Dec. 2, 1941. A. N. GARIN 2,264,836

ELECTRIC DISTRIBUTION SYSTEM AND MULTIWINDING TRANSFORMER THEREFOR

Filed March 14, 1940 2 Sheets-Sheet 1

Fig. 1.

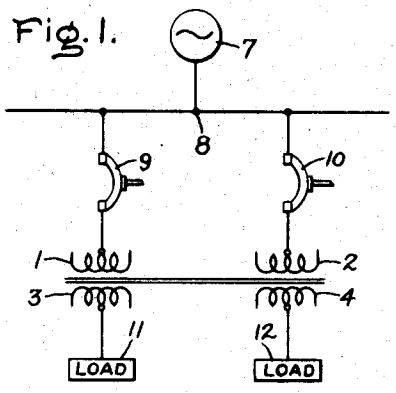


Fig. 2.

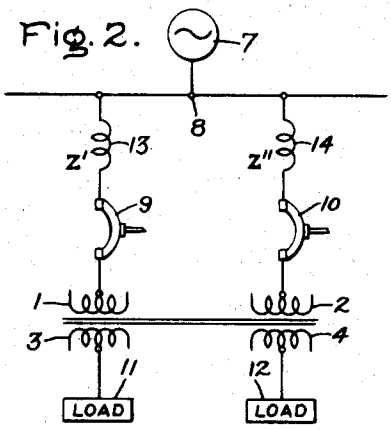


Fig. 3.

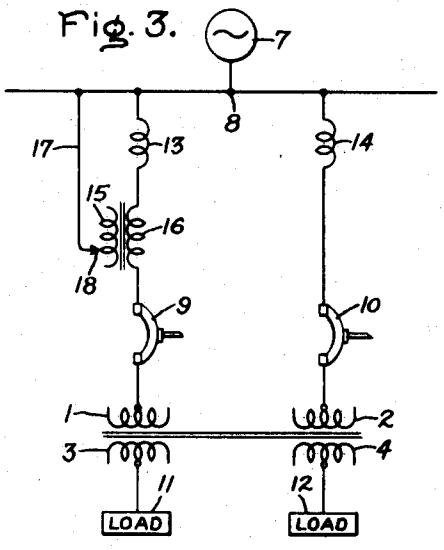


Fig. 4.

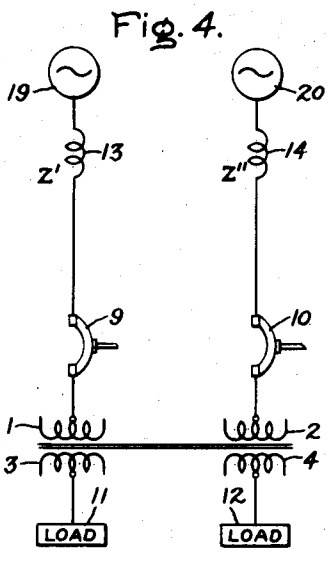
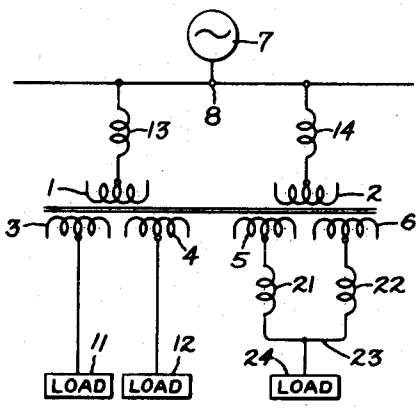


Fig. 5.



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ELECTRIC DISTRIBUTION SYSTEM AND MULTIWINDING TRANSFORMER THEREFOR

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Fig. 6.

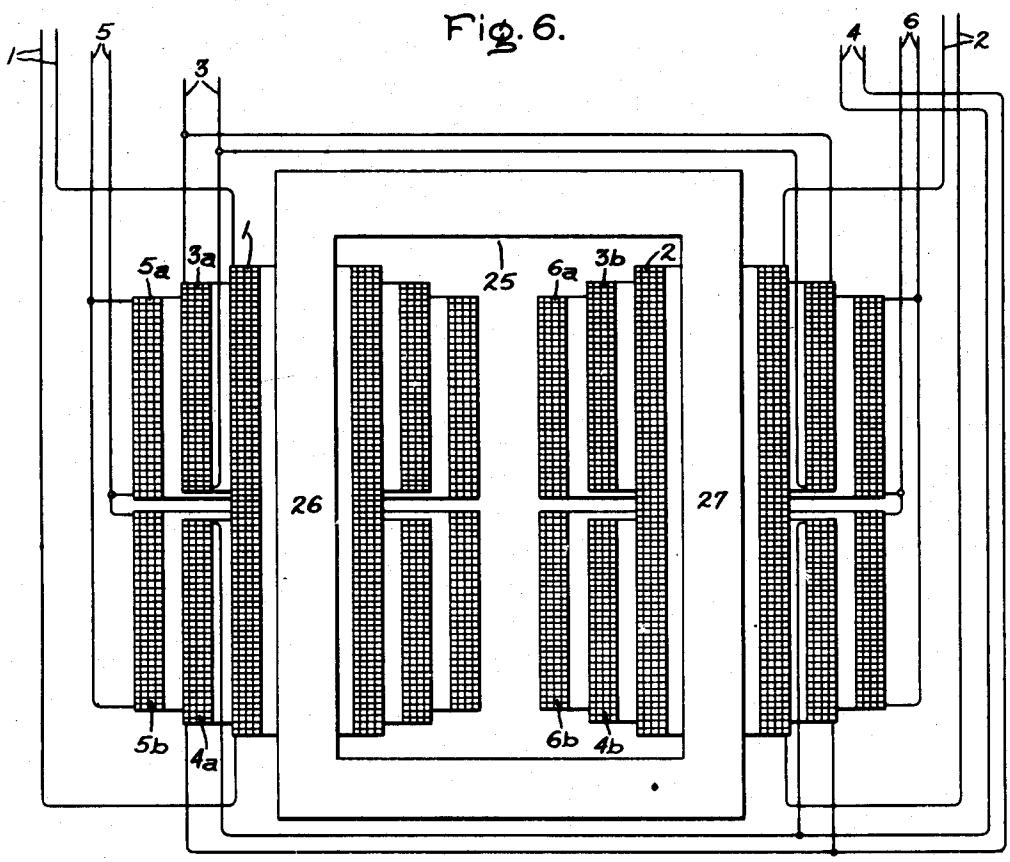
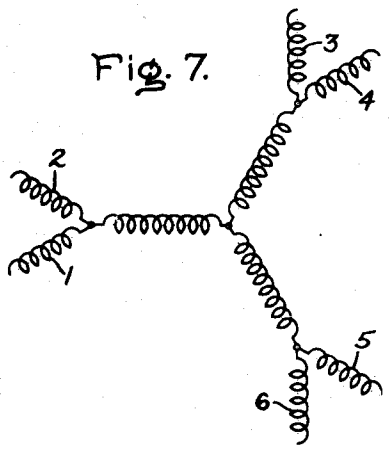


Fig. 7.



