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VARIABLE STANDARD MUTUAL INDUCTANCE CIRCUIT
WITH AIR CORE TRANSFORMER AND TAP CHANGING
CASCADED AUTOTRANSFORMERS

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FIG. 1 Prior Art

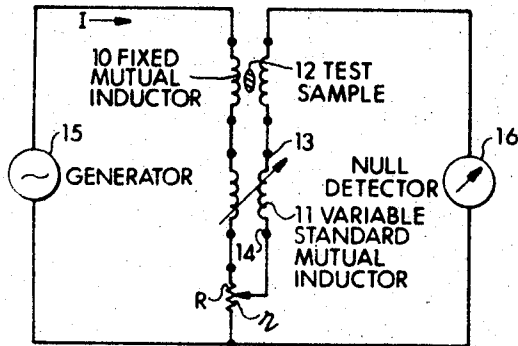


FIG. 2

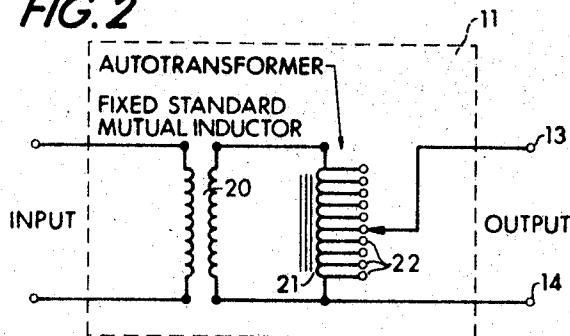


FIG. 3

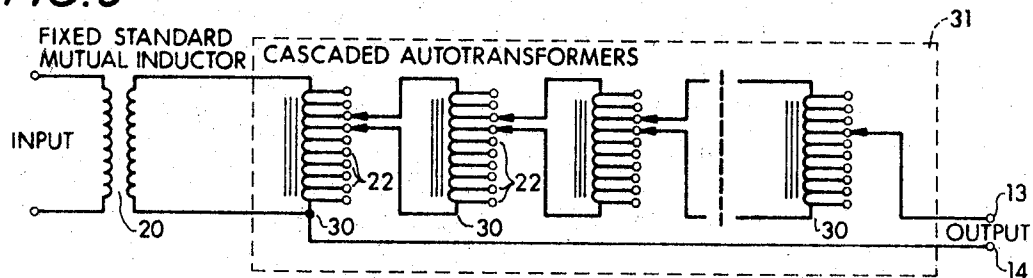
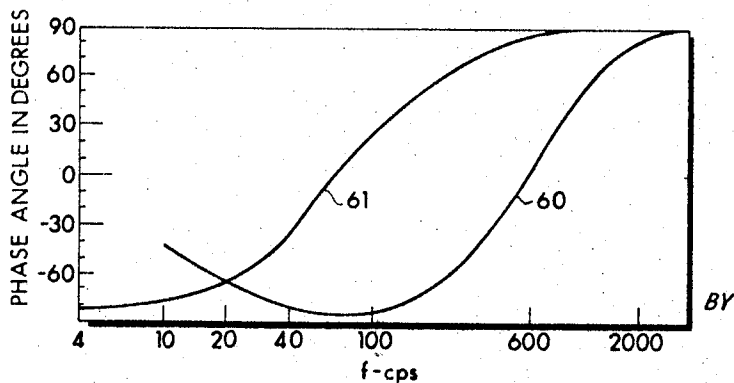


