exciter rate feedback, S_{E}(E_{fdi}) is the field saturation function A_{ei}^{B_{ei}E_{fdi}}, K_{Ei} is the exciter gain, K_{Ai} is the voltage regulator gain, K_{Fi} is the rate feedback gain, T_{Ei} is the exciter time constant, T_{Ai} is the voltage regulator time constant, T_{Fi} is the rate feedback time constant, \theta_i is the bus voltage phase angle, R_{si} is the armature resistance, E_{di}' is the d-axis component of the internal voltage, E_{qi}' is the q-axis component of the internal voltage, I_{di}' is the d-axis component of the internal current, I_{qi}' is the q-axis component of the internal current, X_{di} is the d-axis component of synchronous reactance, X_{qi} is the q-axis component of synchronous reactance, X_{di}' is the d-axis component of transient reactance, X_{qi}' is the q-axis component of transient reactance, T_{d0i}' is the d-axis component of open-circuit time constant, T_{q0i}' is the q-axis component of open-circuit time constant, S_L is the set of non-generator buses, S_B is the set of all buses, S_G \cup S_L, Y \in \mathbb{C}^{|N|\times|N|} is the network admittance matrix, P_{Li} is the active load (demand) at bus i \in S_L, Q_{Li} is the reactive load (demand) at bus i \in S_L.

[0032] In the model of **202-203**, which refers to advantageous step of defining the projected uncertainty moving forward in time as described in Embodiment 3, we consider stationary states of the alternating-current (AC) model of the power flows. Similar to the literature [17], we focus on the steady-states of the power flows in the so-called rectangular voltage model. There, variable is concatenating the real components of voltages with a vector of imaginary component of voltages. **202** suggests that this simplifies the model to constraints on the active and reactive powers and voltage bounds and a constraint on the apparent power (i.e., current). **203** elaborates on the construction of the the network admittance matrix $Y \in \mathbb{C}^{|N|\times|N|}$ in this model. We note that e_k is the kth standard basis vector in $R^{|N|}$.

[0033] Let us remark on Figure 2 that even more general models are possible than the model of Figure 2. There are many more complications: (a) there are a variety of models of rotating machinery, ranging from the Swing Equation through two-axis models to the full model (IEEE 2.2). (b) there are a variety of power electronics. Notably, automatic voltage regulators (AVR) are local feedback control systems that measure the output voltage of a generator, compare it to a set point, and adjust the excitation of the generator using the error signal. (c) The boost to the excitation voltage following disturbance should prevent generator internal voltage from collapsing. Governors have a programable frequency dead band (of the order of tens of mHz). Consequently, there are many models of power systems dynamics possible.

[0034] Figure 3 presents a brief overview of multi-level optimization. In **301**, the problem on N levels is stated succinctly. In **302**, the problem description is expanded. In both cases, we consider convex continuous functions f_i and convex, but not necessarily smooth functions g_i.

[0035] Figure 4 presents a brief overview of the so-called proximal gradient approach to multi-level optimization. In **401**, we repeat the description of the multi-level problem. In **402**, we present the main iteration of the algorithm. In **403**, we fill in the definition of T_{i} missing from 402, which in turn requires the definition of the so-called prox operator. In **404**, we present the conditions on the step sizes α_k that need to be satisfied.

[0036] Figure 5 presents an overview of the data received by the balancing mechanism. TSO 1 wishes to set apparent power limits and voltage magnitude bounds. GenCos 1 and 2 and DisCo 1 and 2 submit their bids.

DETAILED DESCRIPTION OF THE INVENTION

[0037] The present invention is further described by the following examples, which should not be construed as limiting the scope of the invention.

[0038] In Example 1, we consider Embodiment 1 on the Example of Figure 5. We could consider the bi-level polynomial optimization (N = 2 of Figure 3) solved using proximal-gradient algorithm of Figure 4. The inclusion of a solution within the feasible set of a semidefinite-programming relaxation of the polynomial optimization problem corresponding to steady states of optimal power flows (Figure 2, 202-203, corresponding to 503 of Figure 5) could be modelled by a non-smooth, but convex indicator functions g (301 in Figure 3). At the participant level, the extended formulation [32, 25] could be modelled by a non-smooth, but convex indicator functions g (301 in Figure 3).

[0039] In Example 2, we consider Embodiment 2 on the Example of Figure 5. We could consider the bi-level polynomial optimization (N = 2 of Figure 3) solved using proximal-gradient algorithm of Figure 4. The inclusion of a solution within the feasible set of a semidefinite-programming relaxation of the polynomial optimization problem corresponding to steady states of optimal power flows (Figure 2, 202-203, corresponding to 503 of Figure 5) could be modelled by a non-smooth, but convex indicator functions g (301 in Figure 3). At the participant level, the semidefinite-programming relaxation [3] could be modelled by a non-smooth, but convex indicator functions g (301 in Figure 3).

