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[54] **METHOD OF AND MEANS FOR TREATING GRAVITY PROFILES**
 16 Claims, 12 Drawing Figs.

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 [51] Int. Cl. G06g 7/19,
 G01v 7/36
 [50] Field of Search 235/181,
 193; 340/15.5 (C), 172.5; 324/1

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ABSTRACT: A gravity profile containing gravitational anomalies is utilized and treated to provide information as to depths of the sources of the anomalies. The initial measurements of gravity potential along the line of exploration making up the gravity profile are corrected for change in elevation and a plurality of averaged gravity profiles are generated. The generated profiles are subtracted from one another to generate difference profiles in which the several anomalies appear in turn and in greatly enhanced form.

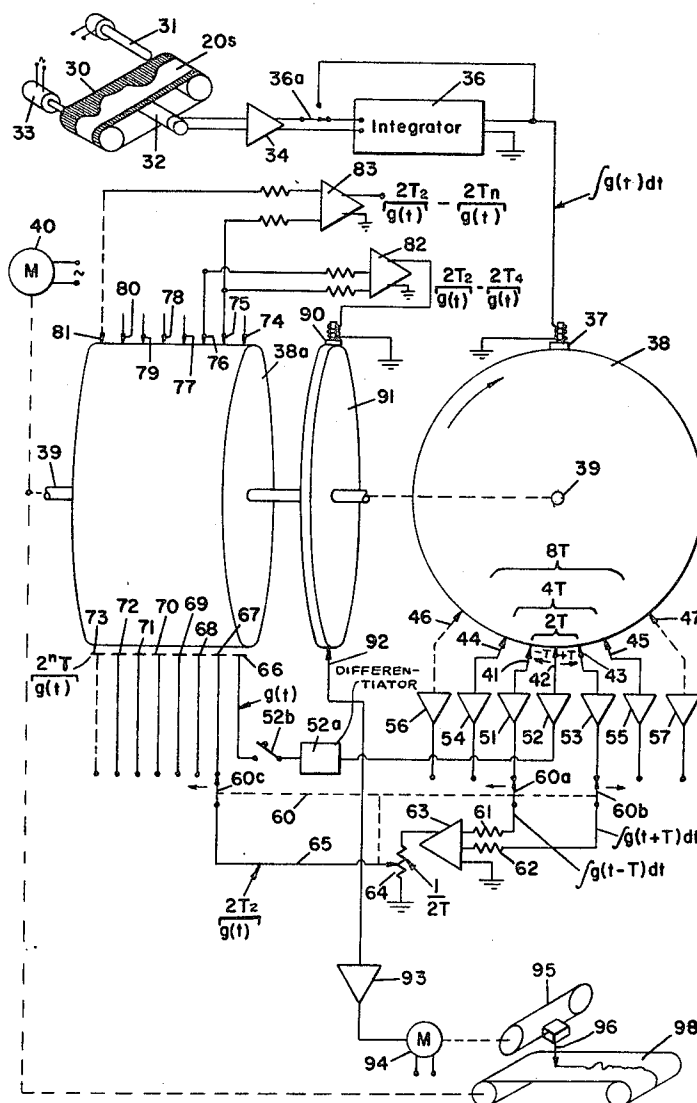


Fig. 1

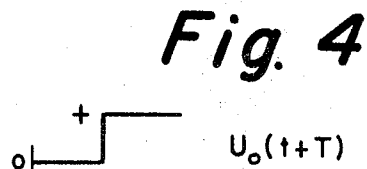
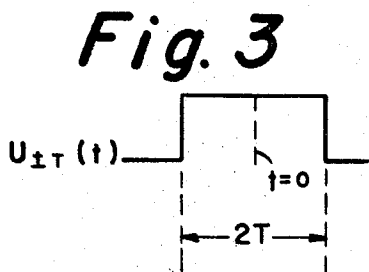
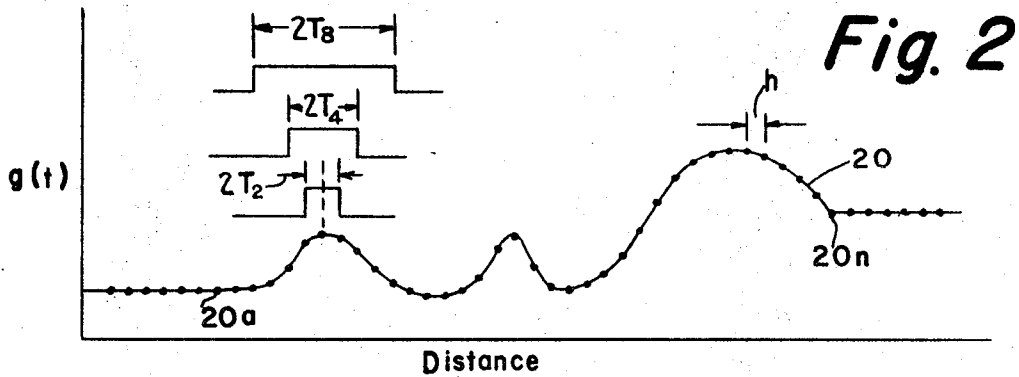
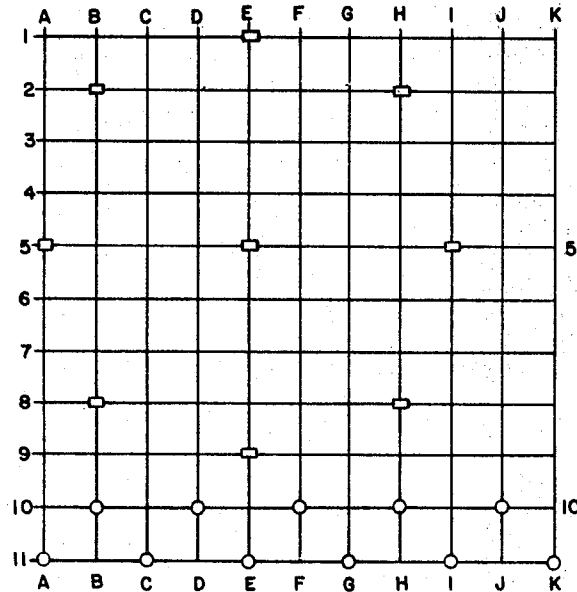


Fig. 6

$$U_{\pm T}(t) * g(t) = \overline{g(t)} = \frac{1}{2T} \left[\int g(t+T) dt - \int g(t-T) dt \right] \quad (1)$$

