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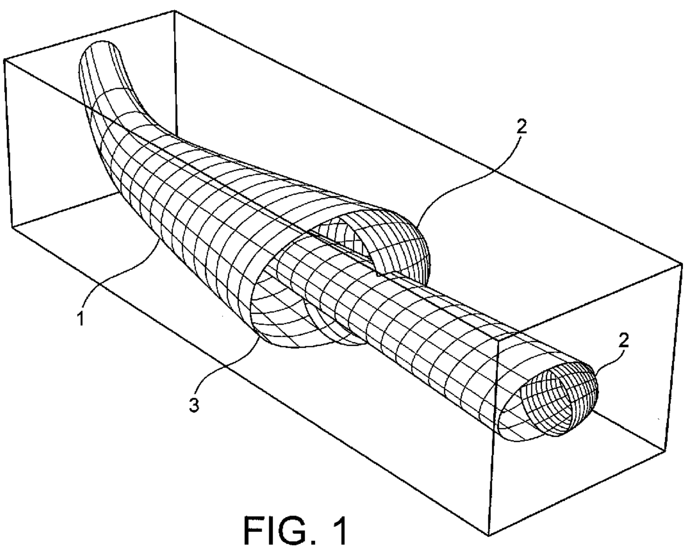


FIG. 1

**(57) Abstract**: A computer-implemented method and a system are provided for determining the relative positions of a wellbore and an object, the wellbore being represented by a first ellipse and the object being represented by a second ellipse. The first ellipse represents the positional uncertainty of the wellbore and the second ellipse represents the positional uncertainty of the object. The method comprises the steps of: receiving input data relating to a measured or estimated position of the wellbore and the object, the position of the wellbore having a first set of parameters defining the first ellipse, and the position of the object having a second set of parameters defining the second ellipse; calculating an expansion factor representing an amount by which one, or both, of the first ellipse and the second ellipse can be expanded with respect to one or both of respective first and second sets of elliptical parameters so that the first and second ellipses osculate, wherein calculating the expansion factor involves determining and solving a quartic equation that is based on the geometry of the ellipses; and determining, based on the calculated expansion factor, position data indicative of the relative positions of the wellbore and the object.

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# Wellbore Positioning System and Method

## Technical Field

The present invention relates to a computer-implemented method and a system for determining the relative positions of a wellbore and an object. Spatial relationships between two ellipses, each of which represents the positional uncertainty of a wellbore, are utilized to determine the conditions governing osculation between the two ellipses, expressing the determination as an expansion scale factor.

## Background

As the drilling of a wellbore in hydrocarbon reservoir (for example, an oil or gas reservoir) proceeds, the positional uncertainty at any point in a well is dependent on a number of factors, including the positional uncertainty of the surface location, the well's geographical location and trajectory and the various instruments used to survey the well. By positional uncertainty is meant positional uncertainty of the well's geographical location, positional uncertainty of its trajectory etc. The expected behaviours of these instrument types are presented as instrument performance models. Application of these models quantifies the uncertainty of the true wellbore position for a stated confidence.

Referring to Figure 1, the positional uncertainty 1 about a point representing the calculated position of the centre of a wellbore is commonly represented as an ellipsoid with its principal axes aligned with the high-side, right-side and along-hole directions. In this context, high-side is the direction normal to the wellbore in the vertical plane and right-side is the direction normal to both the wellbore direction and the high-side and so lies in the horizontal plane. The ellipsoid usually also accounts for the dimensions of the casing or open hole of the wellbore. The size of the ellipsoid varies according to the wellbore trajectory shape, survey instruments used in calculating the position, and selected confidence limits.

Using this model, at any time and point in space, which is to say positions down a wellbore and its surrounding volume, the resulting positional uncertainty about a wellbore along its trajectory is the envelope of the ellipsoids; a curved, continuous cone 2 with a curved end. The interference between two adjacent wells can be visualised as the interference between the two cones. The positional uncertainty can change over time as data is re-processed or more data is acquired. It also changes when the wellbore is resurveyed using a more accurate instrument system, for example when a high accuracy gyroscope is run at a casing point 3. This then narrows the cone, as shown in Figure 1. Therefore, if a new measurement is taken at a subsequent point along the trajectory of the wellbore, the positional uncertainty decreases, then increases as the distance from the measurement point increases.

To a good approximation, at any given point along the wellbore the intersection of a plane that is normal to the along-hole direction of the wellbore with the cone can be represented as an ellipse. Therefore, the problem of calculating the interference between two wells can be reduced to that of calculating the distance between two such ellipses. This simple geometrical model has been adopted by various standards organisations to define minimum acceptable separation distances between two wellbores, for example the Norwegian "Norsk Sokkels Konkurranseposisjon" (NORSOK) D-10.

The separation between wellbores in 3D can be represented in 2D using a collision avoidance plot, also known as a travelling cylinder or normal plane diagram. In this representation the intersection of, for example, an existing and a planned well (or two planned wells) is displayed on a plane, constructed normal to the planned well. The planned well is kept at the centre of the plot and therefore the relative separation between the planned well and the adjacent well is indicated by the locus of points obtained at successive depths. At any point in the subject well the plane also intersects the curved cone and at low or modest angles of incidence between wells the intersections with the cones appears, to good approximation as two ellipses. During drilling the as-drilled and projected positions are shown on the same plot. The planned well is also referred to as the subject or reference well.

If $x$ and $y$ are orthogonal coordinates in the normal plane then the separation $\delta$ between the wells can be calculated using Eqs. 1 to 3, below. In the absence of bias this is also the separation between the error ellipses. Further adjustments can be made if required.

$$\Delta x_0 = x_{0,2} - x_{0,1} \qquad \ldots(1)$$

$$\Delta y_0 = y_{0,2} - y_{0,1} \qquad \ldots(2)$$

$$\delta = \left(\Delta x_0^2 + \Delta y_0^2\right)^{\frac{1}{2}} \qquad \ldots(3)$$

In practical terms the minimum approach distance $\delta_{min}$ between the wellbores must be greater than the sum of the open hole and casing radii, $\delta_{min} > (d_h + d_c) / 2$, where $d_h$ is the hole diameter and $d_c$ is the casing diameter. This criterion automatically satisfies the mathematical constraint $\delta \neq 0$.

Currently, the relationship between two adjacent ellipses is approximated as a "separation factor", $k_s$. In this representation the ellipses are related only by the line passing through their centres. Because of this, the calculation of the characteristic length $s$ for each ellipse may be performed independently of the other. Two common methods are the centre vector method (CVM) and pedal curve method (PCM).

$$k_s = \frac{\delta}{s_1 + s_2} \qquad \ldots(4)$$

Because of mathematical difficulties, existing methods for calculating separation factors are approximations and may be either too optimistic or too conservative, particularly for ellipses with high eccentricities.

For example, Figure 2 shows how the currently used "centre vector method" (CVM) is used to calculate a separation factor between two wellbores. In the CVM the characteristic lengths $s_1$ and $s_2$ are determined from the point of intersection of each ellipse, marked A and B in Figure 2, with the line $\delta$ joining their centres. The separation factor $k_{CVM}$ is calculated using Eq. 4. In this case the ellipses extend beyond their points of intersection and will touch before the separation factor reaches unity. Therefore, separation factors calculated using this method may be too optimistic. Such overly optimistic calculations of the separation factor can lead to safety issues when planning and drilling wells based on computer simulations.

Figure 3 shows an alternative method of calculating a separation factor between two wellbores, the "pedal curve method" (PCM). In the PCM, the characteristic lengths $s_1$ and $s_2$ are determined from the line that is both tangent to the ellipse and is orthogonal to the line $\delta$ joining their centres. The first step is to determine the points of tangency, marked A and B in Figure 3. In this case the tangent lines meet and the separation factor $k_{PCM}$ reaches unity before the ellipses touch. Therefore the separation factors calculated using this method may be too conservative, leading to unnecessary shut-in of wells or missed opportunities.

Although the separation factors calculated by either the centre vector or pedal curve methods are relatively easy to calculate, neither method is a faithful representation of the geometrical relationship between the two ellipses. As shown in Figures 4a and 4b, calculating the separation factor in terms of an "expansion factor", $k$, by the simultaneous and equal expansion ($k > 1$) or contraction ($k < 1$) of both ellipses until they touch, is neither too optimistic nor too pessimistic. This expansion factor calculation can increase the allowable proximity between two adjacent wells whilst satisfying the geometrical and probabilistic constraints. Although iterative methods can be used starting from the elliptical conditions, there is no guarantee that such iterative schemes converge towards the correct expansion factor solution.

<u>Summary</u>

In embodiments of the invention, there is provided a computer-implemented method and a system according to the appended claims.

According to an embodiment of the invention, there is provided a computer-implemented method for determining the relative positions of a wellbore and an object, the wellbore being represented by a first ellipse and the object being represented by a second ellipse, wherein the first ellipse represents the positional uncertainty of the wellbore and the second ellipse represents the positional uncertainty of the object, the method comprising the steps of:

receiving input data relating to a measured or estimated position of the wellbore and the object, the position of the wellbore having a first set of parameters defining the first ellipse, and the position of the object having a second set of parameters defining the second ellipse;

calculating an expansion factor representing an amount by which one, or both, of the first ellipse and the second ellipse can be expanded with respect to one or both of respective first and second sets of elliptical parameters so that the first and second ellipses osculate, wherein calculating the expansion factor involves determining and solving a quartic equation that is based on the geometry of the ellipses; and

determining, based on the calculated expansion factor, position data indicative of the relative positions of the wellbore and the object.

Embodiments of the present invention utilize spatial relationships between two ellipses for determining the conditions governing osculation between the two ellipses (where osculation is the case in which the ellipses touch), expressing the determination as an expansion scale factor. Each expansion factor calculation involves using the smallest positive root of the quartic equation. The explicit schemes of the present invention offer improvements in both calculation efficiency and reliability over known methods of calculating a separation factor and over iterative methods of calculating an expansion factor.

Typically, the wellbore is a first wellbore, and the object is a second wellbore. Alternatively, the object may be a sub-surface hazard that is to be avoided when drilling the wellbore.

Methods are presented for the expansion of either one, or both ellipses. The computer-implemented methods can be used to increase the allowable proximity of two adjacent wellbores whilst satisfying the necessary geometrical and probabilistic constraints. The calculation method is consistent with existing industry wellbore uncertainty models. Since the determination of the osculating condition is exact the calculation is neither too optimistic nor too conservative.

Further features and advantages of the invention will become apparent from the following description of preferred embodiments of the invention, given by way of example only, which is made with reference to the accompanying drawings.

Brief Description of the Drawings

Figure 1 shows a three-dimensional representation of a cone which represents the positional uncertainty of a wellbore;

Figure 2 shows a "centre vector method" for estimating the separation between two ellipses;

Figure 3 shows a "pedal curve method" for estimating the separation between two ellipses;

Figure 4a shows the determination of an "expansion factor" by the simultaneous and equal expansion ($k>1$) of two ellipses;

Figure 4b shows the determination of an "expansion factor" by the simultaneous and equal contraction ($k<1$) of two ellipses;

Figure 5a shows the steps involved in a first method of calculating an expansion factor;

Figures 5b-5e show an expansion of the ellipses carried out in the first method of calculating the expansion factor;

Figure 6a shows the steps involved in a second method of calculating an expansion factor;

Figure 6b shows an expansion of the ellipses carried out in the second method of calculating the expansion factor;

Figures 7a-9b show dual and single sided expansion of various configurations of ellipses;

Figure 10 shows a wellbore positioning system according to the present invention;

Figure 11 shows an example of a planned wellbore in simplified collision avoidance plot;

Figure 12 shows the steps taken in determining the relative position of a wellbore according to the present invention;

Figure 13 shows a schematic diagram of a wellbore being drilled into a formation.

Detailed Description

In directional work an ellipse is generally defined by its centre $(x_0, y_0)$, the lengths of its semi-major and semi-minor axes $a$ and $b$ and the orientation $\theta$ of the major axis direction $\underline{a}$ relative to some reference direction.

Since the dimensions of each ellipse represents some confidence interval that the wellbore lies within its boundary, the equal expansion or contraction of both ellipses until they touch (i.e. osculate) is a measure of a potential collision between the wells. Since the point at which two ellipses touch is a function of both their sizes and orientations, conceptually, the available space can also be calculated by expanding only one ellipse with the other one fixed. Therefore both dual sided and single sided expansion can be applied to calculate a relevant expansion factor $k$.