[0040] In Example 3, we consider Embodiment 3 on the Example of Figure 5, but we consider the more elaborate model of power-system dynamics (201 in Figure 2). We could consider the bi-level polynomial optimization (N = 2 of Figure 3) solved using proximal-gradient algorithm of Figure 4. The inclusion of a solution within the feasible set of steady states of the power-system dynamics (Figure 2, 201, corresponding to 503 of Figure 5) could be discretised and modelled by a non-smooth, but convex indicator functions g (301 in Figure 3). At the participant level, the semidefinite-programming relaxation [3] could be modelled by a non-smooth, but convex indicator functions g (301 in Figure 3).

INDUSTRIAL UTILIZATION

[0041] Present invention is utilizable in the field of clearing markets for electric power and related derivatives. For example, in Europe, its users may be Nominated Electricity Market Operators (NEMOs). Further users may be vendors to the NEMOs. Further users may be participants in such a market, who may be interested in understanding of the market.

CLAIMS

1. A method of balancing electric power systems using multilevel optimization, **characterized in that** it comprises following steps:

- A) using a robust multi-level formulation;
- B) using a convexification of transmission constraints at the transmission-system operator level;
- C) using a convexification of the market-participant level to ensure tractability.

2. The method according to claim 1, wherein the step C) is replaced with the step of using the semidefinite programming (SDP) relaxation of the market-participant level as the convexification of the market-participant level to ensure tractability, wherein the feasible set of the SDP relaxation is semi-algebraic.

3. The method according to claims 1 or 2, wherein it comprises defining the projected uncertainty moving forward in time, preferably followed by cross verifying the uncertainty between the nonconvex polynomial formulation using ellipsoidal tube methods as used in robust model predictive control and SDP techniques tailored specifically to polynomial systems.

4. The method according to any of claims 1 to 3, wherein it comprises using a trajectory of states of the dynamics, by incorporating time and the predicted operation in the future by solving optimal control problems using time-dependent SDPs.

5. The method according to any of claims 1 to 4, wherein it comprises combining the continuous time scale and discrete time scales and analyzing stability of the related systems.

6. A device configured to execute the method of any of claims 1 to 5, **characterized in that** it comprises a network interface, a memory containing a plurality of operating parameters describing the dynamics of an electric power system; and a plurality of participant operating parameters describing operating parameters for a set of at least one

participant selected from the group consisting of producers and consumers and storage providers; a processor, configured to: receive operating parameters from the participants; upon receipt of operating parameters, calculating a plurality of updated operating parameters using an iterative process to determine the updated operating parameters as values of an optimizer using other operating parameters as coefficients in a multi-level optimization problems; and immediately upon calculating, sending participant operating parameters based on the calculated plurality of updated operating parameters.

7. The device according to claim 2, wherein participants comprise of operators of generators.

8. The device according to claim 2, wherein operating parameters received from the participants comprise of distributional forecasts of production and bids, which have not been matched so far.

9. The device according to claim 2, wherein updated operating parameters comprise of information on bids, which have been cleared.

10. The device according to claim 2, wherein updated operating parameters are obtained using multi-level optimization with robustness properties.

Fig. 1

101	Clear as many bids as possible, while minimizing the risk involved,	nominated electricity market operator (NEMO)
102	subject to constraints on the stability of the power systems, governed by the respective TSOs and associations of TSOs and companies serving them (e.g., JAO)	Transmission system operator (TSOs)
103	subject to the solution being optimal from the point of view of GenCos, who provide the supply	Electricity generating companies (GenCos)
104	subject to the solution being optimal from the point of view of the demand-side (DisCos).	Electricity distribution companies (DisCos)

Fig. 2

201

$$\frac{\partial \delta_i}{\partial t} = \omega_i - \omega_s \qquad (1)$$

$$\frac{\partial \omega_i}{\partial t} = \frac{1}{M_i}(T_{M_i} - E'_{qi} - X'_{d_i} I_{d_i}) I_{q_i}) - (E'_{di} - X'_{q_i} I_{q_i}) I_{d_i}) - D_i(\omega_i - \omega_s \qquad (2)$$

$$\frac{\partial E'_{qi}}{\partial t} = \frac{1}{T'_{d0i}}(-E'_{qi} - (X_{di} - X'_{di}) I_{di} + E_{fdi}) \qquad (3)$$

$$\frac{\partial E'_{di}}{\partial t} = \frac{1}{T'_{dq0i}}(-E'_{di} + (X_{qi} - X'_{qi}) I_{qi} \qquad (4)$$