Fig. 7

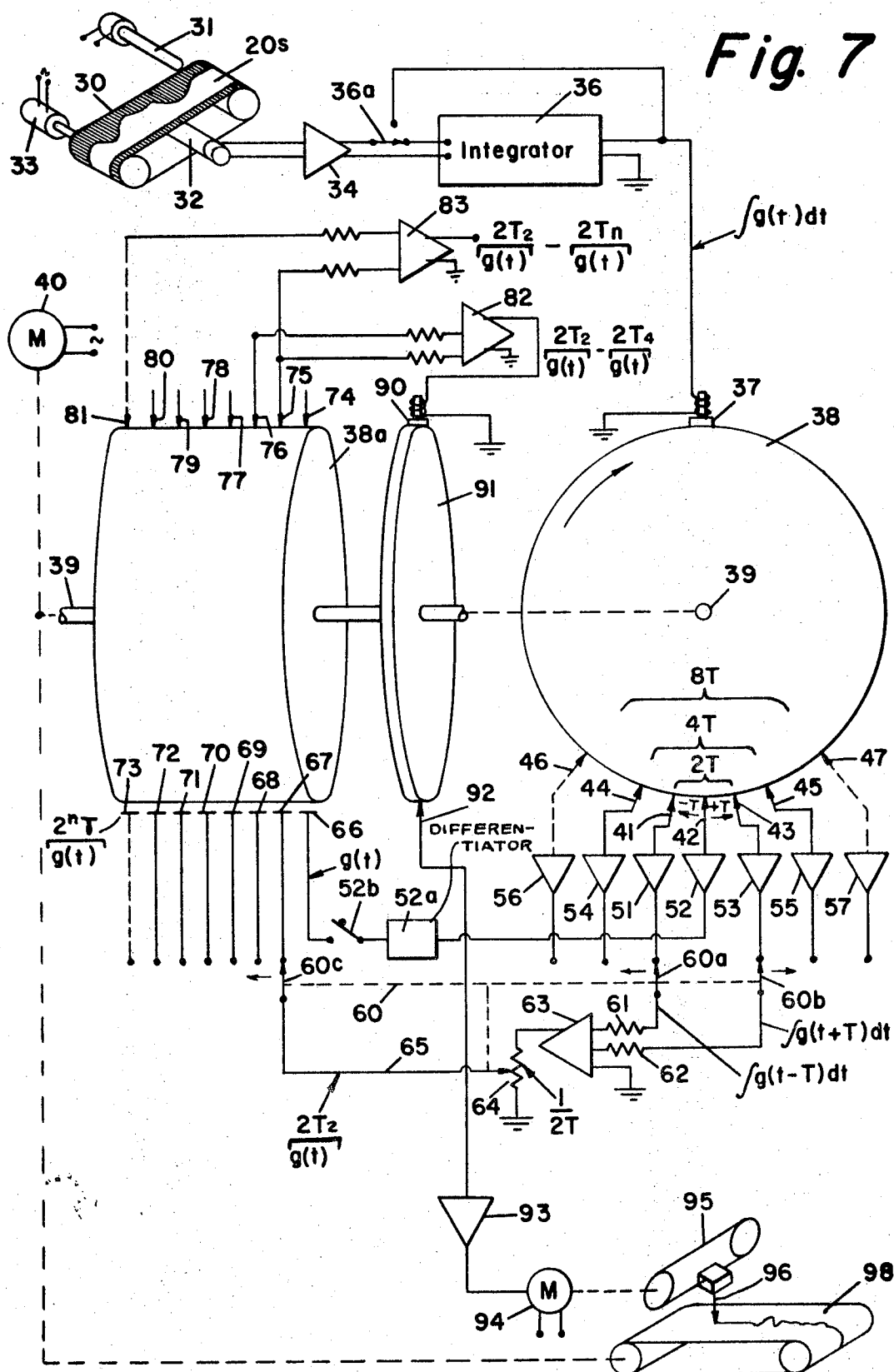


Fig. 8

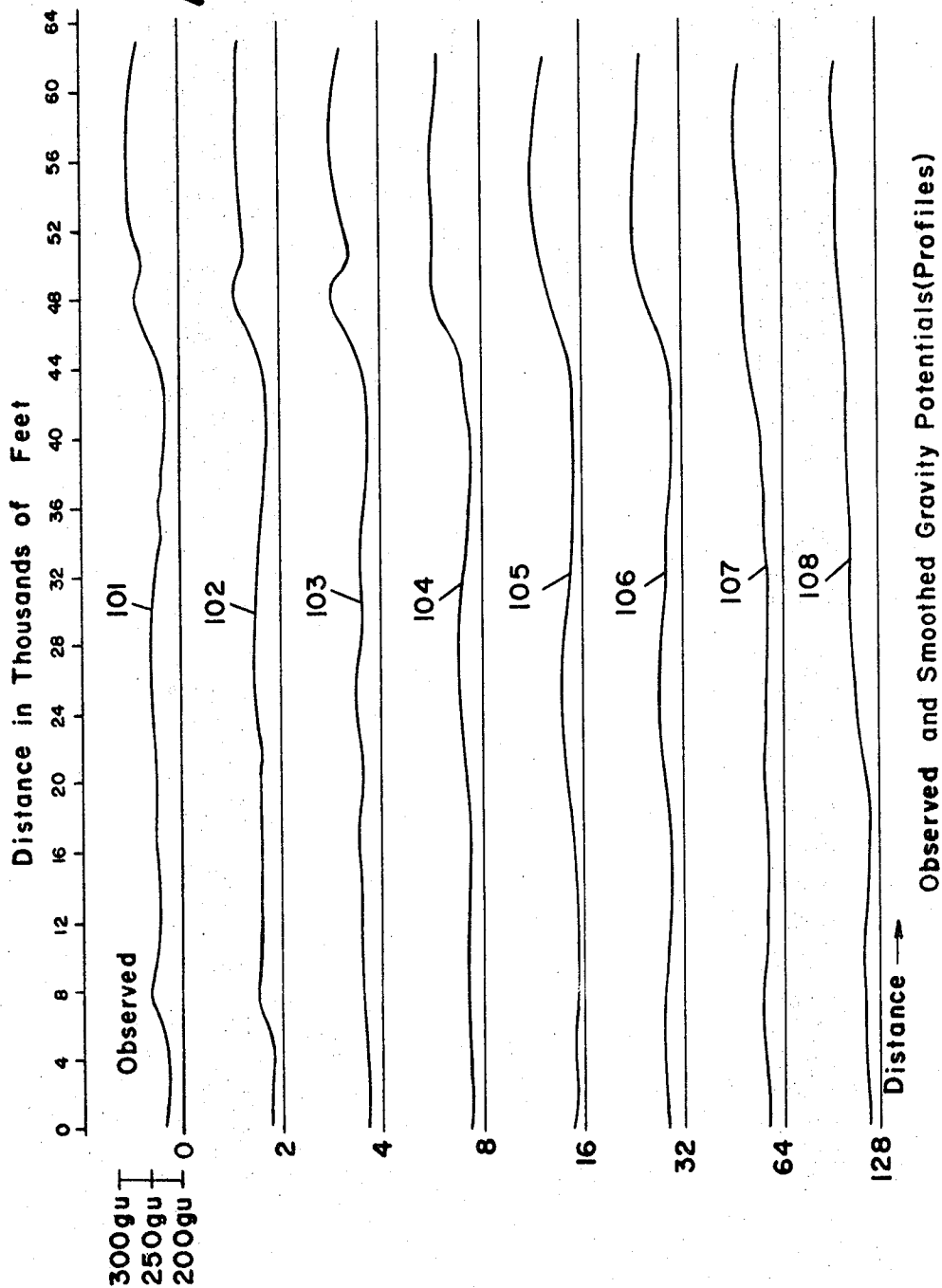
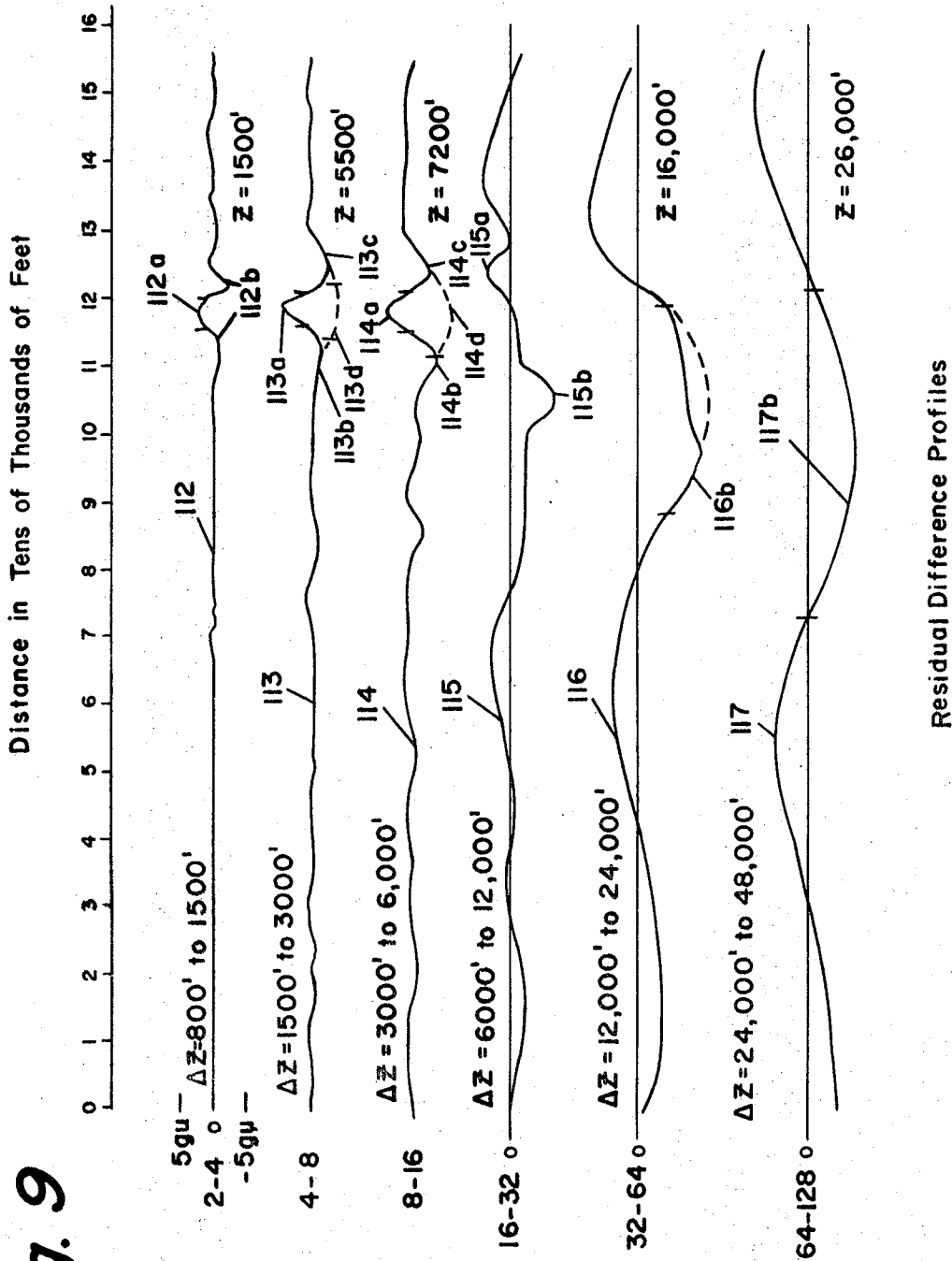