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UNITED STATES PATENT OFFICE

2,264,836

ELECTRIC DISTRIBUTION SYSTEM AND MULTIWINDING TRANSFORMER THEREFOR

Alexis N. Garin, Pittsfield, Mass., assignor to General Electric Company, a corporation of New York

Application March 14, 1940, Serial No. 323,889

18 Claims. (Cl. 171—119)

My invention relates to alternating current distribution systems and more particularly to multiwinding transformers having four or more windings and their arrangements and connections in such systems.

An object of my invention is to provide new and improved arrangements for multiwinding transforming apparatus in systems of distribution for improving the operation of such systems.

It is another object of my invention to obtain in a system of distribution having connected therein a multiwinding transforming means of four or more windings a predetermined most advantageous distribution of load and also to obtain a predetermined most advantageous relationship of terminal voltages of said transforming means under various loading conditions.

It is another object of my invention to provide a new and improved multiwinding transformer having four or more windings.

It is another object of my invention to provide a multiwinding transformer of four or more windings having such characteristics as to secure a predetermined interaction between pairs of windings under load whereby a desired division of current or load between a given pair of windings may be secured for a given relative loading of another pair of windings.

It is another object of my invention to provide a multiwinding transformer of four or more windings having such characteristics as to secure a predetermined relation between terminal voltages of a given pair of windings independently of the relative loading of another pair of windings.

My invention will be better understood from the following description taken in connection with the accompanying drawings, and its scope will be pointed out in the appended claims.

While the use of multiwinding transformers having four or more windings for purposes such as that of supplying a plurality of loads of different voltages is not new, their use for improving system operation by providing, for instance, duplicate windings having the same voltage rating and connected to different parts of the system respectively, is relatively recent and has presented to the art a number of new problems not encountered in the use of two-winding or three-winding transformers and not encountered in the use of n-winding transformers supplying for instance $n-1$ independent loads from a common source.

Whenever a pair of the windings of a multiwinding transformer of four or more windings is connected into a loop, or to two separate sources of power, respectively, or in any other arrangement permissive of exchange of power or circulation of power between that pair of windings, the division of load within that pair of windings, and the relationship of other electrical characteristics of that pair of windings, may be greatly influenced by the condition of relative loading of the other windings of the transformer. As the most favorable value of such an effect depends on the needs and requirements of the particular system to which the multiwinding transformer is to be connected, the present invention is directed to the construction and circuit connections of a multiwinding transformer of four or more windings for securing inherently a predetermined most advantageous distribution of load in such systems and for securing inherently a predetermined most advantageous relationship of terminal voltages under various loading conditions.

In the drawing, Fig. 1 is a conventional schematic single-wire diagram of a four-winding transformer in a very simple system connection involving a loop for the exposition of the physical conceptions and mathematical principles underlying the invention; Fig. 2 is a modification of Fig. 1 to illustrate the effect of impedances external to the transformer windings in such loops; Fig. 3 is a modification of Fig. 2 with the introduction into the loop of a variable voltage external to the transformer and intended to modify the load division between the branches of the loop and their corresponding transformer windings; Fig. 4 illustrates an embodiment in which a pair of the windings of a multiwinding transformer are connected respectively to a pair of independently controlled generators; Fig. 5 illustrates schematically the connections of a six-winding transformer in a system involving two pairs of the windings in loops; Fig. 6 is a diagrammatic illustration of a single-phase multiwinding transformer comprising three pairs of windings so arranged as to make the division of load and terminal voltage balance within each pair independent of the relative loading of any other pair; and Fig. 7 is a schematic equivalent network of the transformer of Fig. 6. Like reference characters indicate similar parts in the several figures of the drawings.

Referring now to Fig. 1 of the drawings, a schematic single-wire diagram is shown as illustrative of a four-winding single-phase transformer and system of connections in accordance with my invention, having a pair of primary windings 1 and 2 and a pair of secondary wind-

ings 3 and 4. Primary winding 1 is supplied from a source of alternating current 7 through a bus 8 and circuit interrupting device 9. Primary winding 2 may, for example, be of the same voltage rating as primary winding 1 and is similarly supplied from the same source 7 and bus 8 but through an independent connection to the bus through circuit interrupting device 10. It is seen that windings 1 and 2 are connected in parallel. Secondary winding 3 is connected to a load 11 and secondary winding 4 to another independent load, 12. As loads 11 and 12 are independent, the voltampere power delivered by winding 3 to load 11 may have any ratio to that delivered by winding 4 to load 12.

The mathematic theory on which the invention is based will now be considered. Let the load current in each winding be designated by I with appropriate subscripts corresponding to the number of the winding, as I_1, I_2, I_3 and I_4, respectively. In the following exposition and equations, these currents may be understood as expressed either in amperes, or ampere-turns, or per unit values, equally well; all three systems of units being well-known in the art. If expressed in amperes, all currents in an equation must be reduced to the basis of the same winding. Expressed in any of these terms, the current in a winding is a direct measure of the volt-ampere load or power in that winding and may therefore be expressed also in volt-amperes, that is, as the current corresponding to the stated volt-amperes. All equations will be understood to be true vectorially.

