March 26, 1935. T. H. LONG 1,995,811

INDUCTION HEATER

Filed May 20, 1930

Fig. 1.

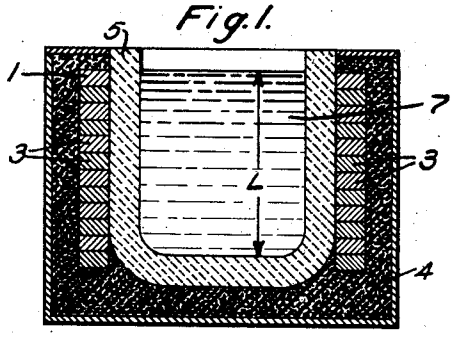


Fig. 3.

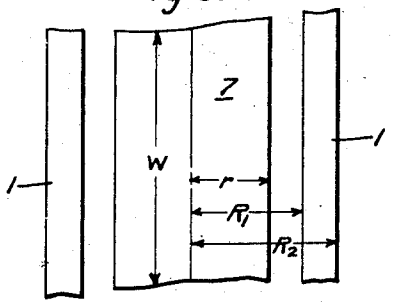


Fig. 2.

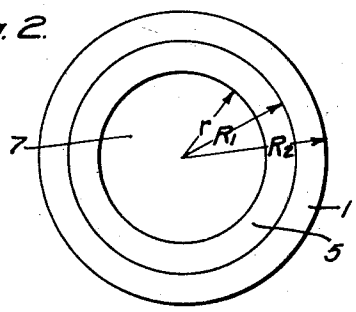


Fig. 4.

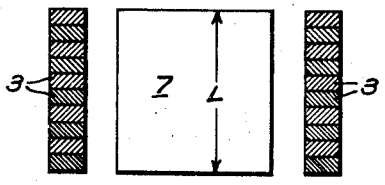
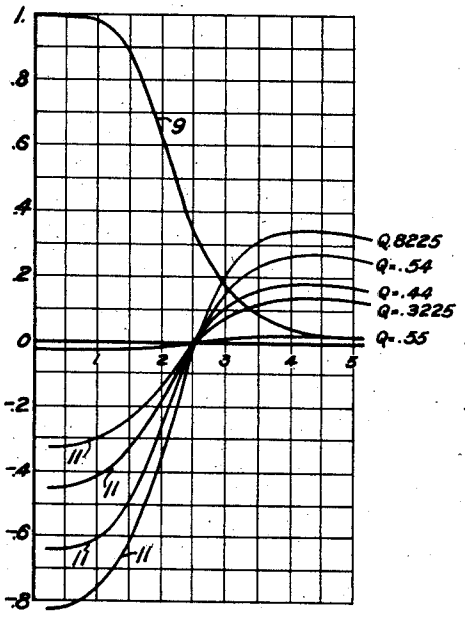


Fig. 5.



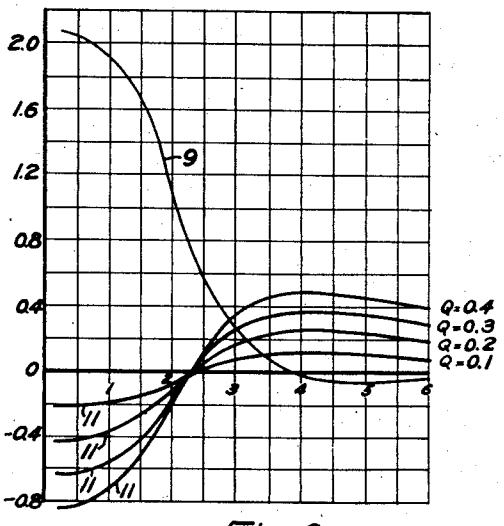


Fig. 6.

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INDUCTION HEATER

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Application May 20, 1930, Serial No. 453,908

8 Claims. (Cl. 219—11)

My invention relates to improvements in heating apparatus and it has particular relation to inductive-heating equipment.

In inductive heating apparatus, constructed and operated according to the teachings of the prior art, it has been customary to use a high-frequency motor-generator set and to correct for low power factor by disposing a plurality of static condensers in series or in parallel with the generator supplying the exciting current to the inducing coils. This method of operating has proved rather expensive, by reason of the cost of the condensers and generator set involved.

On the other hand, it is possible to select the dimensions of the inducing coil and the frequency of the inducing field, in such manner as to give these properties of the equipment values so related to the characteristics of the charge that a maximum, or a considerably improved, power factor is obtained at relatively low frequencies or standard frequencies such as 60 cycles. Inductive-heating equipment having the requisite tractability in its properties has a wide range of usefulness.