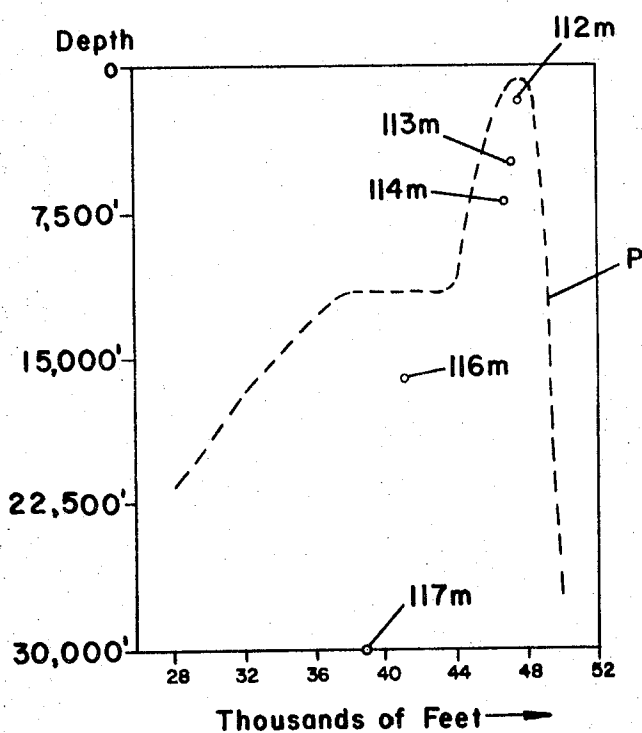
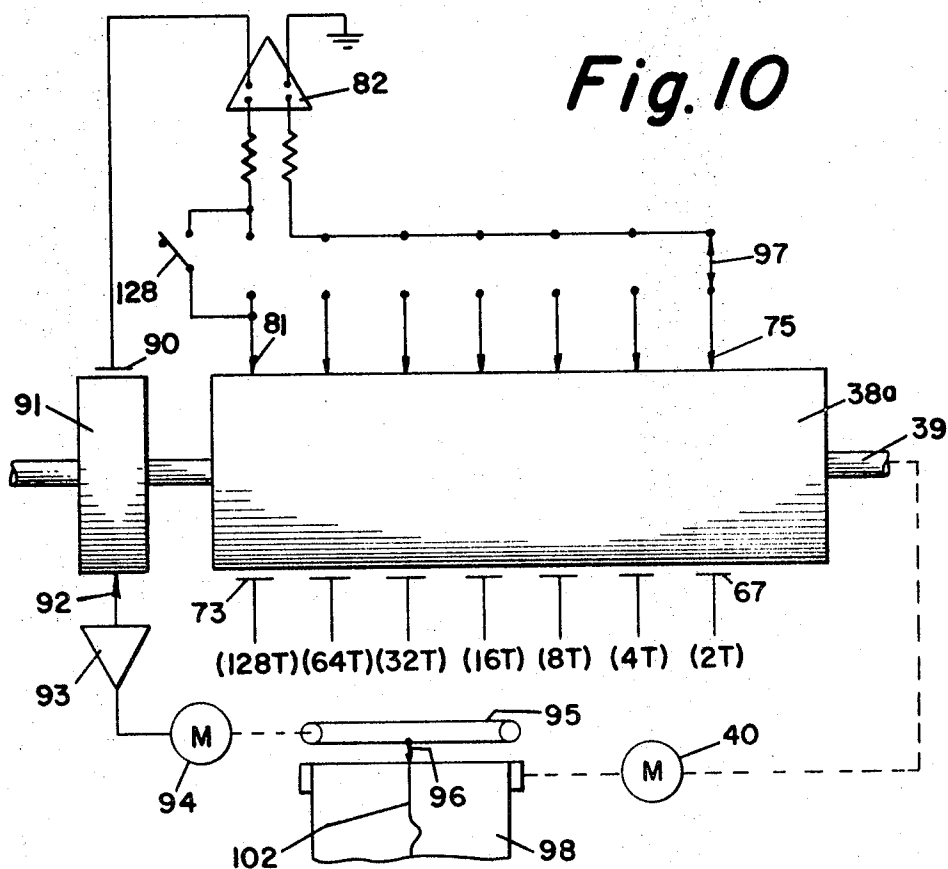
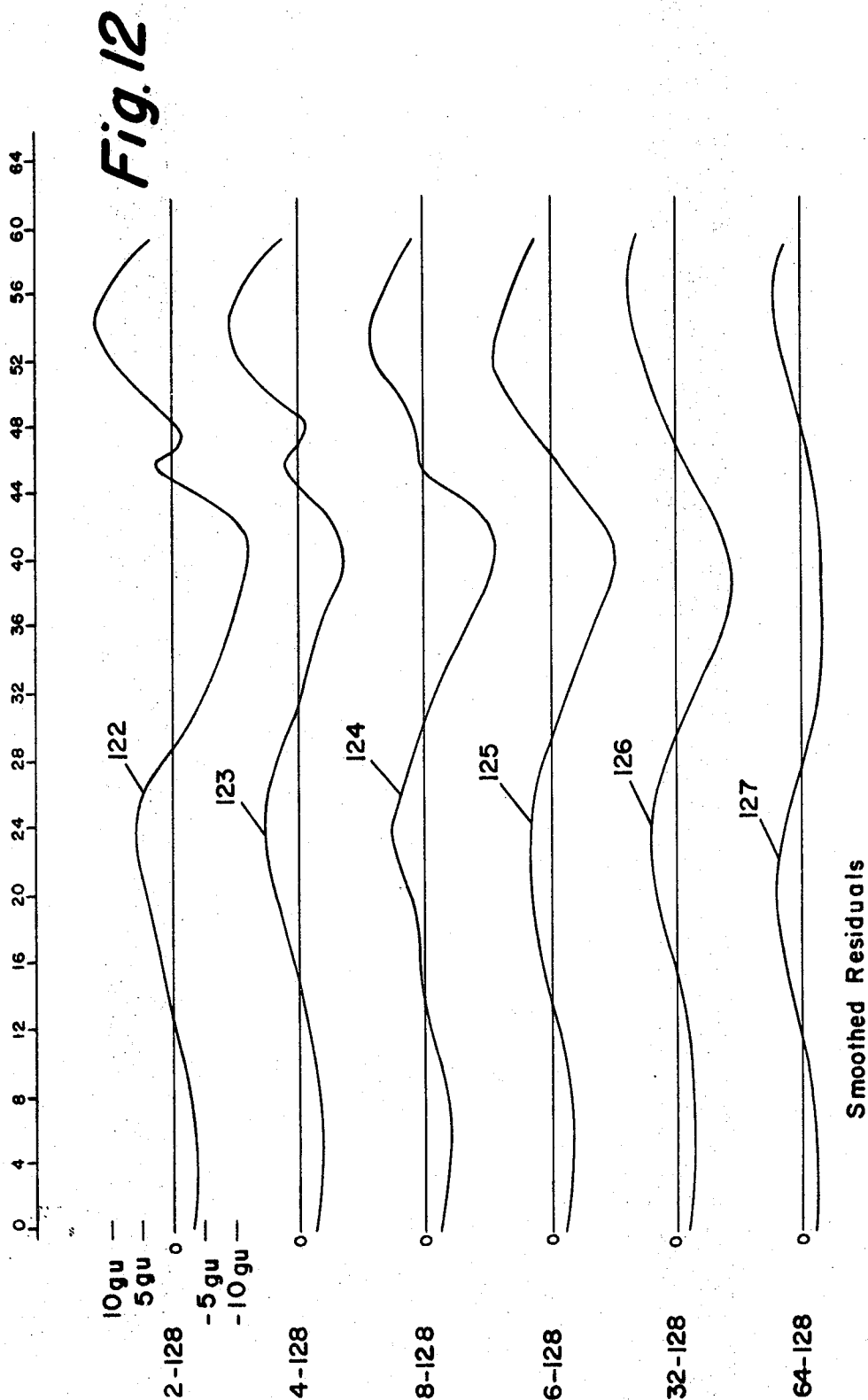