FIG. 4



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FIG. 5

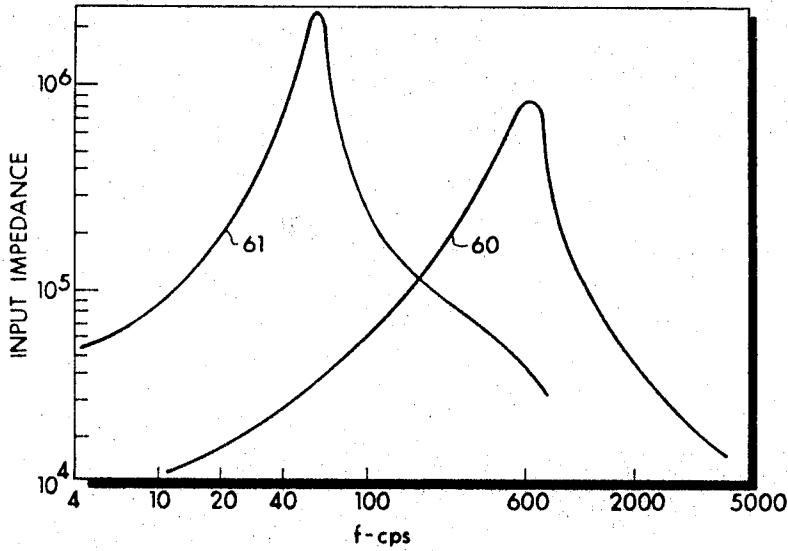


FIG. 6

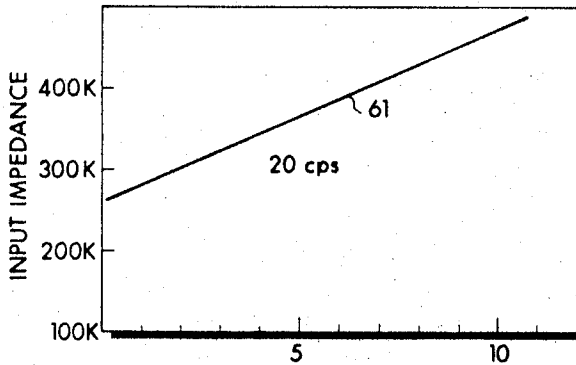
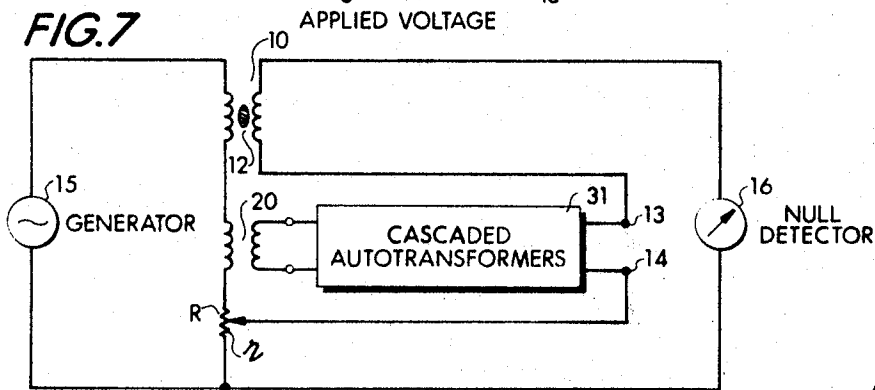


FIG. 7



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VARIABLE STANDARD MUTUAL INDUCTANCE CIRCUIT WITH AIR CORE TRANSFORMER AND TAP CHANGING CASCADED AUTOTRANSFORMERS

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8 Claims

ABSTRACT OF THE DISCLOSURE

A variable standard mutual inductance circuit includes an air (or other low loss medium) core input transformer of high purity with the secondary winding connected to a Kelvin-Varley arrangement of high precision tap changing cascaded autotransformers on high permeability ferromagnetic toroidal cores. The combination may be used in a Hartshorn type bridge circuit including a null detector.

This invention relates to a variable mutual inductor and more in particular to a circuit for providing a variable mutual inductor by the electrical connection of a fixed mutual inductor and a cascade of transformers having a variable voltage ratio.

More specifically this invention is concerned with an improved apparatus for achieving the electrical equivalent of a mutual inductor which is variable in discrete steps, the magnitude of the mutual inductance being predetermined to a very high degree of accuracy and precision over a wide range of frequencies. Thus, by this invention, it is possible to greatly simplify the problem of constructing mutual inductors which are adjustable in decade steps, or in digital steps, or in steps which are any integral multiples of some basic unit.

In order to set forth the novelty of the invention, the state of the art shall be briefly reviewed to indicate more clearly the nature of the improvements contained in the invention. Variable mutual inductors are used in a variety of A.C. bridge circuits, one of the more common of which is the Hartshorn circuit illustrated in FIG. 1. In this circuit the unknown mutual inductor 10, whose mutual inductance is M_x , is compared with adjustable standard mutual inductor 11, whose mutual inductance is M_s . The circuit of FIG. 1 is commonly used to measure the A.C. magnetic susceptibility of various materials, for example, paramagnetic salts, and finds many applications in low temperature physics. In this application the material 12 to be measured is placed within the field of the coils of inductor 10 thereby modifying the magnitude of the mutual inductance M_x . By suitably allowing for the coupling coefficient between the coil system of inductor 10 and the sample material 12, the susceptibility of the sample 12 may be derived from a measurement of the modified value of mutual inductance M_x . If the standard mutual inductor 11 is adjusted so that its secondary voltage cancels that of inductor 10, then $M_s = M_x$. If, however, there are some energy losses in the sample material 12 so that the secondary voltage of inductor 10 is not precisely 90° out of phase with the primary current then a perfect null cannot be attained by adjusting inductor 11 alone. To achieve a complete null, a component of voltage in-phase with the primary current is added by tapping off an appropriate part, r , of the resistor R . The unknown mutual inductance M_x is then considered to be a complex quantity,

$$M_x = M_x' + iM_x''$$

where $M_x' = M_s$ and $M_x'' = r/2\pi f$, f is the frequency and i is $\sqrt{-1}$.