It is, accordingly, an object of my invention to provide inductive-heating apparatus the properties of which are so inter-related that the ratio of the irreversible energy induced in a hypothetical charge, having the properties of conductors most commonly treated in the apparatus, to the reversible energy induced in the charge is substantially a maximum.

It is a further object of my invention to provide a method for heating a charge wherein the characteristics of the elements partaking in the heating operation are so related to the properties of the charge that the power factor of the energy developed in the charge is substantially a maximum.

Still another object of my invention is to provide a method of operating an inductive-heating system according to which the charge is so dimensioned that a maximum power factor is obtained.

An additional object of my invention is to provide inductive-heating apparatus wherein the cooperating elements are so related that the first cost of a high-frequency motor-generator set is eliminated.

A still further object of my invention is to provide inductive-heating apparatus wherein the continued cost arising from the losses engendered in using a high-frequency motor generator set is avoided.

It is still another object of my invention to increase the economy of inductive heating, as applied to industrial processes.

An ancillary object of my invention is to provide inductive-heating apparatus wherein the energy-input distribution in the charge is substantially uniform.

Another ancillary object of my invention is to provide inductive-heating apparatus capable of producing a predetermined temperature distribution, in an element treated therein, with a minimum degree of heat conduction.

More specifically stated, it is an object of my invention to provide inductive-heating apparatus wherein the cross section of a charge, its permeability and its conductivity are so related to the frequency of the inducing magnetic field that a substantially maximum power factor is attained at comparatively low standard frequencies.

According to my invention, I provide an inductive heater dimensioned in compliance with the restrictions defined by a mathematical relation expressing the condition that the value of the power factor in terms of the constants of the apparatus and the properties of an arbitrarily assumed charge, shall have an optimum value. In the following analysis, it will be shown that it is possible to use a comparatively low frequency by properly adjusting the remaining variable properties of the heating apparatus.

Other objects of my invention will become evident from the following description, taken in conjunction with the accompanying drawing, in which:

Figure 1 is a sectional view showing schematically an inductive heater constructed according to my invention;

Fig. 2 is a plan view of the apparatus shown in Fig. 1 with outer casing and insulating material removed;

Fig. 3 is a schematic view illustrating a modification of my invention;

Fig. 4 is a sectional view showing schematically the modification shown in Fig. 3;

Fig. 5 is a graph representing the solution of the power-factor problem, as applied to the apparatus shown in Figs. 1 and 2; and

Fig. 6 is a graph representing the solution of the power-factor problem, as applied to the apparatus shown in Figs. 3 and 4.

The apparatus shown in Figs. 1 and 2 includes a solenoid 1 comprising a plurality of turns of a flattened conductor 3. The cable is surrounded by suitable heat insulating material 4.

Adjacent to the internal surface of the solenoid 1 is a layer of refractory material 5 of tor-

roidal cross section. The charge 7 is disposed within the insulating cylinder 5.

Figs. 3 and 4 represent, diagrammatically, an inductive heater capable of efficiently accommodating a charge that is rectangular in cross section. In the treatment to be hereinafter described, the equipment represented in Figs. 3 and 4 will be regarded as having a length that is large, in comparison with its width and depth; that is to say, I shall regard it as extending indefinitely in the direction W while its width $2r$ and its depth L are definite.

In analyzing the problem, I shall set up and solve the Heaviside-Herz equations for the electro-magnetic field, thus obtaining the differential equation which governs the flux distribution in a charge subject to a magnetic field. In the treatment, it will be implicitly assumed that the wave length, in space, of the inducing current is great in comparison with the dimensions of the apparatus.

I shall further solve the differential equation for the special case of a cylindrical and an elongated rectangular charge, making certain obviously allowable approximations. An equation defining the special properties of the flux distribution in the charge is thus obtained.

From the relation giving the flux distribution, I shall obtain the components of the induced voltage that give rise to the heatless component and the heat component of the power. The relation involving the ratio of these two values, I shall then treat for optimum conditions, solving the equation, thus derived, graphically to obtain an expression relating to properties of the charge and the characteristic of the inductive heater.