Fig. 9







METHOD OF AND MEANS FOR TREATING GRAVITY PROFILES

This is a continuation of Ser. No. 214,973 filed Aug. 6, 1962 and now abandoned.

This invention relates to methods of and means for transforming gravitational potentials existing along a profile into a form which accentuates anomalies and provides information as to the depths of the structures giving rise to them.

Measurements of gravity taken along a line of exploration and thus providing a gravity profile have long been used in geophysical exploration for the reason that the gravity potential along the profile will vary with change in the density of structures located generally below each point of measurement. Gravitational maps and gravity profiles though having a substantial degree of usefulness have nevertheless left much to be desired in providing information as to the depth and character of the masses which give rise to gravitational anomalies in gravity profiles.

In accordance with the present invention, there is utilized a gravity profile containing gravitational anomalies. The gravity profile comprises measured values of gravity or gravity potential along the line of exploration and is generally available in the form of a graph or in a table of gravity measurements either as recorded in the field or as taken from a gravity map in correlation with successive distances from a reference point along the profile. After correction of the measurements for change in elevation along and near the profile, there are generated from the resultant gravity profile a plurality of averaged gravity profiles each differing from the other by reason of the use of a different averaging operator, each successively longer than the last. The generated profiles are then subtracted one from another to generate difference profiles. On these difference profiles the several anomalies appear in turn and in greatly enhanced form. The enhancement, the distinctive character of the anomaly and the increased amplitude all appear as functions of the depths of the sources of the anomalies.

The present invention takes advantage of the fact that the special "period" or length of an anomaly as it appears in the gravity profile is, among other factors, a function of the depth of the source of that anomaly. Accordingly, a reinforced or enhanced peak appearing on one of the difference profiles will be representative of a source located in a depth range which will lie below the earth's surface no greater than a distance between about one-half of the lengths of the averaging operators applied to the two averaged profiles subtracted one from the other to produce the difference profile. Advantage is further taken of a known simple amplitude relationship determined in reference to the occurrence of the maximum amplitude of an anomaly. Thus if half the horizontal distance from the point of maximum amplitude to the point of half amplitude be taken, the amplitude value at that half-distance point will then provide information as to whether the anomaly in question may be considered due to a concentrated mass at a substantial depth or to a shallower distributed mass. For concentrated (cylindrical or spherical) mass anomalies, the difference curves are a direct indication of depth. For distributed masses, the difference curves give the maximum possible depth indication, but the actual depth may be less, depending on the assumed shape.

The present invention may be practiced both by analog and digital procedures and in each case greatly extends the information available to geophysicists in their study of the nature of subsurface formations as revealed by gravity profiles.

For further objects and advantages of the invention together with detailed instructions of how to construct apparatus embodying the invention and to practice the methods of the invention, reference is to be had to the following detailed description taken in conjunction with the accompanying drawings, in which:

FIG. 1 illustrates in part a plurality of stations at which gravitational measurements are made and the manner in which these readings have in the past been averaged;

FIG. 2 illustrates a gravity profile prepared for further processing in accordance with the present invention and illustrating three averaging intervals in connection therewith;

FIG. 3 is a sketch of an averaging operator labeled in accordance with the terms used in equations utilized in explaining the invention;

FIGS. 4 and 5 respectively are illustrations of unit step, or "Heaviside" functions, one positive and one negative, and both including mathematical expressions therefor;

FIG. 6 presents for ready reference the basic equation, the solution of which is provided in accordance with the present invention;

FIG. 7 diagrammatically illustrates an apparatus and system of the analog type embodying the invention;

FIGS. 8, 9 and 12 are graphs plotted with distance as abscissae and gravity potentials in gravity units gu as ordinates illustrative of the nature and character of data provided in accordance with the present invention;

FIG. 10 diagrammatically illustrates an apparatus by means of which there may be subtracted one profile from another; and

FIG. 11 illustrates data obtained in accordance with the invention and the manner in which it is utilized to delineate subsurface formations.

Referring now to FIG. 1, there have been illustrated a plurality of stations at which gravity or gravity potential may be measured. It has been conventional to establish a gravity measuring station at the intersection of vertical lines or columns designated A—K respectively and rows or horizontal lines 1—11. Only selected stations have been illustrated as small rectangles: those needed to emphasize the manner in which gravity measurements in the past have been averaged. Thus, only on rows 10 and 11 have all of the stations thereof been illustrated and these by open circles.

If it be desired to obtain the residual gravity at station E-5, there will be subtracted from the gravity potential at E-5 the average of the gravity readings along a circle of selected size having its center at E-5. Thus, readings at stations A-5, B-2, B-8, E-9, H-2, H-8 and I-5 will be added together and divided by 8. If residual values be desired along a gravity profile such as 5-5, then it will be understood that the foregoing laborious procedure will have to be carried out for the plurality of stations lying along profile 5-5 and further, that the computations will differ for each station since the values around the circle with its center successively at the several stations of profile 5-5 will differ. The averaging and difference procedures just described have been utilized to provide information on anomalies which may appear at a depth approximating the radius of the aforesaid circle. These methods, however, utilize sampling intervals of the order of the periods of anomalies and inject digital processing noise.

In accordance with the present invention, there are provided rapid depth estimates for the anomalies present in a gravity profile with enhanced separation of such anomalies from regional effects as well as separation of basement or sedimentary anomalies from near-surface, erratic density variations and erratic topographic effects. At the same time, processing noise is moved to a frequency well above anomaly frequencies. The manner in which these objectives are achieved will now be set forth in terms of a typical graph 20, obtained from a profile such as FIG. 2, representing a gravity profile with gravity potential $g(t)$ as ordinates plotted against distance as abscissae. It is to be noted that the first point 20a on graph or profile 20 appears at a substantial distance from the origin and that the last point 20n a substantial distance from the terminal end of graph 20. The measurements of the gravity potential at points 20a and 20n have been extended respectively to the left and to the right and each through a distance equal to, or greater than, one-half the length of the longest averaging operator utilized. By so extending the measured values, there is eliminated first-order truncation error and there is minimized distortion in the averaging of the data due to the ends of the line. It may here be noted the data

represented by the extensions just described are eliminated after the computations, later to be described, have been completed.

With the gravity profile 20 available, a sampling interval will be selected which, as shown, may be the interval h corresponding with the horizontal distances between adjacent gravity potentials as appearing on the profile 20. These may correspond with the separation distances between measuring stations, or the interval h may be selected, as for example, as one-quarter mile. In FIG. 2 these quarter-mile points have been illustrated and they appear as dots on a smooth curve illustrating the gravity profile. Station values will frequently be obtained at distances other than the desirable h . In this event, the selected values are obtained from a smooth curve drawn through the observed station values in the manner shown in FIG. 2. This method automatically interpolates gravity values between the observed values and also takes into account any apparent error in a station value because it is inappropriate in that it falls an appreciable distance from the smooth curve. In this connection, the interval h is chosen to be less than one-half the length of the shortest fluctuation in the observed gravity potential, i.e., the distance between the beginning and end of the shortest anomaly as measured along the abscissa. This procedure of selecting h is in accordance with Shannon's sampling theory.