In order to make such measurements with high precision and accuracy the adjustable standard mutual inductor 11 must meet rather severe specifications. It is desirable that it be adjustable in decade steps with sometimes as many as five or six decades variation in its mutual inductance M_s . It is further required that the different steps in each decade of inductor 11 be precisely equal and accurately known, and finally that the secondary voltage of inductor 11 be exactly orthogonal to, i.e., 90° out of phase with, the primary current I . The degree to which orthogonality is achieved is a measure of "purity" of the mutual inductor. In this description of the use of the Hartshorn bridge for measuring complex mutual inductance it was pointed out that the in-phase component M_x' was measured in terms of M_s and the out-of-phase component M_x'' in terms of r . This pre-supposes that M_s is "pure," otherwise errors will be introduced into the measurements of M_x' and M_x'' .

The obvious way of constructing a standard decade mutual inductor is to wind ten identical secondary coils on a single primary coil, the individual secondary coils being connected to switches so that any number of secondary coils may be serially connected to provide a secondary output voltage adjustable in decade steps where the primary coil has a constant amplitude current. If the standard decade mutual inductor has an air-core it is difficult to ensure that the coupling between the primary coil and each of the secondary coils will be the same for all coils inasmuch as the flux is not closely confined as it would be with an iron core. With an iron core, however, there are other problems, such as the variation of mutual inductance with primary current amplitude and the impurity, or non-orthogonality, resulting from the losses in the iron core. Therefore, the earlier practice has been to use air-core mutual inductors in which each of the steps of the decade must be individually adjusted and trimmed.

An alternative approach is to wind a single secondary coil using a ten-strand wire, the individual strands being insulated from each other. Each strand then makes up a separate secondary coil and any number of strands may be connected in series to get a decade variation. Although this construction tends to insure equality of the decade steps, the individual coils of the decade have large mutual capacitances, inasmuch as the strands are in close proximity with one another. This mutual capacitance limits the application of such mutual inductors to low frequencies since the capacitive effects introduce both phase and amplitude errors.

As examples of the problems encountered in constructing standard decade mutual inductors according to the prior art, reference is made to the papers "Mutual Inductance Bridge and Cryostat for Low Temperature Magnetic Measurements," R. A. Erickson, L. D. Roberts and J. W. T. Dabbs, Review of Scientific Instruments, vol. 25, p. 178 (1954), and "Installation of Adiabatic Demagnetization Experiments at the National Bureau of Standards," by D. de Klerk and R. P. Hudson, Journal of Research of the National Bureau of Standards, vol. 25, p. 1178 (1954).

In the bridge application which has been considered, the purpose of the standard mutual inductor 11 is to obtain a secondary voltage which is either in phase or exactly out of phase (depending on relative polarities) with the primary voltage, a voltage which is conductively isolated from the primary, and a voltage which is adjustable in precise discrete steps. These characteristics are diffi-

cult to achieve by means of multiple secondary coils on an air-core mutual inductor.

It is accordingly an object of this invention to provide new type of variable standard mutual inductor having these desirable characteristics.

Other objects and features of this invention will become apparent from the following description of the invention in conjunction with the accompanying drawings, wherein:

FIG. 1 is a schematic of a conventional Hartshorn bridge circuit.

FIG. 2 is a schematic of a variable standard mutual inductor according to this invention.

FIG. 3 is a schematic of a variable standard mutual inductor according to this invention showing a cascade of transformers.

FIG. 4 is a plot of the input phase angle of commercial cascaded decade autotransformers as a function of frequency.

FIGS. 5, 6 are plots of the input impedance of commercial cascaded decade autotransformers as a function of frequency and input voltage at fixed frequency, respectively.

FIG. 7 is a schematic of a Hartshorn bridge circuit using the variable standard mutual inductor of the invention.

It is a feature of this invention that this object is attained by separating the function of conductive isolation from that of adjustability. This feature is illustrated in FIG. 2 which is a schematic version of the invention. A fixed standard mutual inductor 20 of high purity is combined with a high precision auto-transformer voltage-divider 21 to provide at output terminals 13, 14 a predetermined fraction of the secondary voltage of inductor 20 by means of the selection of a tap 22 on transformer 21. The advantages of using this combination of elements are several.