In the analysis, the physical properties of the elements involved are represented as follows:

R=internal radius of the coil in centimeters
r=the radius of the cylindrical charge in centimeters
=one half the width of the rectangular charge
L=the depth of the charge in centimeters
W=the length of the rectangular charge
λ=conductivity of the charge in mho per centimeter cubed
μ=permeability of the charge
H=strength of the magnetic field, threading the charge, in Gilberts per centimeter
$B=\mu H$
β=maximum flux density in lines per square centimeter parallel to the axis of the solenoid
$B_z=\beta e^{j2\pi ft}$ where B_z is the z component of B
f=frequency
t=time variable
J=inducing current
I=current density in the charge
E=induced voltage per centimeter of length
ϕ=total flux

The field equations are

$$\text{(1)}\qquad \text{Curl } H=.4\pi\lambda E$$

$$\text{(2)}\qquad \text{Curl } E=-\frac{\delta B}{\delta t}10^{-8}$$

therefore

$$\text{(3)}\qquad \text{Curl Curl } H=.4\pi\lambda \text{ Curl } E$$

$$\text{(4)}\qquad =-.4\pi\lambda\frac{\delta B}{\delta t}10^{-8}$$

$$\text{(5)}\qquad \text{Curl Curl } B=-.4\pi\lambda\mu\frac{\delta B}{\delta t}10^{-8}$$

or

$$\text{(6)}\qquad \text{Grad Div } B-\Delta^2 B=-.4\pi\lambda\mu\frac{\delta B}{\delta t}10^{-8}$$

but

$$\text{(7)}\qquad \text{Div. } B=0$$

therefore

$$\text{(8)}\qquad \Delta^2 B=.4\pi\lambda\mu\frac{\delta B}{\delta t}10^{-8}$$

The end effect in the solenoid may be safely neglected without appreciably affecting the accuracy of the solution and the induction may be regarded as having a direction parallel to the axis of the solenoid, i. e., using the conventional coordinates, the components of B are given by

$$\text{(9)}\qquad B=B_z$$

and

$$\text{(10)}\qquad B_x=B_y=0$$

therefore

$$\text{(11)}\qquad \Delta^2 B_z=\frac{\delta^2 B_z}{\delta x^2}+\frac{\delta^2 B_z}{\delta y^2}=.4\pi\lambda\mu\frac{\delta B_z}{\delta t}10^{-8}$$

but

$$\text{(12)}\qquad B_z=\beta e^{j2\pi ft}$$

therefore

$$\text{(13)}\qquad \frac{\delta^2 B_z}{\delta x^2}=\frac{\delta^2\beta}{\delta x^2}e^{j2\pi ft}$$

$$\text{(14)}\qquad \frac{\delta^2 B_z}{\delta y^2}=\frac{\delta^2\beta}{\delta y^2}e^{j2\pi ft}$$

$$\text{(15)}\qquad \frac{\delta B_z}{\delta t}=2\pi jf\mu\beta e^{j2\pi ft}$$

therefore (11) becomes

$$\text{(16)}\qquad \Delta^2\beta=.8\pi^2 j\lambda f\mu\beta 10^{-8}$$

For a cylindrical charge, the most satisfactory approach to the problem is made with the help of cylindrical coordinates in which terms

(16) becomes

$$\text{(17)}\qquad \frac{d^2\beta}{dr^2}+\frac{1}{r}\frac{d\beta}{dr}=.8\pi^2 j\lambda f\mu\beta 10^{-8}$$

in view of the fact that β has circular symmetry and does not vary in a vertical direction.

Putting

$$\text{(18)}\qquad -.8\pi^2 j\lambda f\mu 10^{-8}=-jm^2=\alpha^2$$

and

$$\text{(19)}\qquad u=\alpha r$$

I find that

$$\text{(20)}\qquad \frac{d^2\beta}{du^2}+\frac{1}{u}\frac{d\beta}{du}+\beta=0$$

Equation (20) is Bessel's equation of the zeroth order of which the solution is

$$\text{(21)}\qquad \beta=AJ_0(u)+CK_0(u)$$

wherein

$$\text{(22)}\qquad J_0(u)=1-\frac{u^2}{2^2}+\frac{u^4}{2^2 4^2}-\frac{u^6}{2^2 4^2 6^2}\cdots$$

$$\text{(23)}\qquad K_0(u)=J_0(u)\log(u)+\frac{u^2}{2^2}-u^4\left(\frac{1}{1}+\frac{1}{2}\right)\cdots$$

Now it is at once seen that, since β is always finite, it is finite when $r=u=0$. This restriction may be regarded as a boundary condition.

Hence it follows that

$$\text{(24)}\qquad C=0$$

and

(25) $$\beta=AJ_0(u)$$

(26) $$\beta=AJ_0(-jmr)$$

(27) $$=A\left(1+j\frac{m^2r^2}{2^2}-\frac{m^4r^4}{2^24^2}-j\frac{m^6r^6}{2^24^26^2}\cdots\right)$$

(28) $$=A[\text{ber }(mr)+j\text{ bei }(mr)]$$

wherein

(29) $$m=\pi 10^{-4}\sqrt{.8\lambda\mu f}$$

For brevity I shall write

(30) $$\beta=A(\text{ber } x+j\text{ bei } x)$$

where

(31) $$\text{ber } x=1-\frac{x^4}{2^24^2}+\frac{x^8}{2^24^26^28^2}\cdots$$

(32) $$\text{bei } x=\frac{x^2}{2^2}-\frac{x^6}{2^24^26^2}+\frac{x^{10}}{2^24^26^28^210^2}\cdots$$