Mathematically, it may be more convenient to represent the ellipse as a quadratic form, as shown by Eq. 5, incorporating the above elliptical parameters and the expansion factor $k$ within the quadratic form's coefficients, which are shown by Eqs. 6 to 11. Details of both the transform and inverse transform are given in Appendix A.

$$E(x,y,k) = Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Fy + H - a^2b^2k^2 = 0 \quad \ldots(5)$$

Where

$$A = b^2\cos^2\theta + a^2\sin^2\theta \quad \ldots(6)$$

$$B = (b^2 - a^2)\sin\theta\cos\theta \quad \ldots(7)$$

$$C = a^2\cos^2\theta + b^2\sin^2\theta \quad \ldots(8)$$

$$D = -y_0B - x_0A \quad \ldots(9)$$

$$F = -x_0B - y_0C \quad \ldots(10)$$

$$H = x_0^2A + 2x_0y_0B + y_0^2C \quad \ldots(11)$$

The quadratic may be represented in matrix form, as shown by Eq. 12, where $\underline{E}$ is the 3 x 3 symmetric matrix. It is noted that the expansion factor appears in only one of the matrix elements as its square $k^2$.

$$E(x,y,k) = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} A & B & D \\ B & C & F \\ D & F & H - a^2b^2k^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \ldots(12)$$

It is also possible to represent the ellipse in matrix form so that the square symmetric matrix is independent of the ellipse's origin, (Zheng, X., Palffy-Muhoray, P.: "Distance of Closest Approach of Two Arbitrary Hard Ellipses in 2D"). A summary of the representation is included in Appendix A.

## ZPM Expansion Factor

The following calculation, referred to hereinafter as the "ZPM" method, can be used to calculate the expansion factor using dual sided expansion, where each of two ellipses is expanded equally.

Referring to Figure 5a, in step S501 the elliptical parameters $a_1, b_1, \theta_1, x_{0,1}, y_{0,1}$ of ellipse $E_1$ and $a_2, b_2, \theta_2, x_{0,2}, y_{0,2}$ of ellipse $E_2$ are input into a wellbore positioning system, as described below with respect to Figure 10. In step S502, a determination is made as to whether the centres of the ellipses are separated by a distance greater than $\delta_{min}$, as explained above in relation to Eq. 3. If the separation is not greater than $\delta_{min}$, it is determined in step S503 that the wellbores physically interfere and the calculation is stopped in step S504. Alternatively, if the ellipse centres are separated by more than $\delta_{min}$, the distance of closest approach $\delta_{cr}$ is calculated in step S505, as explained below.

The distance of closest approach $\delta_{cr}$ of two arbitrary hard ellipses in 2D can be determined using the method disclosed in Zheng, X., Palffy-Muhoray, P.: "Distance of Closest Approach of Two Arbitrary Hard Ellipses in 2D". Referring to Figure 5b, the ellipse $E_2$ is translated towards $E_1$ in the direction joining their centres until it reaches the position $E_2$* when the ellipses touch externally. The orientations of the two ellipses are maintained throughout. The ellipse $E_1$ is then transformed into a circle $C_1$ and the same mathematical transformation used to obtain the circle is applied to the ellipse $E_2$ (Figure 5c). Note that the circle $C_1$ and the ellipse $E_2$** remain connected at a respective tangent after the transformation. The closest approach between the circle and the ellipse is then found analytically, recovering the closest approach between the ellipses $E_1$ and $E_2$* by applying the inverse of the transformation used to obtain the circle (Figure 5d). Details of the relevant calculations are explained in the paper by Zheng and Palffy-Muhoray, together with the solution of the resulting quartic equation; the quartic equation is given in Appendix B.

The problem's symmetry is then used to determine the expansion factor (Figure 5e). It is noted that the translation of $E_2$ to $E_2$* followed by a magnification of magnitude $k$, of both $E_1$ and $E_2$* together (whilst maintaining their relative position) about the centre of $E_1$ is equivalent to the magnification of magnitude $k$ of each of $E_1$ and $E_2$ about their respective centres, (Snapper, E., Troyer, R.J.: "Metric Affine Geometry", 1971, Academic Press, London, 1, 36-55). Therefore, the dual sided expansion factor $k$ can be calculated from the distance of closest approach using the scaling factor $k=\delta/\delta_{cr}$, and the expansion factor $k$ is output (step S507).

Zheng and Palffy-Muhoray also describe a method for calculating the contact point and provided computer code for both of the closest approach and contact point calculations. Knowledge of the contact point may be used to verify the expansion factor results, checking for each ellipse that $|E(x, y, k)| < \varepsilon$, where $\varepsilon$ is some acceptable tolerance.

## YKC Expansion Factor

A further method, referred to hereinafter as the "YKC" method, can be used to calculate an expansion factor using dual sided expansion, where each of the two ellipses is expanded equally, or single sided expansion, where only one ellipse is expanded while the other remains fixed. However, for dual expansion the ZPM approach is preferred. Tests show that it is more stable computationally, particularly for similarly sized ellipses with centres that are close together.

Referring to Figure 6a, in step S601 the elliptical parameters $a_1, b_1, \theta_1, x_{0,1}, y_{0,1}$ of ellipse $E_1$ and $a_2, b_2, \theta_2, x_{0,2}, y_{0,2}$ of ellipse $E_2$ are input into a wellbore positioning system, as described below with respect to Figure 10. In step S602, a determination is made as to whether dual sided or single sided expansion is preferred. Single sided expansion may be preferred in some cases because of the greater area of space obtained about the expanded wellbore.

***Single Sided Expansion***

For single sided expansion (output "Y" at step S602), the symmetry present in the dual sided expansion is broken and a different approach must be used. Referring to Figure 6a, in this case the size of the first ellipse $E_1$ is fixed and for a solution to exist, the centre $(x_{0,2}, y_{0,2})$ of the second ellipse $E_2$ must lie outside its boundary (step S603). Mathematically this requires the condition that $E_1(x_{0,2}, y_{0,2}, 1) > 0$. In step S604, if the centre of $E_2$ does not lie outside of $E_1$, the system determines that no solution is possible, and the calculation is stopped at step S605.

A characteristic cubic polynomial $P(\lambda) = \det(\lambda \underline{E}_1 - \underline{E}_2) = 0$, which can be used to determine the separation conditions between two ellipses without explicitly calculating the contact point, was derived in Choi, Y.K.: "Collision Detection for Ellipsoids and Other Quadrics", PhD Thesis, University of Hong Kong, March 2008. Choi showed that if $E_1$ and $E_2$ are two ellipses with the characteristic polynomial $P(\lambda)$ (where $\lambda$ is a multiplier) then they are separated if and only if $P(\lambda)$ has two distinct negative roots and they touch each other externally if and only if $P(\lambda)$ has a double negative root. The ellipses are overlapping if $P(\lambda)$ has no negative root.

For the purpose of calculating the expansion factor, the expansion factor can be incorporated in the characteristic polynomial giving $P(\lambda) = \det[\lambda\, \underline{E}_1(k_1) - \underline{E}_2(k_2)] = 0$. For a single sided expansion the first ellipse $E_1$ is fixed so set $k_1 = 1$ and $k_2 = k$. The characteristic, cubic polynomial becomes $P(\lambda) = \det[\lambda\, \underline{E}_1 - \underline{E}_2(k)] = 0$ (step S606). Using Choi's condition, the cubic's discriminant vanishes when the ellipses touch, leaving a quartic equation in $k^2$, as shown by Eq.13. Taking the square root gives the expansion factor $k$.

$$\gamma_4 k^8 + \gamma_3 k^6 + \gamma_2 k^4 + \gamma_1 k^2 + \gamma_0 = 0 \qquad \ldots(13)$$

After lengthy, computer assisted simplification, using a software program such as Mathematica, the coefficients of the quartic equation can be written as Eqs. 14 to 26. Further details are provided in Appendix B.

$$\gamma_4 = a_2^4 b_2^4 \left(z_{12}^2 - 4a_1^2 a_2^2 b_1^2 b_2^2\right) \qquad \ldots(14)$$

$$\gamma_3 = 2a_2^2 b_2^2 \left[6a_1^2 a_2^2 b_1^2 b_2^2 \left(a_2^2 p_1 + b_2^2 q_1\right) + 9a_1^2 a_2^2 b_1^2 b_2^2 z_{12}\ a_2^2 b_2^2 r_1 z_{12} - z_{12}^2 \left(a_2^2 p_1 + b_2^2 q_1\right) - 2z_{12}^3\right] \qquad \ldots(15)$$

$$\begin{aligned}\gamma_2 = &-27a_1^4 b_1^4 a_2^4 b_2^4 + a_2^4 b_2^4 r_1^2 - 6a_1^2 b_1^2 a_2^2 b_2^2 \times \left[2\left(a_2^2 p_1 + b_2^2 q_1\right)^2 + 3a_2^2 b_2^2 r_1\right] \\ &- 2a_2^2 b_2^2 z_{12}\left(a_2^2 p_1 + b_2^2 q_1\right)\left(9a_1^2 b_1^2 - 2r_1\right) + z_{12}^2\left(a_2^2 p_1 + b_2^2 q_1\right)^2 + 12a_2^2 b_2^2 r_1 \left[\left(a_2^2 b_1^2 + a_1^2 b_2^2\right)\right. \\ &\left.\times \cos^2(\theta_1 - \theta_2) + \left(a_1^2 a_2^2 + b_1^2 b_2^2\right)\sin^2(\theta_1 - \theta_2)\right]^2 \end{aligned} \qquad \ldots(16)$$

$$\gamma_1 = 2\left(\left[a_2^2 p_1 + b_2^2 q_1\right]\left\{-a_2^2 b_2^2 r_1^2 + a_1^2 b_1^2 \left[2\left(a_2^2 p_1 + b_2^2 q_1\right)^2 + 9a_2^2 b_2^2 r_1\right]\right\} - r_1 z_{12}\left[\left(a_2^2 p_1 + b_2^2 q_1\right)^2 + 6a_2^2 b_2^2 r_1\right]\right) \qquad \ldots(17)$$

$$\gamma_0 = r_1^2 \left[4a_2^2 b_2^2 r_1 + \left(a_2^2 p_1 + b_2^2 q_1\right)^2\right] \qquad \ldots(18)$$

Where

$$\varphi_1 = \Delta y \cos\theta_1 - \Delta x \sin\theta_1 \qquad \ldots(19)$$

$$\vartheta_1 = \Delta x \cos\theta_1 + \Delta y \sin\theta_1 \qquad \ldots(20)$$

$$\varphi_2 = \Delta y \cos\theta_2 - \Delta x \sin\theta_2 \qquad \ldots(21)$$

$$\vartheta_2 = \Delta x \cos\theta_2 + \Delta y \sin\theta_2 \quad \ldots(22)$$

$$p_1 = \varphi_2^2 - a_1^2 \sin^2(\theta_1 - \theta_2) - b_1^2 \cos^2(\theta_1 - \theta_2) \quad \ldots(23)$$

$$q_1 = \vartheta_2^2 - a_1^2 \cos^2(\theta_1 - \theta_2) - b_1^2 \sin^2(\theta_1 - \theta_2) \quad \ldots(24)$$

$$r_1 = b_1^2 \vartheta_1^2 + a_1^2(\varphi_1^2 - b_1^2) \quad \ldots(25)$$

$$z_{12} = a_1^2\left[a_2^2 \sin^2(\theta_1 - \theta_2) + b_2^2 \cos^2(\theta_1 - \theta_2)\right] + b_1^2\left[a_2^2 \cos^2(\theta_1 - \theta_2) + b_2^2 \sin^2(\theta_1 - \theta_2)\right] \quad \ldots(26)$$

By inspection, this calculation of the closest distance of a point (which may represent an object) to an ellipse is equivalent to the single sided expansion of a unit circle (which is a special case of an ellipse) centred on the point against the ellipse, as shown in Figure 6b. This distance is equal to the expansion factor $k$ (step S607). Once calculated, the expansion factor is output at step S608.

***Dual Sided Expansion***

For dual sided expansion, in step S609 a determination is made as to whether the centres of the ellipses are separated by a distance greater than $\delta_{min}$, as explained above in relation to Eq. 3. If the separation is not greater than $\delta_{min}$, it is determined in step S610 that the wellbores physically interfere and the calculation is stopped in step S611. Alternatively, if the ellipse centres are separated by more than $\delta_{min}$, dual sided expansion can proceed; both ellipses are expanded equally so set $k_1 = k_2 = k$. The characteristic polynomial becomes $P(\lambda) = \det[\lambda\, \underline{E}_1(k) - \underline{E}_2(k)] = 0$ (step S612). This results in another quartic equation in $k^2$; details of the coefficients are provided in Appendix B. When solved (step S613), the smallest positive root of the equation gives the expansion factor $k$, at which point the calculation stops (step S614).