A fixed mutual inductor can be made with higher purity than an adjustable unit because it avoids complex geometrical configurations and because it may be an air-core or other low-loss medium device. It is also easier to make it astatic (as by using toroidal geometry) so as to minimize coupling with other circuit elements. Mutual capacitance effects which limit the upper frequency limit are also minimized.

By using a separate auto-transformer which preferably has an iron-core for voltage division, much greater precision is achieved than is possible in a conventional multiple-secondary standard mutual inductor. The art of constructing a cascade of voltage-divider auto-transformers 31 is well known and in actual practice they consist of a number of decade voltage divider auto-transformers 30 arranged in a Kelvin-Varley circuit 31 as in FIG. 3. Suitable voltage-divider transformers are commercially available, as for example the Ratiotran manufactured by Gertsch Products, Inc. and Dekatran manufactured by Electro Scientific Industries, Inc., or their equivalents. A high degree of precision in voltage division is achieved by using high permeability ferromagnetic toroidal core material in the transformers 30, which practically eliminates leakage flux and consequent errors in voltage division or phase shift. The magnitude of the output voltage is controlled entirely by the turns-ratio at the selected tap 22 position; and because of the tight coupling, the phase shift between the input and output voltages is negligibly small.

It is important to note that the use of a ferromagnetic core in the voltage-divider causes no errors in voltage division whereas in a mutual inductor a ferromagnetic core would be undesirable as pointed out earlier. In the voltage-divider auto-transformer the magnitude of the various self and mutual impedances vary with the amplitude and frequency and frequency of the applied voltage but the output-to-input voltage-ratio is independent of these variables over a rather wide range of these variables. Another desirable feature is the fact that the input impedance of transformer 21, 31 may be of the order of hundreds of

thousands of ohms and therefore very high compared to the output impedance of mutual inductor 20, while the transformer output impedance is intrinsically low, typically of the order of ten ohms. Therefore, transformer 21, 31 does not load the secondary of inductor 20 to any significant extent while its low output impedance is a desirable feature as it makes for greater flexibility in coupling to the detector in the bridge circuit.

Thus, by separating the voltage divider function from the mutual induction function, a combination of elements has been achieved in which these functions are realized in a manner superior to that of the ordinary adjustable mutual inductor.

Although commercially available decade transformers have high ratio accuracy, high input impedance, low output impedance, and low phase shift over their operating ranges, it is not apparent from certain other of their characteristics that they could be successfully combined with a fixed air-core mutual inductor to provide a solution to the problem of a variable standard mutual inductor.

One of these characteristics is a self-resonance in the input impedance of the transformers in the middle of the operating ranges at about 400 c.p.s. on the curve 60 for the higher frequency type of transformer and 60 c.p.s. on the curve 61 for the lower frequency type as illustrated in FIG. 5. The input impedance is of the order of one megohm at resonance and falls off rapidly on either side of resonance. The input phase angle curves 60, 61 for these transformers, shown in FIG. 4, illustrate the typical $\pm 90^\circ$ phase shift of a resonant circuit. Further, the input signal amplitude dependence of the input impedance of the transformer is exhibited in FIG. 6. That transformers having such input characteristics could be successfully combined with air-core mutual inductors to provide a voltage at the output terminals of such a transformer which is 90° out of phase with the current at the input terminals of the fixed mutual inductor is less than obvious. Only the relatively high input impedance of the transformer compared to the output impedance of the fixed mutual inductor to which it is connected, allows the combination to be used since under these impedance conditions the change in impedance and phase angle at the input of the transformer does not materially affect the phase of the voltage at the output of the fixed mutual inductor which is in turn not affected in its transfer through the transformer to the output.

To illustrate the small loading errors, a typical voltage-divider auto-transformer has an input impedance of the order of a megohm at 60 cycles per second. When used in combination with a mutual inductor such as the Leeds and Northrup Type 1540 having a mutual inductance of 50 mh., the corresponding phase error is approximately 10^{-5} radians and the amplitude error 0.01 percent. These figures are merely representative of what can be accomplished with readily available components and do not necessarily represent the ultimate performance possible with the invention.

