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(54) Title: FORCE AND RATE LIMITED CONTROL

(57) Abstract

A control system in which a velocity error is compared to a limit that is a non–linear function of the position error. If the position error is greater than the velocity limit, the velocity limit is used for the velocity command; otherwise the position error is used. The velocity command is compared with the velocity feedback to generate the velocity error.

FORCE AND RATE LIMITED CONTROL

This application claims priority from provisional application 60/087,923, filed on June 4, 1998.

TECHNICAL FIELD

This invention relates to position controls.

BACKGROUND OF THE INVENTION

In control systems, there is a class of control problems that exhibit some sort of velocity limit which changes independently of the position controller. Electro-mechanical position controllers use motors have an inherent velocity limit in the back EMF of the motor verses the supply voltage to the system. As the supply voltage varies the velocity limit of the system varies. Hydraulic systems are limited by the pump characteristic and other loads on the hydraulic pump when the actuator is operated. In a single gimbal control moment gyroscope (CMG) controlled satellite application, the velocity limit becomes the available angular momentum from the CMG array.

The problem with a velocity limited force limited system is that there is inconsistent performance when the velocity limit changes. Using a linear control results in good performance with a low velocity limit but a system that over shoots if there are high velocity limit. Conversely, good performance with a high velocity limit and produces a long settling time with a smaller velocity limit.

Fig. 1 exemplifies the prior art and the limit problem. A controller 10 provides a position command X_c which is summed with the velocity and position of an object or "mass" 12. A force limited actuator 14, such as a CMG, is controlled by the position error 16 and moves the mass 12. The function 18 represents the inherent actuator force limit as a function of input position error at 16. The function 18 output is effectively summed with a velocity limit VL from a velocity limit 11 for the actuator, such as a power supply limit or CMG angular momentum saturation. The actuator has inherent feedback and transfer characteristics 14a-b, in addition to the force limitation that is represented by transfer function 18. The performance of this system for a small .1 inch step in X_c, (commanded position) followed by a large 2.4 inch step to 2.5 inches would show that for the small step the mass moves precisely with no overshoot because the

velocity limit of the actuator is not reached. However, for the large step the system is
limited by velocity limit, which results in a large over shoot and ringing.

Following classic control theory, two approaches have been used on this
common problem. One centers on finding a compromise set of gains that give good

5   performance with some intermediate velocity limit with some modest over shoot at high
velocity limits and some long tails (decay or settling time) at low velocity limits. This
solution works well if the variation in the velocity limit is not very large. A second
approach pre-calculates a new set of gains for each transient but suffers from
miscalculation when the initial and actual conditions are different, for instance a second

10   position change is commanded during a transient. The controller can be confused by
this change and produce incorrect gains. For that reason, this gain solution is not widely
used.

BRIEF DESCRIPTION OF THE DRAWING

15   Fig. 1 is a functional block diagram of a prior art control system.

Fig. 2 is a functional block diagram of a control system that employs the present
invention.

DISCLOSURE OF THE INVENTION

20   According to the invention, a velocity error is compared to a limit that is a non-
linear function of the position error. If the position error is greater than the velocity
limit, the velocity limit is used for the velocity command; otherwise the position error is
used. The velocity command is compared with the velocity feedback to generate the
velocity error.

25   According to the invention, in a single axis position control in a velocity limited
system, the position error is used to produce a commanded or desired velocity value
which is applied to a non-linear velocity limit defined by the following functions:

$$\dot{x} = k \cdot \sqrt{-2 \cdot a \cdot x} \quad \text{for } x < 0$$

Similarly for errors greater than zero

30       $$\dot{x} = -k \cdot \sqrt{2 \cdot a \cdot x} \quad \text{for } x > 0$$

According to the invention, in a multi-axis position control, such as a controlled
momentum gyro system in a satellite, where the unknown system velocity limit is

function of the aggregate limits along the multiple axis, the second commanded velocity in axis is applied to a non-linear velocity limit using the following function:

$$x_i = \sqrt{\dfrac{2x_i^2}{\sqrt{\dfrac{x_1^2}{A_1^2} + \dfrac{x_2^2}{A_2^2} + \dfrac{x_3^2}{A_3^2}}}}$$

A benefit of the invention is that any transient settles in the same way, i.e., has the same overshoot, undershoot. Another benefit is that the performance is independent of the actuator (plant) velocity limit. An actuator's velocity limits do not have to be known. Using the "n" axis function as the basis for the velocity limit for each axis, a nonlinear control can be built based on ellipsoidal torque and velocity limits in an n-dimensional control, such as a CMG set in a satellite.