Mathematically, the smoothing operator, illustrated in FIG. 3, must be some integer multiple of the sample interval h for discrete operators. If the midpoint of the operator be taken as $t=0$, where t is distance ($t=ih$, where i =any integer), then the operator may be expressed as $U \pm T(t)$. The total length of the operator will be $2T$. This concept is particularly helpful since $U \pm T(t)$ may be decomposed into two unit step functions, namely, $\pm U_0(t \pm T)$. FIG. 4 illustrates that $U_0(t+T)$ is a unit step function; similarly, $-U_0(t-T)$ is a unit step function, the former being positive and the latter negative in sign. Thus these two unit functions, when algebraically added, define the smoothing operator of FIG. 3. Further extensions of this technique will be later explained.

If a function such as gravity potential $g(t)$ be convolved with a unit step function, the result will be the integral of $g(t)$. Accordingly, there may be written the following equation and for convenience presented in FIG. 6:

$$U_{\pm T}(t) \times g(t) = \frac{2T}{g(t)} = \frac{1}{2T} \left[\int g(t+T) dt - \int g(t-T) dt \right] \quad (1)$$

In the foregoing equation, the expression $2T/g(t)$ is the average gravity potential taken over the averaging length $2T$. As will be later explained, the function $U \pm T(t)$ will be utilized for obtaining running averages of the several values appearing throughout the length of the gravity profile 20. Though an analog method of performing that averaging has been illustrated in FIG. 7, it is clear that one may utilize instantaneous values of gravity potential along the profile 20, and average these values as the averaging operator is moved along profile 20. This can conveniently be done by converting the instantaneous values to binary numbers and the averages obtained by a conventional averaging program for a digital computer. In the program for the computer, the foregoing Equation (1) applies with the substitution of (ih) for the term (t) and an operational summation sign for each of the integral signs. The interval h is substituted for (dt) .

The result of averaging the gravity values over the profile 20 will be the production of a new gravity profile then representative of average values of $g(t)$ for the first averaging interval $2T$ which, as above suggested, may correspond with a distance of one-half mile if $T=h$ and h =one-quarter mile.

The foregoing operations are then repeated with successive different averaging intervals. It is preferred that these intervals each differ from the other by an octave (approximately) or that the second and each successive averaging interval have approximately twice the length of the preceding one. Thus, if the first smoothing interval $2T$ corresponds with $T=h$ of FIG. 2, the second smoothing interval $4T$ will have twice the length

of interval $2T$ and the third interval $8T$ will have twice the length of interval $4T$. The smoothing intervals will progressively change and have values of $16T$, $32T$, $64T$, $128T$ and more, if desired.

Although the averaging operators illustrated herein are $2^n T$ long (where $n=1, 2, 3, 4, \dots$ etc.), it is often desirable to use operators $(2^n - 1) T$ long, that is, $3T$, $7T$, $15T$, $31T$, $63T$ and $127T$ length smoothing intervals, since such smoothed curves, in digital operations, are centered about the observed gravity values.

The several resultant averaged gravity profiles are then subtracted one from the other to generate difference profiles. The difference profiles will contain enhanced representation of anomalies as a function of the depths of the masses or sources giving rise to the respective anomalies. These multiple operations may be rapidly performed, thus to provide in conveniently usable form new and improved data from gravity profiles.

In the analog embodiment of the invention, as illustrated in FIG. 7, the gravity profile 20 of FIG. 2 has been converted to a reproducible signal 20s as a continuously variable area record on film 30. This film or recording medium 30, though illustrated as of the photographic type, could just as well be a magnetic tape having recorded thereon the changes in gravity potential as shown by curve 20 of FIG. 2 and with an associated pickup head for generating signals representative of continuous gravity profile 20. For the photographically reproducible signals, a light source 31 in conventional manner (lenses and light slits having been omitted to simplify the drawing) directs a plane of light across the film 30. Light passing through the continuously variable area record is received by a photoresponsive cell 32. The film is driven at a constant speed by motor 33 and has been illustrated as of the endless type for convenient repetitive playback. Since the length of the profile is related to distance and since it is being driven at constant speed, it will be understood that there will have been established proportionality between distance and time.

The output from the photoresponsive means 32 is amplified by an amplifier 34 and applied to an integrator 36. This integrator is preferably of high quality and may be of the kind illustrated at pages 12-20 in the book by Korn and Korn entitled "Electronic ANALOG Computers," Second Edition (1956). The integrator 36 performs the integrating steps as called for by Equation (1) as appearing in FIG. 6. The output from the integrator, $\int g(t) dt$, with further amplification if desired, is applied to a magnetic recording head 37 disposed in recording relationship with a magnetic recording head 37 disposed in recording relationship with a magnetizable medium, such as magnetic tape, carried by a drum 38 supported on a shaft 39 driven by a motor 40, or if desired, from the motor 33.

As shown, there are provided in association with the drum 38 a plurality of pickup heads 41-47.

The pickup or reproducing heads 41-47 have respectively associated therewith amplifiers 51-57. Though only two pickup heads 41 and 43 need be utilized with these heads adjustable circumferentially on the drum 38, in practice, it will be preferred to utilize a multiplicity of pickup heads all circumferentially adjustable on the drum 38. Each of the respective pairs will have separation distances corresponding with the differing selected smoothing intervals. Thus, if a reference point be chosen on the drum 38, such for example, as the position of the pickup head 42, the purpose of which will be later explained, then the pickup head 41 will have a distance from the reference position at pickup head 42 corresponding with the interval $-T$ of FIGS. 2 and 3. With clockwise rotation of drum 38, pickup head 41 will be lagging, that is to say, the integral of the gravity potential arrives at pickup head 41 after the time occurrence t by the interval T . Similarly, it will be seen that the pickup head 43 is in a leading position, that is, spaced from the reference position of pickup head 42 by the interval T . Thus, the time occurrence t of the events on the

profile 20 of FIG. 2 arrive at this pickup head 43 in time advanced by the interval T . In terms of the expressions appearing in FIGS. 4 and 5, pickup head 41 corresponds with the $(t-T)$ expression, and pickup head 43 corresponds with the expression $(t+T)$. The negative sign of the expression of FIG. 5 $(-U_0)$ is taken into account by the polarity of the connection from pickup head 41 to amplifier 51 or, of course, that polarity can be established at the output of the amplifier.

As the observed gravity profile, after integration and after extension of its initial and final values as described in connection with FIG. 2, is recorded on the drum 38 it then appears at pickup head 43. This pickup head reproduces the initial value until the drum not only has moved the recorded signal to pickup head 41 but also through a distance corresponding with the greatest sampling interval $128T$. Thus as the first of the fixed values 20a, FIG. 2, arrives at pickup head 43, that value will be combined with successive values appearing at pickup head 43. Finally, as the end of the record arrives at pickup head 43 corresponding with the last fixed value 20n, FIG. 2, pickup head 43 continues to generate signals corresponding with value 20n until the end of the extended record. The foregoing, of course, applies equally to the pickup heads having separation distances equal to $128T$. It is in this manner that there is avoided the truncation error already described.

From the foregoing analysis, it will now be clear that the pickup head 41 generates at the output of amplifier 51 signals corresponding with the expression $[-\int g(t-T)dt]$ and that the pickup head 43 generates at the output of amplifier 53 signals corresponding with the expression $[\int g(t+T)dt]$. The negative sign ahead of the first mentioned expression has already been taken care of by the connection to or from the amplifier 51 opposite to the connection to or from the amplifier 53.