A pair of independent but simultaneous load currents, I_3 in winding 3 and I_4 in winding 4, may be resolved into two components; one component having identical numerical value and phase in both windings; the second component having the same value in both windings but opposite phase to each other, as follows:

$$I_3=\left(\frac{I_3+I_4}{2}\right)+\left(\frac{I_3-I_4}{2}\right) \quad (1)$$

$$I_4=\left(\frac{I_3+I_4}{2}\right)-\left(\frac{I_3-I_4}{2}\right) \quad (2)$$

Through mathematical investigations, I have found that the currents I_1 and I_2 in windings 1 and 2, respectively, corresponding to the currents I_3 and I_4, in windings 3 and 4, respectively, involve three important components, as represented by the three terms on the right of each one of the equations as follows:

$$I_1=-\left[\left(\frac{I_3+I_4}{2}\right)+k'\left(\frac{I_3+I_4}{2}\right)+k''\left(\frac{I_3-I_4}{2}\right)\right] \quad (3a)$$

$$=-\left[(1+k')\left(\frac{I_3+I_4}{2}\right)+k''\left(\frac{I_3-I_4}{2}\right)\right] \quad (3b)$$

$$I_2=-\left[\left(\frac{I_3+I_4}{2}\right)-k'\left(\frac{I_3+I_4}{2}\right)-k''\left(\frac{I_3-I_4}{2}\right)\right] \quad (4a)$$

$$=\left[-(1-k')\left(\frac{I_3+I_4}{2}\right)-k''\left(\frac{I_3-I_4}{2}\right)\right] \quad (4b)$$

The negative signs in front of the brackets arise from the consideration that the vector sum of all of the load currents of a transformer must add up to zero.

In these equations, as in 1 and 2, the first term on the right, that is $-(I_3+I_4)/2$, represents one half of the total output of the transformer, and hence, the balanced share of each primary winding in the total load of the transformer.

The second term in Equations 3*a* and 4*a*, namely,

$$k'\left(\frac{I_3+I_4}{2}\right)$$

negative in winding 1 and positive in winding 2, is an exchange or circulating current between those two windings, and, as indicated, is a function of the total output of the transformer independent of its division between windings 3 and 4.

The third term in Equations 3*a* and 4*a*, namely,

$$k''\left(\frac{I_3-I_4}{2}\right)$$

with opposite signs in windings 1 and 2, is also an exchange or circulating current between those two windings, but this circulating current is a function of the difference between the currents of windings 3 and 4, and is independent of the total output of the transformer, while the second component was seen to be a function of the total output independent of its division between windings 3 and 4. It is thus seen that an unequal division of load current between windings 1 and 2 may be conceived of as the resultant of the superposition of a balanced load current having the same sign, that is, flowing in the same direction (in or out) in both windings, and a circulating current, with opposite signs (flowing into one, and out of the other, winding); and that this circulating current in its turn can be resolved into two components; one dependent on total load and independent of the unbalance in the secondaries; the other, dependent on the unbalance in the secondaries and independent of the total load.

It may be observed that the third term, associated with k'' in Equations 3 and 4, is exclusively characteristic of transformers having at least four windings and is absent in those with fewer windings.

The significance of the foregoing conceptions for the most advantageous operation of electrical systems involving multiwinding transformers of four or more windings may be realized in accordance with my invention by considerations as follows:

(A) If in Fig. 1, the circuits of windings 1 and 2 have equal volt-ampere capacities and it is desired to operate them with minimum total loss and minimum heating of the windings, the constants k' and k'' of the network should be zero, in which case, I_1 and I_2 will be alike independent of any unbalance in I_3 and I_4.

(B) If, as further illustration, windings 1 and 2 have unequal capacities, or, if for any other reason, it is more advantageous to maintain I_1 and I_2 unequal but at a constant ratio to each other independent of the ratio of I_3 to I_4, then it is necessary that the constant k'' be zero and k' an appropriate finite value.

(C) If, as a still further illustration, it is desired to reduce the short-circuit difficulties on the system by requiring that winding 1 exclusively shall furnish the load in winding 3, and winding 2 exclusively that in 4, then it will be seen that the constant k' must be zero and k'' must be unity.

(D) Finally, in the general case, the most advantageous operation of the system may demand that the unbalance between I_1 and I_2 for balanced loads in windings 3 and 4 be a certain finite amount, represented by $k'=c$, and for maximum unbalanced loads an amount represented by $k'=c$ and $k''=d$.

Now, I have discovered the mathematical laws according to which the windings of a multiwinding transformer of four or more windings may be arranged so as to secure inherently any predetermined most advantageous ratio of power between the circuits of any pair of windings such as 1 and 2 for any specified or expected ratio of loads in another pair of windings, such as 3 and 4, in arrangements such as is shown in Fig. 1 and the other figures of the accompanying drawing and their various possible modifications.

I find that if it is desired that I_1 and I_2 in the system shown in Fig. 1 be such as correspond to a value of k' in Equations 3 and 4 equal to c, and k'' equal to d, this result can be secured inherently by adjusting the leakage impedances, Z_{12}, Z_{13}, Z_{14}, Z_{23} and Z_{24} of the various pairs of the four windings of the transformer corresponding to the digits of the subscripts of the Z's in accordance with equations as follows:

$$\frac{(Z_{23}-Z_{13})+(Z_{24}-Z_{14})}{2Z_{12}}=k'=c \qquad (5)$$

$$\frac{(Z_{23}-Z_{13})-(Z_{24}-Z_{14})}{2Z_{12}}=k''=d \qquad (6)$$

I use the term "leakage impedance" in its well-recognized sense in the transformer literature, and wish to imply particularly the effective value thereof measurable at the circuit terminals if subject to modifying elements not normally a part of said leakage impedance. The reactances or impedances referred to herein are those reactances and impedances which the transformer offers to load currents but not those which apply to magnetizing currents. These impedances to load current are sometimes referred to as load reactance or load impedance but herein they are referred to as leakage reactance or leakage impedance in accordance with the common practice in transformer literature. It is well known that the power or kilovolt-amperes flowing between the windings 1 and 2, for example, must overcome the leakage impedance Z_{12}. The various leakage impedances herein referred to are thus identified by Z with the digits of the subscripts corresponding to the windings involved. The values of the various leakage impedances in these and other equations may be expressed in ohms. If so, all of them must be reduced to the same voltage or turn basis, as well understood in the art. They may also be, and are preferably, expressed in "per unit" values, based on a common reference load, a very convenient method of expression now in considerable use in transformer literature.