Examples

Some examples of elliptical configurations are shown in Figures 7a to 9b. The configurations of two ellipses on Figures 7a, 8a and 9a correspond to the configurations in Figures 7b, 8b and 9b, respectively. In Figures 7a, 8a and 9a, the dashed ellipses represent the expanded, osculating ellipses when a dual expansion method is used. Figures 7b, 8b and 9b the dashed ellipse represents the expansion of one of the ellipses in a single sided expansion.

In using the ZPM dual sided expansion method and the YKC single sided expansion method, the expansion factors for these configurations are calculated as follows:

Figure 7a – the dual sided expansion for the ellipses $E_1(3, 2, 0°, 0, 0)$ and $E_2(4, 2,$

90°, 8, 0) gives the expansion factor $k = 1.6$;

Figure 7b – the single sided expansion for the ellipses $E_1(3, 2, 0°, 0, 0)$ and $E_2(4, 2, 90°, 8, 0)$ gives the expansion factor $k = 2.5$;

Figure 8a – the dual sided expansion for the ellipses $E_1(3, 2, 30°, -3, -2)$ and $E_2(2, 1, 135°, 1, 1)$ gives the expansion factor $k = 1.25568$;

Figure 8b – the single sided expansion for the ellipses $E_1(3, 2, 30°, -3, -2)$ and $E_2(2, 1, 135°, 1, 1)$ gives the expansion factor $k = 2.01033$;

Figure 9a – the dual sided expansion for the ellipses $E_1(7, 1, 135°, -2, 0)$ and $E_2(5, 1, 150°, 1, 0)$ gives the expansion factor $k = 0.814767$; and

Figure 9b – the single sided expansion for the ellipses $E_1(7, 1, 135°, -2, 0)$ and $E_2(5, 1, 150°, 1, 0)$ gives the expansion factor $k = 0.695637$.

Referring to the configurations of Figures 7a, 8a and 9a, Table 1 shows a comparison of the CVM and PCM separation factors ($k_{CVM}$ and $k_{PCM}$, respectively) with the dual sided expansion factor ($k_{ZPM}$) for the three elliptical configurations.

| Ellipses | | Factor | | |
|---|---|---|---|---|
| $E_1$ | $E_2$ | $k_{CVM}$ [-] | $k_{PCM}$ [-] | $k_{ZPM}$ [-] |
| 3,2,0°,0,0 | 4,2,90°,8,0 | 1.60000 | 1.60000 | 1.60000 |
| 3,2,30°,-3,-2 | 2,1,135°,1,1 | 1.25593 | 1.24452 | 1.25568 |
| 7,1,135°,-2,0 | 5,1,150°,1,0 | 0.91190 | 0.32055 | 0.81477 |

From the separation and expansion factors of Table 1, it can be seen that the factors are calculated to be the same value (1.6) for the configuration of Figure 7a; calculations agree only in a special case where the major or minor axes of the ellipses are collinear. The differences in the calculated factors in any particular case may be much more pronounced as eccentricities increase.

System

As shown in Figure 13, wellbore drilling systems generally comprise drilling equipment 4 arranged to drill a wellbore 5 into the one or more hydrocarbon-bearing reservoirs in a formation 6. The drilling system typically comprises a controller 7 arranged to control the drilling equipment. An existing wellbore 8 is also shown.

In order to determine optimum settings of the various components of the wellbore drilling system, the wellbore positioning system 100 comprises suitable computer-implemented models, software tools and hardware, as shown in Figure 10. A reservoir model 121 may be employed. As known in the art, a reservoir model is a conceptual 3-dimensional construction of a reservoir that is constructed from incomplete data with much of the inter-well space estimated from data obtained from nearby wells or from seismic data. In conjunction with this, a trajectory model 123, that is, a computer model that constructs 2D and/or 3D representations of the geographical locations and/or trajectories of wellbores may be employed. The trajectory model may comprise or make use of a collision avoidance plot, also known as a travelling cylinder or normal plane diagram. An expansion factor calculation tool 111, as described further below, can calculate the expansion factor as explained above. Using the reservoir model 121, the trajectory model 123 can use information such as the volume and shape of the reservoir 3 (including the arrangement of overlying rock formations and the locations of any faults or fractures in the rock formations and sub-surface hazards), the porosity of the oil-bearing rock formations, the location of existing production well(s) and injection well(s), in combination with the results of the expansion factor calculation tool 111, to provide an indication as to the possible trajectory of a planned wellbore.

In one arrangement, referring to Figure 10, the expansion factor calculation tool 111 and optionally the reservoir model 121, the trajectory model 123 and an optimisation tool 125 are executed by the wellbore positioning system 100. The wellbore positioning system 100, which is for example a control system on a platform, can comprise conventional operating system and storage components such as a system bus connecting a central processing unit (CPU) 105, a hard disk 103, a random access memory (RAM) 101, and I/O and network adaptors 107 facilitating connection to user input/output devices and interconnection with other devices on a network N1. The Random Access Memory (RAM) 101 contains operating system software 131 which controls, in a known manner, low-level operation of the wellbore positioning system 100. The server RAM 101 contains the software tools and models 111, 121, 123 and 125 during execution thereof. Each item of software is configurable with measurement and/or predetermined data stored in a database or other storage component which is operatively coupled or connected to the wellbore positioning system 100; in the system of Figure 2, storage component DB1 stores all such data relating to the expansion factor calculation tool 111 and is accessible thereby, while storage component DB2 stores all other data for use by the other components of the system 100.

Input data received by receiving means of the system 100 comprise the elliptical parameter values and are based on a measured position of an existing wellbore or an estimated (i.e. modelled or simulated) position of a planned wellbore. Such estimated input data can be modelled or estimated upon planning a wellbore, for example upon an initial assessment or appraisal of a reservoir when developing a new field. Alternatively, in the case where the position of a planned wellbore is being determined in order to avoid an object other than another wellbore, such as a sub-surface hazard, the input data includes measurement data relating to the position of the object.

The measurement data may comprise specific measured values as directly measured by suitably positioned measurement equipment such as survey instruments 12, or may comprise values derived from a number of separate positional measurements. Therefore, the raw measured data may, if necessary or preferred, be manipulated by appropriate software and executed by the CPU 105 of the system 100, in order to generate measurement or estimated position data that are suitable for inputting into the expansion factor calculation tool 111. Such manipulation may comprise using the reservoir and/or trajectory models to determine the parameter values of the two ellipses.

The expansion factor calculation tool 111 may comprise a software program such as Mathematica. This program can be used in a number of ways during the calculation of the expansion factor. Firstly by making use of its symbolic manipulation, the substitutions, for example, for A, B, C, D, F, G (which is equivalent to $H - a^2b^2k^2$ – see Appendix A), H can be made. The determinants can then be expanded and the equations simplified using this program. Additionally, Mathematica™ is preferably employed to program the resulting quartic coefficients and solve the quartic equation. Alternatively, the expressions can be programmed in, for example, Visual Basic™ within an EXCEL™ spreadsheet.

An optimisation tool 125 may be provided to assist in the planning and drilling of wellbores. The optimisation tool may be used in conjunction with the trajectory model 123 to compute an optimal position for the wellbore in 2D or an optimal trajectory in 3D, based on input data including the calculated expansion factor and the measured or estimated input data that relates to the position of one or more existing wellbores or objects. In the case where a number of positions or trajectories are possible, the optimisation tool 125 may be programmed with rules that take into account additional data representing, for example, threshold values representing practical limits to the degree of curvature of the wellbore trajectory. In this way, the optimisation tool 125 can determine an optimum alignment of the trajectory, as explained further below with reference to Figure 11.

Figure 11 shows a simplified collision avoidance plot which may be produced by the trajectory model 123 upon calculation of the expansion factor; the x and y axes represent length in metres. In Figure 11 the dashed ellipse represents the tolerable errors, including an acceptable operational margin, for a planned wellbore at some point in space. The solid ellipses represent the tolerable errors surrounding three adjacent, drilled wellbores. By inspection, at this position in the wellbore the planned wellbore is heavily constrained and its position at this point cannot be moved within the collision avoidance plot without infringing the space in which the other wellbores may lie. The centre vector method is generally excluded in such a scenario as it is overly optimistic. Using the pedal curve method, a well planner would conclude that the planned wellbore could not be threaded through this point. A well planner using the expansion factor calculation method, which honours the geometry, would conclude that the well could, with care, pass through this point. This is confirmed by the common sense approach that, visually, the dashed ellipse fits comfortably within the available space.

The use of the expansion factor in the wellbore positioning method and system of the invention is advantageous in the planning and drilling of wellbores, as it provides more space in which to plan and optimise the trajectories of wellbores. However, if a planner concluded that it was not possible to drill through the gap of Figure 11, then the wellbore would have to be planned around the existing wellbores. Such activities add to the tortuosity of the wellbore's trajectory, which increases torque and drag forces, and/or may be difficult to achieve with the available tools. In some cases the detour may not be possible. In subsurface terms, the detour may make it difficult to achieve optimum alignment to a target. If so, oil and gas reserves and production may be adversely affected.

The wellbore positioning system 100 is preferably operatively connected to a controller 133 of the wellbore drilling system, for example via the network N1. The controller 133 of the wellbore drilling system is automatically configured with the one or more operating modes determined by the system 100, the controller 133 being arranged to apply the one or more operating modes.

Method

Referring to Figure 12, the steps involved in a first embodiment of a computer-implemented method for determining one or more operating modes for the wellbore drilling system are shown.

In step S1201, the input data is received by the wellbore positioning system 100.

At step S1202, the input data are input into the expansion factor software tool 111, the calculations of which are described above in relation to Figures 5a-5e, 6a and 6b. The expansion factor calculation tool is then run in step S1203, and generates, at step S1204, position data indicative of a relative position or proximity of the planned wellbore to the existing or simulated wellbore or object. This data may be output in various forms, for example, as coordinates of a 2D or 3D simulation of a reservoir, or as a collision avoidance plot.

At step S1205, the generated position data are used to determine one or more operating modes of the wellbore drilling system. The operating mode can represent an instruction or suggested setting for the drilling system, which can subsequently be applied to the drilling system. The determination can include the step of comparing, in accordance with a predetermined set of rules (which can be set using a collision avoidance plot implemented by the trajectory model 123), the calculated position data to predetermined known or threshold position data that is accessible from the database DB2. For example, the determination may be based on a known position of an existing wellbore or a sub-surface hazard.

Software executed by the CPU 105 of the system 100 determines, on the basis of the determined position data, the one or more operating modes of the wellbore drilling system. The expansion factor calculation tool 111, the reservoir model 121 and/or the trajectory model 123 may be configured to determine the operating mode(s) upon generation of the position data, or a separate software component may be provided. Additional technical and physical constraints determined by the reservoir model 121 or the trajectory model 123 may be taken into account in order to determine the operating mode, and can be stored and accessed from the databases DB1 and DB2 as necessary.

For example, the operating mode can comprise an instruction to go ahead with the drilling of a planned wellbore or not, this determination being based on a determination by the trajectory model 123 that the trajectory of the planned wellbore under consideration is drillable. Alternatively or additionally, the operating mode can comprise one or more specific configuration settings for the wellbore drilling system, such as a drilling speed or trajectory.

The software component used to determine the operating mode is configured to use a predetermined set of rules in conjunction with input data such as the calculated expansion factor, in order to determine the operating mode. These rules are stored in and accessible from the database DB1 and DB2 as necessary.

The computer-implemented method can further include an optional step, S1206, of applying or inputting the determined operating mode into a controller of the wellbore drilling system.

The above embodiments are to be understood as illustrative examples of the invention. It is to be understood that any feature described in relation to any one embodiment may be used alone, or in combination with other features described, and may also be used in combination with one or more features of any other of the embodiments, or any combination of any other of the embodiments. Furthermore, equivalents and modifications not described above may also be employed without departing from the scope of the invention, which is defined in the accompanying claims.

## Appendix A

### Ellipse Representations

The derivations using the YKC conditions depend on the ability to translate freely between the ellipse representations. The first and second quadratic forms are mathematically equivalent.