To determine A, I apply a second boundary condition

(33) When $mr=mr_1=x_1,\beta=\beta_1=A(\text{ber } x_1+j\text{ bei } x)$ and the equation (30) becomes

(34) $$\beta=\beta_1\frac{\text{ber } x+j\text{ bei } x}{\text{ber } x_1+j\text{ bei } x_1}$$

which defines the distribution of flux throughout the cylindrical charge.

The total flux is obtained by integrating β over the transverse cross-section of the cylinder.

(35) $$\phi=\int_0^{r_1}\beta 2\pi r dr$$

$$=\frac{2\pi\beta_1}{\text{ber } x_1+j\text{ bei } x_1}\int_0^{r_1}(\text{ber } mr+j\text{ bei } mr)rdr$$

$$\phi=\frac{2\pi x_1\beta_1}{m^2}\frac{\text{ber } x_1\text{ bei}'\ x_1-\text{bei } x_1\text{ ber}'\ x_1-j\ (\text{ber } x_1\text{ ber}'\ x_1+\text{bei } x_1\text{ bei}'\ x_1)}{\text{ber}^2\ x_1+\text{bei}^2\ x_1}$$

where

(36) $$\text{ber}'=\frac{d\text{ ber}}{dx};\ \text{bei}'=\frac{d\text{ bei}}{dx}$$

For brevity I shall now write

(37) $$F(x)=x\frac{\text{ber } x\text{ bei}'\ x-\text{bei } x\text{ ber}'\ x}{\text{ber}^2\ x+\text{bei}^2\ x}$$

(38) $$f(x)=x\frac{\text{ber } x\text{ ber}'\ x+\text{bei } x\text{ bei}'\ x}{\text{ber}^2\ x+\text{bei}^2\ x}$$

Hence

(39) $$\phi=\frac{2\pi\beta_1}{m^2}[F(x_1)-jf(x_1)]$$

It is seen that ϕ comprises two components namely

$$\frac{2\pi\beta_1}{m^2}F(x_1)$$

in phase with the inducing current J and

$$\frac{2\pi\beta_1}{m^2}f(x_1),$$

90° out of phase with this current. These two components represent the flux in the charge. In addition, there is a magnetic field in the gap between the coil and the charge, the intensity of which may be taken as equivalent to

$$\frac{\beta_1}{\mu}$$

for all practical purposes. This field is in phase with the inducing current J.

Hence the total flux in phase with J is given by

(40) $$\phi_J=\frac{2\pi\beta_1}{m^2}F(x)+\frac{\pi\beta_1}{\mu}(R^2-r^2)$$

where x may be now regarded as written for $x1$, since it is my purpose to vary $x1$ as will be seen hereinafter.

The flux 90° out of phase with J is given by

(41) $$\phi K=\frac{2\pi\beta_1}{m^2}f(x)$$

The heatless component of the voltage is essentially proportional to ϕJ, while the heat component of the voltage is proportional to ϕK and the constants of proportionality do not vary perceptibly with r.

Let θ=the angle the cosine of which is the power factor.

Hence the power factor is measured by

(42) $$\cot\theta=\gamma\frac{\phi K}{\phi J}=\gamma\frac{f(x)}{F(x)+\frac{m^2}{2\mu}(R^2-r^2)}$$

(43) $$\tan\theta=\frac{1}{P}=\frac{1}{\gamma}\frac{F(x)+\frac{m^2}{2\mu}(R^2-r^2)}{f(x)}$$

For a maximum power factor, the derivative with respect to x of the fraction in x and r in equation (43) is 0; that is,

(44) $$\frac{dS}{dx}=0$$

where

(45) $$S=\frac{F(x)+\frac{m^2}{2\mu}(R^2-r^2)}{f(x)}$$

Put

(46) $$Q=\frac{R^2-r^2}{r^2}$$

Q is constant if the assumption is made that the gap between the coil and the charge is proportional to the radius of the charge.