As the calculation of and design for specified leakage impedances is common knowledge among transformer engineers, these equations are sufficient to enable any one skilled in the art to design and build transformers having the characteristics under consideration.

It will be observed that Equations 5 and 6 offer only two limitations to the relationships of the five leakage impedances involved. They thus disclose not only how the desired result may be secured, but also the maximum degree of liberty in design which the designer may enjoy in securing those desired results; for, by a well-known algebraic theorem, two equations involving more than two independent variables (the leakage impedances in the present case) admit of an infinite number of solutions; and, therefore, there are an infinite variety of arrangements of four windings permitting their leakage impedances to satisfy the conditions of Equations 5 and 6.

It follows from Equations 3 and 4 and 5 and 6 that, if the powers in windings 1 and 2 are to be balanced under all conditions of loading of windings 3 and 4, the leakage impedances are to be adjusted in the relationships simultaneously as follows:

$$(Z_{23}-Z_{13})+(Z_{24}-Z_{14})=0 \qquad (7)$$

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})=0 \qquad (8)$$

As another example, if the volt-ampere capacity of winding 1 is twice that of winding 2, and it is desired that the power be divided between them proportional to their capacity and in-phase with each other, irrespective of the division of output between windings 3 and 4, then by Equation 3b k'' is zero and k' corresponds to

$$\frac{I_1}{I_2}=\frac{1+k'}{1-k'}=2$$

Hence, by elementary algebra, $k'=\frac{1}{3}$, and by Equations 5 and 6 the leakage impedance must be in the relationship

$$\frac{(Z_{23}-Z_{13})+(Z_{24}-Z_{14})}{2Z_{12}}=\frac{1}{3} \qquad (9)$$

$$\frac{(Z_{23}-Z_{13})-(Z_{24}-Z_{14})}{2Z_{12}}=0 \qquad (10)$$

Obviously the two requirements of Equations 7 and 8, or 9 and 10, can be met by the leakage impedances involved in an infinite variety of ways, and thus these equations disclose the maximum amount of freedom which the designer may enjoy in securing the characteristics mentioned.

Fig. 2 illustrates a modification of Fig. 1 in which the connections of windings 1 and 2 to bus 3 are made through lengths of line or cables offering appreciable impedances, or through special impedors, such as for limiting short circuits or assisting in parallel operation or for other purposes, and represented in Fig. 2 by impedance 13 offering an impedance Z' in series with winding 1, and impedance 14 offering an impedance Z'' in series with winding 2. The impedances Z' and Z'' may have any vector value. It will be seen that the connection is equivalent to parallel operation of windings 1 and 2 with the introduction of external impedances into the paralleling loop.

I have discovered that in this modification, a desired division of power between the circuits of windings 1 and 2, corresponding to specified values of k' and k'' in Equations 3 and 4, may be secured by adjusting the leakage impedances of the windings in accordance with the following equations:

$$\frac{(Z_{23}-Z_{13})+(Z_{24}-Z_{14})+2(Z''-Z')}{2Z_{12}+2(Z''+Z')}=k' \qquad (11)$$

$$\frac{(Z_{23}-Z_{13})-(Z_{24}-Z_{14})}{2Z_{12}+2(Z''+Z')}=k'' \qquad (12)$$

Evidently, if the internal impedances Z_{12}, Z_{13}, etc. are expressed as per unit values, then the external impedances should also be so expressed.

Fig. 3 illustrates, again schematically, in a conventional one-wire diagram, a modification of Fig. 2 in which an additional voltage, which we may designate here as ΔE, is introduced into the loop by means of an auxiliary transformer, shown here as having two windings 15 and 16, to further influence the division of power in the two branches of the loop, corresponding to wind-

ings 1 and 2. Winding 16 of the auxiliary transformer is a secondary winding and is connected in series with the loop 9—8—10. Winding 15 of the auxiliary transformer is a primary and is excited from bus 8 through line 17 and a conventional tap-changing mechanism 18 shown in the figure very schematically as a movable arrow. It will be apparent to those skilled in the art that any known method and mechanism for changing the magnitude, or the phase, or both the magnitude and the phase, of the voltage ΔE introduced into the loop through winding 16, may be utilized without departing from my invention in its broader aspects. The equivalent of ΔE may also be obtained by unbalancing the turns of windings 1 and 2.

Voltage ΔE will circulate current between windings 1 and 2, independently of the loads I_3 and I_4, as represented by the fourth term on the right-hand sides of the following equations:

$$I_1=-\left[\left(\frac{I_3+I_4}{2}\right)+k'\left(\frac{I_3+I_4}{2}\right)+k''\left(\frac{I_3-I_4}{2}\right)+\frac{\Delta E}{Z_{12}+(Z''+Z')}\right] \quad (13)$$

$$I_2=-\left[\left(\frac{I_3+I_4}{2}\right)-k'\left(\frac{I_3+I_4}{2}\right)-k''\left(\frac{I_3-I_4}{2}\right)-\frac{\Delta E}{Z_{12}+(Z''+Z')}\right] \quad (14)$$

In these equations, if the I's are expressed in amperes, the Z's will be in ohms and the ΔE in volts, all expressed in terms of the same windings; or, as indicated above, all may be expressed preferably in per-unit values. In the general case, ΔE also will be a vector.

It will be seen from these equations that any desired ratio of I_1 to I_2, not otherwise inherent in the network, may be forced by suitable choice of ΔE. In such a case, the present invention is directed to such an adjustment of the leakage impedances of the windings as leads to the use of the minimum or otherwise most advantageous value of ΔE and most economical equipment to furnish that ΔE to the system. Thus, given the most advantageous values of I_1 and I_2 corresponding to the load conditions I_3 and I_4 to which windings 3 and 4 may be adapted to be subjected, we may determine by Equations 11 and 12 the leakage impedance values which will give this without the benefit of ΔE. If the required combination of impedance values is disadvantageous from the standpoint of other requirements of the system, a set of compromise values are chosen for the impedances and with their aid k' and k'' are calculated by Equations 11 and 12. Thus, having the values of the (I)s, of the (Z)s, and of the (k)s of Equations 13 and 14, the value of the necessary ΔE, as the single unknown defined by those equations, is readily calculated by elementary mathematical methods. Without the benefit of this method, unnecessarily large values of ΔE and equipment to furnish it will have to be provided to make certain that a desired division of power can be secured. Alternatively, given the desired relationship of I_1 and I_2, and the available ΔE, these equations define the necessary relationship of the various impedances to secure the desired results.