### First Quadratic Form

The ellipse $E_0(x, y) = b^2 x^2 + a^2 y^2 - a^2b^2 = 0$ with semi-major axis $a$ and semi-minor axis $b$, aligned with the $x$ and $y$ axes and centred on the origin can be represented as a quadratic form, Eq. A-1.

$$E_0(x,y)=\begin{bmatrix}x & y & 1\end{bmatrix}\begin{bmatrix}b^2 & 0 & 0\\ 0 & a^2 & 0\\ 0 & 0 & -a^2b^2\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix} \qquad \ldots\text{(A-1)}$$

Writing the ellipse $E_0(x, y) = \underline{x}\,\underline{E}_0\,\underline{x}^T$ gives

$$\underline{E}_0=\begin{bmatrix}b^2 & 0 & 0\\ 0 & a^2 & 0\\ 0 & 0 & -a^2b^2\end{bmatrix} \qquad \ldots\text{(A-2)}$$

The matrix $\underline{T}$ translates a point on the ellipse by an amount $x_0$ in the $x$ direction and $y_0$ in the $y$ direction. The rotation matrix $\underline{R}$ rotates a point by an amount $\theta$ clockwise about the origin. The scaling matrix $\underline{S}$ scales a point by a factor $k$ relative to the origin.

$$\underline{T}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -x_0 & -y_0 & 1\end{bmatrix} \qquad \ldots\text{(A-3)}$$

$$\underline{R}=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \qquad \ldots\text{(A-4)}$$

$$\underline{S}=\begin{bmatrix}k^{-1} & 0 & 0\\ 0 & k^{-1} & 0\\ 0 & 0 & 1\end{bmatrix} \qquad \ldots\text{(A-5)}$$

Note that the translation, rotation and scaling operations do not commute and therefore the order in which these operations are performed is important. Combining these transformations as shown in Eq. A-6 gives the ellipse $E(x, y, k)$ in the body of the paper, Eqs. 5 and 12.

$$\underline{E} = \underline{x}\,\underline{T}\,\underline{R}\,\underline{S}\,\underline{E}_0\,\underline{S}^T\,\underline{R}^T\,\underline{T}^T\,\underline{x}^T \qquad \text{...(A-6)}$$

## *Inverse Transform*

The coordinates of the ellipse's centre, semi-major and semi-minor axes and orientation may be recovered from the first quadratic form using the inverse transform, Eqs. A-7 to A-11. Although the inverse transform is not used in either the separation or expansion factor calculations it provides an effective means of testing the correctness of the transform. Note that the constant $G$ is equivalent to $H - a^2b^2k^2$.

$$x_0 = \frac{CD - BF}{B^2 - AC} \qquad \text{...(A-7)}$$

$$y_0 = \frac{AF - BD}{B^2 - AC} \qquad \text{...(A-8)}$$

$$a = \sqrt{\frac{2\left(AF^2 + CD^2 + GB^2 - 2BDF - ACG\right)}{\left(B^2 - AC\right)\left[\sqrt{(A-C)^2 + 4B^2} - (A+C)\right]}} \qquad \text{...(A-9)}$$

$$b = \sqrt{\frac{2\left(AF^2 + CD^2 + GB^2 - 2BDF - ACG\right)}{\left(B^2 - AC\right)\left[-\sqrt{(A-C)^2 + 4B^2} - (A+C)\right]}} \qquad \text{...(A-10)}$$

$$\theta = \begin{cases} 0 & B = 0, A < C \\ \pi/2 & B = 0, A > C \\ \frac{1}{2}\tan^{-1}\left(\frac{2B}{A-C}\right) & B \neq 0, A < C \\ \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{2B}{A-C}\right) & B \neq 0, A > C \end{cases} \qquad \text{...(A-11)}$$

Second Quadratic Form

The ellipse can also be represented so the symmetric matrix is independent of the ellipse's origin, Eq. A-12 and A-13, (Zheng and Palffy-Muhoray, 2010). Here $\underline{I}$ is the identity matrix and the vector $\underline{\theta}$ = [sin$\theta$, cos$\theta$]. In Zheng and Palffy-Muhoray's paper these authors assume $k = 1$ throughout.

$$\left[x-x_0, y-y_0\right]\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix} = k^2 \qquad \ldots\text{(A-12)}$$

Where

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \frac{1}{b^2}\left[\underline{I} + \left(\frac{b^2}{a^2} - 1\right)\underline{\theta}^T \underline{\theta}\right] \qquad \ldots\text{(A-13)}$$

Appendix B

Expansion Factors

## YKC Expansion Factor (Single Sided)

For a single sided expansion the characteristic polynomial becomes $P(\lambda) = \det[\lambda \underline{E}_1 - \underline{E}_2(k)] = 0$, Eq. B-1. For conciseness substitute $\chi = k^2$. Expanding the determinant gives a cubic polynomial which coefficients are functions of the coefficients of the quadratic forms and the square of the expansion factor, Eq. B-2.

$$\det\left\{ \lambda \begin{bmatrix} A_1 & B_1 & D_1 \\ B_1 & C_1 & F_1 \\ D_1 & F_1 & H_1 - a_1^2 b_1^2 \end{bmatrix} - \begin{bmatrix} A_2 & B_2 & D_2 \\ B_2 & C_2 & F_2 \\ D_2 & F_2 & H_2 - a_2^2 b_2^2 \chi \end{bmatrix} \right\} = 0 \qquad \ldots\text{(B-1)}$$

$$w_3(\chi)\lambda^3 + w_2(\chi)\lambda^2 + w_1(\chi)\lambda + w_0(\chi) = 0 \qquad \ldots\text{( B-2)}$$

Choi, 2008 showed that the cubic discriminant equals zero when the ellipses touch externally, Eq. B-3.

$$w_2^2 w_1^2 - 4w_2^3 w_0 + 18 w_3 w_2 w_1 w_0 - 4 w_3 w_1^3 - 27 w_3^2 w_0^2 = 0 \qquad \ldots\text{(B-3)}$$

By inspection, the coefficients $w_j$ are of the form $w_j = u_j + v_j\chi$

Making this substitution, Eq. B-3 then becomes a quartic equation in $\chi$, Eq. B-4.

$$\begin{aligned}
&\left(v_1^2 v_2^2 - 4v_1^3 v_3\right)\chi^4 + \left(2u_2 v_1^2 v_2 + 2u_1 v_1 v_2^2 - 4u_0 v_2^3\right. \\
&\left. - 12 u_1 v_1^2 v_3 + 18 u_0 v_1 v_2 v_3\right)\chi^3 + \left(u_2^2 v_1^2 + 4u_1 u_2 v_1 v_2\right. \\
&+ u_1^2 v_2^2 - 12 u_0 u_2 v_2^2 - 12 u_1^2 v_1 v_3 + 18 u_0 u_2 v_1 v_3 \\
&\left. + 18\, u_0 u_1 v_2 v_3 - 27 u_0^2 v_3^2\right)\chi^2 + \left(2 u_1 u_2^2 v_1 + 2 u_1^2 u_2 v_2\right. \\
&\left. - 12 u_0 u_2^2 v_2 - 4 u_1^3 v_3 + 18 u_0 u_1 u_2 v_3\right)\chi
\end{aligned}$$

$$+u_1^2 u_2^2 - 4u_0 u_2^3 = 0 \qquad \text{...(B-4)}$$

Note that both $u_3 = 0$ and $v_0 = 0$ and

$$u_0 = -a_1^4 b_1^4 \qquad \text{...(B-5)}$$

$$u_1 = a_1^2 b_1^2 \{ -a_2^2 b_1^2 \cos^2(\theta_2 - \theta_1) - a_1^2 \left[ a_2^2 \sin^2(\theta_2 - \theta_1) + b_2^2 \cos^2(\theta_2 - \theta_1) \right] + b_2^2 \left[ -b_1^2 \sin^2(\theta_2 - \theta_1) + (\Delta x_0 \cos\theta_2 + \Delta y_0 \sin\theta_2)^2 \right] + a_2^2 (\Delta y_0 \cos\theta_2 - \Delta x_0 \sin\theta_2)^2 \} \qquad \text{...(B-6)}$$

$$u_2 = a_2^2 b_2^2 \left[ -a_1^2 b_1^2 + b_1^2 (\Delta x_0 \cos\theta_1 + \Delta y_0 \sin\theta_1)^2 + a_1^2 (\Delta y_0 \cos\theta_1 - \Delta x_0 \sin\theta_1)^2 \right] \qquad \text{...(B-7)}$$

$$v_1 = -a_1^2 b_1^2 a_2^2 b_2^2 \qquad \text{...(B-8)}$$

$$v_2 = -a_2^2 b_2^2 \{ a_1^2 \left[ a_2^2 \sin^2(\theta_1 - \theta_2) + b_2^2 \cos^2(\theta_1 - \theta_2) \right] + b_1^2 \left[ a_2^2 \cos^2(\theta_1 - \theta_2) + b_2^2 \sin^2(\theta_1 - \theta_2) \right] \} \qquad \text{...(B-9)}$$

$$v_3 = -a_2^2 b_2^2 \qquad \text{...(B-10)}$$

Then substitute Eqs. B-5 to B-10 into Eq. B-4. Simplification gives the coefficients $\gamma_i$ of the quartic equation, Eq. 13 given in the specific description.

<u>YKC Expansion Factor (Dual Sided)</u>

The derivation of the dual sided expansion with the characteristic polynomial $P(\lambda) = \det[\lambda \underline{E}_1(k) - \underline{E}_2(k)] = 0$ proceeds in the same way. Again for conciseness substitute $\chi = k^2$.

$$\det\left\{ \lambda \begin{bmatrix} A_1 & B_1 & D_1 \\ B_1 & C_1 & F_1 \\ D_1 & F_1 & H_1 - a_1^2 b_1^2 \chi \end{bmatrix} - \begin{bmatrix} A_2 & B_2 & D_2 \\ B_2 & C_2 & F_2 \\ D_2 & F_2 & H_2 - a_2^2 b_2^2 \chi \end{bmatrix} \right\} = 0 \qquad \text{...(B-11)}$$

The coefficients $\gamma_i$ of the quartic equation may be calculated as Eq. B-12 to B-24.

$$\varphi_1 = \Delta y \cos\theta_1 - \Delta x \sin\theta_1 \qquad \text{...(B-12)}$$

$$\vartheta_1 = \Delta x \cos\theta_1 + \Delta y \sin\theta_1 \qquad \text{...(B-13)}$$

$$\varphi_2 = \Delta y \cos\theta_2 - \Delta x \sin\theta_2 \qquad \text{...(B-14)}$$

$$\vartheta_2 = \Delta x \cos\theta_2 + \Delta y \sin\theta_2 \qquad \text{...(B-15)}$$

$$r_1 = \varphi_1^2 a_1^2 + \vartheta_1^2 b_1^2 \qquad \text{...(B-16)}$$

$$r_2 = \varphi_2^2 a_2^2 + \vartheta_2^2 b_2^2 \qquad \text{...(B-17)}$$

$$p_1 = a_1^2\left[a_2^2 \sin^2(\theta_1-\theta_2) + b_1^2 + b_2^2\cos^2(\theta_1-\theta_2)\right] + b_1^2\left[a_2^2\cos^2(\theta_1-\theta_2) + b_2^2\sin^2(\theta_1-\theta_2)\right] \qquad \text{...(B-18)}$$

$$p_2 = a_2^2\left[a_1^2 \sin^2(\theta_1-\theta_2) + b_1^2\cos^2(\theta_1-\theta_2)\right] + b_2^2\left[a_1^2\cos^2(\theta_1-\theta_2) + a_2^2 + b_1^2\sin^2(\theta_1-\theta_2)\right] \qquad \text{...(B-19)}$$

$$\begin{aligned}\gamma_4 = \tfrac{1}{2}\{ & b_1^2 b_2^2 \sin^2(\theta_1-\theta_2) + a_2^2\left[b_1^2\cos^2(\theta_1-\theta_2) - b_2^2\right] \\ & + a_1^2\left[a_2^2\sin^2(\theta_1-\theta_2) - b_1^2 + b_2^2\cos^2(\theta_1-\theta_2)\right]\}^2 \\ & \times\Big(2a_1^4\left[a_2^2\sin^2(\theta_1-\theta_2) + b_2^2\cos^2(\theta_1-\theta_2)\right]^2 \\ & + 2b_1^4\left[a_2^2\cos^2(\theta_1-\theta_2) + b_2^2\sin^2(\theta_1-\theta_2)\right]^2 \\ & + a_1^2 b_1^2\{a_2^2 b_2^2\left[-5+\cos 4(\theta_1-\theta_2)\right] \\ & + 4\left(a_2^4+b_2^4\right)\sin^2(\theta_1-\theta_2)\cos^2(\theta_1-\theta_2)\}\Big)\end{aligned} \qquad \text{...(B-20)}$$

$$\gamma_3 = 2\left(-9a_1^2b_1^2a_2^2b_2^2 r_1 p_2 + 6a_1^2b_1^2 r_2 p_2^2 - 9a_1^2b_1^2a_2^2b_2^2 r_2 p_1 - r_1 p_2^2 p_1 + 6a_2^2b_2^2 r_1 p_1^2 - r_2 p_2 p_1^2\right) \qquad \text{...(B-21)}$$

$$\gamma_2 = 18a_1^2b_1^2a_2^2b_2^2 r_1 r_2 - 12a_1^2b_1^2 r_2^2 p_2 + r_1^2 p_2^2 - 12a_2^2b_2^2 r_1^2 p_1 + 4r_1 r_2 p_1 p_2 + r_2^2 p_1^2 \qquad \text{...(B-22)}$$

$$\gamma_1 = 2\left(2a_2^2b_2^2 r_1^3 + 2a_1^2b_1^2 r_2^3 - r_1^2 r_2 p_2 - r_2^2 r_1 p_1\right) \qquad \text{...(B-23)}$$

$$\gamma_0 = r_1^2 r_2^2 \qquad \text{...(B-24)}$$

<u>ZPM Expansion Factor</u>

Zheng and Palffy-Muhoray approach leads to a quartic equation in the variable $Q$, Eq. (B-25).

