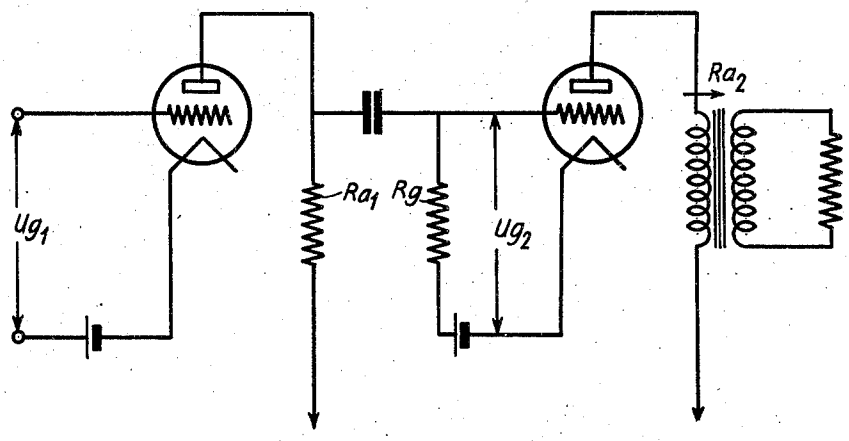
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CASCADE AMPLIFIER CIRCUIT WITHOUT UPPER HARMONICS

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CASCADE AMPLIFIER CIRCUIT WITHOUT UPPER HARMONICS

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1 Claim. (Cl. 179—171)

As is known, nonlinear distortions in transmission systems of any type can be compensated in that a system presenting upper harmonics is so connected with a second system having upper harmonics that the upper harmonics compensate each other. To this end, to a tube circuit a second tube circuit may for instance be connected which contains a tube whose characteristic represents the mirror image of the characteristic of the first tube, or two tubes having the same characteristic can be used whereby however the circuit must be such that the input voltage of the second tube is displaced in phase at 180°.

The invention is based upon the last mentioned suggestion. Previous attempts have been made in producing such a circuit, but it was thereby assumed that the characteristic of the tubes used would always follow the same mathematical law, and that furthermore almost the same working point would have to be chosen, and that finally the quotient of load resistance and internal resistance of the first tube would have to be lower than that of the second tube.

The invention is worked out with particular regard to the teaching that it is of first importance to compensate the second harmonics and that the higher harmonics can by additional consideration in the choice of the working point, be easily reduced to such a value that they no longer disturb the transmission.

The single figure of the drawing shows a two-stage amplifier of conventional type (although it will be understood that the invention is not limited to a particular number of stages) which may be operated in accordance with the invention if the steepness and the coupling organs are so chosen that the following equation will be fulfilled:

$$\frac{T_1 S_2}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}=\frac{R_{a1}S_1^2T_2}{\left(1+\frac{R_{a2}}{R_{i2}}\right)^2}$$

wherein S is the steepness; T, the differentiation of the steepness with respect to the control potential; R_a is the load resistance of each tube; and R_i is the internal resistance of each tube. Index 1 designates the respective constants of the first tube and index 2 the respective constants of the second tube. In matching the end tube with the consumer in order to avoid reflexions and to utilize the end tube as favorably as possible, the equation assumes the following simplified form:

$$\frac{T_1 S_2}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}=\frac{1}{4}R_{a1}S_1^2T_2$$

It is obvious that a proper choice of the quantities involved will result in satisfying this equation, and it will be shown further on that the satisfying of this equation is the condition for eliminating second harmonic distortion in the case where the last tube's impedance is matched by its load impedance. In contrast to this conclusion, it was thought in the prior art that in order to eliminate second harmonic distortion in a circuit of the type here considered it would be necessary to sacrifice a proper matching of the impedances of the load and of the last tube.

Very small upper harmonics can thereby be maintained inasmuch as the working point of a given tube characteristic can be so selected that the expression T becomes as small as possible.

The drawing shows a circuit of a cascade amplifier in accordance with the invention, in connection with which the above stated relation will now be derived in detail.