(47) $$S=\frac{F(x)+\frac{x^2}{2\mu}Q}{f(x)}$$

(48) $$F'(x)=x-2\frac{F(x)f(x)}{x}$$

(49) $$f'(x)=x\left(\frac{bei'^2x+ber'^2x}{ber^2x+bei^2x}-2\frac{f^2(x)}{x^2}\right)$$

Therefore

(50) $$S'=\frac{F'(x)+\frac{x}{\mu}Q}{f(x)}-f'(x)\frac{F(x)+\frac{X}{2\mu}Q}{f^2(x)}$$

and

(51) $$\left(1+\frac{Q}{\mu}\right)x-\frac{2F(x)f(x)}{x}-\left\{F(x)+\frac{Q}{2\mu}x^2\right\}\left\{\phi(x)-\frac{2f'(x)}{x}\right\}=0$$

where

(52) $$\phi(x)=\frac{\text{ber}'^2\ x+\text{bei}'^2\ x}{\text{ber } x\text{ ber}'\ x+\text{bei } x\text{ bei}'\ x}=\frac{f'(x)}{f(x)}+\frac{2f(x)}{x}$$

Equation (51) may be simplified to a form

(53) $$\frac{F(x)\phi(x)}{x}-1=\frac{Q}{\mu}\left[1+f(x)-\frac{x}{2}\phi(x)\right]$$

This equation may be solved graphically for x and its solution for several values for parameter Q, is shown in Fig. 5.

The curve 9, above the axis of abcissæ, corresponds to the left-hand side of the Equation 53 and the curves 11, that cross the axis, correspond to the right-hand side of the equation. The latter are plotted for a multiplicity of values of Q, as is indicated.

It is evident from the curve that, for Q as infinite, the right-hand side of the Equation 53 reduces to the vertical line through the point $x=2.5$. Hence, for a maximum power factor x, which is equivalent to

$$\pi 10^{-4}\sqrt{.8\lambda\mu f}r, \tag{62}$$

is greater than 2.5 and has a value determined by

$$\phi=\frac{\beta_1 W}{m}\frac{\sinh 2mr_1+\sin 2mr_1-j(\sinh 2mr_1-\sin 2mr_1)}{\cosh 2mr_1+\cos 2mr_1} \tag{63}$$

the gap between the coil and the charge. In furnaces constructed according to the teachings of the prior art, x has a value greater than 7.03.

The actual value of x that is chosen for any given case is not arbitrary within the limits but depends on the most convenient value that may be given to Q; that is, to the distance between the coil and the charge.

In practice, the smallest value that Q has is .05. It is seen from the curve that a value of 5.25 for x corresponds to this value of Q. The relation that dominates the dimensions of the furnace and the adjustment of the frequency of the current, therefore, reduces to

$$2.5<\pi 10^{-4}\sqrt{.8\lambda\mu f}r<5.25 \tag{54}$$

Under ordinary circumstances, μ has a value of unity at the temperatures at which the heater is used, while the maximum value of λ is $4\cdot10^4$. Hence, a more specific expression for the Equation 54 is

$$2.5<.056\sqrt{fr}<5.25 \tag{55}$$

I shall now consider the case wherein the charge has the cross-section of a rectangle the length of which extends indefinitely. In practice, the cross-section of a charge may be simply an elongated rectangle. It has also been found that the solution which will be obtained hereinafter may, with reasonable accuracy, be applied to a charge that has a cross-section in the form of an ellipse having a long major axis and a short minor axis.

Since the cross-section of the charge is indefinitely long, the effect at the ends of the charge may be neglected. Hence, if β is expressed with reference to the rectangular coordinates w, in the direction of the length, and r, in the direction of the width, it is seen that β is a function of r only, that is,

$$\beta=\beta(r) \tag{56}$$

Equation 16 holds for any geometric form of charge and, when written in terms of r and w, reduces to

$$\frac{d^2\beta}{dr^2}=.8\pi^2 j\lambda f\mu\beta 10^{-8} \tag{57}$$

$$\frac{d^2\beta}{dr^2}=2jm^2\beta \tag{58}$$

where

$$m^2=.4\pi^2\lambda f\mu 10^{-8} \tag{59}$$

Equation 58 is a linear differential equation having constant coefficients, and its solution may easily be shown to be

$$\beta=\beta_1\frac{\cosh mr\cos mr+j\sinh mr\sin mr}{\cosh mr_1\cos mr_1+j\sinh mr_1\sin mr_1} \tag{60}$$

wherein $\beta=\beta_1$ for $r=r_1$, $2r_1$ being the width of the charge.

Again

$$\phi=2W\int_0^{r_1}\beta dr \tag{61}$$

and when the indicated operation is performed it is found that

$$\phi=\frac{2\beta_1 W}{(1+j)m}\frac{\sinh mr_1\cos mr_1+j\cosh mr_1\sin mr_1}{\cosh mr_1\cos mr_1+j\sinh mr_1\sin mr_1}$$

which in more simple form appears as

$$\phi=\frac{\beta_1 W}{m}\frac{\sinh 2mr_1+\sin 2mr_1-j(\sinh 2mr_1-\sin 2mr_1)}{\cosh 2mr_1+\cos 2mr_1}$$

By analogy to the relations on the cylindrical charge, Equation 63 may be written as

$$\phi=\frac{\beta_1 W}{m}[F(x)-jf(x)] \tag{64}$$

where x is now $2mr$, m being now defined by the Equation (59).