$$0 = E'_{di} - V_i \sin(\delta_i - \theta_i) - R_{si} I_{di} + X'_{qi} I_{qi} \qquad (5)$$

$$0 = E'_{qi} - V_i \cos(\delta_i - \theta_i) - R_{si} I_{qi} + X'_{di} I_{di} \qquad (6)$$

$$\frac{\partial E'_{fdi}}{\partial t} = \frac{1}{T_{E_i}}(-(K_{E_i} + S_E(E_{fdi})) E_{fdi} + V_{Ri} \qquad (7)$$

$$\frac{\partial V_{R_i}}{\partial t} = \frac{1}{T_{A_i}}(-(V_{R_i} + K_{A_i} R_{F_i} - \frac{K_{A_i} K_{F_i}}{T_{F_i}} E_{fdi} + K_{A_i}(V_{refi} - V_i))) \qquad (8)$$

$$\frac{\partial R_{F_i}}{\partial t} = \frac{1}{T_{F_i}}(-(R_{F_i} + \frac{K_{F_i}}{T_{F_i}} E_{fdi}) \qquad (9)$$

$$\text{for generator } i : 0 = I_{di} V_i \sin(\delta_i - \theta_i) + I_{qi} V_i \cos(\delta_i - \theta_i) + P_{L_i} - \sum_j V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij}) \qquad (10)$$

$$\text{for generator } i : 0 = I_{di} V_i \cos(\delta_i - \theta_i) + I_{qi} V_i \sin(\delta_i - \theta_i) + Q_{L_i} - \sum_j V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) \qquad (11)$$

$$\text{for load } i : P_{L_i} = \sum_j V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij}) \qquad (12)$$

$$\text{for load } i : Q_{L_i} = \sum_j V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) \qquad (13)$$

201

$$P_{Li}^{\min} \leq \operatorname{tr}(Y_k x x^T) + P_{Li} \leq P_{Li}^{\max} \qquad \forall i \in S_B$$

$$Q_{Li}^{\min} \leq \operatorname{tr}(\bar{Y}_k x x^T) + Q_{Li} \leq Q_{Li}^{\max} \qquad \forall i \in S_B$$

$$(V_k^{\min})^2 \leq \operatorname{tr}(M_k x x^T) \leq (V_k^{\max})^2 \qquad \forall i \in S_B$$

$$(\operatorname{tr}(Y_{lm} x x^T))^2 + (\operatorname{tr}(\bar{Y}_{lm} x x^T))^2 \leq (S_{lm}^{\max})^2 \qquad \forall (l, m) \in S_B \times S_B$$

203

$$y_k = e_k e_k^T y,$$

$$y_{lm} = (j \frac{\bar{b}_{lm}}{2} + g_{lm} + j b_{lm}) e_l e_l^T - (g_{lm} + j b_{lm}) e_l e_m^T,$$

$$Y_k = \frac{1}{2} \begin{bmatrix} \Re(y_k + y_k^T) & \Im(y_k^T - y_k) \\ \Im(y_k - y_k^T)) & \Re(y_k + y_k^T) \end{bmatrix},$$

$$\bar{Y}_k = -\frac{1}{2} \begin{bmatrix} \Im(y_k + y_k^T) & \Re(y_k - y_k^T) \\ \Re(y_k^T - y_k) & \Im(y_k + y_k^T) \end{bmatrix},$$

$$M_k = \begin{bmatrix} e_k e_k^T & 0 \\ 0 & e_k e_k^T \end{bmatrix},$$

$$Y_{lm} = \frac{1}{2} \begin{bmatrix} \Re(y_{lm} + y_{lm}^T) & \Im(y_{lm}^T - y_{lm}) \\ \Im(y_{lm} - y_{lm}^T) & \Re(y_{lm} + y_{lm}^T) \end{bmatrix},$$

$$\bar{Y}_{lm} = -\frac{1}{2} \begin{bmatrix} \Im(y_{lm} + y_{lm}^T) & \Re(y_{lm}^T - y_{lm}) \\ \Re(y_{lm}^T - y_{lm}) & \Im(y_{lm} + y_{lm}^T) \end{bmatrix}.$$

Fig. 3

301

$$\begin{cases} \min\limits_{x\in X_{N+1}^*} \ \omega(x) \\ X_{i+1}^* = \underset{x\in X_i{}^*}{\arg\min}[f_i(x)+g_i(x)], \quad i\in\{0,1,...,N\} \\ X_0^* = \mathbb{R}^n, \end{cases}$$