Other features and benefits of the invention will be apparent to one skilled in the art from the drawing and following discussion.

BEST MODE FOR CARRYING OUT THE INVENTION

In Fig. 2, the reference numerals refer to the same functions and elements in Fig. 1, but the difference is found in the controller 20. Fig. 2 also shows a single axis control; that is, the mass moves in a straight line and neither it or the actuator 18 are rotating. It receives the position signal X_c which is summed with object position at 22, the position error being applied to a gain 24, producing a velocity output at 26 that is applied to a non-linear limitation function 28 for the one or single axis based on the position error. The velocity command output at 30 is summed at 32 with the mass velocity, and the error is applied to a gain 34 to produce a force with a saturation that is applied to the actuator 18. The following explains the derivation of equations 1 and 2, which define the function 28 for a "single axis" control problem.

Single axis derivation of non-linear function

$$\ddot{x}_c = 6.5 \cdot (x_c - x) - 7.0 \cdot \dot{x} \tag{1}$$

$$F_c = M \cdot \ddot{x}_c \tag{2}$$

By taking equation 1 and splitting it into two:

$$\dot{x}_c = 6.5 \cdot (x_c - x) / 7.0 \tag{3}$$

$$\ddot{x}_c = 7.0 \cdot (\dot{x}_c - \dot{x}) \tag{4}$$

Now a nonlinear velocity limit can be placed after equation 3. The equation is based on constant acceleration at the limiting force.

$$a = \frac{\text{force\_limit}}{\text{mass}} \tag{5}$$

The equations for the velocity limit are based on position error so the requirement is to drive to the origin. If the position error is negative the velocity limit of interest is positive and the acceleration is negative.

$$\ddot{x} = -a \tag{6}$$

Integrating equation 6:

$$\dot{x} = -a \cdot t + v_0 \tag{7}$$

Integrating equation 7:

$$x = -\frac{a \cdot t^2}{2} + v_0 \cdot t - x_0 \tag{8}$$

At the final time t_f both the velocity and the position must come to zero. Thus:

$$x_0 = \frac{a \cdot t_f^2}{2} \quad \text{and} \quad v_0 = a \cdot t_f \tag{9}$$

Solving equation 8 for time t

$$t = t_f - \sqrt{\frac{-2 \cdot x}{a}} \tag{10}$$

Substituting equations 9 and 10 into equation 7 produces:

$$\dot{x} = \sqrt{-2 \cdot a \cdot x} \qquad \text{for} \qquad x < 0 \tag{11}$$

Similarly for errors greater than zero

$$\dot{x} = -\sqrt{2 \cdot a \cdot x} \qquad \text{for} \qquad x > 0 \tag{12}$$

Derivation of non-linear function for n -dimensions

The above is the solution in controlling the position of an object in one dimension. The following assumes that in Fig. 2, commanded position, feedback and

position are quantified in three (n = 3) dimensions. In a satellite using CMGs for attitude control the three dimensions are coupled, i. e., if roll is at maximum velocity there is no angular momentum to transfer to pitch or yaw. Furthermore, the peak rotational acceleration in each of the three axis are different because different inertia and angular momentum is available from the CMG array for different commanded attitude. Defining roll, pitch, and yaw as axis 1,2, and 3 respectively, the acceleration limits become:

$$A_1, \ A_2, \ A_3, \qquad\qquad \frac{a_1^2}{A_1^2}+\frac{a_2^2}{A_2^2}+\frac{a_3^2}{A_3^2}=1 \qquad\qquad (13)$$