It is to be noted that the constant m, as defined by the Equation (59), is the m of the Equation (18) divided by

$$\sqrt{2}.$$

In Equation (64)

$$x=2mr=\pi 10^{-4}\sqrt{1.6\lambda\mu f}r \tag{65}$$

$$F(x)=\frac{\sinh x+\sin x}{\cosh x+\cos x} \tag{66}$$

$$f(x)=\frac{\sinh x-\sin x}{\cosh x+\cos x} \tag{67}$$

The total flux in phase with the inducing current J is now given by

$$\phi_J=\frac{\beta_1}{m}WF(x)+\frac{2\beta_1 W(R-r)}{\mu} \tag{68}$$

$$\phi_k=\frac{\beta_1}{m}WF(x) \tag{69}$$

and

$$S=\frac{\phi_J}{\phi_K}=\frac{F(x)+\frac{Q}{\mu}x}{f(x)} \tag{70}$$

where

$$Q=\frac{R-r}{r} \tag{71}$$

and is assumed to remain constant

$$S'=\frac{F'(x)+\frac{Q}{\mu}}{f(x)}-\frac{f'(x)}{f^2(x)}\left[F(x)+\frac{Qx}{\mu}\right] \tag{72}$$

$$F'(x)+\frac{Q}{\mu}-f'(x)\frac{F(x)+\frac{Q}{\mu}x}{f(x)} \tag{73}$$

which may be written

$$F(x)\phi(x)-1=\frac{Q}{\mu}\left\{1+x[f(x)-\phi(x)]\right\} \tag{74}$$

where

$$\phi(x)=\frac{\cosh x-\cos x}{\sinh x-\sin x} \tag{75}$$

The solution of the Equation (74) for various

values of Q is shown in Fig. 6. It is seen that, in this case, the dominating inequality is

$$(76) \qquad 2.25<\pi 10^{-4}\sqrt{1.6\lambda\mu f}r<9.91$$

a more restricted inequality being

$$(77) \qquad 2.25<\pi 10^{-4}\sqrt{1.6\lambda\mu f}r<3.9$$

The values 2.25 and 3.9 are limiting values obtained by putting $Q=\infty$ and $Q=0$ in Equation (74). The value 9.91 is equivalent to

$$7.03\sqrt{2}$$

and is the value determined by the prior art.

It is seen that the rectangular (or elliptic) charge yields a maximum power factor for a width smaller than the diameter of the corresponding cylindrical charge. This property of the system is of advantage in situations where it is desirable to use a long and narrow charge. It can be applied with facility to apparatus wherein steel strip is normalized.

After the strip passes between a pair of rollers, it traverses an elongated induction heater where it is raised to the temperature that it had before it came under the action of the rollers. It is then rapidly cooled. Steel treated in this manner has more desirable properties than annealed steel.

For $\mu=1$ and for a maximum value of $4\cdot10^4$ for λ (76) becomes

$$(78) \qquad 2.25<.0795\sqrt{f}\,r<9.91$$

for $f=60$ cycles (78) becomes

$$(79) \qquad 2.25<.661\,r<9.91.$$

When magnetic charges are heated, the permeability is ordinarily of the order of 200.

For such a case, (76) becomes

$$(80) \qquad 2.25<1.12\sqrt{f}\,r<9.91,$$

λ having been taken equal to $4\cdot10^4$.

I do not wish to be restricted to the specific structural details or arrangement of parts herein set forth, as various modifications thereof may be effected without departing from the spirit and scope of my invention. In particular, I may point out that, while the treatment here has been applied to a solid cylindrical or rectangular charge, it equally well applies to individual particles of metal disposed in a cylindrical or rectangular container.

One further matter that is ancillary to my invention and that is within the scope of equivalents thereof is the dimensioning of the apparatus in such manner that a predetermined energy-input distribution for the charge is obtained for any given situation.