$$\tan^2\phi\left(\zeta+1-Q^2\right)\left(\frac{Q}{b_2'}+1\right)^2 = \left(Q^2-1\right)\left(\frac{Q}{b_2'}+1+\zeta\right)^2 \qquad \text{...(B-25)}$$

This can be written in the standard form, Eq. (B-26)

$$\psi_4 Q^4 + \psi_3 Q^3 + \psi_2 Q^2 + \psi_1 Q + \psi_0 = 0 \qquad \text{...(B-26)}$$

Where the quartic coefficients $\psi_i$ are

$$\psi_4 = -\frac{1}{b_2'^2}\left(1+\tan^2\phi\right) \quad \text{...(B-27)}$$

$$\psi_3 = -\frac{2}{b_2'^2}\left(1+\zeta+\tan^2\phi\right) \quad \text{...(B-28)}$$

$$\psi_2 = -\tan^2\phi - (1+\zeta)^2 + \frac{1}{b_2'^2}\left[1+(1+\zeta)\tan^2\phi\right] \quad \text{...(B-29)}$$

$$\psi_1 = \frac{2}{b_2'}\left(1+\tan^2\phi\right)(1+\zeta) \quad \text{...(B-30)}$$

$$\psi_0 = \left(1+\zeta+\tan^2\phi\right)(1+\zeta) \quad \text{...(B-31)}$$

To avoid a clash of symbols, note that the nomenclature used here differs from that used by Zheng and Palffy-Muhoray. The variables $\zeta$ and $\phi$ used here are defined in their paper.

Appendix C

Nomenclature

$a$ = Ellipse semi-major axis length, L, ft
$\underline{a}$ = Unit vector in the major axis direction
$b$ = Ellipse semi-minor axis length, L , ft
$d$ = Diameter, L, ft
$k$ = Expansion scale factor, dimensionless
$m$ = Element of a transformation matrix
$p$ = Substituted variable
$r$ = Substituted variable
$s$ = Characteristic length for a separation factor, L, ft
$u$ = Substituted variable
$v$ = Substituted variable
$w$ = Coefficients of the YKC cubic equation
$x$ = Ordinate in the normal plane, L, ft
$y$ = Ordinate in the normal plane, L, ft
$z$ = Substituted variable
$A$ = First ellipse quadratic form coefficient
$B$ = Second ellipse quadratic form coefficient
$C$ = Third ellipse quadratic form coefficient
$D$ = Fourth ellipse quadratic form coefficient
$E$ = Ellipse
$F$ = Fifth ellipse quadratic form coefficient
$G$ = Sixth ellipse quadratic form coefficient
$H$ = Modified sixth ellipse quadratic form coefficient
$\underline{E}$ = Ellipse matrix representation
$\underline{R}$ = Rotation matrix
$\underline{S}$ = Scaling matrix
$\underline{T}$ = Translation matrix
$\underline{I}$ = Unit matrix
$P$ = Polynomial
$Q$ = Independent variable of the ZPM quartic equation

Greek Symbols

$\delta$ = Centre to centre distance between ellipses, L, ft
$\gamma$ = Coefficients of the YKC quartic equation
$\varphi$ = Substituted or temporary variable
$\vartheta$ = Substituted or temporary variable
$\psi$ = Coefficients of the ZPM quartic equation
$\chi$ = Square of the expansion factor, dimensionless
$\theta$ = Ellipse orientation angle to major axis, radians
$\phi$ = First variable defined by ZPM
$\zeta$ = Second variable defined by ZPM
$\lambda$ = Multiplier
$\Delta$ = A difference in a parameter

<u>Subscripts and Superscripts</u>

$0$ = Condition at an origin

' = Transformed condition

1,2,3,4 = First, second etc.

$c$ = Casing

$cr$ = Critical, or closest approach

$h$ = Hole

$i$ = Index

$j$ = Index

$s$ = Separation factor

Claims

1. A computer-implemented method for determining the relative positions of a wellbore and an object, the wellbore being represented by a first ellipse and the object being represented by a second ellipse, wherein the first ellipse represents the positional uncertainty of the wellbore and the second ellipse represents the positional uncertainty of the object, the method comprising the steps of:

receiving input data relating to a measured or estimated position of the wellbore and the object, the position of the wellbore having a first set of parameters defining the first ellipse, and the position of the object having a second set of parameters defining the second ellipse;

calculating an expansion factor representing an amount by which one, or both, of the first ellipse and the second ellipse can be expanded with respect to one or both of respective first and second sets of elliptical parameters so that the first and second ellipses osculate, wherein calculating the expansion factor involves determining and solving a quartic equation that is based on the geometry of the ellipses; and

determining, based on the calculated expansion factor, position data indicative of the relative positions of the wellbore and the object.

2. The method of claim 1, wherein the first and second ellipses are expanded equally, and wherein the calculation of the expansion factor further comprises:

solving the quartic equation to determine the distance between the centres of the ellipses when the second ellipse is translated towards the first ellipse along a line joining the centres of the ellipses so that the ellipses osculate; and

calculating the expansion factor based on the determined distance and a scale factor.

3. The method of claim 1, wherein either the first and second ellipses are expanded equally, or one of the first and second ellipses is expanded, so that the ellipses osculate, and wherein the calculation of the expansion factor further comprises:

applying the first and second sets of elliptical parameters to a polynomial equation, the solution of which represents a separation condition of the ellipses;

determining the quartic equation from the polynomial equation; and solving the quartic equation to calculate the expansion factor.

4. The method of claim 3, wherein the ellipses osculate when the polynomial equation has a double root.

5. The method of any of claims 1 to 4, wherein the wellbore is a first planned or drilled wellbore, and the object is a second planned or drilled wellbore.

6. The method of any of claims 1 to 4, wherein the object is a sub-surface hazard.

7. The method of any preceding claim, wherein the input elliptical parameters are derived from a measured or estimated position of the wellbore and the object.

8. The method of any preceding claim, further comprising the step of determining a trajectory of the wellbore in a three-dimensional simulation.

9. The method of any preceding claims, further comprising the step of optimising the position of the wellbore relative to the object.

10. A computer-implemented method for determining one or more operating modes of a wellbore drilling system, the wellbore drilling system being arranged to drill a wellbore in a rock formation, the method comprising the steps of:

receiving position data determined according to the method of claim 1;

inputting said position data into a wellbore trajectory model;

operating the wellbore trajectory model so as to generate trajectory data indicative of a trajectory of the wellbore; and

determining, on the basis of the trajectory data, said one or more operating modes of the wellbore drilling system.

11. The method of claim 10, further comprising the steps of:

automatically configuring a controller of the wellbore drilling system with the one or more operating modes determined by the wellbore positioning system; and

applying the one or more operating modes.

12. A wellbore positioning system arranged to determine the relative positions of a wellbore and an object, the wellbore being represented by a first ellipse and the object being represented by a second ellipse, wherein the first ellipse represents the positional uncertainty of the wellbore and the second ellipse represents the positional uncertainty of the object, the system comprising:

data receiving means arranged to receive input data relating to a measured or estimated position of the wellbore and the object, the position of the wellbore having a first set of parameters defining the first ellipse, and the position of the object having a second set of parameters defining the second ellipse;

expansion factor calculation means arranged to calculate an expansion factor representing an amount by which one, or both, of the first ellipse and the second ellipse can be expanded with respect to one or both of respective first and second sets of elliptical parameters so that the first and second ellipses osculate, wherein calculating the expansion factor involves determining and solving a quartic equation that is based on the geometry of the ellipses; and

position determining means arranged to determine, based on the calculated expansion factor, position data indicative of the relative positions of the wellbore and the object.

13. The wellbore positioning system of claim 12, further comprising operating mode determining means arranged to determine, on the basis of the position data, one or more operating modes of a wellbore drilling system.

14. The wellbore positioning system of claim 12 or 13, the system being operatively connected to a controller of the wellbore drilling system such that the controller of the wellbore drilling system is automatically configured with the one or more operating modes determined by the wellbore positioning system, the controller being arranged to apply the one or more operating modes.

15. A computer program product comprising a set of instructions which, when executed by a computing device, is configured to cause the computing device to carry out the method according to any one of claims 1 to 11.