Fig. 4 shows a modification of Fig. 2 in which windings 1 and 2 are connected to independent generators 19 and 20 respectively instead of a common bus. For the purpose of the present invention, the system of Fig. 4 is completely equivalent to that of Fig. 3, and the same equations as apply to Fig. 3 apply also to Fig. 4 irrespective of whether generators 19 and 20 have similar or different voltage ratings, provided that the various quantities involved are expressed in "per unit" values and provided it is understood that

$(E_{19}-E_{20})$ corresponds to ΔE,
E_{20} corresponds to the common voltage of circuits 1 and 2 of Fig. 3,
Z' and Z'' correspond to the total external impedances respectively in series with 1 and 2, inclusive, of the corresponding generator impedances.

This system arrangement is equivalent to the parallel operation of two elements; the combination of a generator and its connected transformer winding as one element, and another generator and its connected transformer winding as the second element. Therefore, in speaking of parallel operation, I do not wish to be understood as restricted to circuits of identical voltage rating conductively connected in parallel, but as meaning broadly any arrangement in which the circuits under consideration are able to interchange or circulate power, vectorially if necessary.

Evidently, if the generators 19 and 20 are of the same voltage rating and their busses connected together ahead of the transformer windings 1 and 2, the resulting circuit will be equivalent to that of Fig. 1 or Fig. 2; and therefore, Figs. 3 and 4 may also be considered as different operating connections of one and the same system arranged in a flexible bus-sectionalizing scheme. Viewed particularly in this light, it will be seen that the adjustment of the leakage impedances of the four-winding transformer may be not only very desirable but also necessary to secure a specified result of the kind described desirable under various bus-operating schemes.

The application of the present invention is not limited to the use of a four-winding transformer, but includes also those with higher number of windings as well.

Fig. 5 illustrates schematically the application of a six-winding transformer, as a modification of Fig. 2 by the addition thereto of windings 5 and 6, their respective series impedances 21 and 22, their common bus 23 and common load 24. The load currents in windings 1 and 2 are now the resultant of the superposition of their respective load currents corresponding to (I_3, I_4) and to (I_5, I_6); that is, first calculate the duty in windings 1 and 2, considering the windings 1, 2, 3 and 4 as a four-winding transformer, and then again with the windings 1, 2, 5 and 6 as a four-winding transformer; and combine the two results. If the distribution of load in windings 5 and 6 are desired, similar calculations as described above are made treating windings 5 and 6 as if they were the windings 1 and 2 of the foregoing equations, taking them first with one pair of the other windings (say 3 and 4) as a four-winding transformer and then with the remaining pair.

In the light of the foregoing explanations, the generalization of the method to the n-winding transformer will be clear to those skilled in the art.

As an n-winding transformer involves

$$n(n-1)/2$$

leakage impedances, a six-winding transformer involves fifteen leakage impedances, and the ar-

rangement of the windings to secure a desired result of the type described tends to increase in freedom and complexity with increasing number of windings. Therefore, as an example of a very desirable case, I shall describe now a specific arrangement of the windings of a transformer having three pairs of windings to secure inherently equal power or load division within each pair of windings when the pair in question is connected into a loop or the equivalent of a loop, as follows.

Fig. 6 shows in section a single-phase transformer with concentric windings, comprising a core 25 with two winding legs 26 and 27, and three pairs of windings 1 and 2, 3 and 4, and 5 and 6 respectively. Winding 1 is wound around and distributed along core leg 26 exclusively. Winding 2 is wound around and distributed along leg 27 exclusively. Winding 3 comprises two coils connected in parallel, 3a wound around and distributed along the upper half of core leg 26, and 3b wound around and distributed along the upper half of core leg 27. Winding 4 comprises two coils in parallel, 4a wound around and distributed along the lower half of core leg 26, and 4b similarly on the lower half of core leg 27. Winding 5 comprises two coils in parallel, 5a wound around and distributed along the upper half, and 5b around the lower half, of core leg 26; while winding 6 is similarly arranged on core leg 27. In every case, coils connected in parallel are wound around the core in such directions as to be suitable for parallel connection, as is well known in the art.

This arrangement of the three pairs of windings assures that the division of load between the two branches of a loop connected to any one of the pairs of windings, as paired above, will be equal and independent of the division within either one of two pairs of circuits connected to the remaining pairs of the windings. This can be verified as follows:

By symmetry,

$$Z_{13}=Z_{14}=Z_{23}=Z_{24} \tag{15}$$

and therefore k' and k'', by Equations 5 and 6 are zero for the two pairs of windings 1 and 2, and 3 and 4. Therefore, the division of load in 1 and 2 will be equal and independent of any unbalance of load between 3 and 4.

Considering windings 1 and 2 in combination with windings 5 and 6, it will be evident from symmetry again that

$$Z_{25}=Z_{16} \tag{16}$$

and

$$Z_{26}=Z_{15} \tag{17}$$

hence, Equation 5 with its subscripts 3 and 4 changed to 5 and 6, respectively, becomes Equation 18:

$$k'=\frac{(Z_{25}-Z_{15})+(Z_{26}-Z_{16})}{2Z_{12}}=0 \tag{18}$$

That k'' (Equation 6) is also zero for the present case may be verified as follow. Considering the reactance components of the various impedances,

$$\overline{X}_{15}\cong\overline{X}_{25} \tag{19}$$

because

$$\overline{X}_{15}$$

is substantially proportional to the insulation distance S_{15} between windings 1 and 5, which is evidently equal substantially to the insulation distance S_{13a} between 1 and 3a, plus the insulation distance S_{3a5a} between 3a and 5a:

$$\overline{X}_{15}\sim S_{15}\cong S_{13a}+S_{3a5a} \tag{20}$$

The leakage reactance

$$\overline{X}_{25}$$

between windings 2 and 5 is substantially proportional to the sum of the insulation distances S_{23b} (between 2 and 3b, which distance is the same as S_{13a} by symmetry) and S_{3a5a} (between 3a and 5a), and therefore

$$\overline{X}_{15}\cong\overline{X}_{25} \tag{21}$$

The foregoing statement about

$$\overline{X}_{25}$$

follows from the fact that a transformation of power from 2 to 5 takes place practically entirely by a double transformation, namely, first from winding 2 to windings 3b and 4b by induction and from them to 3a and 4a respectively by conduction, and second from 3a and 4a to 5a and 5b respectively by induction. Stated otherwise, windings 3 and 4 couple windings 2 and 5 which otherwise have a very high leakage reactance between themselves.