The two expressions within the brackets of Equation (1), FIG. 6, are now algebraically added together by applying the outputs from the amplifiers 51 and 53 through contacts 60a and 60b by way of summing resistors 61 and 62 to a summing amplifier 63. Thus, the output of amplifier 63 provides a solution for the two terms within the brackets of Equation (1) of FIG. 6. By applying the output of amplifier 63 to a potentiometer 64 and by suitably setting the movable contact associated with this voltage divider, the output at line 65 will be the output of the amplifier 63 modified by the expression $1/2T$: the reciprocal of the smoothing interval $2T$. Thus, the movable contact of potentiometer 64 may be adjusted as the selector switch 60 is operated to change the connections to the amplifier for solving the equation with a different smoothing interval.

The solution of Equation (1) of FIG. 6 which has thus been provided appears as the output from potentiometer 64 and corresponds with the expression $2T/g(t)$. It is this signal which may be applied directly to the amplifier 93 of the recorder. However, and for reasons which will later become apparent, that signal is applied by way of contact 60c of selector switch 60 to a recording head 67 of a recording drum 38a carrying a recording medium, such as magnetic tape. In practice, the mechanical connection shown by the broken line connection from the movable contact of potentiometer 64 will ordinarily be omitted, and instead there will be provided summing amplifiers for each pair of pickup heads 47-47, etc., and with several potentiometers corresponding with potentiometer 64 set in terms of the respective smoothing intervals. Thus, as the drum 38 is rotated, there will concurrently be recorded by recorder head 67-73, etc., averaged gravity profiles, each averaged with a different length interval. Thus, consistent with the assumptions earlier made, the spacing between pickup heads 41 and 43 will be equal to $2T$, the spacing between heads 44 and 45 equal to $4T$, the spacing between heads 46 and 47 equal to $8T$, and so on until the final smoothing interval, which may be $128T$, will have been set by pickup heads associated with drum 38. Since not all of these pickup heads have been illustrated in association with drum 38, the final recording head 73 carries the label of $2^n T/g(t)$, where $(n=1, 2, 3, 4, 5, 6, 7)$.

Though the subtraction of one averaged gravity profile from another may be accomplished by suitable switching between the plurality of pickup heads 74-81, it is to be understood that all of the desired difference profiles may be concurrently generated by providing a plurality of summing amplifiers. For purposes of simplicity, only two amplifiers 82 and 83 are shown in FIG. 7. One of them, the amplifier 82 has its output connected to the associated recording head 90 in recording relationship with a recording medium, such as magnetic tape on a drum 91. In practice, the drum 91 will be wide and will have recording heads and pickup heads in number respectively equal to the number of difference gravity-profiles selected for an embodiment of the present invention.

The pickup head 75 generates a signal corresponding with the expression $2T_2/g(t)$, and the pickup head 76 generates a signal corresponding with the expression $4T_4/g(t)$. Thus, if the connections to pickup head 76 as applied to the input of the summing amplifier 82 be reversed relative to the polarity of the conductors of pickup head 75, it will be seen that the summing amplifier 82 will provide an output corresponding with the difference between the aforesaid averaged gravity profiles, i.e., $2T_2/g(t)-2T_4/g(t)$. This difference gravity profile is now recorded on the tape carried by the drum 91. It is reproduced by the pickup head 92, amplified by an amplifier 93, and applied to control the rotation of a motor 94 which through a belt or violin string 95 positions a recording head or pen 96 laterally of a recording medium 98. This recording medium, which can be a recording chart, is driven from shaft 39 which carries the drums 38, 38a and 91. Recorders of the type referred to are well known to those skilled in the art and are highly suited to the generation of the difference gravity profile as detected by pickup head 92. In practice, a multiplicity of recorders may be utilized, but for reasons of economy it will sometimes be preferred to utilize switching as between the respective pickup heads 74-81 for successive generation of the difference gravity profiles by means of which valuable information may be obtained from the original gravity profile illustrated in FIG. 2. As further illustrated in FIG. 7, the second difference gravity profile may be generated by applying the output from amplifier 83 to the recording head 90. This difference profile, for illustration of the broad principle, has been shown in FIG. 7 as $2T_2/g(t)-2T_n/g(t)$. Thus, there have been disclosed several embodiments by means of which the invention may be practiced.

Now that there has been explained the manner in which an analog solution has been provided for the equation of FIG. 6, it is again emphasized that the equation may be solved by utilizing the smoothing operators as set forth above, digitizing the values from the gravity profile 20 with the extensions at the ends as already described and that after averaging with the several length operators, a computer may be readily programmed to generate outputs representative of difference profiles and that such binary outputs representative of difference profiles and that such binary outputs from computers may be, through a decoder, applied to the amplifier 93 to generate the difference profiles in manner just explained.

Referring now to FIGS. 8 and 9, there will be considered the multiplicity of averaged profiles and difference profiles all generated as described above.

In FIG. 8 the curve 101 has been plotted from observed data, i.e., from gravity measurements taken along a line of exploration to provide the gravity profile. This gravity profile is converted into the reproducible signal 20s, FIG. 7, integrated at 36 and the smoothing interval $2T_2$ established to produce outputs applied to the amplifier 63. After modification by the potentiometer 64 there is produced at conductor 65 an output equal or proportional to the gravity potential of curve 101 smoothed by the operator $2T_2$. This is the signal recorded by recording head 67 on drum 38a.

As shown in FIG. 10, the signal recorded by recording head 67 is detected by the pickup or playback head 75 and through a multiple point switch 97 is applied to the amplifier 82 with its output connected to the recording head 90 of drum 91. The pickup or playback head 92 applies the resultant output to

amplifier 93 and thence to the motor 94 which generates on the recording medium 98 the gravity profile 102 of FIG. 8.

By operating the switch 97 to the successive positions with concurrent change in positions of selector switch 60 of FIG. 7 there will be generated the smoothed gravity potential curves 102—108, FIG. 8, each respectively smoothed with smoothing operators 2T, 4T, 8T, 16T, 32T, 64T and 128T. The result of these smoothing operations is the progressive removal of higher (spacial) frequencies (shorter length oscillations) present in the original observed gravity profile 101, these high frequency components being due to gravity anomalies closer to the surface. With the smoothing operator proportional to samples about a quarter of a mile apart, the 128T interval (T=one-quarter mile) will largely average out anomalies less than about 16 miles across (these periodicities then being less than about 32 miles).

By now taking the differences between the foregoing progressively smoothed curves 101—108, anomalies appearing in the form of wavelets of differing frequency will be accentuated. Thus as explained in detail in connection with FIG. 7 (and in which there was directly obtained the curves now to be described) the curve or difference gravity potential 112 of FIG. 9 is the result of subtracting the gravity potential 103 of FIG. 8 from the gravity potential 102. For convenience, each curve of FIG. 9 has in the left-hand margin a reference to the operations which produced it, i.e. 2—4, being indicative of the manner in which the curve 112 was obtained. Thus the difference profile 113 resulted from the subtraction of curve 104 from 103; difference profile 114 was obtained from subtracting curve 105 from 104. The difference profile 115 was obtained from subtracting profile 106 from 105. The difference profile 116 was obtained by subtracting profile 107 from 106. The difference profile 117 was obtained by subtracting profile 108 from 107. Thus, the profile of longer smoothing interval is in each case subtracted from a profile of a shorter smoothing interval.

Referring again to the difference profile 112 and with the foregoing assumed selection of samples of one-quarter of a mile, anomalies ranging in period from about one-quarter to about one-half of a mile wide will be enhanced. Thus there appears on gravity potential 112 a wavelet having a positive-going component 112a and a negative-going component 112b. In accordance with the present invention, advantage is taken of the fact that the anomaly period or length is a function of the depth of the source of the anomaly. Since the width of the anomaly on say the positive component 112a can have a width at half maximum amplitude which can range from between 800 ft. to 1500 ft., it is known at once that the wavelets 112a and 112b will be representative of anomalies within or near the foregoing range. If the width of the positive component 112a at 50 percent of maximum amplitude be measured, it will be found that the depth Z will approximate 1500 feet. It is for this reason that the difference profile 112 has on it the label ΔZ (the range of depth from 800 feet to 1500 feet). Similarly, the remaining difference profiles 113—117 have been labeled with their respective depth ranges and also with depths as determined from the width of the wavelets at their halfway points. In this connection it is to be noted that on difference profile 113 the depth of the anomaly is given as 5500 feet which is beyond the range of ΔZ for this gravity profile. The same disparity exists for the depth Z of 7200 feet for the difference profile 114. In both profiles, however, it will be noted that the two troughs 113b and 113c indicate that the anomaly 113a has both a positive and negative portion. The negative-going portion can be illustrated by the broken line 113d. Thus the fact that two anomalies superimpose explains the disparity between the predicted range of depth and that determined by the measurements described above. Accordingly, the negative going wave can be ascertained as indicated by the broken-line representation 114d.