The desire is to bring the error to zero in each of the axes at the same time. This means that the deceleration time for each axis needs to be the same.

$$\frac{x_1^2}{b_1^2} = \frac{x_2^2}{b_2^2} = \frac{x_3^2}{b_3^2} = 1 \qquad\qquad (14)$$

where

$$b_n = \frac{1}{2}a_n \cdot t^2 \qquad\qquad (15)$$

Solving for t^4:

$$\frac{4 \cdot x_1^2}{a_1^2} = \frac{4 \cdot x_2^2}{a_2^2} = \frac{4 \cdot x_3^2}{a_3^2} = t^4 \qquad\qquad (16)$$

Now the individual accelerations (a) can be solved for.

$$a_1^2 = \frac{4x_1^2}{t^4} \qquad a_2^2 = \frac{4x_1^2}{t^4} \qquad a_3^2 = \frac{4x_1^2}{t^4} \qquad\qquad (17)$$

Plugging in the values of a into equation 13 and solving for t^2

$$t^2 = 2 \cdot \sqrt{\frac{x_1^2}{A_1^2}+\frac{x_2^2}{A_2^2}+\frac{x_3^2}{A_3^2}} \qquad\qquad (18)$$

Differentiating the square root of equation 16:

$$t = \frac{\dot{x}_1}{a_1} = \frac{\dot{x}_2}{a_2} = \frac{\dot{x}_3}{a_3} \qquad\qquad (19)$$

Squaring the equation

$$\dot{x}_i^2 = a_i^2 t^2 \qquad\qquad (20)$$

Substituting equation 17 for a_I

$$\dot{x}_i^2 = \frac{4\,x_i^2}{t^4}\,t^2 \tag{21}$$

Simplifying, substituting equation 18 for t^2, and taking the square root

$$\dot{x}_i = \sqrt{\frac{2\,x_i^2}{\sqrt{\dfrac{x_1^2}{A_1^2} + \dfrac{x_2^2}{A_2^2} + \dfrac{x_3^2}{A_3^2}}}} \tag{22}$$

Equation 13 produces a non-linear velocity limit as a function of position and maximum acceleration in three coupled axis where the only variable is position (e.g., position of one axis of the satellite) Using equation 22 for function 28 based as the basis for the velocity limit for the value 30 for each axis when the velocity error exceeds a certain maximum plus/minus value, a nonlinear control can be built based on ellipsoidal torque and velocity limits, where the velocity limit does not to be known be the control.

Although the invention has been shown and described with respect to exemplary embodiments thereof, it should be understood by those skilled in the art that various changes, additions and combinations of the features, components and functions disclosed herein may be made without departing from the spirit and scope of the invention.

1.   A control system characterized by:

an actuator;

an object connected to the actuator;

a velocity feedback loop that provides a velocity signal that represents a velocity

5   of the object;

a position feedback loop that provides a position signal that represents a change

in position of the object;

a position control that provides a force signal to each of said actuators to

produce a commanded force on the object from the actuator in response to a difference

10   between a velocity limit signal and said velocity signal;

means for producing a velocity error signal as a function of the difference

between a commanded position signal and the position signal provided by the position

feedback loop; and

means for producing said velocity limit signal with a non-linear value that is a

15   function of the position error signal if said velocity error signal is greater than a first

magnitude and is said velocity error signal if the magnitude of said velocity error signal

is less than said first magnitude.

2.   The control system described in claim 1, characterized in that:

20   said means for producing the velocity limit signal comprises a function

generator that outputs said velocity limit signal by performing the function comprising

$\dot{x} = -k\sqrt{2ax}$ for a first value of x and $\dot{x} = k\sqrt{-2ax}$ for a second value of x

where $\dot{x}$ is said velocity limit signal, x is said position error signal, a is an acceleration

limit for the actuator and k is a constant.

25

3.   The control system described in claim 1, characterized in that:

said means for producing the velocity limit signal comprises a function

generator that outputs said velocity limit signal by performing a function using the

square root of the position error signal.