In general, the resistance components of transformer impedances are small compared with their reactance components, and

if

$$X_{15}=X_{25} \tag{22}$$

then

$$Z_{15}=Z_{25} \tag{23}$$

It may be shown by similar reasoning that Z_{16} is substantially equal to Z_{26}, and the conclusion follows that

$$k''=\frac{(Z_{25}-Z_{15})-(Z_{26}-Z_{16})}{2Z_{12}}=0 \tag{24}$$

Division of load between these windings 1 and 2 when connected in parallel or in a loop, or operated in any manner making it possible for them to exchange current or power, is therefore on a basis of equality independent of any unbalance of load in the other pairs of windings.

The law governing the division of load between a pair of windings of a transformer is dependent on their ability to exchange or circulate power and not on their function as primary or secondary; so, in Fig. 5, in which 1 and 2 are primaries, 5 and 6 secondaries, the division of load within each pair is determined by the same mathematical law. Accordingly, the analysis made in the foregoing, showing the inherent tendency of the transformer to balance the volt-amperes in the pair of windings 1 and 2 when allowed to interchange power, may be repeated similarly for the pair 3 and 4, or 5 and 6, of Fig. 6, and the same conclusions will be reached, namely, that the transformer will inherently tend to maintain equal volt-amperes in each branch of any one of the pairs of windings as paired, regardless of load unbalance within either one of the other pairs.

If the inductively coupled windings of the transformer of Fig. 6 are shown by an equivalent conductively connected network of impedances representing the leakage impedances between pairs of windings, they take the form shown in Fig. 7 which shows very clearly that no current (or power) circulating within any pair tends to set up a circulating current (or power) within any other pair. It is to be noted that Fig. 7 is a

connection diagram like Figs. 1 to 5, inclusive, and not a vector diagram.

Another important characteristic of the system represented by Fig. 7 is that no circulating current (or power) within any pair of the windings tends to unbalance the terminal voltage of any other pair; that is, unbalanced load in one of the pairs of the windings of Fig. 6 does not tend to unbalance the terminal voltages of another pair of the paired windings when the latter is not free to circulate power.

In the general case, Equations 5 and 6 are modified as follows to give the components of voltage unbalance in 1 and 2, caused by arbitrary currents in 3 and 4, when circulation of power between 1 and 2 is prevented:

$$-\frac{(Z_{23}-Z_{13})+(Z_{24}-Z_{14})}{2}\times\frac{(I_3+I_4)}{2}=\Delta E' \quad (25)$$

$$-\frac{(Z_{23}-Z_{13})-(Z_{24}-Z_{14})}{2}\times\frac{(I_3-I_4)}{2}=\Delta E'' \quad (26)$$

Here, $\Delta E'$ is the voltage unbalance caused exclusively by the balanced components, and $\Delta E''$ that caused exclusively by the unbalanced components, of the currents in 3 and 4.

Frequently, it is desirable that such voltage unbalancing effect be absent, while sometimes it is desired that the voltage of one of the circuits increase with respect to that of another under specified conditions of loading, and Equations 25 and 26 show how either kind of result can be obtained quantitatively.

It will be evident further that there is a direct correspondence between circulating volt-amperes between a pair of windings when free to circulate power and the voltage unbalance when not free to circulate power; and this comment applies to both components of the unbalance distinguished in the foregoing.

It will be further evident that the various disclosures made in the foregoing for single-phase transformers are applicable to each phase of a polyphase transformer or bank of transformers.

While I have described the invention in the foregoing at considerable length and in terms of transformers with four or more separate windings for clarity of exposition of its principles, it will be evident to those skilled in the art that the various windings need not be separately insulated but may have parts in common as autotransformers, and that the various equations and relationships specified in the foregoing apply equally in all cases if the values of the various impedances involved in those equations and relationships are the values effective at the terminals of the respective circuits. Therefore, in all my references to transformer windings I wish to be understood broadly as including autotransformer windings as well as those which are separately insulated.

Furthermore, while I have shown and described my invention by means of particular representative embodiments thereof, it will be obvious to those skilled in the art that many changes and modifications may be made in them without departing from my invention; and I, therefore, aim to cover in the appended claims all such changes and modifications as fall within the true spirit and scope of my invention.

What I claim as new and desire to secure by Letters Patent of the United States is:

1. A multiwinding alternating current transformer having at least four windings designated respectively 1, 2, 3, and 4 in which the leakage impedances of said windings are prearranged to have the relationship

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{23} and Z_{24} represents respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscripts of the letter Z, so as to cause a predetermined division of current flow between one pair of windings independently of the relative loading of the other windings when said one pair of windings is connected for parallel operation and said other windings are connected to be loaded simultaneously and independently of each other.

2. A multiwinding alternating current transformer having at least four windings designated respectively 1, 2, 3, and 4 in which the leakage impedances of said windings are prearranged to have simultaneously the two relationships

$$(Z_{23}-Z_{13})+(Z_{24}-Z_{14})\cong 0$$

and

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 2Z_{12}$$

wherein Z_{12}, Z_{13}, Z_{14}, Z_{23} and Z_{24} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscripts of the letter Z, so as to cause the current in one of the windings of one pair of said windings to be substantially proportional to the current in one of the windings of another pair of said windings and the current in the other winding of said one pair to be substantially proportional to the current in the other winding of said other pair when said one pair of windings is connected for parallel operation and the windings of said other pair are connected to be loaded simultaneously and independently of each other.