Finally as an example of how the invention may be applied to a bridge circuit of the Hartshorn type reference is made to FIG. 7. This circuit is basically that of FIG. 1 where the variable standard mutual inductor 11 is replaced by the inductor circuit of FIG. 3. The generator 15 provides the same current I to the primary windings of inductors 12 and 20. The voltage induced in the secondary of inductor 10 contains components in phase and 90° out-of-phase with the current I . The out-of-phase component is nulled by adjusting the voltage at terminals 13, 14 of transformer 31. The in-phase component is nulled by adjustment of the tap of resistor R to the value r . The null detector 16, which may be any sensitive A.C. detector suitable for the frequency of the generator 15, determines when a null condition exists. When the voltage ratio of transformer 31 is expressed as the decimal fraction a at the null condition, the following conditions exist, $M_x' = aM_s$ and $M_x'' = r/2\pi f$, where M_s is the mutual inductance of the fixed mutual inductor 20.

Although the invention has been described as it used in a Hartshorn bridge circuit, it will be apparent to those skilled in the art that the variable standard mutual inductor constructed according to this invention can be universally employed wherever such an inductor is required. In addition, the decade autotransformer may be replaced by a conventional isolated primary and secondary winding type of transformer where the secondary winding may be a plurality of windings or a tapped single winding to provide a decade output voltage capability. In addition, the voltage output may be continuously controlled, as in a Variac, if the reduced accuracy and precision is not objectionable as in the last transformer of the cascade. Numerous other modifications may be made by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. A variable mutual inductance circuit comprising
 - a high-purity fixed mutual inductor having a primary and a secondary winding, each winding having a pair of terminals,
 - a high-precision voltage transformer having a pair of input terminals and a plurality of output terminals, said transformer input impedance at said input terminals being high relative to said inductor output impedance at said secondary winding, means for selecting a pair of output terminals, both terminals of said inductor secondary winding being directly connected to both input terminals of said transformer to provide a mutual inductance between said inductor primary winding and a selected pair of said transformer output terminals.
2. The apparatus as in claim 1 wherein said mutual inductor primary and secondary windings are inductively coupled to each other through a low-loss medium, said transformer output terminals are connected to selected turns of a winding of said transformer, said winding being wound on a high-permeability ferromagnetic core to provide low leakage flux between the turns of said winding, the ratio of the turns between said input terminals and the turns between said output terminals determining the magnitude of the mutual inductance.
3. The apparatus as in claim 2 wherein said transformer is an autotransformer having one winding with a plurality of taps on its turns, said input and output terminal pairs being connected to selected pairs of said taps.
4. The apparatus of claim 1 wherein said transformer is a cascade connection of autotransformers in a Kelvin-Varley arrangement, said autotransformers having a plurality of output terminals connected to the tapped winding of the autotransformer, means for selecting the desired output terminals of each autotransformer, whereby the mutual inductance may be provided as desired.
5. A variable mutual inductor comprising
 - a fixed mutual inductor having a separate primary and secondary winding inductively coupled to each other through a low-loss medium,
 - a transformer variable voltage divider having input and output terminals, said input terminal impedance being substantially higher than the secondary winding impedance of said inductor,
 - said inductor secondary winding being connected to said transformer input terminals to provide a mutual in-

ductance between the inductor primary winding and the transformer output terminals which is a linear function of the voltage division provided by said voltage divider.

6. In a bridge network in which one of the elements is a variable standard mutual inductor, an improved variable mutual inductor comprising,
 - a fixed mutual inductor having a separate primary and secondary winding inductively coupled to each other through a low-loss medium,
 - a decade transformer variable voltage divider having input and output terminals,
 - said transformer being capable of providing a precise ratio of input to output voltage at said terminals,
 - said input terminal impedance being substantially higher than the secondary winding impedance of said inductor,
 - said inductor secondary winding being connected to said transformer input terminals to provide a mutual inductance between the inductor primary winding and the transformer output terminals which is a linear function of the voltage division provided by said voltage divider.
7. A mutual inductance circuit comprising
 - a mutual inductor having a primary and secondary winding coupled to each other through a low-loss medium,
 - a cascaded connection of voltage transformers to provide a known voltage ratio between the input terminals of the cascade and the output terminals of the cascade,
 - the impedance at said input terminals being high relative to the output impedance of said secondary winding,
 - said inductor secondary winding being connected to the input terminals of said transformer cascade to provide a mutual inductance between said inductor primary winding and said output terminals determined by the mutual inductance value of said mutual inductor and said voltage ratio,
 - and means for changing said voltage ratio.
8. The circuit of claim 7 wherein each transformer of the cascade has a pair of input terminals and a plurality of output terminals, the input terminals of one transformer being connected to a selected pair of output terminals of the preceding transformer, said means for changing said voltage ratio comprising means for selecting the output terminals of the transformers.

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U.S. Cl. X.R.

323—47, 75; 324—34, 63