4.   A control system characterized by:

a plurality of actuators;

an object connected to each of said plurality of actuators;

5   a velocity feedback loop that provides a velocity signal that represents a velocity

of the object in each of n axis;

a position feedback loop that provides a position signal that represents a change

in position of the object in each of said n axis;

a position control that provides a force signal to the actuator to produce a

10   commanded force on the object from the actuator in response to a difference between a

velocity limit signal and said velocity signal;

means for producing a velocity error signal manifesting the difference between

a commanded position signal and the position signal provided by the position feedback

loop; and

15   means for producing said velocity limit signal with a non-linear value that is a

function of the position error signal in each of said n axis if said velocity error signal is

greater than a first magnitude and is said velocity error signal if the magnitude of said

velocity error signal is less than said first magnitude.

20   5.   The control system of claim 4, characterized in that:

said function comprises the square root of the said position error signal for one

of each of said n axis to a first power divided by a value based on an acceleration limit

for an actuator.

25   6.   The control system of claim 4, characterized in that: said means for

producing the velocity limit signal comprises a function generator that outputs said

velocity limit signal by performing the function $\dot{x}_i = \sqrt{\dfrac{2x_i^2}{\sqrt{\dfrac{x_1^2}{A_1^2} + \dfrac{x_2^2}{A_2^2} + \dfrac{x_3^2}{A_3^2}}}}$,