3. A multiwinding alternating current transformer having at least four windings designated respectively 1, 2, 3, and 4 in which the leakage impedances of said windings are prearranged to have simultaneously the two relationships

$$(Z_{23}-Z_{13})+(Z_{24}-Z_{14})\cong 0$$

and

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{23} and Z_{24} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscript of the letter Z, so as to cause an equal division of load between one pair of windings independently of the relative loading of the other windings of said transformer when said one pair of windings is connected for parallel operation and when at least two other of said windings are loaded unequally.

4. In combination, a multiwinding alternating current transformer having at least four windings generally designated 1, 2, 3 and 4 and having the leakage impedances designated Z_{12}, Z_{13}, Z_{14}, Z_{23} and Z_{24} representing respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscripts of the letter Z prearranged to cause a division of load between windings 1 and 2 when operated in parallel to be in the ratio of $(1+k')$ to $(1-k')$ for a predetermined loading of windings 3 and 4, and a different division of load corresponding to the superposition of a circulating power in said windings 1 and 2 equal to k'' times one-half the difference between the loads of windings 3 and 4 when operated with unbalanced loads, said transformer

leakage impedances being proportioned in accordance with the simultaneous equations

$$\frac{(Z_{23}-Z_{13})+(Z_{24}-Z_{14})}{2Z_{12}}=k'$$

and

$$\frac{(Z_{23}-Z_{13})-(Z_{24}-Z_{14})}{2Z_{12}}=k''$$

5. A multiwinding alternating current transformer having at least four windings designated respectively 1, 2, 3, and 4 in which the leakage impedances of said windings are prearranged to have simultaneously the two relationships

$$(Z_{23}-Z_{13})+(Z_{24}-Z_{14})\cong 0$$

and

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{23} and Z_{24} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscript of the letter Z, so as to maintain the terminal voltages of two of its windings balanced independently of the relative loading of the remaining windings when said remaining windings are loaded.

6. A multiwinding alternating current transformer having at least four windings and generally designated respectively 1, 2, 3 and 4, in which the leakage impedances of said windings have simultaneously the two relationships

$$(Z_{23}-Z_{13})+(Z_{24}-Z_{14})\cong 0$$

and

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{23} and Z_{24} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscript of the letter Z.

7. A multiwinding alternating current transformer having at least four windings designated respectively 1, 2, 3, and 4 in which the leakage impedances are prearranged to have the relationship

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{23} and Z_{24} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscripts of the letter Z, so as to render the difference between the terminal voltages of a pair of said windings independent of any load unbalance in the remaining windings when said remaining windings are loaded.

8. A multiwinding alternating current transformer having at least four windings and generally designated respectively 1, 2, 3 and 4, in which the leakage impedances of said windings have the relationship

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{23} and Z_{24} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscript of the letter Z.

9. A multiwinding alternating current transformer having at least four windings and generally designated respectively 1, 2, 3 and 4 in which the leakage reactances of said windings have the relationship

$$(X_{23}-X_{13})-(X_{24}-X_{14})\cong 0$$

wherein X_{13}, X_{14}, X_{23} and X_{24} represent respectively the leakage reactance of each of the winding pairs corresponding to the digits of the subscript of the letter X.

10. A multiwinding alternating current transformer having at least four windings designated respectively 1, 2, 3, and 4 in which the leakage impedances are prearranged to have simultaneously the two relationships

$$(Z_{23}-Z_{13})+(Z_{24}-Z_{14})\cong 0$$

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\neq 0$$

wherein Z_{13}, Z_{14}, Z_{23} and Z_{24} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscripts of the letter Z, so as to cause the difference between terminal voltages of two of its windings to vary proportionally to the difference in the loads of two of its other windings when said other windings are loaded.

11. A multiwinding alternating current transformer having at least four windings and generally designated respectively 1, 2, 3 and 4 in which the leakage impedances of said windings have the relationship

$$(Z_{23}-Z_{13})+(Z_{24}-Z_{14})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{23} and Z_{24} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscript of the letter Z.

12. In an alternating current distribution system, a generating source, a bus member connected to said generating source, a pair of distribution branch circuits connected to said bus member, the total series impedance of said branch circuits being Z' and Z'' respectively, a multiwinding transformer having two windings generally designated 1 and 2 which are connected respectively to said branch circuits and having two additional windings generally designated 3 and 4 which are connected to be loaded simultaneously and unequally, said two branch circuits having a most advantageous division of power in the ratio of $(1+k')$ to $(1-k')$ when said windings 3 and 4 are equally loaded, and a different most advantageous division of power corresponding to the superposition of a circulating power in said two branch circuits equal to k'' times one-half the difference in the load of said windings 3 and 4, the leakage impedances Z_{12}, Z_{13}, Z_{14}, and Z_{23}, and Z_{24} which represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscripts of the letter Z having the two relationships

$$\frac{(Z_{23}-Z_{13})-(Z_{24}-Z_{14})}{2Z_{12}+2(Z''+Z')}=k''$$

and

$$\frac{(Z_{23}-Z_{13})+(Z_{24}-Z_{14})+2(Z''-Z')}{2Z_{12}+2(Z''+Z')}=k'$$

13. In combination, a multiwinding alternating current transformer having at least four windings generally designated 1, 2, 3 and 4 and having leakage impedances designated Z_{12}, Z_{13}, Z_{14}, Z_{23} and Z_{24} representing respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscript of the letter Z, an advantageous division of load between windings 1 and 2 when operated in parallel being in the ratio of $(1+k')$ to $(1-k')$ for equal loading of windings 3 and 4, and a different advantageous division of load corresponding to the superposition of a circulating power in said windings 1 and 2 equal to k'' times one-half the difference between the loads of windings 3 and 4 when operated with unbalanced loads, an impedance Z' connected in series with winding 1, and an imped-

ance Z'' connected in series with winding 2, said transformer leakage impedances and said series impedances Z' and Z'' being proportioned in accordance with the simultaneous equations

$$\frac{(Z_{23}-Z_{13})+(Z_{24}-Z_{14})+2(Z''-Z')}{2Z_{12}+2(Z''+Z')}=k'$$

and

$$\frac{(Z_{23}-Z_{13})-(Z_{24}-Z_{14})}{2Z_{12}+2(Z''+Z')}=k''$$

14. A multiwinding alternating current transformer having three pairs of windings designated respectively 1 and 2, 3 and 4, and 5 and 6, in which the leakage impedances of said windings are proportioned to have the following relationships

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 0$$

and

$$(Z_{25}-Z_{15})-(Z_{26}-Z_{16})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{15}, Z_{23}, Z_{24}, Z_{25}, and Z_{26} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscripts of the letter Z, so as to make the voltage relation of the terminal voltages of any one of said pairs of windings independent of the condition of relative loading of any one of the other pairs of windings when said other windings are loaded.