The energy input per unit time for the cylindrical charge is given by the relation

$$(81) \qquad W=\frac{E^2\lambda}{2}$$

but by Equation 1

$$(82) \qquad E=\frac{\text{Curl } H}{.4\pi\lambda}=\frac{\text{Curl } B}{.4\pi\lambda\mu}$$

$$=\frac{d\beta}{dr}\,\frac{1}{.4\pi\lambda\mu}$$

and, by substituting in (81), it is seen that the absolute value of the energy developed is given by

$$(83) \qquad \overline{W}=\frac{m^2\beta_1^2}{.8\pi^2\lambda\mu^2}\,\frac{\text{ber}'^2x+\text{bei}'^2x}{\text{ber}^2x_1+\text{bei}^2x_1}$$

For a rectangular charge, the absolute value of the energy input is given by

$$(84) \qquad \overline{W}=\frac{m^2\beta_1^2}{.4\pi^2\lambda\mu^2}\,\frac{\cosh x-\cos x}{\cosh x_1+\cos x_1}$$

These relations are of particular interest in certain fields of industry e. g., in rolling mills where it is necessary to reheat an ingot after it has been removed from the mold and prior to the rolling operation.

When the ingot is poured, the entire mass of metal may be regarded as having a uniform temperature. As a result of the thermal capacity of the walls of the mold, the ingot, when removed from the mold, is not of uniform temperature throughout but has a maximum temperature in some interior region and has a minimum temperature on its bounding surface.

In reheating the ingot before rolling, it is desirable that it be brought back to substantially a uniform temperature throughout, the value of which will be approximately the same as the temperature at its hottest part when it was removed from the mold.

In general, it is possible to determine the theoretical temperature-distribution characteristic of any ingot of given dimensions, and to so adjust the properties of the inductive reheating apparatus in which the ingot is treated that the power input in each individual region of the ingot is substantially proportional to the temperature through which the metal in these regions is to be heated. For example, in a cylindrical ingot, the temperature distribution varies parabolically from the center to the periphery. It is obvious that the constants of the heating apparatus may be so selected that W, in Equation (83), varies substantially as a parabola similar to the inverted temperature-distribution curve.

As a result of such specific arrangement of the apparatus, the heating energy is liberated in individual regions in proportion as it is needed, and it is not necessary to allow time for heat flow from an abnormally heated region to a cooler region. I have found that this process reduces the reheating time from several hours to approximately 10 minutes.

I desire, that only such limitations shall be imposed upon my invention as are indicated in the appended claims.

I claim as my invention:

1. Inductive electrical heating apparatus comprising a coil to be magnetically coupled to a cylindrical charge, a substantial air gap intervening between said coil and charge and an alternating source of current for energizing said coil, the characteristics of said coil and said source being so related to each other and to the properties of said charge that they substantially obey the relation

$$\frac{F(x)\phi(x)}{x}-1=\frac{Q}{\mu}\left[1+f(x)-\frac{x}{2}\phi(x)\right]$$

where

$$F(x)=x\frac{\text{ber } x \text{ bei}' x-\text{bei } x \text{ ber}' x}{\text{ber}^2 x+\text{bei}^2 x}$$

$$\phi(x)=\frac{\text{ber}'^2 x+\text{bei}'^2 x}{\text{ber } x \text{ ber}' x+\text{bei } x \text{ bei}' x}$$

$$f(x)=x\frac{\text{ber } x \text{ ber}' x+\text{bei } x \text{ bei}' x}{\text{ber}^2 x+\text{bei}^2 x}$$

$$Q=\frac{R^2-r^2}{r^2}$$

$$x=\pi 10^{-4}\sqrt{.8\lambda\mu f}r$$

r being the radius of the charge
μ being its permeability
λ being its conductivity
R being the radius of the inducing coil and
f being the frequency of the current.

2. Inductive electrical heating apparatus comprising a coil adapted to be magnetically coupled to a rectangular charge and a source of alternating current for feeding said coil, the characteristics of said elements being so related to each other and to the properties of the charge as to substantially attain the relation

$$F(x)\phi(x)-1=\frac{Q}{\mu}\{1+x[f(x)-\phi(x)]\}$$

wherein

$$F(x)=\frac{\sinh x+\sin x}{\cosh x+\cos x}$$

$$f(x)=\frac{\sinh x-\sin x}{\cosh x+\cos x}$$

$$\phi(x)=\frac{\cosh x-\cos x}{\sinh x-\sin x}$$

$$Q=\frac{R-r}{r}$$

$$x=2\pi 10^{-4}\sqrt{.4\lambda\mu f}r$$

$2r$ being the width of the charge
λ being its conductivity
μ being its permeability
f being the frequency of the current
2R being the width of the heating coil.

3. The method of heating a furnace charge, which is in the form of a circular cylinder, from an initial condition in which it has a transverse temperature distribution function which is substantially a parabola with an axis coinciding with the central axis of the cylinder, which consists in generating heat within the cylindrical mass, the rate of heat generation having a substantially parabolic distribution relative to said central axis.