where $\dot{x}_i$ is said velocity limit signal in one of the axis i, x_i is the position error signal in said one axis and A is an acceleration limit for said actuator.
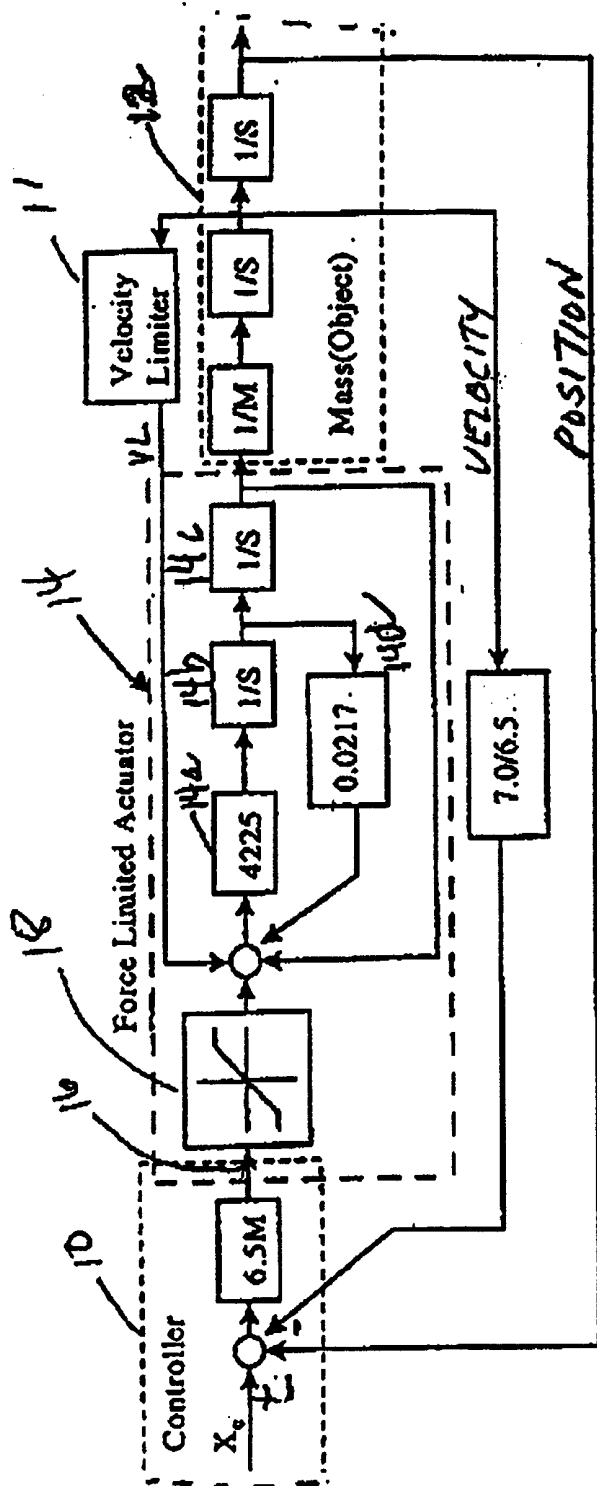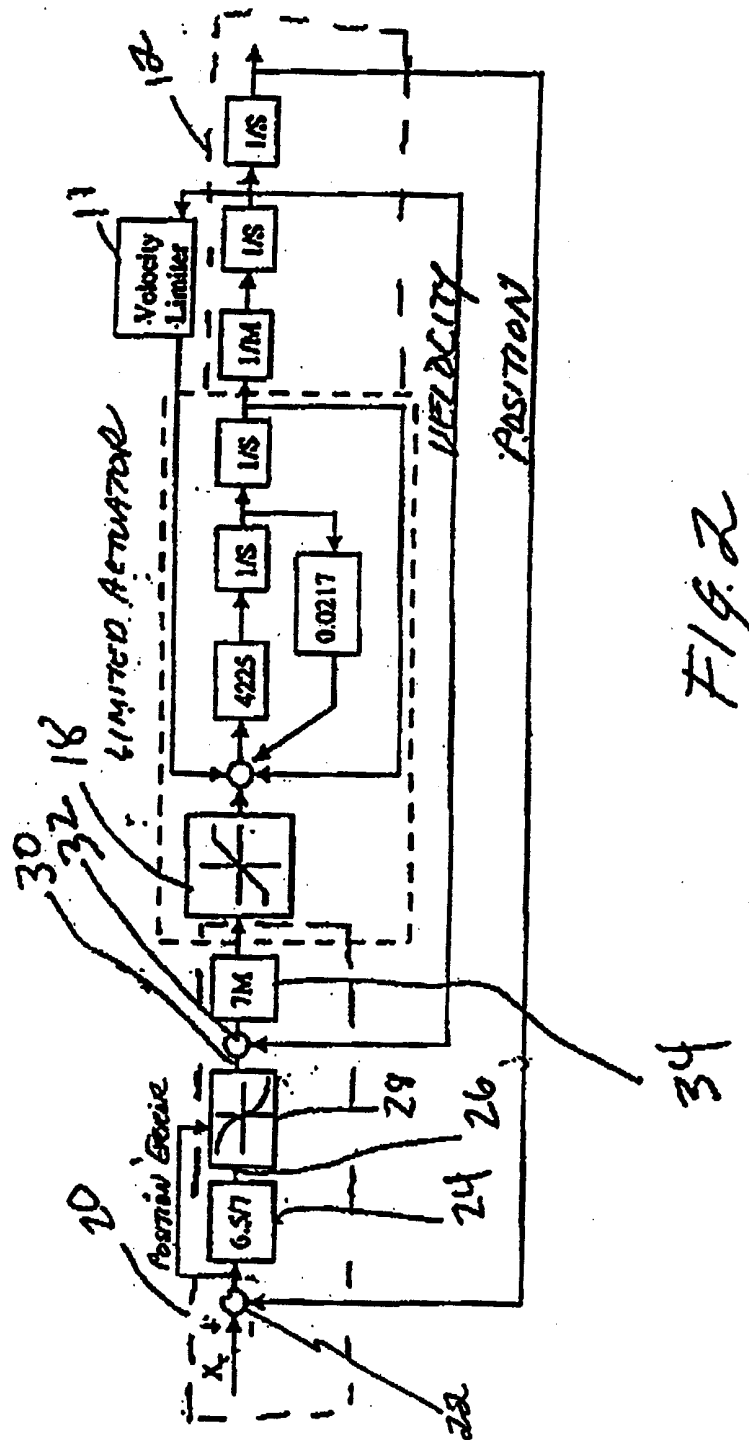
FIG. 1

FIG.2