302

$$\begin{cases} \underset{x\in X_{N+1}^*}{\arg\min}\,\omega(x), \\ X_1^* = \underset{x\in\mathbb{R}^n}{\arg\min}[f_0(x)+g_0(x)], \\ X_2^* = \underset{x\in X_1^*}{\arg\min}[f_1(x)+g_1(x)], \\ . \\ . \\ . \\ X_N^* = \underset{x\in X_{N-1}^*}{\arg\min}[f_{N-1}(x)+g_{N-1}(x)], \\ X_{N+1}^* = \underset{x\in X_N^*}{\arg\min}[f_N(x)+g_N(x)]. \end{cases}$$

Fig. 4

401

$$\begin{cases} \min\limits_{x\in X_{N+1}^*} \ \omega(x) \\ X_{i+1}^* = \underset{x\in X_i{}^*}{\arg\min}[f_i(x)+g_i(x)], \quad i\in\{0,1,...,N\} \\ X_0^* = \mathbb{R}^n, \end{cases}$$

402

$$x^{k+1} = {\alpha_k}^{(0)}T_0(x^k) + {\alpha_k}^{(1)}T_1(x^k) + ... + {\alpha_k}^{(N)}T_N(x^k),$$

403

$$T_{i+1}(x) = \mathrm{prox}_{t_i g_i}[x - t_i\nabla f_i(x)],\ \forall i\in\{0,1,2,...,N\},$$

$$\mathrm{prox}_g(x) = \underset{u\in\mathbb{R}^n}{\arg\min}\left\{g(u)+\frac{1}{2}\|u-x\|^2\right\}$$

404

for all $i\in\{0,1,...,N\}$, $\alpha_k^{(i)}\geq 0$ and for all $k\in\mathbb{N}$, one has $\sum\limits_{i=1}\alpha_k^{(i)} = 1$.

$\lim\limits_{k\to\infty}\alpha_k^{(0)} = 0$, $\sum\limits_{k=1}^{+\infty}\alpha_k^{(0)} = \infty$ and, $\limsup\limits_{k}\frac{1}{\alpha_k^{(0)}}\sum\limits_{i=2}^{+\infty}\alpha_k^{(i)} = \delta^*\in[0,+\infty)$.

$X_{N+1}^*\neq\emptyset$, $\lim\limits_{k\to\infty}\alpha_k^{(1)} = 1$ and for all $j\in\{0,2,...,N\}$ one has $\lim\limits_{k\to\infty}\alpha_k^{(j)} = 0$.

Fig. 5

501

From 9.20 till 9.45, GenCo 1 can produce 100 MW of power @ €100 / MWh with 99% certainty.

From 9.46 till 10.00, GenCo 1 can produce 100 MW of power @€100 / MWh with 97% certainty.

From 9.20 till 10.00, GenCo 2 can produce 20 MW of power @ €40 / MWh with 60% certainty.

502

From 9.20 till 9.30, DisCo 1 needs 100 MW of power @ up to €100 / MWh.

From 9.20 till 10.00, DisCo 2 needs 20 MW of power @ up to €200 / MWh.

503

TSO 1 wishes to see apparent power at all branches within 100 MW by 9.30 and within 90 MW from 9.30 till 10.00 to account for the higher ambient temperature. Voltage magnitude bounds need to be kept within 0.95 and 1.05 p.u. until 10.00.

INTERNATIONAL SEARCH REPORT

International application No
PCT/CZ2023/000027

A. CLASSIFICATION OF SUBJECT MATTER
INV. G06Q50/06 G06Q10/04 H02J3/00
ADD.

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
G06Q H02J

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

EPO-Internal

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	DU YAN ET AL: "A Hierarchical Real-Time Balancing Market Considering Multi-Microgrids With Distributed Sustainable Resources", IEEE TRANSACTIONS ON SUSTAINABLE ENERGY, IEEE, USA, vol. 11, no. 1, 1 January 2020 (2020-01-01), pages 72-83, XP011762354, ISSN: 1949-3029, DOI: 10.1109/TSTE.2018.2884223 [retrieved on 2019-12-16]	1,5
Y	the whole document ----- -/--	2-4,6-10

☒ Further documents are listed in the continuation of Box C. ☐ See patent family annex.

* Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier application or patent but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance;; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance;; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search 20 December 2023	Date of mailing of the international search report 08/01/2024
Name and mailing address of the ISA/ European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Fax: (+31-70) 340-3016	Authorized officer Bartal, P

1

C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
Y	BISSAN GHADDAR ET AL: "Optimal Power Flow as a Polynomial Optimization Problem", ARXIV.ORG, CORNELL UNIVERSITY LIBRARY, 201 OLIN LIBRARY CORNELL UNIVERSITY ITHACA, NY 14853, 14 April 2014 (2014-04-14), XP081344430, DOI: 10.1109/TPWRS.2015.2390037 the whole document -----	2-4,6-10

1