15. A multiwinding alternating current transformer having three pairs of windings designated respectively 1 and 2, 3 and 4, and 5 and 6, in which the leakage impedances of said windings are proportioned to have the following relationships

$$(Z_{23}-Z_{13})-(Z_{24}-Z_{14})\cong 0$$

and

$$(Z_{25}-Z_{15})-(Z_{26}-Z_{16})\cong 0$$

wherein Z_{13}, Z_{14}, Z_{15}, Z_{23}, Z_{24}, Z_{25} and Z_{26} represent respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscripts of the letter Z, so as to maintain the terminal voltages of a given pair of windings balanced independently of the condition of load unbalance within any one of the other pairs of windings.

16. A multiwinding alternating current transformer comprising a magnetic core having two winding legs, a pair of similar windings arranged one on each winding leg and distributed along substantially the full winding length of said winding leg, a second pair of windings arranged symmetrically on said two winding legs, each winding including two parts in parallel, one of said parts of one of said pair of windings being distributed along one-half of one of said winding legs and the second part along one-half of the other leg, the two parts of the second winding of said second pair of windings being distributed respectively on the remaining halves of said winding legs; and a third pair of windings arranged one on each one of said winding legs, each winding including two parts in parallel and distributed respectively along the two parts of the second pair of windings on the same core leg; said first and third pairs of windings being disposed on opposite sides of said second pair of windings.

17. An alternating current transformer comprising a magnetic core element, a pair of windings so disposed on said core element as to provide high leakage reactance between said pair of windings when not coupled through other windings, a second pair of windings on said core element and so disposed as to provide high leakage reactance between themselves and also between each one of them and each one of said first pair of windings, when not coupled through other windings, and a third pair of windings arranged on said core element and so disposed as to provide high leakage reactance between each one of said third pair of windings and one of the first pair of windings when not coupled through other windings, said second pair of windings so disposed as to couple closely said first pair of windings with each other and said third pair of windings with each other, and said third pair of windings so disposed as to couple closely said second pair of windings.

18. In an alternating current distribution system, two circuits in said system, a multiwinding transformer having at least four windings generally designated 1, 2, 3 and 4 and having leakage impedances designated Z_{12}, Z_{13}, Z_{14}, Z_{23} and Z_{24} representing respectively the leakage impedance of each of the winding pairs corresponding to the digits of the subscript of the letter Z, an advantageous division of load between windings 1 and 2 when operated in parallel being in the ratio of $(1+k')$ to $(1-k')$ for equal loading of windings 3 and 4, and a different advantageous division of load corresponding to the superposition of a circulating power in said windings 1 and 2 equal to k'' times one-half the difference between the loads of windings 3 and 4 when operated with unbalanced loads, said windings 1 and 2 being connected to said two circuits in a manner to permit a circulation of power between said windings 1 and 2 and said circuits, variable impedances Z' and Z'' connected external to said winding pair 1 and 2, respectively, to be traversed by said circulating power, means for loading said windings 3 and 4, means operatively connected with winding pair 1 and 2 to provide a variable voltage ΔE for modifying the power in said windings 1 and 2 irrespective of the condition of loading of said windings 3 and 4, said transformer leakage impedances being so proportioned and said variable impedances Z' and Z'' being so adjusted that the currents in windings 1 and 2 will have the following relation

$$I_1=-\left[\frac{(I_3+I_4)}{2}+k'\frac{(I_3+I_4)}{2}+k''\frac{(I_3+I_4)}{2}+\frac{\Delta E}{Z_{12}+(Z''+Z')}\right]$$

and

$$I_2=-\left[\frac{(I_3+I_4)}{2}-k'\frac{(I_3+I_4)}{2}-k''\frac{(I_3-I_4)}{2}-\frac{\Delta E}{Z_{12}+(Z''+Z')}\right]$$

where I_1 and I_2 are the currents of windings 1 and 2, I_3 and I_4 are the currents of windings 3 and 4, Z is the leakage impedance of the winding pairs indicated and ΔE is the voltage derived from the means to provide said variable voltage, and wherein

$$k'=\frac{(Z_{23}-Z_{13})+(Z_{24}-Z_{14})+2(Z''-Z')}{2Z_{12}+2(Z''+Z')}$$

and

$$k''=\frac{(Z_{23}-Z_{13})-(Z_{24}-Z_{14})}{2Z_{12}+2(Z''+Z')}$$

ALEXIS N. GARIN.

Certificate of Correction

Patent No. 2,264,836. December 2, 1941.

ALEXIS N. GARIN

It is hereby certified that errors appear in the printed specification of the above numbered patent requiring correction as follows: Page 1, second column, line 21, for "drawing" read *drawings*; page 2, first column, line 16, for "mathematic" read *mathematical*; page 5, first column, line 65, for "follow" read *follows*; line 69, for "$\bar{x}_{15} \cong \bar{x}_{25}$" read $\underline{\bar{x}}_{15} \cong \underline{\bar{x}}_{25}$; and line 72, for "$\bar{x}_{15}$" read $\underline{\bar{x}}_{15}$; and second column, line 4, for "$\bar{x}_{15}$" read $\underline{\bar{x}}_{15}$; line 6, for "$\bar{x}_{25}$" read $\underline{\bar{x}}_{25}$; line 14, for "$\bar{x}_{15} \cong \bar{x}_{25}$" read $\underline{\bar{x}}_{15} \cong \underline{\bar{x}}_{25}$; and line 16, for "$\bar{x}_{25}$" read $\underline{\bar{x}}_{25}$; page 6, second column, line 5, claim 1, for "represents" read *represent*; and that the said Letters Patent should be read with these corrections therein that the same may conform to the record of the case in the Patent Office.

Signed and sealed this 20th day of January, A. D. 1942.

[SEAL]

HENRY VAN ARSDALE,
Acting Commissioner of Patents.

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