In the gravity profile 115 "picks" can be made at the 50 percent points for the negative-going component 115b and for the positive-going component 115a. However, knowing that the

negative-going anomaly has been developing as explained in connection with gravity potentials 113 and 114, the difference profile 115 may be considered as lacking in sufficient definiteness to indicate a simple anomaly and thus attention may be directed to the difference profile 116. Here it will be seen that the negative-going character of the anomaly is now quite pronounced and picks may be readily made as indicated by the vertical lines intersecting the negative-going component 116b which provide a depth for the anomaly at 16,000 feet. The broken-line portion of the component 116b can be reliably drawn since it is known that there still remains some effect from the positive-going components which appeared in the earlier difference profiles 112—115. In the final difference profile 117 there is obtained a very smooth negative component 117b providing even greater reliability in the pick of the 50 percent points for determination of the depth Z approximating 30,000 feet.

Inasmuch as the profiles of FIG. 8 and 9 are all plotted with distance along the line of exploration as abscissae, information as to the location along the profile of the anomaly just described is also indicated.

Thus, the midpoints of the wavelets of FIG. 9 on which the pick-lines have been illustrated by the vertical lines described above may now be plotted, FIG. 11, with distance for abscissae and depth as ordinates. The small circles 112m—114m, 116m and 117m are respectively indicative of depths to the center of the respective anomalies determined by the gravity profiles 112—114, 116 and 117 of FIG. 9. It will be observed that there is migration to the left with increase of depth of the center points of the respective anomalies. The information obtained as to the distribution of the anomaly at each point, since it is gradually increasing in volume, makes it possible to draw, as indicated by the dashed line, the outline of the assumed total anomaly. This dashed curve P shows that the anomaly comprises a narrow spine extending upwardly toward the surface from one side of a much larger volume, larger with increasing depth. This upwardly rising spine suggests the presence of a salt dome and thus indicates the desirability of a seismic survey over the region in question. It is in this manner that the present invention greatly extends the information available from gravity surveys and to a point where complex subsurface anomalies may be identified, such as the salt dome illustrated in FIG. 11.

Further in accordance with the invention, the nature of residual effects of gravity represent information needed in the interpretation of gravity data. In accordance with the present invention and particularly the arrangements of FIGS. 7 and 10 a set of smoothed residuals of gravity potential may be readily obtained as illustrated in FIG. 12. Thus, if a switch 97 be in its illustrated position, FIG. 10, and switch 128 closed with the polarity of the signals from the pickup or playback head 81 reversed from pickup head 75, it will be seen that there will be applied to the summing amplifier 82 the difference between signals detected by playback head 75 and those from playback head 81. More specifically, signals representative of the smoothed gravity profile 108 of FIG. 8 will be subtracted from the smoothed gravity profile 102. These, it will be recalled, have sampling intervals respectively equal to 128T and 2T. The result of the described operations is the production on the recording medium 98 of the smoothed residual gravity potential 122 of FIG. 12. As the selector switch 97 is moved to the left through its switching positions there will be successively generated the smoothed residual gravity potential curves 123—127 of FIG. 12, each curve of profile having an additional labeling indicating the several subtractions which take place, the final smoothed gravity potential 108 of FIG. 8 in each case being subtracted in turn from the remaining gravity potentials of FIG. 8.

It is to be noted that in FIG. 8 the potential curves 101—108 have been plotted with each unit of magnitude of ordinates equal to 50 gravity units. In one embodiment of the invention this unit was 1 inch. In FIGS. 9 and 12 the same unit of magnitude for the ordinates for the same embodiment of the in-

vention are each made equal to five gravity units. The distance scale is 10,000 feet per unit. In the present drawings, the scales for the illustrated data have been selected for its display in much the same form as in said described embodiment.

Now that there has been described an analog system embodying the invention together with the manner in which information can be generated as to depth of anomalies, it will be understood that other modifications may be made and that certain features of the invention can be utilized without other features. Moreover, additional information can be generated from gravity profiles, particularly where there is considerable interest in distinguishing between near-surface anomalies of relatively great area and somewhat deeper anomalies of greater density and less area.

It has been found that if the gravity profile 101 of FIG. 8 be differentiated prior to recording on the drum 38, shallow distributed masses will produce components on the resultant gravity curve having sharp edges which disappear in the later records. Thus one can distinguish between the concentrated deeper masses and the shallow distributed masses. To differentiate signals applied to the recording head 37, FIG. 7, a differentiator may be included in circuit with the recording head 37. Without such a differentiator, however, it is known that the pickup head 42 reproduces the quantity $\int g(t) dt$. By applying this quantity to a differentiator 52a and by closing the switch 52b there will be obtained and applied to the recording head 66 the function $g(t)$. However, since the function $g(t)$ forms the input to the integrator 36 it will be seen that by operating the switch 36a to its uppermost position the integrator 36 will be bypassed and the function $g(t)$ will be directly applied to the recording head 37. Hence, switch 52b may remain open. In this way the differential of the function above described may be utilized in generating smoothed gravity profiles as explained for profiles 102-108 of FIG. 8 and also utilized in the production of difference profiles such as shown in FIG. 9 and also for smoothed residuals of the kind already described in connection with FIG. 12.

Referring again to FIGS. 2-6, it will be remembered that the methods and procedures outlined in connection with FIG. 7 produce a convolution of a function $g(t)$ with the operator of FIG. 3. While in this case the function $g(t)$ is a function of gravity potential, it is to be understood that the method may also be applied to other functions such as $x(t)$ which varies in amplitude along a scale. The convolution will then be with an operator which may comprise at least the two step functions of FIGS. 4 and 5, respectively of positive and negative sign and respectively occurring at different positions along the abscissa scale of the function $x(t)$. This convolution is, according to the right-hand side of the equation of FIG. 6, performed by algebraically adding together at least two functions respectively representing summations of integrals of $x(t)$, these integrals being displaced one from the other along said scale by amounts corresponding with the displacements one from the other of said step functions.

I claim:

1. The method of treating a gravity profile containing gravitational anomalies extending along a distance scale which comprises the following steps each executed by automatic computing apparatus:

generating and storing machine responsive signals representing the amplitudes of gravity potentials represented by said gravity profile,
algebraically and progressively adding together along said profile the stored machine responsive signals representing amplitudes spaced apart by an interval,
dividing the algebraic sum of said machine responsive signals by a factor proportional to the reciprocal of said interval to produce quotient machine responsive signals, and
recording the quotient machine responsive signals representative of the averaged profile in correlation with the scale of said gravity profile.

2. The method of treating gravity profiles containing gravitation anomalies which comprises the following steps each executed by automatic computing apparatus:

generating and storing machine responsive signals representing said gravity profiles,
generating from a stored machine responsive signal representing a gravity profile a plurality of averaged machine responsive signals representing gravity profiles smoothed by differing smoothing intervals extending along said gravity profile, and
subtracting said plurality of averaged machine responsive signals one from the other to generate machine responsive signals representing difference profiles thereby to enhance the appearance of the anomalies on the several difference profiles.

3. The method of claim 2 in which said averaged machine responsive signals are generated by integrating with respect to distance along said gravity profile,

algebraically adding together values from the integrated profile corresponding with

$$\int g(t+T)dt - \int g(t-T)dt$$

where:

$g(t)$ = gravity values along said gravity profile,

T = one-half the smoothing interval, and

t = distance along said gravity profile, to produce a resultant sum machine responsive signal, and

modifying said sum machine responsive signal by the factor $1/2T$.

4. The method of claim 2 in which said smoothing intervals each differ one from the other by the same order of change in length.

5. The method of claim 3 in which said smoothing intervals each differ one from the other by an octave.

6. The method of claim 5 in which anomalies on said difference profiles are related to zones of depth approximately respectively one-half of the smoothing intervals of two generated machine responsive signals subtracted one from the other.