16. The computer program product of claim 15, comprising a computer readable storage medium.

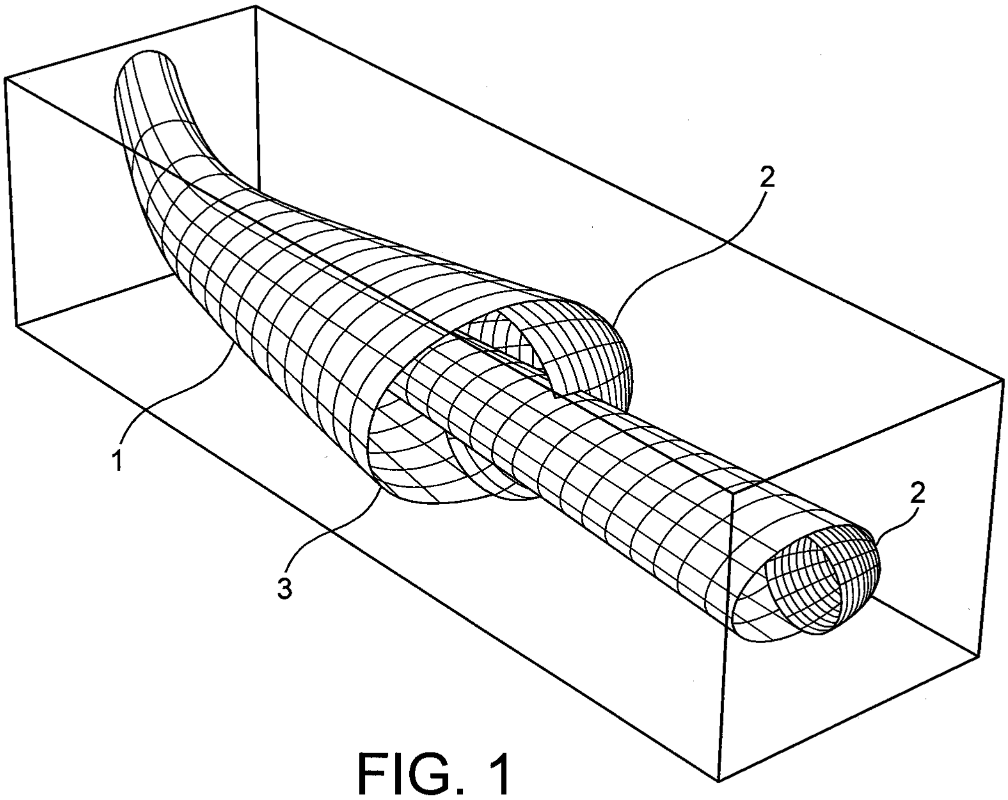


FIG. 1

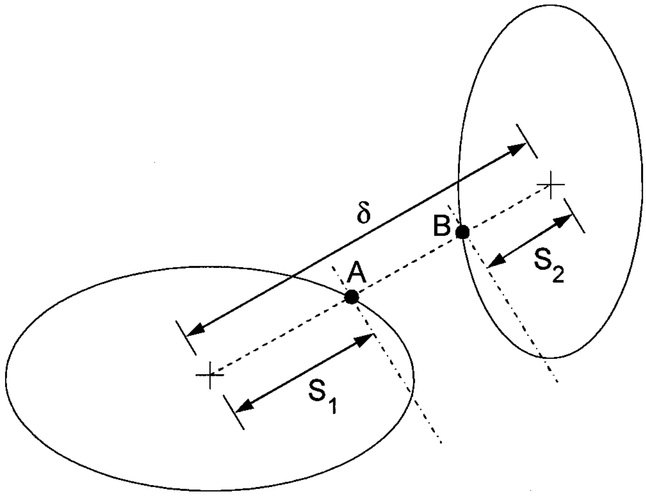


FIG. 2

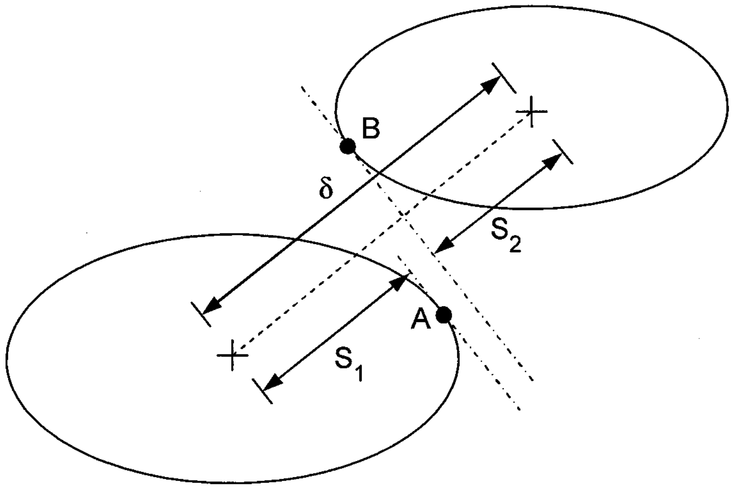


FIG. 3

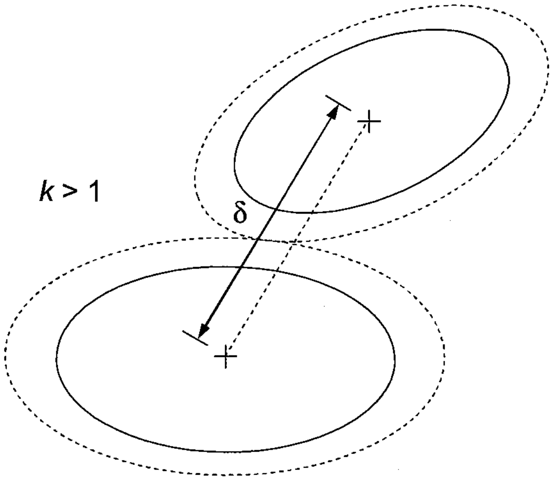


FIG. 4a

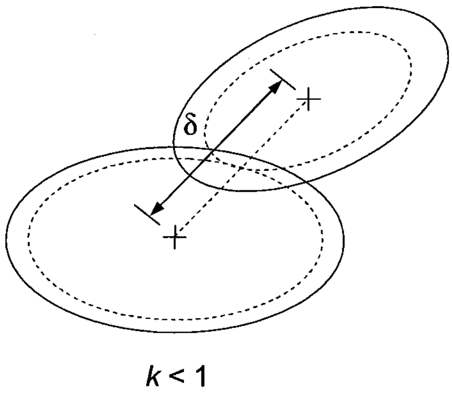


FIG. 4b

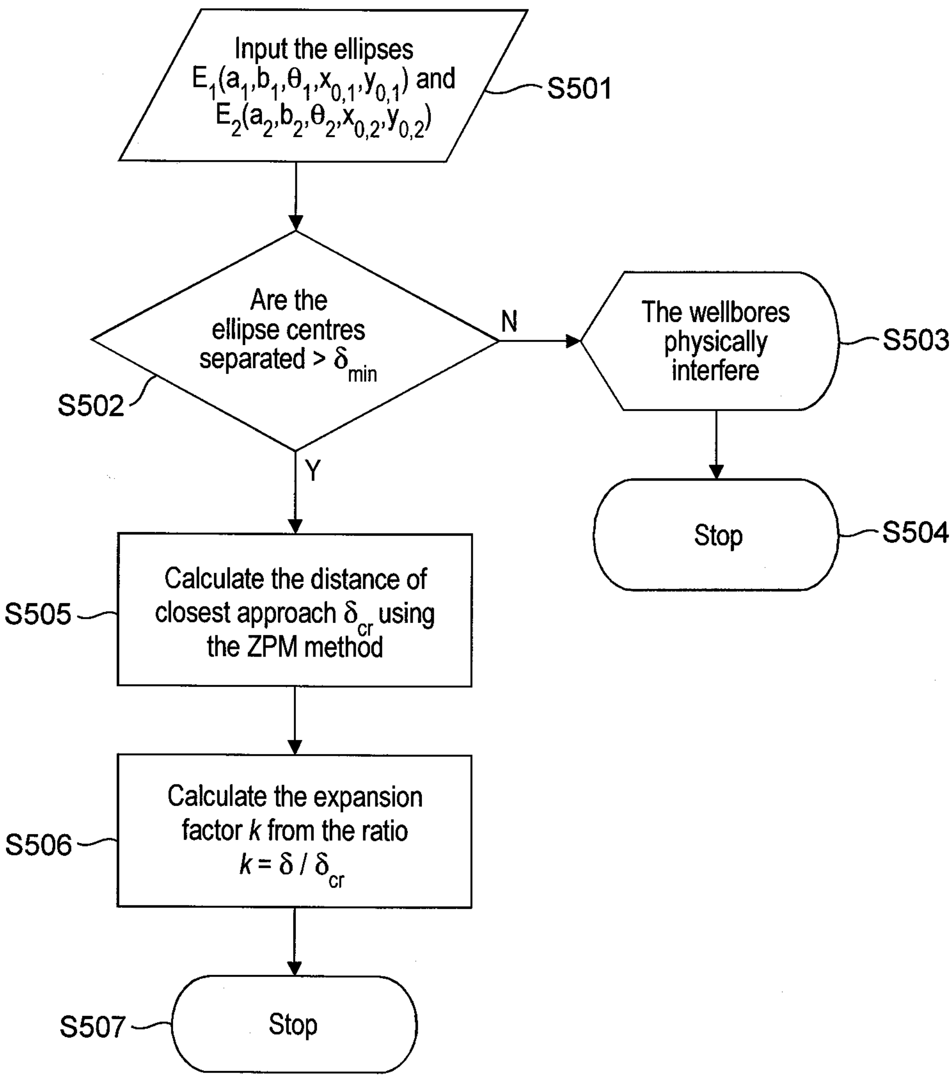
Input the ellipses
$E_1(a_1,b_1,\theta_1,x_{0,1},y_{0,1})$ and
$E_2(a_2,b_2,\theta_2,x_{0,2},y_{0,2})$
S501
Are the ellipse centres separated > $\delta_{min}$
S502
N
The wellbores physically interfere
S503
Stop
S504
Y
Calculate the distance of closest approach $\delta_{cr}$ using the ZPM method
S505
Calculate the expansion factor $k$ from the ratio $k = \delta / \delta_{cr}$
S506
Stop
S507