At the grid of the first tube a purely sinusoidal voltage is assumed namely:

$$u_{g1}=U_{g1}\sin\omega t, \qquad (1)$$

u being the instantaneous value of grid voltage, and U being the amplitude. from which follows the resulting control voltage expressed in terms of the instantaneous grid voltage, plate resistance and load resistance:

$$u_{st1}=u_{g1}\frac{1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)} \qquad (2)$$

It is this voltage u_{st} which when multiplied by the steepness S, commonly called mutual conductance or transconductance, gives the alternating plate current. For the chosen working point of the first tube there follows:

$$i_{a1}=f(u_{st1})=f(U_{st1}\sin\omega t) \qquad (3)$$

In developing the Equation (3) according to Taylor's theorem (see Barkhausen "Lehrbuch der Elektronen Roehren" 2nd volume, "Verstaerker", 4th edition, page 57, Formula 51) there will be

obtained by neglecting the D. C. portion and the quantities of an order greater than the second order:

$$i_{a1}=S_1U_{st1}\sin\omega t=\frac{1}{4}T_1U_{st1}^2\cos 2\omega t+\ldots \quad (4)$$

wherein,

$$S_1=\frac{\partial i_{a1}}{\partial u_{st1}};\ T_1=\frac{\partial^2 i_{a1}}{\partial u_{st1}^2}=\frac{\partial S_1}{\partial u_{st1}} \quad (5)$$

S_1U_{st1} is the amplitude of the amplified fundamental for the plate resistance R_{a1}.

$$-\frac{1}{4}T_1U_{st1}$$

is the amplitude of the amplified second harmonics related to the plate resistance $R_{a1}=0$.

Since however, at finite R_{a1} an anode reaction occurs also for the second harmonics, the actually appearing amplitude of the second harmonics (Barkhausen pages 58–59) is:

$$-\frac{1}{4}T_1U_{st1}^2\frac{1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}$$

in which case R_{a1} designates the plate resistance for the second harmonics, and which in the case of a purely ohmic coupling, is equal to the plate resistance of the fundamental wave. Thus for the actual plate current flowing through resistor R_{a1} there is:

$$i_{a1}=S_1U_{st1}\sin\omega t-\frac{1}{4}T_1U_{st1}^2\frac{1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}\cos 2\omega t+\ldots \quad (6)$$

According to Equation (2) there is:

$$u_{st1}=u_{g1}\frac{1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}$$

and hence according to Equation (1):

$$U_{st1}=U_{g1}\frac{1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}\ldots \quad (2a)$$

By inserting this value in Equation (6) the equation reads:

$$i_{a1}=S_1\frac{1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}U_{g1}\sin\omega t-\frac{1}{4}T_1\frac{1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)^3}U_{g1}^2\cos 2\omega t+\ldots \quad (7)$$

This equation represents the anode current i_{a1} produced by the grid potential U_{g1}.

The alternating grid potential of the second tube u_{g2} is given for $R_g \gg R_{a1}$ by the equation:

$$u_{g2}=-R_{a1}i_{a1}\ldots \quad (8)$$

and in considering the Equation (7) it is given by the equation:

$$u_{g2}=\underbrace{-\frac{R_{a1}S_1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}U_{g1}\sin\omega t}_{u_{g2}(\omega)}+\underbrace{\frac{1}{4}\frac{R_{a1}T_1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)^3}U_{g1}^2\cos 2\omega t+\ldots}_{u_{g2}(2\omega)} \quad (9)$$

The alternating grid potential u_{g2} forms two parts, namely the fundamental wave given by $u_{g2}(\omega)$ and the second harmonics represented by $u_{g2}(2\omega)$.

The amplification of the alternating grid potential u_{g2} by the second tube will be separately treated according to the parts $u_{g2}(\omega)$ and $u_{g2}(2\omega)$ in the following course of the calculation.

For the part $u_{g2}(\omega)$ there follows analogous to Equation (7):

$$i_{a2}(\omega)=+S_2\frac{1}{\left(1+\frac{R_{a2}}{R_{i2}}\right)}U_{g2}(\omega)\sin\omega t-\underbrace{\frac{1}{4}T_2\frac{1}{\left(1+\frac{R_{a2}}{R_{i2}}\right)^3}U_{g2}(\omega)^2\cos 2\omega t}_{I_{a2}(2\omega/\omega)} \quad (10)$$

whereby according to Equation (9)

$$U_{g2}(\omega)=-S_1\frac{R_{a1}}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}U_{g1}\ldots \quad (11)$$

The amplitude of the plate current of the second harmonics produced in the second tube by the fundamental wave $u_{g2}(\omega)$ will be according to Equations (10, 11):

$$I_{a2}(2\omega/\omega)=-\frac{1}{4}S_1^2T_2\frac{R_{a1}^2}{\left(1+\frac{R_{a1}}{R_{i1}}\right)^2\left(1+\frac{R_{a2}}{R_{i2}}\right)^3}U_{g1}^2\ldots \quad (12)$$