7. The method of claim 2 in which each of said difference profiles is recorded and the separation distances at half amplitude of anomalies are recorded in terms of depth of their occurrence.

8. The method of claim 2 in which said gravity profile is extended at and without change from both its initial and final values by amounts equal to the maximum value of said smoothing intervals.

9. The method of treating gravity profiles containing gravitational anomalies which comprises the following steps each executed by automatic computing apparatus recording on a reproducible medium signals representative of a gravity profile with the initial and final values of the gravity potential of said profile extended a distance along said medium proportional to a distance equal to the maximum of the smoothing interval defined by the smoothing function hereinafter set forth, generating from said medium electrical signals,

convolving with said electrical signals representative of said gravity profile $g(t)$ a smoothing function $U \pm T(t)$ for generating averaged gravity profiles $2T/g(t)$ where each change of T is of the same order as the preceding change, and subtracting in succession said profiles one from the other, the longer-smoothed from the shorter-smoothed profiles, for generating difference profiles thereby to enhance the appearance of the anomalies on the several difference profiles.

10. The method of claim 9 in which said electrical signals representative of said averaged gravity profiles are recorded in succession prior to said subtraction thereof one from the other.

11. The method of processing gravity profiles representing gravitational potential along a line on the earth to accentuate anomalies therein comprising the following steps each executed by automatic computing apparatus:

recording said gravity profiles along a length of a reproducible medium corresponding with a distance along said line on the earth,

converting the recording on said reproducible medium to a first electric signal representing said gravitational potential as a function of time,

integrating said first electric signal to produce a second electric signal representing integrated gravity potential as a function of time,

continuously subtracting a first component of said second electric signal from a second component of said electric signal which is separated in time from said first component by an interval which is an integer multiple of a sampling interval to generate a third electric signal representing an average gravity profile,

continuously subtracting a third component of said second electric signal from a fourth component of said second electric signal which is separated in time from said third component by an interval which is a different integer multiple of said sampling interval to generate a fourth electric signal representing an average gravity profile,

continuously subtracting said third electric signal from said fourth electric signal to generate an electric signal representing a difference profile, and

recording said electric signal representing said difference profile on a reproducible medium to produce a record in which the appearance of said anomalies is enhanced.

12. A system for treating a gravity profile containing gravitational anomalies extending along a distance scale in which system elements are interconnected to comprise:

means for generating electrical signals representing the amplitudes of gravity potentials represented by said gravity profile,

storage means for storing the generated electrical signals, means for extracting from said storage means a plurality of components of the signals stored therein, each time-shifted one from the other by a different smoothing interval related to said scale,

summation means, pairs of said components being applied to said summation means to produce at the output thereof signals representing the difference between the pairs of components applied to the input,

dividing means, the output of said summation means being applied to said dividing means, said dividing means being set to divide the output of said summation means by the smoothing interval separating the pair of components applied to the input of said summation means, the output of said divider being an electrical signal representing a gravity profile averaged over a smoothing interval,

means for storing a plurality of the electrical signals each of which represents a gravity profile averaged over a smoothing interval,

difference means,

means for extracting from storage pairs of said electrical signals representative of averaged gravity profiles averaged over different smoothing intervals, said last-named pairs of signals being applied to said difference means, to generate at the output of said difference means an electrical signal representing a difference gravity profile, and

means for recording the output of said difference means on a reproducible medium to produce a gravitational record in which the appearance of said anomalies is enhanced.

13. A system for treating a gravity profile containing gravitational anomalies extending along a distance scale in which system elements are interconnected to comprise:

means for generating electrical signals varying in proportion to the amplitudes of gravity potentials represented by said gravity profile,

means for storing said generated signals on a reproducible recording medium on a scale proportional to said distance scale,

pickup means associated with said last-named recording medium respectively spaced one from the other by amounts respectively equal to a plurality of smoothing intervals of increasingly greater length,

an adder for algebraically adding together the signals detected by said pickup means,

voltage dividing means,

means for applying the algebraic sum of said detected signals to said voltage dividing means for modifying the sum by a factor proportional to the reciprocal of the smoothing interval,

means for recording the output of said last-named means as representative of an averaged gravity profile in correlation with said scale of said stored first-named gravity profile, and

signal storing means interposed between said recording means and said voltage dividing means for storing electrical signals, together with means for reproducing the signals stored therein for application to said recording means, the electrical signals from the output of said voltage dividing means being applied to said signal storing means.

14. A system for treating a gravity profile containing gravitational anomalies extending along a distance scale in which system elements are interconnected to comprise:

means for generating electrical signals varying in proportion to the amplitudes of gravity potentials represented by said gravity profile,

means for storing said generated signals on a reproducible recording medium on a scale proportional to said distance scale,

pickup means associated with said last-named recording medium respectively spaced one from the other by amounts respectively equal to a plurality of smoothing intervals of increasingly greater length,

an adder for algebraically adding together the signals detected by said pickup means,

voltage dividing means,

means for applying the algebraic sum of said detected signals to said voltage dividing means for modifying the sum by a factor proportional to the reciprocal of the smoothing interval,

means for recording the output of said last-named means as representative of an averaged gravity profile in correlation with said scale of said stored first-named gravity profile,

said means for recording having a plurality of recording and reproducing means for storage signals representative of a plurality of said averaged profiles,

difference means associated with said reproducing means, and

means for applying pairs of said signals representative of a plurality of said averaged profiles to said difference means, said difference means generating a plurality of output signals each representing the difference between different pairs of signals recorded by said recording means.

15. A system for treating a gravity profile containing gravitational anomalies extending along a distance scale in which system elements are interconnected to comprise:

means for generating electrical signals varying in proportion to the amplitudes of gravity potentials represented by said gravity profile,

means for storing said generated signals on a reproducible recording medium on a scale proportional to said distance scale,

pickup means associated with said last-named recording medium respectively spaced one from the other by amounts respectively equal to a plurality of smoothing intervals of increasingly greater length,

an adder for algebraically adding together the signals detected by said pickup means,

voltage dividing means,

means for applying the algebraic sum of said detected
signals to said voltage dividing means for modifying the
sum by a factor proportional to the reciprocal of the
smoothing interval,
means for recording the output of said last-named means as
representative of an averaged gravity profile in correla-
tion with said scale of said stored first-named gravity
profile,
said means for recording having a plurality of recording and
reproducing means for storing signals representative of a
plurality of said averaged profiles,
difference means associated with said reproducing means,
means for applying pairs of said signals representative of a
plurality of said averaged profiles being applied to said
difference means, said difference means generating a plu-
rality of output signals each representing the difference
between different pairs of signals recorded by said
recording means, and
recording means for recording as difference profiles said

output signals.

16. The method of processing data representing physical
properties of the earth including anomalies extending along a
distance scale which comprises the following steps each ex-
ecuted by automatic computing apparatus:

generating and storing machine responsive signals
representing the amplitudes of said physical properties
along a profile,

algebraically and progressively adding together along said
profile the stored machine responsive signals representing
amplitudes spaced apart by an interval,

dividing the algebraic sum of said machine responsive
signals by a factor proportional to the reciprocal of said
interval, and

recording the quotient of said last-named step as machine
responsive signals representative of the averaged profile
in correlation with the scale of said physical properties.

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UNITED STATES PATENT OFFICE
CERTIFICATE OF CORRECTION

Patent No. 3,576,985

Dated May 4, 1971

Inventor(s) Philip L. Lawrence

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

Column 1, line 14, after "having" cancel "s" and substitute therefor --a--. Column 2, line 40, after "B-8" insert --
Column 3, formula (1), that portion of the formula reading

$U \pm T(t) \times g(t)$ should read $U \pm T(t) - \times g(t)$

Column 4, lines 49 and 50, cancel "magnetic recording head disposed in recording relationship with a". Column 5, line "47-47" should read -- 41-47 --. Column 6, lines 54 and cancel "representative of difference profiles and that such binary outputs". Column 8, line 66, "of" should read --or. Column 10, line 63 (Claim 9), start a new subparagraph beginning with the word "subtracting".

Signed and sealed this 24th day of August 1971.

(SEAL)
Attest:

EDWARD M. FLETCHER, JR.
Attesting Officer

WILLIAM E. SCHUYLER, JR.
Commissioner of Patents

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