FIG. 5a

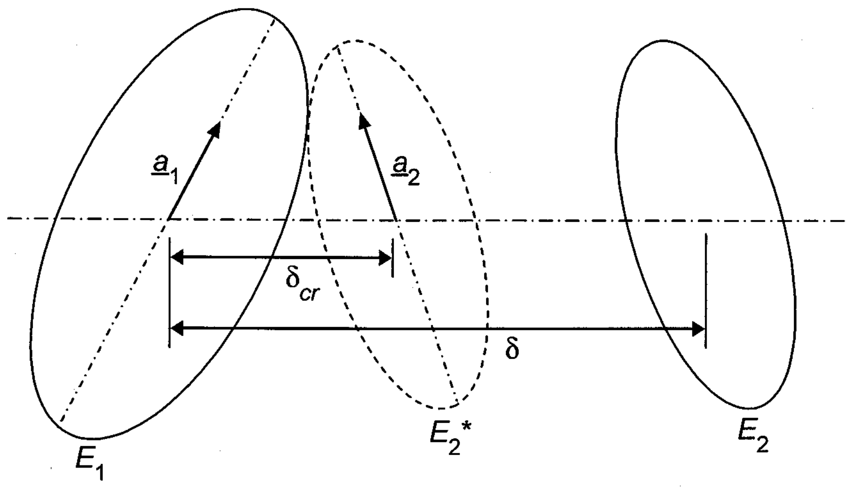


FIG. 5b

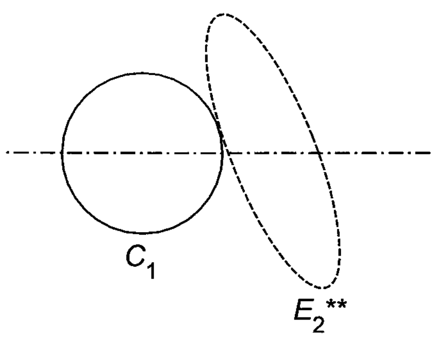


FIG. 5c

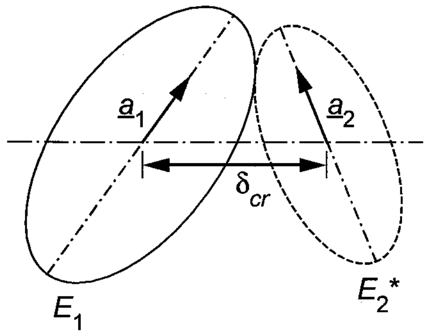


FIG. 5d

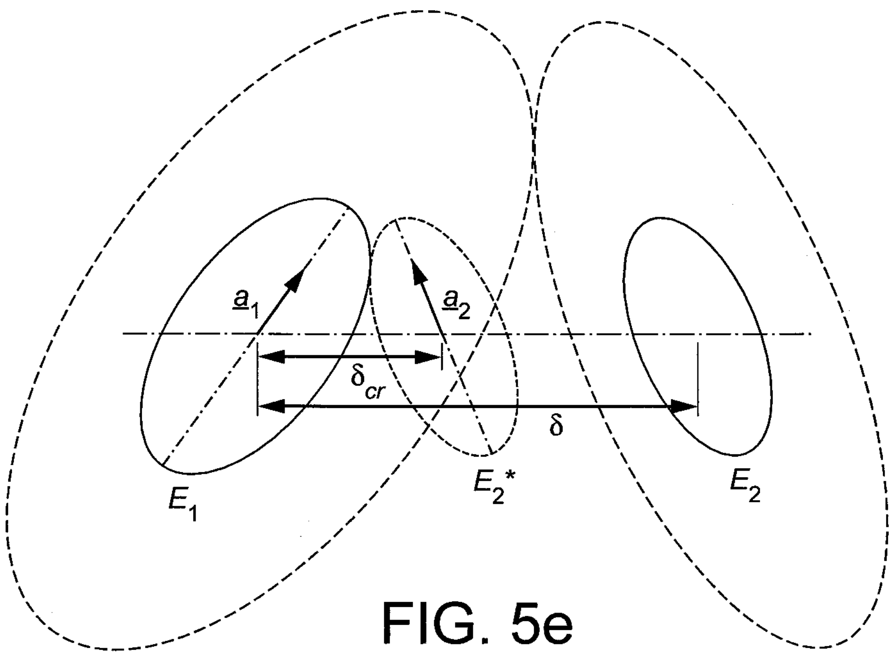


FIG. 5e

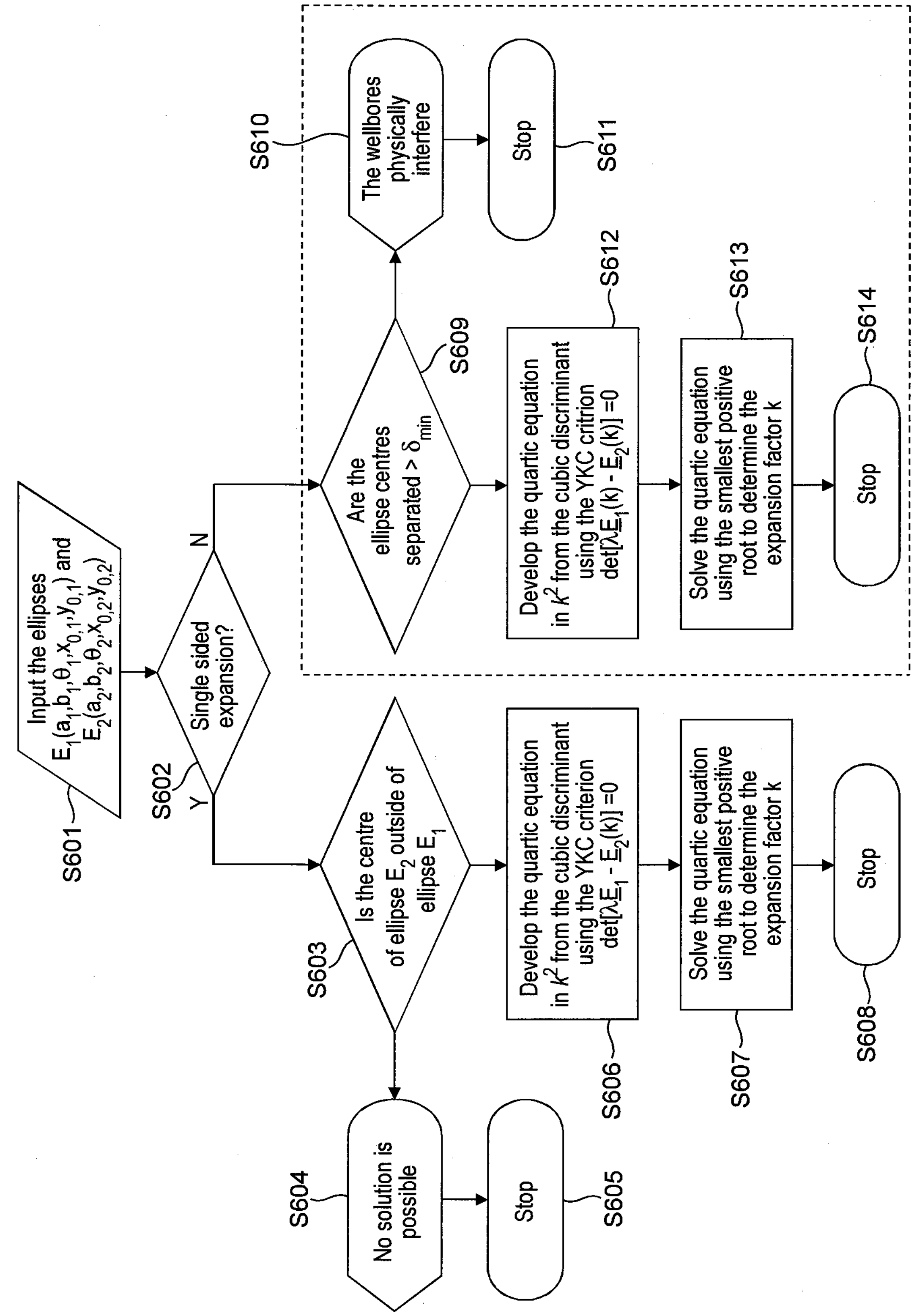
S601
Input the ellipses
E1(a1,b1,θ1,x0,1,y0,1) and
E2(a2,b2,θ2,x0,2,y0,2)
S602
Single sided
expansion?
Y
N
S603
Is the centre
of ellipse E2 outside of
ellipse E1
S604
No solution is
possible
Stop
S605
Develop the quartic equation
in k2 from the cubic discriminant
using the YKC critrion
det[λE1 - E2(k)] =0
S606
Solve the quartic equation
using the smallest positive
root to determine the
expansion factor k
S607
Stop
S608
S609
Are the
ellipse centres
separated > δmin
S610
The wellbores
physically
interfere
Stop
S611
Develop the quartic equation
in k2 from the cubic discriminant
using the YKC critrion
det[λE1(k) - E2(k)] =0
S612
Solve the quartic equation
using the smallest positive
root to determine the
expansion factor k
S613
Stop
S614