4. The method of heating a furnace charge, which is in the form of a circular cylinder, from an initial condition in which it has a transverse temperature distribution function which is substantially a parabola with an axis coinciding with the central axis of the cylinder, which consists in generating heat within the cylindrical mass, the rate of heat generation having a distribution which is a parabola substantially similar to the first-mentioned parabola and is coaxial therewith.

5. The method of heating a furnace charge, which is in the form of a circular cylinder and which has cooled from a substantially uniform elevated temperature condition by heat outflow from its side walls, which consists in generating heat by electromagnetic induction within said cylindrical mass by means of an alternating magnetic field such that the rate $\overline{W}$ of heat generation fulfills the equation

$$\overline{W}=\frac{m^2\beta_1^2}{.8\pi^2\lambda\mu_2}\frac{\text{ber}'^2 x+\text{bei}'^2 x}{\text{ber}^2 x_1+\text{bei}^2 x_1}$$

wherein

$$x=\pi 10^{-4}\sqrt{.8\lambda\mu f}r$$

r being the radial distance from the axis of said mass of a point at which $\overline{W}$ is the energy input
λ being the conductivity of said mass
μ being the permeability of said mass
f being the frequency of said magnetic field

$$x_1=\pi 10^{-4}\sqrt{\lambda\mu.8f}r_1$$

r_1 being the radius of said mass
β_1=the amplitude of the flux density at the periphery of the mass and

$$m=\pi\sqrt{10^{-4}\lambda\mu.8f}$$

6. The method of heating a rectangular conducting mass, having the form of a rectangular parallelopiped, which has cooled from a substantially uniform elevated temperature condition by heat outflow through two opposite side walls, which consists in generating heat within said mass by electromagnetic induction through the agency of an alternating magnetic field such that the rate $\overline{W}$ of heat generation fulfills the equation

$$\overline{W}=\frac{m^2\beta_1^2}{.4\pi^2\lambda\mu_2}\frac{\cosh x-\cos x}{\cosh x_1+\cos x_1}$$

wherein

$$x=2\pi 10^{-4}\sqrt{.4\lambda\mu f}r$$

r being the distance in said mass from the center line parallel to its longest side at which $\overline{W}$ is the energy input

$$x=2\pi 10^{-4}\sqrt{.4\lambda\mu f}r_1$$

$2r_1$ being the width of said mass and

$$m=\pi 10^{-4}\sqrt{.4\lambda\mu f}$$

7. The method of heating a furnace charge, which is in the form of a circular cylinder and which has cooled from a substantially uniform elevated temperature condition by heat outflow from its side walls, which consists in generating heat by electromagnetic induction within said cylindrical mass by means of an alternating magnetic field such that the rate $\overline{W}$ of heat generation fulfills the equation

$$\overline{W}=\frac{m^2\beta_1^2}{.8\pi^2\lambda\mu^2}\frac{\text{ber}'^2 x+\text{bei}'^2 x}{\text{ber}^2 x_1+\text{bei}^2 x_1}$$

wherein

$$x=\pi 10^{-4}\sqrt{.8\lambda\mu f}r$$

r being the radial distance from the axis of said mass of a point at which $\overline{W}$ is the energy input
λ being the conductivity of said mass
μ being the permeability of said mass
f being the frequency of said magnetic field

$$x_1=\pi 10^{-4}\sqrt{.8\lambda\mu f}r_1$$

r_1 being the radius of said mass
β_1=the amplitude of the flux density at the periphery of the mass and

$$m=\pi 10^{-4}\sqrt{.8\lambda\mu f}$$

the constants of the heating apparatus being so selected that $\overline{W}$ in the above equation varies substantially as a parabola similar to the inverted temperature-distribution curve.

8. The method of heating a rectangular conducting mass, having the form of a rectangular parallelopiped, which has cooled from a substantially uniform elevated temperature condition by heat outflow through two opposite side walls,

which consists in generating heat within said mass by electro-magnetic induction through the agency of an alternating magnetic field such that the rate $\overline{W}$ of heat generation fulfills the equation

$$\overline{W}=\frac{m^2\beta_1^2}{.4\pi^2\lambda\mu^2}\,\frac{\cosh x-\cos x}{\cosh x_1+\cos x_1}$$

wherein

$$x=2\pi 10^{-4}\sqrt{.4\lambda\mu f}\,r$$

r being the distance in said mass from the center line parallel to its longest side at which $\overline{W}$ is the energy input

$$x=2\pi 10^{-4}\sqrt{.4\lambda\mu f}\,r_1$$

$2r_1$ being the width of said mass and

$$m=\pi 10^{-4}\sqrt{.4\lambda\mu f}$$

the constants of the heating apparatus being so selected that $\overline{W}$ in the above equation varies substantially as a parabola similar to the inverted temperature-distribution curve.

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