For the grid voltage part $u_{g2}(2\omega)$ representing the second harmonics, substantially only the linear amplification need be considered since owing to the smallness of this part, the higher harmonics determined thereby are of an order that is smaller than the second order. Therefore, for the amplification of $u_{g2}(2\omega)$ an anode current is obtained:

$$i_{a2}(2\omega)=S_2\frac{1}{\left(1+\frac{R_{a2}}{R_{i2}}\right)}U_{g2}(2\omega)\cos 2\omega t\ldots \quad (13)$$

so that according to Equation (9):

$$U_{g2}(2\omega)=\frac{1}{4}\frac{R_{a1}T_1}{\left(1+\frac{R_{a1}}{R_{i1}}\right)^3}U_{g1}^2\ldots \quad (14)$$

The amplitude of the plate current of the second harmonics produced in the second tube by the second harmonics $u_{g2}(2\omega)$, will be in accordance with Equations (13) and (14):

$$I_{a2}(2\omega/2\omega)=+\frac{1}{4}T_1S_2\frac{R_{a1}}{\left(1+\frac{R_{a1}}{R_{i1}}\right)^3\left(1+\frac{R_{a2}}{R_{i2}}\right)}U_{g1}^2\ldots \quad (15)$$

From Equation (12) and (15) there follows for the resulting amplitude of the second harmonics of the plate current i_{a2}:

$$I_{a2}(2\omega)=I_{a2}(2\omega/2\omega)+I_{a2}(2\omega/\omega)=\frac{1}{4}R_{a1}U_{g1}^2\left[-\frac{R_{a1}S_1^2T_2}{\left(1+\frac{R_{a1}}{R_{i1}}\right)^2\left(1+\frac{R_{a2}}{R_{i2}}\right)^3}+\frac{T_1S_2}{\left(1+\frac{R_{a1}}{R_{i1}}\right)^3\left(1+\frac{R_{a2}}{R_{i2}}\right)}\right]\ldots \quad (16)$$

Due to the phase displacement of 180° between the two parts $I_{a2}(2\omega/\omega)$ and $I_{a2}(2\omega/2\omega)$, also indicated by the different signs of the amplitudes, the second harmonics of the plate current can be made to disappear. This occurs when the quantity in the brackets becomes equal to zero in other words the following condition of the invention is fulfilled, namely:

$$\frac{T_1S_2}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}=\frac{R_{a1}S_1^2T_2}{\left(1+\frac{R_{a2}}{R_{i2}}\right)^2}\ldots \quad (17)$$

This condition will in general be fulfilled for

the disappearance of the second harmonics of a resistance-capacity coupled cascade amplifier. In matching in accordance with a further feature of the invention, the output of the cascade with the consumer the Formula (17) assumes the simplified expression:

$$\frac{T_1 S_2}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}=\frac{1}{4}R_{a1}S_1^2T_2 \quad . \quad . \quad . \qquad (18)$$

As tests have shown, an amplifier circuit can be produced by the use of the idea of the invention which has a noise factor of less than 0.05%. The direct grid potential which determines the working point and hence the steepness S may in this case even fluctuate within the limits of ±5%, a tolerance not even utilized in ordinary grid batteries.

The application of the invention moreover, is not limited to the circuit shown in the drawing. In like manner it may be utilized also in cascade circuits with more than two tubes. Furthermore, two cascades as shown in the figure may also be combined to a push-pull circuit.

What I claim is:

A cascade amplifier comprising a first tube, a coupling resistance R_{a1} in the anode alternating current path of the first tube, a second tube coupled to the first tube and fed with the entire voltage developed across said coupling resistance, a load resistance R_{a2} in the anode circuit of the second tube, the resistance R_{a1} satisfying the equation

$$\frac{T_1 S_2}{\left(1+\frac{R_{a1}}{R_{i1}}\right)}=\frac{R_{a1}S_1^2T_2}{\left(1+\frac{R_{a2}}{R_{i2}}\right)^2}$$

where S_1, S_2 are the rates of change of plate current with respect to grid voltage of the first and second tubes respectively, T_1, T_2 are the rates of change of S with respect to grid voltage of the first and second tubes respectively and R_{i1}, R_{i2} are the internal plate resistances of the first and second tubes respectively, whereby the output of the second tube is substantially free of second harmonics of the signals applied to the input of said amplifier.

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