FIG. 6a

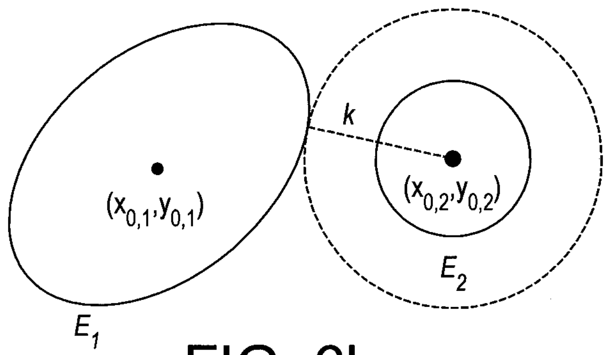


FIG. 6b

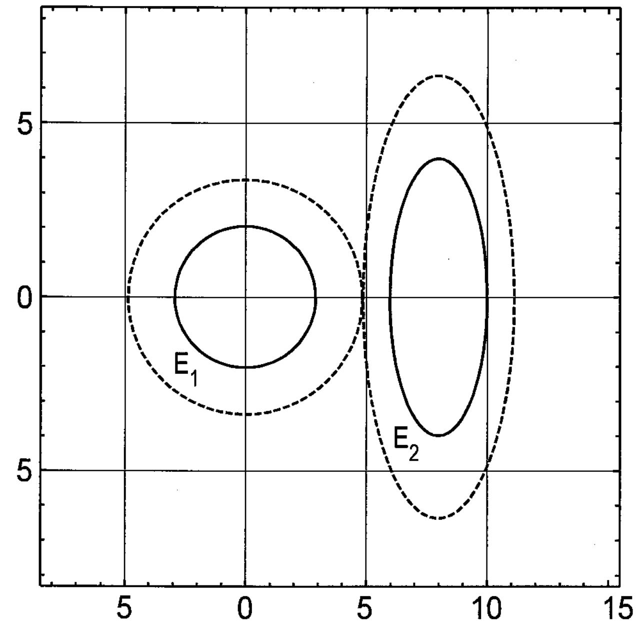


FIG. 7a

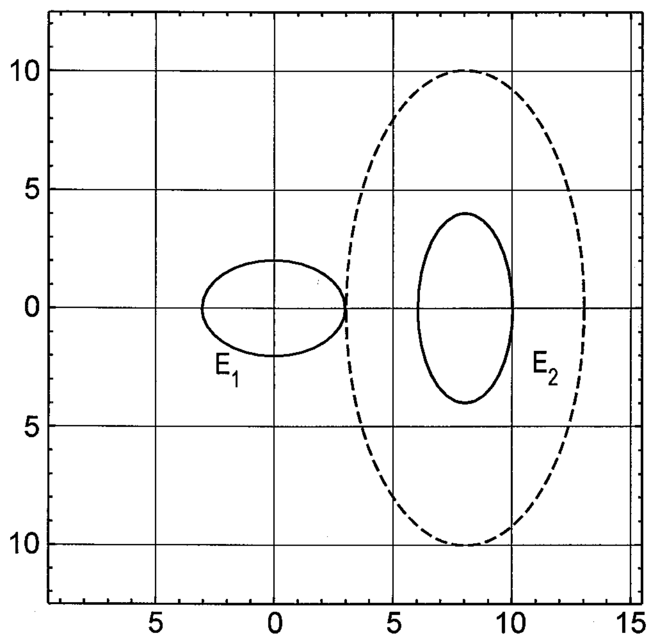


FIG. 7b

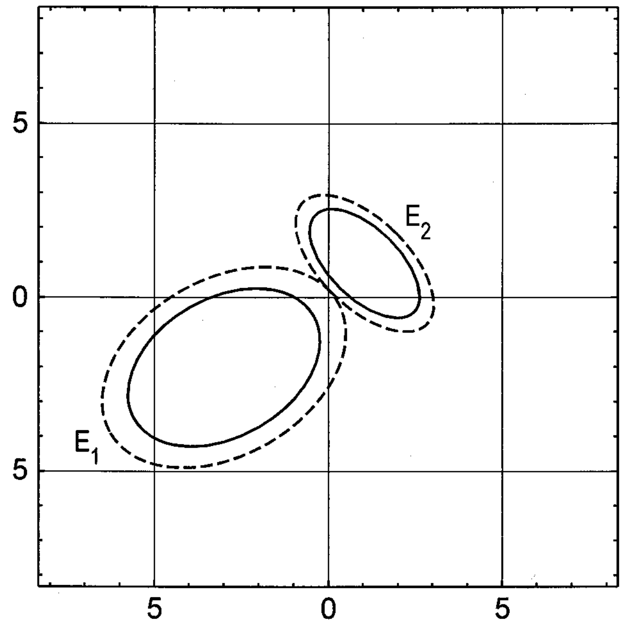


FIG. 8a

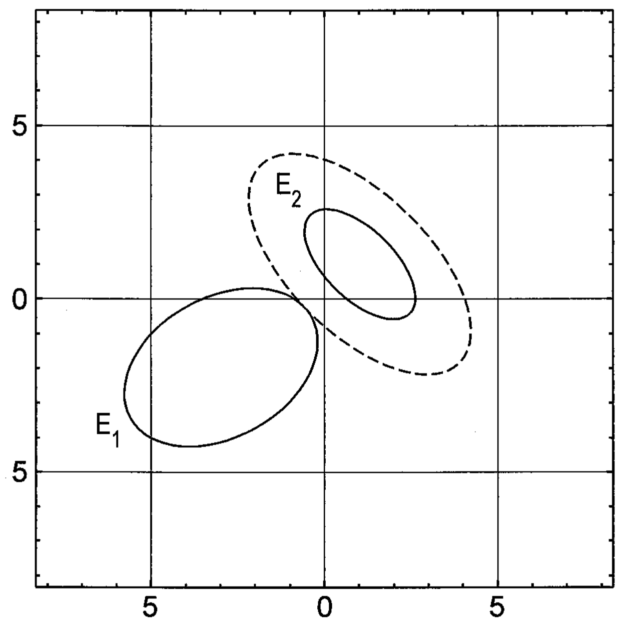


FIG. 8b

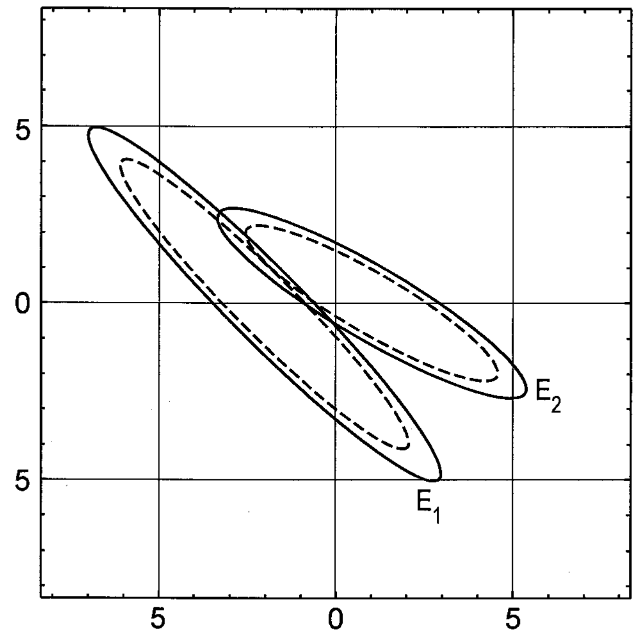


FIG. 9a

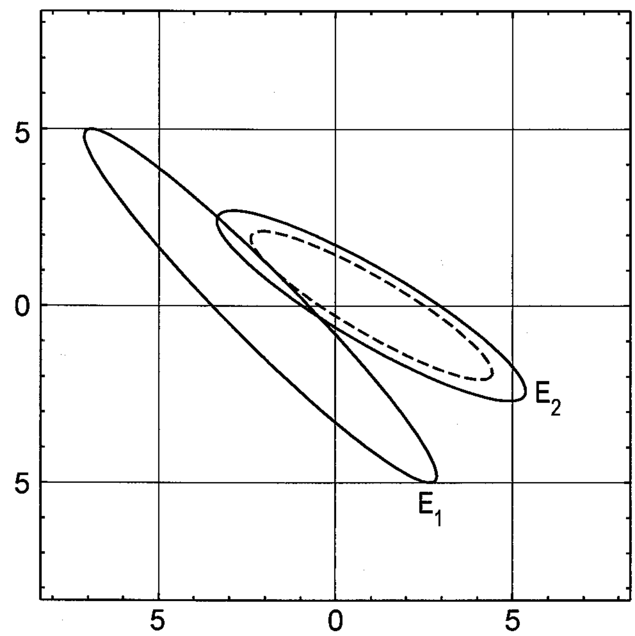


FIG. 9b

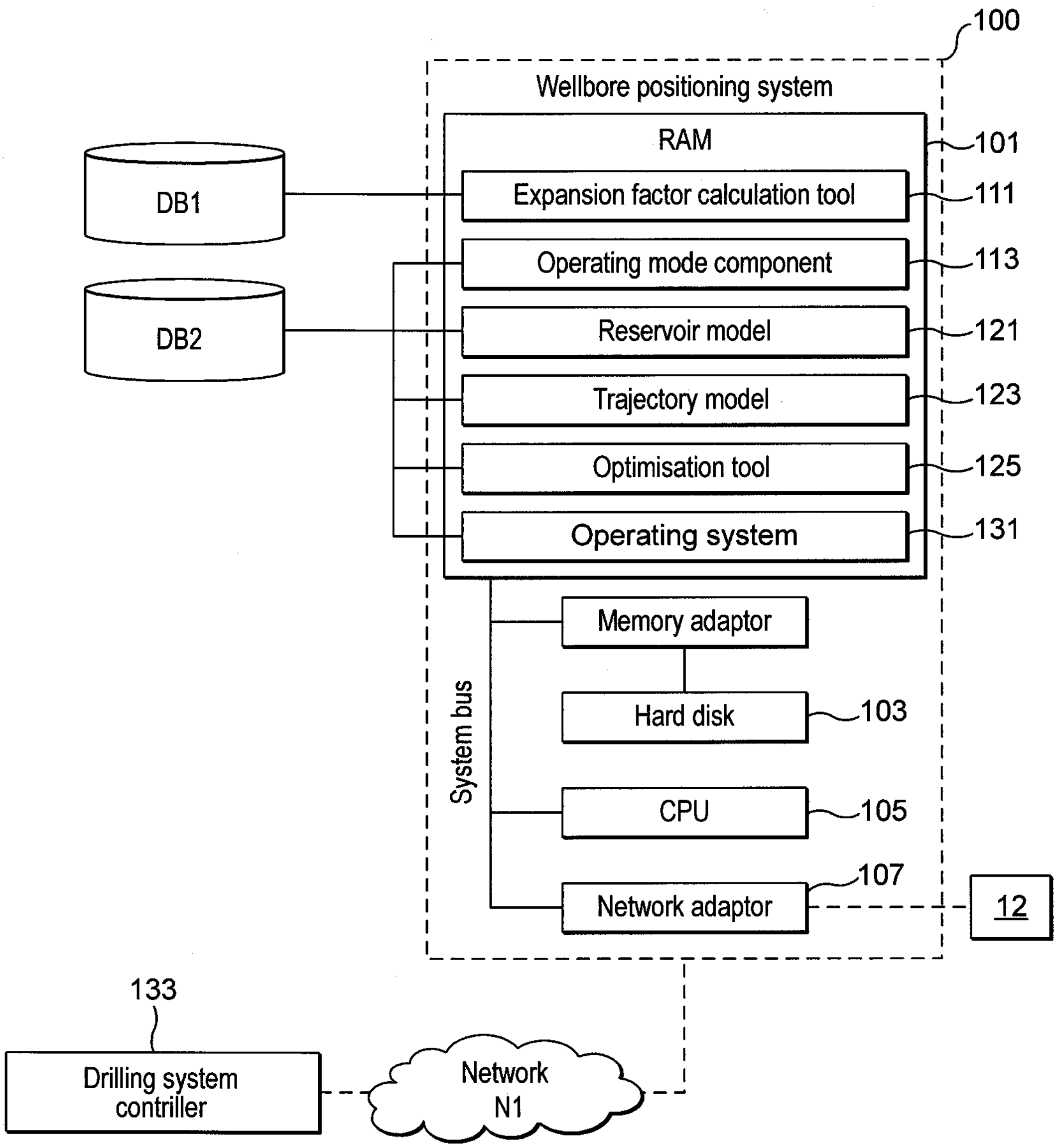
100
Wellbore positioning system
RAM
101
DB1
Expansion factor calculation tool
111
Operating mode component
113
DB2
Reservoir model
121
Trajectory model
123
Optimisation tool
125
Operating system
131
Memory adaptor
System bus
Hard disk
103
CPU
105
107
Network adaptor
12
133
Drilling system contriller
Network N1

FIG. 10

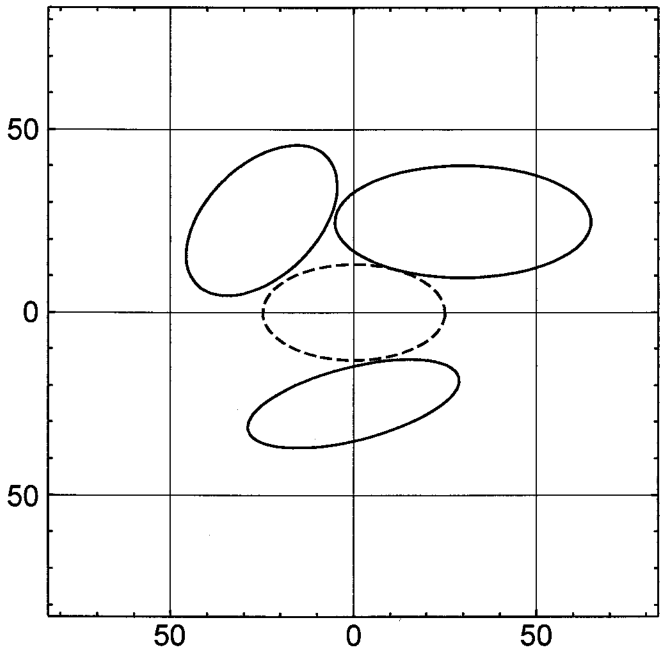


FIG. 11

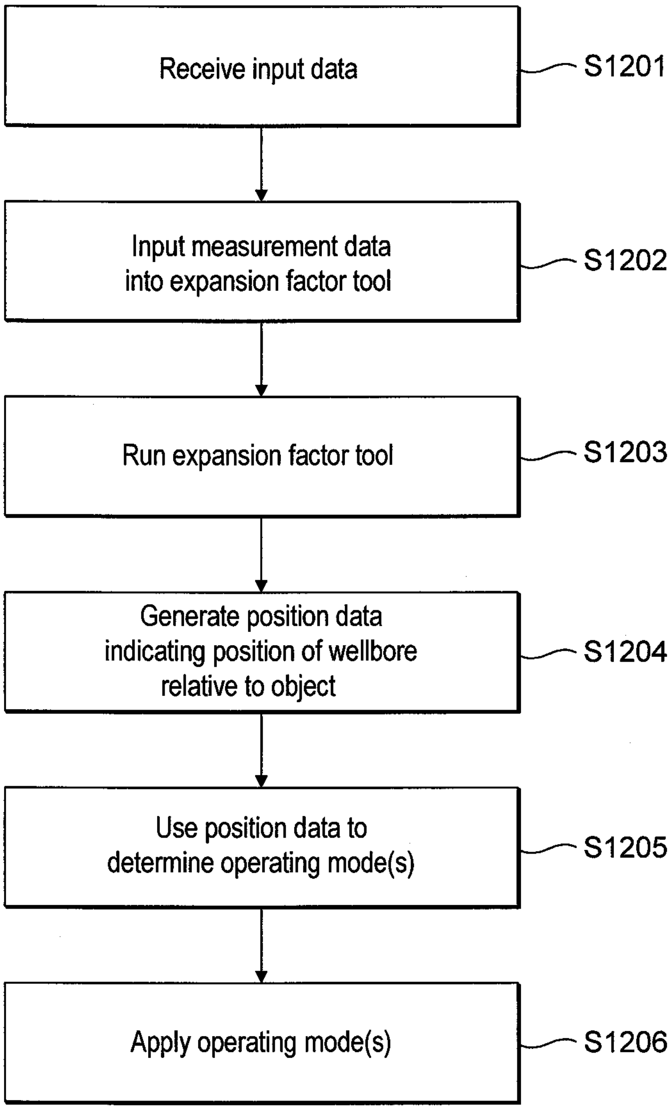
Receive input data
S1201
Input measurement data
into expansion factor tool
S1202
Run expansion factor tool
S1203
Generate position data
indicating position of wellbore
relative to object
S1204
Use position data to
determine operating mode(s)
S1205
Apply operating mode(s)
S1206

FIG. 12

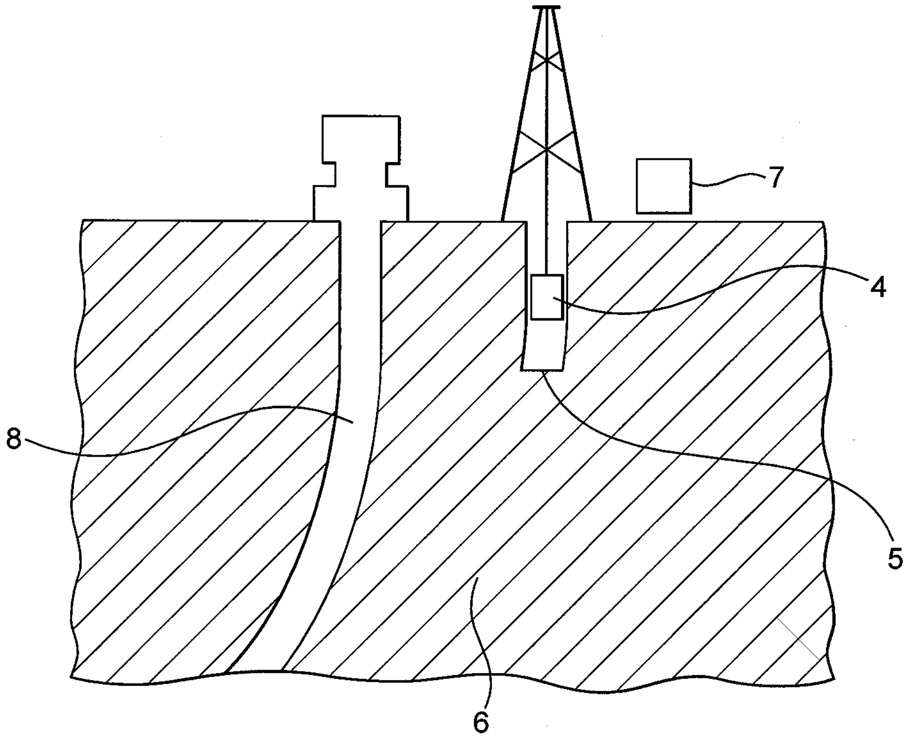


FIG. 13

# INTERNATIONAL SEARCH REPORT

International application No
PCT/EP2013/050863

**A. CLASSIFICATION OF SUBJECT MATTER**
INV. E21B47/022
ADD.

According to International Patent Classification (IPC) or to both national classification and IPC

**B. FIELDS SEARCHED**

Minimum documentation searched (classification system followed by classification symbols)
E21B G01V

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

EPO-Internal, WPI Data

**C. DOCUMENTS CONSIDERED TO BE RELEVANT**

| Category* | Citation of document, with indication, where appropriate, of the relevant passages | Relevant to claim No. |
|---|---|---|
| A | US 6 834 732 B2 (HAARSTAD IVAR [NO]) 28 December 2004 (2004-12-28) column 2, line 35 - line 52 column 9, line 23 - line 39 ----- | 1-14 |
| A | US 2009/120690 A1 (PHILLIPS WAYNE J [US]) 14 May 2009 (2009-05-14) paragraphs [0018], [0033], [0042]; figure 3 ----- | 1-14 |
| A | WO 96/35859 A1 (SYSDRILL LTD [GB]; ROPER DAVID JOHN [GB]; HENLY ADAM GREGORY [GB]) 14 November 1996 (1996-11-14) page 21, paragraphs 2,3 page 23, paragraph 1 page 24 page 26, paragraphs 1,2 figure 1 ----- | 1-14 |

-/--

[X] Further documents are listed in the continuation of Box C.

[X] See patent family annex.

* Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier application or patent but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

| Date of the actual completion of the international search | Date of mailing of the international search report |
|---|---|
| 29 April 2013 | 07/05/2013 |

| Name and mailing address of the ISA/ | Authorized officer |
|---|---|
| European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Fax: (+31-70) 340-3016 | Schneiderbauer, K |

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Form PCT/ISA/210 (second sheet) (April 2005)

# INTERNATIONAL SEARCH REPORT

International application No
PCT/EP2013/050863

C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

| Category* | Citation of document, with indication, where appropriate, of the relevant passages | Relevant to claim No. |
|---|---|---|
| A | US 2010/241410 A1 (MCELHINNEY GRAHAM A [GB] ET AL) 23 September 2010 (2010-09-23) paragraphs [0034], [0035], [0036] ----- | 1-14 |
| A | US 2008/289877 A1 (NIKOLAKIS-MOUCHAS CHRISTOS [US] ET AL) 27 November 2008 (2008-11-27) paragraphs [0016], [0069]; claim 1; figure 4 ----- | 1-14 |

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# INTERNATIONAL SEARCH REPORT

Information on patent family members

International application No
PCT/EP2013/050863

| Patent document cited in search report | | Publication date | Patent family member(s) | | | Publication date |
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| | | | US | 2010241410 | A1 | 23-09-2010 |
| | | | WO | 2010107856 | A2 | 23-09-2010 |
| US 2008289877 | A1 | 27-11-2008 | CA | 2685290 | A1 | 27-11-2008 |
| | | | GB | 2462227 | A | 03-02-2010 |
| | | | RU | 2009147272 | A | 27-06-2011 |
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| | | | WO | 2008144710 | A1 | 27-11-2008 |